Particles with negative and zero energies in black holes and cosmological models

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The problem of the sign of the energy in relativistic case has principal meaning and can lead to different physical phenomena for example to the Penrose effect in rotating black holes. In nonrelativistic case the potential energy of the particle is defined up to the additive constant and depending on its definition one can have different classification of motion of particles with negative energies. For example if the energy of the particle at rest on space infinity in nonrelativistic case is taken to be zero then the sign of the sum of the potential and kinetic energies in case of the Kepler problem defines the bounded and unbounded orbits. However taking it as in relativistic case to be mc^2 for particle with mass m (c is the velocity of light) one obtains for the full negative energy of the nonrelativistic particle moving with the velocity v on the distance r from the attracting massive body with the mass M

$$mc^2 + \frac{mv^2}{2} - G\frac{mM}{r} < 0 \quad \Rightarrow \quad r < \frac{GM}{c^2} = \frac{r_g}{2}, \tag{1}$$

where G is the Newton gravitational constant, r_g is the gravitational radius. So negative energy in this case can be only on distances smaller than the gravitational radius.

However it can show that negative (and zero) particle energies are possible not only in strong gravitational fields but in the case of absence of the gravitational field in different reference frame, for example in rotating coordinates (A.A.Grib, Yu.V.Pavlov, Gen. Relativ. Gravit. (2017) **49** 78).

Here we consider the cases Schwarzschild black hole, flat space-time in Milne's coordinates, and Gödel cosmological model.

Negative energy in nonrotatitng black hole

Nonrotating black hole of mass M in **Schwarzschild coordinates** is described by metric

$$ds^2 = \left(1-\frac{r_g}{r}\right) c^2 dt^2 - \frac{dr^2}{1-\frac{r_g}{r}} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (2)$$

where $r_g = 2GM/c^2$ is the gravitational radius of the black hole, G is gravitational constant, c is the light velocity. Geodesic complete space-time of the nonrotating black hole one can be described in **Kruskal–Szekeres coordinates**, $\{u, v\} \in (-\infty, +\infty)$, which in region $u > |v| \geq 0$ are connected with Schwarzschild coordinate in $r > r_g$ in the following way

$$\begin{aligned} u &= \sqrt{\frac{r}{r_g} - 1}\, \exp\left(\frac{r}{2r_g}\right) \cosh\frac{ct}{2r_g}, \\ v &= \sqrt{\frac{r}{r_g} - 1}\, \exp\left(\frac{r}{2r_g}\right) \sinh\frac{ct}{2r_g}. \end{aligned} \qquad (3)$$

For $r < r_g$ and $v > |u| \geq 0$ the transformation from Schwarzschild coordinate in the Kruskal–Szekeres coordinates has the form

$$\begin{aligned} u &= \sqrt{1 - \frac{r}{r_g}}\, \exp\left(\frac{r}{2r_g}\right) \sinh\frac{ct}{2r_g}, \\ v &= \sqrt{1 - \frac{r}{r_g}}\, \exp\left(\frac{r}{2r_g}\right) \cosh\frac{ct}{2r_g}. \end{aligned} \qquad (4)$$

Geodesic equations in Schwarzschild coordinates in the plane $\theta = 0$ are

$$\frac{dt}{d\lambda} = \frac{r}{r - r_g} \cdot \frac{E}{c^2}, \tag{5}$$

$$\left(\frac{dr}{d\lambda}\right)^2 = \frac{E^2}{c^2} + \frac{r_g - r}{r^3} J^2 + \frac{r_g - r}{r} m^2 c^2, \qquad \frac{d\varphi}{d\lambda} = \frac{J}{r^2},$$

where E is the energy of the moving particle, J is the conserved projection of the particle angular momentum on the axis orthogonal to the plane of motion, m is the particle mass, λ is affine parameter on geodesic. For massive particle $\lambda = \tau/m$, where τ is the proper time.

In external region of the black hole ($r > r_g$) for any physical object the time coordinate t is always increasing and so the energy E of the particle is positive (see (5)). Inside the horizon of the black ($r < r_g$), where t is space like ($g_{tt} < 0$) one has movement as in increasing as in decreasing t. As it is seen from the first formula in (5) for a particle moving inside the horizon in the direction of decreasing of the coordinate t the energy E of the particle will be positive while for increasing coordinate t the energy E is negative. For constant t inside the black hole $E = 0$ due to formula (5).

For the observer outside of the black hole the conserved E along all trajectories of the free fall is equal to

$$E = mc^2 \sqrt{\left(1 - \frac{r_g}{r}\right) \Big/ \left(1 - \frac{\mathbf{v}^2}{c^2}\right)}, \tag{6}$$

where $\mathbf{v}$ is the velocity measured by the observer at rest in the Schwarzschild coordinates. So we can call E inside the black hole as "energy at infinity".

On Fig. 1 the trajectories for radial movement with **positive**, **zero** and **negative** energies in Kruskal–Szekeres coordinates are represented by **red**, **green** and **blue** lines.

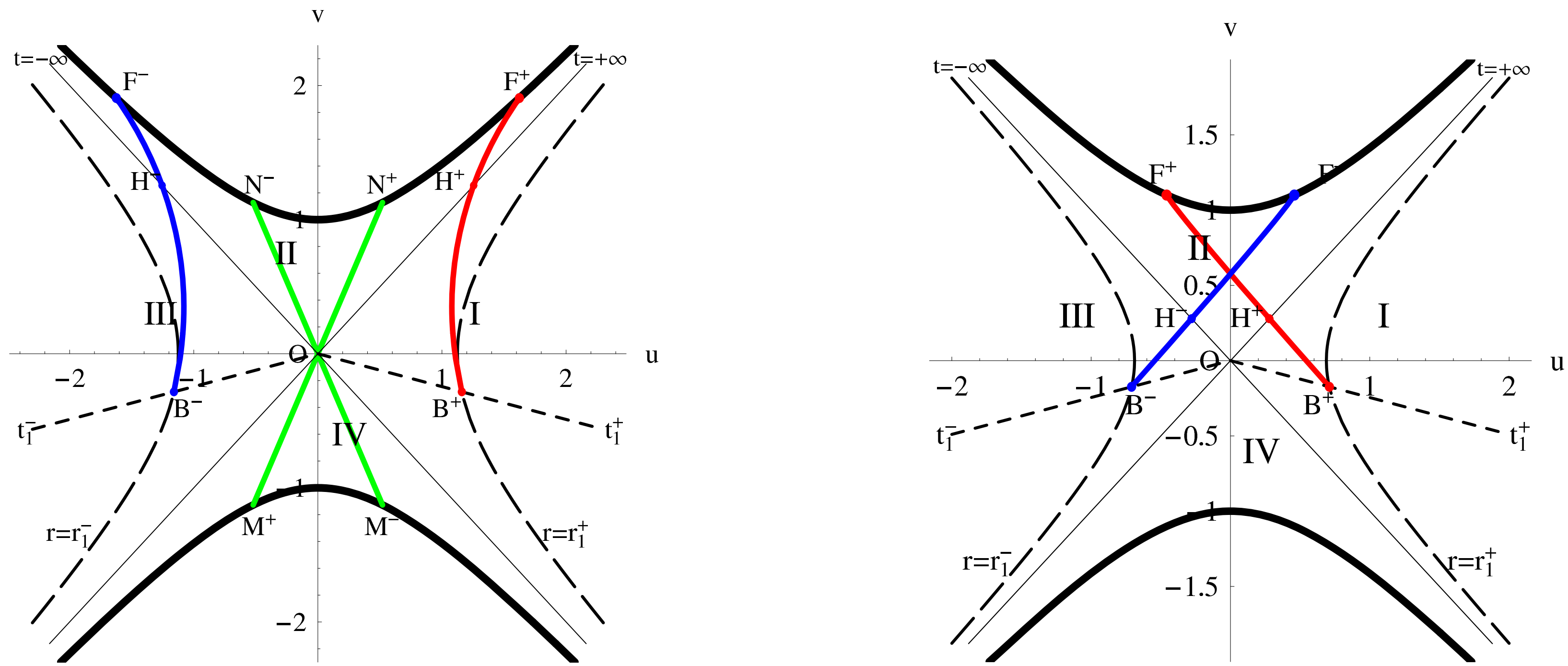

Figure 1: Trajectories of particles with positive ($B^+H^+F^+$), zero ($M^\pm ON^\pm$) and negative ($B^-H^-F^-$) energy, $t_1^\pm = \mp 0.5 r_g/c$: On the left one can see falling from the rest at $|E| = 0.5mc^2$, on the right with $|E| = mc^2$ with the corresponding initial velocity from the point $r = 1.15 r_g$. On lines ($M^\pm ON^\pm$) the coordinate $t = \pm r_g/c$.

As one can see from (3), (4) the coordinate lines of constant t in Kruskal–Szekeres coordinates are straight lines through the origin of coordinates. In region II coordinates t decreases for moving from H^+ to F^+ (positive E) and increases for moving from H^- to F^- (negative value of E). Direct lines ($M^\pm ON^\pm$) correspond to constant $t = \pm r_g/c$ and therefore $E = 0$.

Let us consider the problem of back influence of falling particles on metric of the black hole space-time. For macroscopic bodies with 4-velocity (u^i), with the energy density ε and pressure p in space-time with metric g_{ik} the energy-momentum density tensor is

$$T_{ik} = (\varepsilon + p) u_i u_k - p g_{ik}, \tag{7}$$

$i, k = 0, 1, 2, 3.$

The trace of the energy-momentum tensor

$$T_i^i = \varepsilon - 3p \tag{8}$$

is invariant and it will be negative for $\varepsilon - 3p < 0$, in particular, for dust like matter $(p = 0)$ with negative energy $\varepsilon < 0$. The back influence of falling particles with negative energy will be determined by the such energy-momentum tensor in the right hand side of Einstein equations. Notion of the existence of particles with negative energies as it is known was used by S. Hawking to predict Hawking effect for black holes.

Negative and zero energies in flat space-time

The geodesic lines equations can be obtained for space-time with metric g_{ik} from the Lagrangian

$$L = \frac{g_{ik}}{2}\frac{dx^i}{d\lambda}\frac{dx^k}{d\lambda}, \tag{9}$$

where λ is the affine parameter on the geodesic. The energy of the particle E is equal to the zero covariant component of the momentum (p_i) multiplied on the light velocity

$$p_i = \frac{\partial L}{\partial\left(\frac{dx^i}{d\lambda}\right)} = g_{ik}\frac{dx^k}{d\lambda}, \tag{10}$$

$$E = cp_0 = cg_{0k}\frac{dx^k}{d\lambda}. \tag{11}$$

Defining the affine parameter for the massive particle as $\lambda = \tau/|m|$, where τ is the proper time of the moving particle one obtains

$$p_i p^i = m^2 c^2 \tag{12}$$

and the energy of the particle is

$$E = |m| c g_{0k}\frac{dx^k}{d\tau}. \tag{13}$$

Here we suppose generally that the mass can be negative.

Using notation $(\zeta^i) = (1,0,0,0)$ for the translation in time coordinate generator one can write (11) for the energy of the particle as

$$E = c(p,\zeta). \tag{14}$$

If the metric components don't depend on the time coordinate x^0, then ζ is the time like Killing vector and the energy E is conserved on the geodesic.

For time like vector ζ and massive particle one has

$$\sqrt{(\zeta,\zeta)} \le \frac{E}{|m|c^2} < +\infty \tag{15}$$

and the energy (14) is positive.

For space like vector ζ, as it take place in the ergosphere of rotating black hole, the **arbitrary positive and negative values are possible.**

Note that in spite of the invariance of the scalar product (14) the value (13) of the energy depends on the choice of the reference frame. This occurs due to the fact that by changing the reference frame in which the physical measurements are made the observer is changing vector ζ. The analysis of the situation in rotating coordinate system in flat space-time is made in

A.A.Grib, Yu.V.Pavlov. Comparison of particle properties in Kerr metric and in rotating coordinates. **Gen. Relativ. Gravit. (2017) 49 78**

A.A.Grib, Yu.V.Pavlov. Static limit and Penrose effect in rotating reference frames. **Theor. Math. Phys. 2019. Vol. 200 1117–1125.**

In Minkowski space-time in Galilean coordinate system or any other coordinate system with $g_{0\alpha} = 0$, $(\alpha = 1, 2, 3)$ the energy (11) is

$$E = c^2 \frac{dt}{d\lambda} \tag{16}$$

and it is always positive in movement "forward" in time because in the future light cone one has $dt/d\lambda > 0$. For massive particle as with positive as negative mass m the energy is

$$E = |m| c^2 \frac{dt}{d\tau} > 0. \tag{17}$$

Now let us give an example showing that in flat space-time the energy of the relativistic particle can be negative and zero in case of the special choice of the coordinate frame.

Negative Energies in Milne's Universe

Consider the coordinate system in which metric of flat space-time has the form of the metric of the expanding homogeneous isotropic Universe — Milne universe

E.A. Milne, Relativity, Gravitation and World-Structure (Clarendon Press, Oxford, 1935)

$$ds^2 = c^2dt^2 - c^2t^2\left(d\chi^2 + \sinh^2\chi d\Omega^2\right), \tag{18}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$, coordinate χ is changing from 0 to $+\infty$.

In new coordinates

$$T = t\cosh\chi, \quad r = ct\sinh\chi, \quad cT > r > 0 \tag{19}$$

the interval (18) coincides with Minkowski interval

$$ds^2 = c^2dT^2 - dr^2 - r^2\, d\Omega^2. \tag{20}$$

This space-time with coordinate $t \geq 0$, $\chi \geq 0$ is corresponds to the future cone in coordinates cT, r.

The **radial distance** from points $\chi = 0$ and χ in metric (18) is $D = ct\chi$. Taking D as the radial coordinate one obtains the interval as

$$ds^2 = \left(1 - \frac{D^2}{c^2t^2}\right)c^2dt^2 + 2\frac{D}{t}dtdD - dD^2 - c^2t^2\sinh^2\left(\frac{D}{ct}\right)d\Omega^2. \tag{21}$$

From the condition $ds^2 \geq 0$ one obtains that if D is larger than $D_s = ct$, then no physical object can be at rest in coordinates t, D, θ, ϕ. The value D_s **corresponds to** $\chi = 1$ and it **plays the role of the static limit** for the rotating black hole in Boyer-Lindquist coordinates.

The energy of the particle with mass m in coordinates t, D, θ, ϕ is

$$E = cg_{0k}\frac{dx^k}{d\lambda} = mc^2\frac{dt}{d\tau}\left(1 - \frac{D^2}{c^2t^2} + \frac{D}{c^2t}\frac{dD}{dt}\right) = mc^2\frac{dt}{d\tau}\left(1 + \chi t\frac{d\chi}{dt}\right). \tag{22}$$

From (18) one obtains for any physical object the inequality

$$t\left|\frac{d\chi}{dt}\right| \leq 1. \tag{23}$$

So particle can have **negative energy only for $\chi > 1$, i.e. out of the static limit, if**

$$\frac{d\chi}{dt} < -\frac{1}{\chi t}. \tag{24}$$

Note that the components of metric (21) depend on time and the energy (22) in general is not conserved on the geodesics. If the energy is zero then particle is moving noninertial according to the law

$$E = 0 \;\Leftrightarrow\; \frac{d\chi}{dt} = -\frac{1}{\chi t} \;\Leftrightarrow\; \chi = \sqrt{\chi_0^2 - 2\log(t/t_0)}, \qquad t \in \left[t_0,\; t_0\exp((\chi_0^2-1)/2)\right]. \tag{25}$$

The trajectory of such movement for case $\chi_0 = 2$, $t_0 = 0.11$ is represented by the curve on Fig. 2 in coordinates T, r (see (19)). In case of the inertial movement trajectory in these coordinates

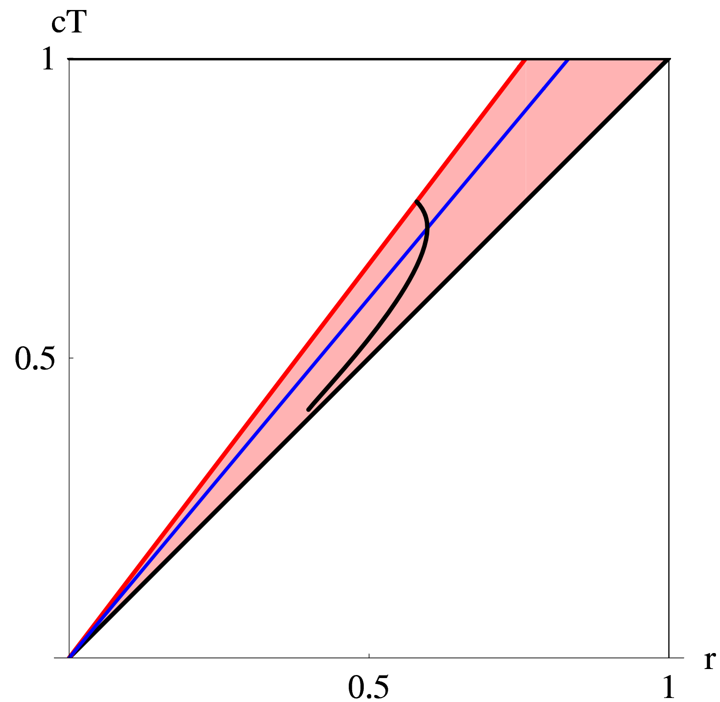

Figure 2: Possible region of movement of particle with negative and zero energies in the reference frame t, D in flat coordinate T, r.

is the direct line. Possible region of movement of particles with negative and zero energies in the reference frame t, D is defined in the coordinate T, r by conditions $1 \leq cT/r \leq \coth 1 \approx 1.313$.

Velocities of movement in coordinates T, r and t, χ satisfy condition

$$t\frac{d\chi}{dt} = \frac{\dfrac{dr}{dT} - c\tanh\chi}{c - \dfrac{dr}{dT}\tanh\chi}. \tag{26}$$

So for

$$\chi\tanh\chi \geq 1 \tag{27}$$

particles at rest in inertial frame T, r will have negative energies in the frame t, D. This region can be seen on Fig. 2 as the region above the blue line in red district. Zero energy of the particle at rest in T, r coordinates is possible only the blue line defined by the root of equation $\chi\tanh\chi = 1$, i.e. $\chi \approx 1.1997$.

So one can see that for specific choice of coordinates one can obtain negative and zero energies for particles at rest in inertial frame.

Note that for small distances $(D/(ct) = \chi \ll 1)$ the metric (21) becomes the metric of comoving spherical coordinate system of Minkowski space-time

$$ds^2 = c^2dt^2 - dD^2 - D^2d\Omega^2, \tag{28}$$

and the energy (22) will be equal to usual energy in inertial system of flat space-time

$$E_u = mc^2\frac{dt}{d\tau} \approx mc^2\frac{dT}{d\tau}, \tag{29}$$

because for $\chi \ll 1$ one has $t \approx T$.

The decay of the body on two bodies, one with negative energy and the other with the positive energy being larger than the energy of the initial decaying one corresponds to the Penrose process. This process occurs out of the static limit on distance $D > ct$. However later these two products of the decay move inside the static limit and during flight in the direction of the origin where the metric is that of Minkowski space change their energies in such manner that the result will be the same as in inertial frame. Really due to (22),

$$E = E_u + mc^2 \chi t \frac{d\chi}{d\tau}. \tag{30}$$

Here E_u is the energy in the reference frame t, χ, such that $g_{0i} = 0$, $i \neq 0$, and g_{00} does not depend on time. So E_u is conserved. At the point of decay both energies E and E_u are conserved. When the body 2 with the positive energy arrives to the coordinate origin $\chi = 0$ its energy E (30) will be equal to E_u and no growth of the energy will be observed.

Body 1 with the negative energy E due to (22) after decay will have the negative value of velocity $d\chi/dt$ larger (in absolute value) than that of the body 2. This means that it will arrive to the origin before the arrival of body 2. It's energy in the origin of the coordinate frame will be also positive and the full energy of 1 and 2 will be equal to that of decaying body. So at the origin one will not observe any effect which makes this situation similar to the situation for Kerr's black hole.

Negative energy in Gödel universe

Metric of the Gödel cosmological model of the rotating Universe proposed in 1949 can be written as

$$ds^2 = c^2 dt^2 - dx_1^2 + \frac{\exp\left(2\sqrt{2}\omega x_1/c\right)}{2} dx_2^2 + 2\exp\left(\sqrt{2}\omega x_1/c\right) c\,dt\,dx_2 - dx_3^2, \tag{31}$$

where ω is constant. Such metric is the exact solution of Einstein's equation with background matter as ideal liquid without pressure and negative cosmological constant Λ

$$R_{ik} - \frac{1}{2} R g_{ik} + \Lambda g_{ik} = -8\pi \frac{G}{c^4} T_{ik}, \tag{32}$$

were

$$-\Lambda = \left(\frac{\omega}{c}\right)^2 = 4\pi \frac{G}{c^2}\rho, \qquad T_{ik} = \rho c^2 u_i u_k, \tag{33}$$

$u^i = \delta^i_0$. Here ω has the sense of the angular velocity of rotation of the vector of fluid of the background matter u^i. Taking instead of t, x_1, x_2 new coordinates t', r, ϕ:

$$\exp\left(\sqrt{2}\omega x_1/c\right) = \cosh 2r + \cos\phi \sinh 2r, \qquad \omega x_2 \exp\left(\sqrt{2}\omega x_1/c\right) = \sin\phi \sinh 2r, \tag{34}$$

$$\tan\frac{1}{2}\left(\phi + \omega t - \sqrt{2}t'\right) = \exp(-2r)\tan\frac{\phi}{2}, \tag{35}$$

one writes the interval (31) in the form:

$$ds^2 = 2\left(\frac{c}{\omega}\right)^2 \left(dt'^2 - dr^2 + (\sinh^4 r - \sinh^2 r) d\phi^2 + 2\sqrt{2}\sinh^2 r\, d\phi\, dt'\right) - dx_3^2, \tag{36}$$

where $-\infty < t' < \infty$, $0 \le r < \infty$, $0 \le \phi < 2\pi$ with identifying $\phi = 0$ and $\phi = 2\pi$.

Now consider the general case of space-time t', r, ϕ, z with the interval

$$ds^2 = a^2\Big[(dt' + \Phi(r)d\phi)^2 - dr^2 - dz^2 - R^2(r)d\phi^2\Big], \tag{37}$$

where a is constant, $-\infty < t < \infty$, $0 \le r < \infty$, $-\infty < z < \infty$, $0 \le \phi \le 2\pi$ with identifying $\phi = 0$ and $\phi = 2\pi$. Let's $\Phi(r) > 0$ and $R(r) > 0$ for $r > 0$.

For Gödel universe $a = \sqrt{2}c/\omega$, $z = x_3/a$ and

$$\Phi(r) = \sqrt{2}\sinh^2 r, \qquad R(r) = \sinh r \cosh r. \tag{38}$$

The metrical tensor is

$$(g_{ik}) = a^2 \begin{pmatrix} 1 & \Phi & 0 & 0 \\ \Phi & \Phi^2 - R^2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad (g^{ik}) = \frac{1}{a^2R^2(r)} \begin{pmatrix} R^2 - \Phi^2 & \Phi & 0 & 0 \\ \Phi & -1 & 0 & 0 \\ 0 & 0 & -R^2 & 0 \\ 0 & 0 & 0 & -R^2 \end{pmatrix}, \tag{39}$$

where indexes $i, k = 0, 1, 2, 3$ correspond to t', ϕ, r, z. Note that for any $r > 0$ the metrical tensor is not degenerate $\det(g_{ik}) = -a^8R^2(r) < 0$. The degeneration for $r = 0$ in Gödel universe is coordinate degeneracy. The eigenvalues of g_{ik} tensor are

$$\lambda_{1,2} = \frac{a^2}{2}\left(\Phi^2 - R^2 + 1 \pm \sqrt{(\Phi^2 - R^2 + 1)^2 + 4R^2}\right), \qquad \lambda_{3,4} = -a^2. \tag{40}$$

For $r > 0$ one has

$$\lambda_1 \ge a^2, \qquad 0 > \lambda_2 \ge -a^2R^2. \tag{41}$$

Note that inspite $g_{\phi\phi}$ is positive for $\Phi(r) > R(r)$ the signature of g_{ik} for all $r > 0$ is the standard $(+, -, -, -)$.

Possible movement of particles is defined by $ds^2 \geq 0$ so for the interval (37) one has

$$dt'^2 + \left(\Phi^2(r) - R^2(r)\right) d\phi^2 + 2\Phi(r) d\phi dt' - dr^2 - dz^2 \geq 0. \tag{42}$$

It is important that for any coordinate system with interval (37) the physical body for any values of r, ϕ, z can be at rest, i.e. there is no static limit! However in (37) there is nondiagonal term $dt' d\phi$ like in Kerr's metric. But differently from the case of rotating coordinate system there is the possibility of the change of the sign before $d\phi^2$.

From (42) one obtains

$$\left(\frac{dt'}{d\phi}\right)^2 + 2\Phi(r)\frac{dt'}{d\phi} + \Phi^2(r) - R^2(r) \geq 0. \tag{43}$$

The solution of this inequality is the union of two intervals

$$\frac{dt'}{d\phi} \in (-\infty, -(\Phi(r) + R(r))] \cup [R(r) - \Phi(r), +\infty) \,. \tag{44}$$

Considering cases of different signs of $d\phi$, one obtains the following sets of solutions of (43):

$$d\phi \geq 0 \Rightarrow \; dt' \geq (R - \Phi) d\phi \; \vee \; dt' \leq -(R + \Phi) d\phi, \tag{45}$$

$$d\phi \leq 0 \Rightarrow \; dt' \geq -(R+\Phi)d\phi \;\vee\; dt' \leq (R-\Phi)d\phi. \tag{46}$$

These sets define light "cones" of future and past for the metric (37).

The form of these cones in cases $\Phi \ll R$, $\Phi = R$ and $\Phi > R$ is shown in Fig. 3 for the Gödel universe with

$$\Phi(r) > R(r) \quad \Leftrightarrow \quad r > r_0 = \log(1+\sqrt{2}). \tag{47}$$

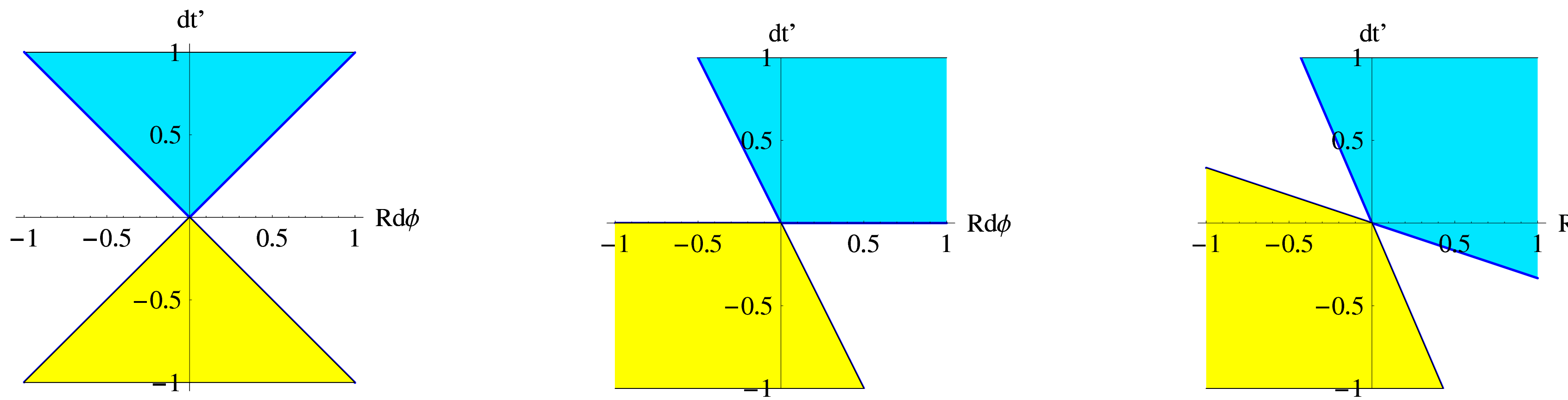

Figure 3: Light "cones" of future (blue color) and past (yellow color) for Gödel universe in coordinates t', ϕ for cases $r = 10^{-3}$ (left), $r = r_0$ (center) and $r = 2r_0$ (right).

Let us find limitations on possible values of the energy of particles moving in such universe.

The coordinate t' is dimensionless, so the "physical energy" of the particle is expressed through the time component of the momentum as

$$E = p_0 \frac{c}{a} = g_{0k} \frac{c}{a} \frac{dx^k}{d\lambda}. \tag{48}$$

For the frame with coordinates (37) the covariant t', ϕ, z components are conserved, because the component of metric depend only on r. So the conserved energy on the geodesic for the interval (37) is

$$E = ca \left(\frac{dt'}{d\lambda} + \Phi(r) \frac{d\phi}{d\lambda} \right). \tag{49}$$

From (45), (46) for the case of movement "forward" in time, i.e. in the future light cone one obtains

$$dt' + \Phi d\phi \geq R|d\phi|, \tag{50}$$

so

$$E \geq caR \frac{|d\phi|}{d\lambda}. \tag{51}$$

It means that for particle moving in the future cone in Gödel universe the energy is not negative.

For movement “back in time” the energy is limited from above by

$$E \leq -caR\frac{|d\phi|}{d\lambda} \tag{52}$$

and so it can be less or equal zero. However such movement physically is inconsistent.

The “time machine” effect in Gödel universe corresponds to continuous movement in the future cone. So for $r > r_0$, where $\Phi(r) > R(r)$ closed loops (they are not geodesic lines) $r = \text{const}$, $z = \text{const}$, called Gödel cycles, from $\phi = 0$ to $\phi = 2\pi$ are closed time-like curves. Particle moving along such cycle is moving “forward” in time but due to identification of values ϕ different on 2π it occurs in the past after the whole cycle. It’s energy is positive due to (51). Such “time machine” is different from that moving in the past by the sign of particle energy.

Conclusion

Three different cases are investigated concerning the possibility of existence of particles with negative and zero energies.

1. **Schwarzschild black hole.** Trajectories of particles with negative and zero energies exist inside of the horizon of black hole which can be shown in Kruskal–Szekeres coordinates.

2. **Flat space-time in Milne's coordinates.** Here one also has the possibility of existence of particles with negative and zero energies if nonsynchronous system of coordinates is used.

3. **Gödel cosmological model with rotation.** Here we proved that in this model in Gödel's coordinates particles with negative and zero energies don't exist.

Thank You for attention !