Arising of mass in scalar quantum field theories.

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Schwinger-Dyson equation

- $$D^{-1} = \Delta^{-1} - \Sigma$$

where D is a full propagator, Δ is a bar propagator, Σ is a self-energy operator.

In the minimal subtraction (MS) scheme it holds:

$$\Delta(p) = \frac{1}{p^2}$$

where *p* is a momentum.

- The inverse full propagator has the following characteristic:

$$\begin{cases} D^{-1}(p)\Big|_{p^2=-m^2} = 0 \\ \dfrac{\partial}{\partial(p^2)} D^{-1}(p)\Bigg|_{p^2=-m^2} = \dfrac{1}{A} \end{cases}$$

In the main approximation it holds

$$D(p) = \frac{A}{p^2 + m^2}$$

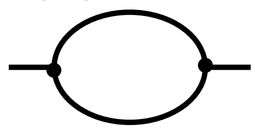
where A is an amplitude, m is a mass.

We consider scalar theories ϕ^3, ϕ^4 ϕ^6 in logarithmic dimensions in Euclidian space.

Theory φ^3.

- $$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{3!}\phi^3, \qquad d = 6 - 2\varepsilon.$$

In one-loop approximation we have:

Σ = ½  + ½ 

where line is the full propagator (skeleton equation).

Substituting the ansatz D(p)=A/(p^2+m^2) we obtain

$$\Sigma = \frac{\lambda^2}{2}\frac{\mu^{2\varepsilon}}{(2\pi)^d}\frac{A}{0^2+m^2}\int dk\,\frac{A}{k^2+m^2} +$$

$$+\frac{\lambda^2}{2}\frac{\mu^{2\varepsilon}}{(2\pi)^d}\int dk\,\frac{A}{k^2+m^2}\,\frac{A}{(p-k)^2+m^2}.$$

Using the formulas

$$\frac{1}{AB} = \int_0^1 \frac{dt}{[At + B(1-t)]^2} \quad \text{and}$$

$$\int \frac{dk}{(k^2 + m^2)^\alpha} = \pi^{d/2} \frac{\Gamma(\alpha - d/2)}{\Gamma(\alpha)} m^{d-2\alpha}$$

we obtain

$$\Sigma = \frac{\lambda^2 A^2 \mu^{2\varepsilon}}{2(4\pi)^{3-\varepsilon}} \left(\Gamma(-2+\varepsilon) m^{2-2\varepsilon} + \Gamma(-1+\varepsilon) \int_0^1 [m^2 + p^2 t(1-t)]^{1-\varepsilon} dt \right)$$

Applying the R-operation we obtain

$$R\Sigma = \frac{gA^2}{2}\left[\frac{m^2}{4}\left(3 - 2\gamma_E - 2\ln\frac{m^2}{4\pi\mu^2}\right) + \int_0^1 [m^2 + p^2 t(1-t)]\left(\gamma_E - 1 + \ln\frac{m^2 + p^2 t(1-t)}{4\pi\mu^2}\right) dt\right]$$

where $g = \frac{\lambda^2}{(4\pi)^3}$, γ_E is the Euler's constant.

The Schwinger-Dyson equation takes the form:

$$D^{-1} = p^2 - \frac{gA^2}{2}\frac{m^2}{4}\left(3 - 2\gamma_E - 2\ln\frac{m^2}{4\pi\mu^2}\right) - \frac{gA^2}{2}\int_0^1 [m^2 + p^2 t(1-t)]\left(\gamma_E - 1 + \ln\frac{m^2 + p^2 t(1-t)}{4\pi\mu^2}\right) dt$$

To calculate mass we put $p^2 = -m^2$. We also differentiate the equation in p^2 and put $p^2 = -m^2$, and we obtain the system of 2 equations:

- $$\begin{cases} 0 = -m^2 - \dfrac{gA^2m^2}{2}\left[\dfrac{1}{3}\left(\gamma_E + \ln\dfrac{m^2}{4\pi\mu^2}\right) + \dfrac{\pi\sqrt{3}}{6} - \dfrac{41}{36}\right] \\ \dfrac{1}{A} = 1 - \dfrac{gA^2}{2}\left[\dfrac{1}{6}\left(\gamma_E + \ln\dfrac{m^2}{4\pi\mu^2}\right) + \dfrac{3\pi\sqrt{3}-17}{18}\right] \end{cases}.$$

Solving it in frames of perturbation theory we obtain:

$$\begin{cases} A = \dfrac{2}{3} + \dfrac{2\pi\sqrt{3}-9}{243}g + \mathcal{O}(g^2) \\ \ln\dfrac{m^2}{4\pi\mu^2} = -\dfrac{27}{2g} + \dfrac{23-2\pi\sqrt{3}}{12} - \gamma_E + \mathcal{O}(g) \end{cases}.$$

So it holds

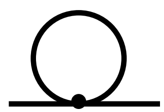
$$m^2 \sim \exp\left(-\frac{27}{2g}\right).$$

Theory φ^4.

- $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4!}(\phi^2)^2, \qquad \lambda > 0, d = 4 - 2\varepsilon,$

ϕ is an *n*-component scalar field

In one-loop approximation we have:

$\Sigma = \frac{1}{2}\frac{n+2}{3}$ 

Substituting the ansatz D(p)=A/(p^2+m^2) we obtain

$$\Sigma = \frac{\lambda}{2}\frac{n+2}{3}\frac{\mu^{2\varepsilon}}{(2\pi)^d}\int dk \frac{A}{k^2+m^2} =$$

$$= \frac{\lambda A(n+2)\mu^{2\varepsilon}m^{2-2\varepsilon}\Gamma(-1+\varepsilon)}{6(4\pi)^{2-\varepsilon}}$$

After the R-operation the Schwinger-Dyson equation takes the form:

$$D^{-1}(p) = p^2 - \frac{gAm^2(n+2)}{6}\left(-1 + \gamma_E + \ln\frac{m^2}{4\pi\mu^2}\right)$$

where $g = \frac{\lambda}{(4\pi)^2}$, γ_E is the Euler's constant.

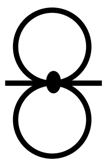
Substituting $p^2 = -m^2$ and differentiating D^{-1} with p^2 we receive:

$$\begin{cases} A = 1 \\ \ln\dfrac{m^2}{4\pi\mu^2} = -\dfrac{6}{(n+2)g} + 1 - \gamma_E \end{cases}.$$

Theory φ^6.

- $$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{6!}\phi^6, \qquad \lambda > 0, d = 3 - 2\varepsilon,$$

In the main approximation in coupling constant we have:

$\Sigma = \frac{1}{8}$ 

Substituting the ansatz D(p)=A/(p^2+m^2) we obtain

$$\Sigma = \frac{\lambda}{8}\left[\frac{\mu^{2\varepsilon}}{(2\pi)^d}\int dk \frac{A}{k^2 + m^2}\right]^2 = \frac{\lambda A^2 \mu^{4\varepsilon}}{8(4\pi)^{3-2\varepsilon}}\left[m^{1-2\varepsilon}\Gamma\left(-\frac{1}{2} + \varepsilon\right)\right]^2$$

For ε=0 we obtain:

$$\Sigma = \frac{gA^2m^2}{2}$$

where $g = \frac{\lambda}{64\pi^2}$.

- The Schwinger-Dyson equation takes the form:

$$D^{-1}(p) = p^2 - \frac{gA^2m^2}{2} \; .$$

Substituting $p^2 = -m^2$ and differentiating D^{-1} with p^2 we receive:

$$\begin{cases} m^2\left(1+\dfrac{gA^2}{2}\right) = 0 \\ \dfrac{1}{A} = 1 \end{cases} .$$

The solution is: m = 0, A = 1.

That is, the mass does not appear.

Now, we apply the Pauli-Villars regularization:

$$D(p) = \frac{A}{p^2 + m^2} - \frac{A}{p^2 + \alpha m^2}, \alpha \to \infty$$

in the dimension $d = 3$.

$$\Sigma = \frac{\lambda}{8}\left[\frac{1}{(2\pi)^3}\int dk\left(\frac{A}{p^2 + m^2} - \frac{A}{p^2 + \alpha m^2}\right)\right]^2$$

$$= \frac{gA^2m^2}{2}(1 - \alpha)^2 .$$

After the R-operation we obtain the same result

$$R\Sigma = \frac{gA^2m^2}{2}.$$

Conclusion.

We have investigated the scalar models φ^3, φ^4 and φ^6 in the logarithmic dimension of space. For the theories φ^3 and φ^4 the mass appears in the first order of perturbation theory whereas for the φ^6-theory the mass does not appear in the first order. The result does not depend on the way of regularization.

Thank you for your attention!!!