

# Electrodynamics against the background of Null Cosmic Strings

D.V. Fursaev, I.G.Pirozhenko

BLTP JINR and Dubna State University

October 11, 2022

MQFT, St.-Petersburg, 10-15 October 2022

# Null Strings

A null cosmic string is a one-dimensional object whose points move along trajectories of light rays, orthogonally to the string itself.

[A. Schild. Classical Null Strings. Phys. Rev. D, 16:1722, 1977.](#) (Schild equations)

Null strings are characterised by their optical properties [a complex optical scalar which is determined by an analogue of the Sachs' optical equation].

[D.V. Fursaev, Phys. Rev. D103 \(2021\) no.12, 123526](#)

The origin of null strings may be related to physics of fundamental strings at the Planckian energies.

[F. Xu, JHEP 10 \(2020\) 045](#)

- The study is motivated by possible effects of null strings in cosmology. World-sheets of null strings develop caustics accumulating energy.

[E.A. Davydov, D.V. Fursaev, V.A.Tainov, Phys. Rev. D105 \(2022\) no.8, 083510](#)

# Null strings (massless, tensionless)

From a massive cosmic string at rest along z-axis

$$ds^2 = -dt^2 + dz^2 + dr^2 + (1 - 4G\mu)^2 r^2 d\Theta^2, \quad r^2 = x^2 + y^2$$

→ Aichelburg-Sexl boost (Penrose limit)

$$\cosh \chi = (1 - v^2/c^2)^{-1/2} \rightarrow \infty, \quad E = mc^2 \cosh \chi \rightarrow \text{finite}$$

→ Kerr-Schild metric

$$ds^2 = -dudv + \omega|y|\delta(u)du^2 + dy^2 + dz^2, \quad \omega \equiv 8\pi G\varepsilon$$

$\varepsilon$  - energy per unit length,  $u = t - x$ ,  $v = t + x$ .

C. Barrabes, P.A. Hogan, W. Israel, Phys.Rev. D66 (2002) 025032.

The problem of particle (wave) in the field of null cosmic strings is related to the movement in the impulsive (shock) gravitational wave background.

R. Penrose, *Part of General relativity : Papers in honour of J.L. Synge*, 101-115 (1972)

The gravitational shock wave of a massless particle attracted a considerable interest in the context of black hole formation in high energy particle collisions.

T. Dray and G. 't Hooft. *NPB*, 253:173–188, 1985. G. 't Hooft., *Phys. Lett. B*, 198:61–63, 1987.

- To our knowledge the field effects in the background of a null cosmic string are not yet comprehensively studied.

Classical and quantum FT for **scalar fields only** on general shock wave background was given by C. Klimcik [PLB'1988], and in the scattering matrix context by C.O. Lousto, N. G. Sanchez, *Nucl.Phys.B* 355 (1991) 231-249.

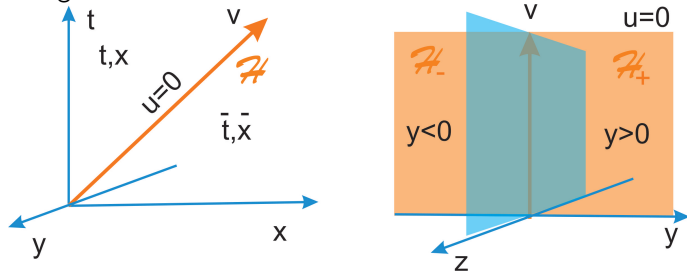
- Our aim is to derive some physical effects using the **holonomy property** of the null string spacetime.

# Null string dynamics

We consider a null straight string moving in the Minkowski spacetime in the direction of  $x$  axes and parallel to  $z$  axes,

$$ds^2 = -dudv + \omega|y|\delta(u)du^2 + dy^2 + dz^2, \quad \omega \equiv 8\pi GE.$$

The delta-function in the metric indicates a singularity of the coordinate chart along the hypersurface  $\mathcal{H} : u = 0$ , which is the **event horizon** of the string.



- The string equations of motion are  $t - x = 0$ ,  $y = 0$ .
- Null string world surface is  $u = y = 0$ .

# Null Holonomies

- The null string yields the nontrivial **null holonomy**. The parallel transport of a vector  $V$  along a closed contour around the string results in a **null rotation**,

$$V' = M(\omega)V, \quad \omega \equiv 8\pi GE.$$

**Null holonomy** belongs to the parabolic subgroup of the Lorentz transformations (null rotations),  $x'^{\mu} = M_{\nu}^{\mu}(\omega)x^{\nu}$ , and acts on  $u, v, y, z$  coordinates in  $R^{1,3}$  as follows:

$$u' = u, \quad v' = v + 2\omega y + \omega^2 u, \quad y' = y + \omega u, \quad z' = z.$$

The group parameter of the holonomy,  $\omega \equiv 8\pi GE$ , is determined by the energy of a string per unit length,  $E$ .

These transformations **do not move points on the string world surface**.

M. van de Meent, Geometry of Massless Cosmic Strings, Phys. Rev. D87 (2013) no.2, 025020, e-Print: arXiv:1211.4365 [gr-qc].

# The Holonomy Method

- We decompose the space-time into two parts by the **light surface**  $u = 0$ . This surface is the string **event horizon**  $\mathcal{H}$ , as the events that occur in the half-space  $u > 0$  do not affect the events in the area  $u < 0$ , and vice versa.

For particles and light rays we define **the ingoing trajectories** (  $u < 0$  ) and **the outgoing trajectories** (  $u > 0$  ).

We consider two parts of the event horizon, left and right with respect to the string:  $\mathcal{H}_L$  ( $y < 0$ ), and  $\mathcal{H}_R$  ( $y > 0$ ).

- To describe **the outgoing fields**, we introduce a coordinate chart which has a cut on  $\mathcal{H}_L$ .
- The string horizon is considered as a Cauchy hypersurface where initial data for **outgoing trajectories** are determined. The 'right' trajectories ( $y > 0$ ) behave smoothly across  $\mathcal{H}$  while the 'left' trajectories ( $y < 0$ ) are **shifted along the  $v$  coordinate and change their direction under the null rotation**.
- At  $y < 0$ ,  $u = 0$ , the coordinate transformations are reduced to

$$v' = v + 2\omega y, \quad y < 0.$$

# Cauchy data for fields on the string horizon

The holonomy method allows one to study different fields (fibre bundles) over the null string geometry.

We introduce Cauchy data on the string horizon  $\mathcal{H}$  for fields in  $u > 0$ ,

$$\phi(x) |_{\mathcal{H}_{\pm}} = \hat{\phi}_{\pm}(\mathbf{x}) , \quad \mathbf{x} \equiv (v, y, z),$$

where

$$\hat{\phi}_{+}(\mathbf{x}) = \bar{\phi}(\bar{x}) |_{\mathcal{H}_{+}} , \quad \hat{\phi}_{-}(\mathbf{x}) = S(\omega) \bar{\phi}(\bar{x}) |_{\mathcal{H}_{-}} ,$$

and  $\bar{\phi}$  is the (ingoing) field at  $u < 0$ . The element  $S(\omega)$  belongs to some representation of the Lorentz group.

These transition conditions guarantee that physical quantities measured by 'left' observers behave smoothly across the horizon while the factor  $S(\omega)$  ensures the required holonomy.

Here  $\phi$  stands as a vector gauge field or a symmetric second rank tensor field corresponding to a gravity wave.

# Cauchy data for fields on the string horizon

The equations of motion for  $\phi$  (under appropriate gauge conditions) are

$$\square\phi(x) = j(x) \quad ,$$

where  $\square = \partial_\mu \partial^\mu$  and  $j(x)$  is an external source.

For waves propagating in the string spacetime the problem is **homogeneous** ( $j = 0$ ), and with the given Cauchy data the solution **in the domain  $u > 0$**  is given by

$$\phi(x) = \int_{y' > 0} d\mathbf{x}' D(u, \mathbf{x} - \mathbf{x}') \hat{\phi}_+(\mathbf{x}') + \int_{y' < 0} d\mathbf{x}' D(u, \mathbf{x} - \mathbf{x}') \hat{\phi}_-(\mathbf{x}') \quad ,$$

where the  $D$ -function is the solution to the following problem:

$$\square D(x) = 0 \quad , \quad D(u, \mathbf{x})|_{u=0} = \delta^{(3)}(\mathbf{x}) \quad ,$$

$$D(u, \mathbf{x}) = \frac{1}{(2\pi)^3} \int d^3 p e^{ip \cdot x} \quad , \quad D(x) = \frac{1}{\pi} \frac{\partial}{\partial v} \delta(x^2) \quad ,$$

where  $x^2 = x^\mu x_\mu = -uv + y^2 + z^2$ ,  $p \cdot x = p_\mu x^\mu$ .

The solution to the **inhomogeneous problem** in domain  $u > 0$  can be written as

$$\phi(x) = \phi_1(x) + \phi_2(x) .$$

Here  $\phi_1(x)$  coincides with a standard solution in  $R^{1,3}$  (without the string) taken at  $u > 0$ . Let  $\hat{\phi}_1(\mathbf{x}) = \phi_1(u = 0, \mathbf{x})$  and  $\hat{\phi}_{1,\pm}$  are corresponding data on  $\mathcal{H}_{\pm}$ .

Then  $\phi_2(x)$  is a solution to the **a homogeneous problem**

$$\square \phi_2(x) = 0 \quad , \quad \phi_2(u = 0, \mathbf{x}) = \hat{\phi}_2(\mathbf{x}) \quad , \quad \hat{\phi}_{2,\pm}(\mathbf{x}) = \hat{\phi}_{\pm}(\mathbf{x}) - \hat{\phi}_{1,\pm}(\mathbf{x}) \quad .$$

The Cauchy data for  $\phi_2(x)$  ensure the required data for the solution  $\phi(x)$ .

- It is clear from the above construction that **most physical effects are determined by partial refraction of waves on the string horizon  $\mathcal{H}_-$  as seen by observers crossing  $\mathcal{H}_+$ .**

# Refraction of EM waves on the string horizon

Consider the scattering of electromagnetic waves by null strings. The corresponding equations and gauge-fixing conditions are

$$\square A_\mu = 0 \quad , \quad \partial A = 0 \quad ,$$

We use the Lorentz gauge conditions since they are invariant under holonomy transformations on  $\mathcal{H}$  and, therefore, can be imposed in the whole spacetime of a null string.

Outside the string world-sheet the spacetime is Minkowsky

$$ds^2 = -dvdu + dy^2 + dz^2 \quad , \quad \square = -4\partial_u\partial_v + \partial_y^2 + \partial_z^2.$$

Before scattering on the string a monochromatic plane wave has a form

$$\bar{A}_\mu(\bar{x}) = \Re (\bar{E}_\mu e^{i\bar{k}\cdot\bar{x}}) \quad , \quad u < 0,$$

where  $\bar{E}_\mu$  is some complex polarization vector,  $\bar{k}^\mu \bar{E}_\mu = 0$ . The incoming data is denoted with the bar.

The Cauchy data on  $\mathcal{H}$  are defined by ingoing fields,

$$\hat{A}_{+,\mu}(\mathbf{x}) = \bar{A}_\mu(\bar{x})|_{\mathcal{H}_+} \quad , \quad \hat{A}_{-,\mu}(\mathbf{x}) = M_\mu{}^\nu(\omega) \bar{A}_\nu(\bar{x})|_{\mathcal{H}_-} \quad ,$$

where  $M_\mu{}^\nu = \eta_{\mu\mu'} \eta^{\nu\nu'} M^{\mu'}_{\nu'}$ .

On the right coordinate chart there is no transformation of wave crossing  $\mathcal{H}_+$ . If the holonomy transformations are applied to  $\bar{A}_\mu(\bar{x})$  on  $\mathcal{H}_-$  the wave leaves  $\mathcal{H}_-$  with the transformed momentum

$$k_-^\mu = M^\mu{}_\nu(\omega) \bar{k}^\nu \quad .$$

The transformed momentum  $k_-$  is introduced to satisfy the condition  $\bar{k} \cdot \bar{x}|_{\mathcal{H}_-} = k_- \cdot x_-$ .

Since the velocity of **the left observers** change in the corresponding way, the observers **do not see the transformation of the wave**.

**For the right observers the wave on  $\mathcal{H}_-$  changes its energy and looks refracted.**

# Refraction of EM waves on the string horizon

If  $E$  and  $\vec{k}$  are, respectively, the energy and the momentum of the incoming wave, the angle  $\varphi_{\text{refr}}$  of the refraction and the energy of the refracted wave are

$$\cos \varphi_{\text{refr}} = \frac{(\vec{k}_- \vec{k})}{E_- E} = \frac{1}{EE_-} \left[ E^2 + \frac{\omega^2}{2} (Ek^x - (k^x)^2) + \omega Ek^y \right]$$
$$E_- = \left( 1 + \frac{\omega^2}{2} \right) E - \frac{\omega^2}{2} k^x + \omega k^y \quad .$$

The waves traveling along the string axis  $z$  **are not refracted**.

A general solution describing physical effects caused by the refraction of waves on the string horizon can be used for EM or gravity waves. In case of the monochromatic waves each component of  $A_\mu$  or  $h_{\mu\nu}$  can be treated as a scalar wave. Therefore, if one neglects effects related to polarizations common features of scattering problem can be understood by studying a scalar field theory.

# Interference wedge

Consider a real massless scalar field  $\phi$ , with equation

$$\square\phi = 0 \quad ,$$

which behaves as a monochromatic wave at  $u < 0$ ,  $\bar{\phi}(\bar{x}) = e^{i\bar{k}\cdot\bar{x}}$ .

In the domain  $u > 0$  the scattered wave is a superposition of waves,

$$\phi(x) = \phi_+(x) + \phi_-(x) \quad ,$$

coming from  $\mathcal{H}_+$  and  $\mathcal{H}_-$ , respectively.

$$\phi_{\pm}(x) = [\theta(\pm f_{\pm}) + \varepsilon(\pm f_{\pm})G(g_{\pm})] \exp(ik_{\pm}x) \quad , \quad k_v > 0 \quad (1)$$

$$\phi_{\pm}(x) = [\theta(\mp f_{\pm}) + \varepsilon(\mp f_{\pm})G^*(-g_{\pm})] \exp(ik_{\pm}x) \quad , \quad k_v < 0 \quad . \quad (2)$$

where  $\theta$  and  $\varepsilon$  are the step and the sign functions, and

$$f(k, x) = uk_y + 2k_v y \quad , \quad g(k, x) = \frac{f^2(k, x)}{4k_v u} \quad ,$$

where  $f, g$  are dimensionless functions,  $f_{\pm} = f(k_{\pm}, x)$ ,  $g_{\pm} = g(k_{\pm}, x)$ .

# Near-field and far-field effects

Near the string worldsheet,  $u \rightarrow 0$ ,  $g \rightarrow 0$  (a 'near-field zone') the scattered wave is not monochromatic due to the presens of the  $G$ -factor.

In a 'far-field zone',  $g \gg 1$ , the wave has a simple form. For  $k_v > 0$ , it is

$$\begin{aligned}\phi(x) &= \theta(f_+) \exp(ik_+x) + \theta(-f_-) \exp(ik_-x) + \phi_{\text{tail}}(x) \ , \\ \phi_{\text{tail}}(x) &= \varepsilon(f_+) \frac{1}{\sqrt{4\pi g_+}} e^{ik_+x+i\varphi_+} + \varepsilon(-f_-) \frac{1}{\sqrt{4\pi g_-}} e^{ik_+x+i\varphi_-} + O(g_{\pm}^{-3/2}) \ ,\end{aligned}$$

where  $\varphi_{\pm} = g_{\pm} + \pi/4$ . The 'tails'  $\phi_{\text{tail}}(x)$  are determined by the  $G$ -factor whose amplitudes are suppressed by factors  $g_{\pm}^{-1/2}$ .

$|g_{\pm}| \sim L/\lambda$  where  $\lambda$  is a wave length and  $L$  is a distance related to position of the observer with respect to the string trajectory.

Physical effects in the far-field zone are interesting for the distant observers. Right observers crossing  $\mathcal{H}_+$  interpret the solution as a refraction of the left wave on  $\mathcal{H}_-$ .

# The boundaries of diffraction

The surfaces  $f_{\pm}(x) = 0$ , determine boundaries of diffraction of right and left parts of the wave behind the string. The normal vectors  $n_{\pm}$  to these surfaces,  $df_{\pm} = n_{\mu}dx^{\mu}$ , are orthogonal to the wave vectors

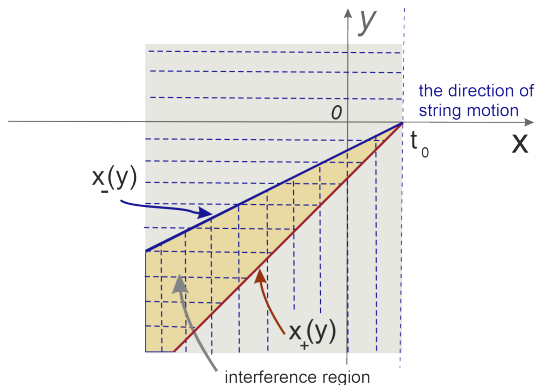
$$(n_{\pm} \cdot k_{\pm}) = 0 \quad .$$

The surfaces  $f_{\pm}(x) = 0$  intersect at the string world-sheet.

The domains of the diffraction overlap. In the overlap region,  $f_{+} > 0$ ,  $f_{-} < 0$ ,  $u > 0$ , the left and right waves interfere since the wave vectors  $k_{\pm}$  are related by the nontrivial null rotation,  $k_{-} = M(\omega)k_{+}$ .

Thus, the null string leaves behind an **interference wedge**. This physical effect is similar to the effect of massive and null strings which leave behind the regions of overdensities of non-relativistic matter.

# The boundaries of diffraction



The interference wedge is shown for the string at the moment  $t = t_0$ . The string is stretched along the  $z$  axis, orthogonal to figure plane and is located at  $x = t_0$ ,  $y = 0$ . The parameters for the wave are such that  $k_y > 0$ ,  $k_{-y} = k_y - 2\omega k_v > 0$ .

# The overlap region

To demonstrate the existence of the overlap region we fix the moment  $t = t_0$ , put  $k_+ = k$ , and suppose that  $k_y > 0$ ,  $k_- = k_y - 2\omega k_v > 0$ . In coordinates  $x$  and  $y$  conditions  $f_+ > 0$ ,  $f_- < 0$ ,  $u > 0$  look as

$$x_-(y) < x < x_+(y) \quad , \quad x < t_0 \quad ,$$

$$x_+(y) = t_0 + \frac{2k_v y}{k_y} \quad , \quad x_-(y) = t_0 + \frac{2k_v y}{k_y - 2\omega k_v} \quad .$$

These conditions hold for  $y < 0$ . The angle of the interference wedge,  $\varphi_{\text{intf}}$ , is the angle between the lines  $x = x_{\pm}(y)$

$$\cos \varphi_{\text{intf}} = \frac{k_y(k_y - 2\omega k_v) + 4k_v^2}{(k_y^2 + 4k_v^2)^{1/2}((k_y - 2\omega k_v)^2 + 4k_v^2)^{1/2}} \quad .$$

At small  $\omega$ ,  $\varphi_{\text{intf}} = O(\omega^2)$ . The interference wedge exists at  $k_y = 0$  when  $\cos \varphi_{\text{intf}} = (1 + \omega^2)^{-1/2}$ .

## The energy density of a real scalar field

$$\mathcal{E}_\omega(x) = T_{00}(x) = (\partial_u \varphi(x))^2 + (\partial_v \varphi(x))^2 + \frac{1}{2}(\partial_y \varphi(x))^2 + \frac{1}{2}(\partial_z \varphi(x))^2.$$

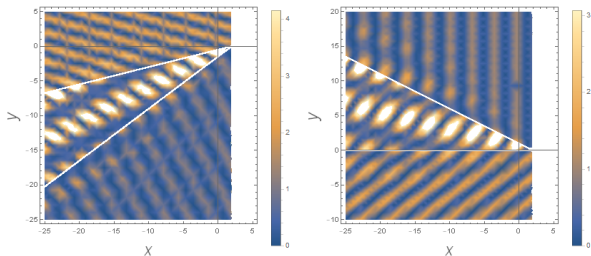


Fig.: instant energy density in the  $(x, y)$  plane orthogonal to the string located at  $x = t_0 = 2$ ,  $y = 0$ ,  $\omega = 0.5$ ; coordinates are given in dimensionless units ( $k_v = 1$ ). To get an observable intensity map one has to compute the time average of the energy density. Left: the parameters for the wave are such that  $k_v > 0$ ,  $k_{-y} = k_y - 2\omega k_v > 0$ . Right:  $k_y = 0$ ,  $\cos \phi_{int} = (1 + \omega^2)^{-1/2}$ .

# Electrodynamics near null strings

Consider electromagnetic fields created by charges moving near null strings. The Maxwell equations in the region below the string horizon,  $u < 0$ , are

$$\square \bar{A}^\mu = \bar{j}^\mu \quad ,$$

$$\bar{j}^\mu(x) = \bar{\rho}(x) \bar{u}^\mu(x) \quad ,$$

where  $\bar{u}^\mu$  are 4-velocities of charges distributed with a density  $\bar{\rho}$ ; we use the Lorentz gauge  $\partial \bar{A} = 0$ .

Above the string horizon,  $u > 0$ , one has the boundary problem (for the  $R$  coordinate chart)

$$\square A^\mu = j^\mu \quad ,$$

$$\hat{A}_+^\mu(\mathbf{x}) = \bar{A}^\mu(\bar{x})|_{\mathcal{H}_+} \quad , \quad \hat{A}_-^\mu(\mathbf{x}) = M^\mu{}_\nu(\omega) \bar{A}^\nu(\bar{x})|_{\mathcal{H}_-} \quad ,$$

$$\hat{j}_+^\mu(\mathbf{x}) = \bar{j}^\mu(\bar{x})|_{\mathcal{H}_+} \quad , \quad \hat{j}_-^\mu(\mathbf{x}) = M^\mu{}_\nu(\omega) \bar{j}^\nu(\bar{x})|_{\mathcal{H}_-} \quad ,$$

The conditions for the current  $\hat{j}_{\pm,\mu}$  on  $\mathcal{H}_\pm$  ensure continuity of the current on the horizon from the point of view of observers crossing  $\mathcal{H}$ .

# Electrodynamics near null strings

The electric charge  $Q$  on null surfaces  $u = C$ ,

$$Q = \int_{u=C} d\Sigma_\mu j^\mu(u, \mathbf{x}) = \int_{u=C} j^u(u, \mathbf{x}) dv dy dz \quad ,$$

conserves across  $\mathcal{H}$ , since  $j^u$  component does not change under null rotations.

The solution to is

$$A^\mu(x) = A_1^\mu(x) + A_2^\mu(x) \quad ,$$

where  $A_1^\mu(x)$  is a solution if inhomogenises problem with corresponding data  $\hat{A}_{1,\pm}^\mu$  on  $\mathcal{H}_\pm$ , and  $A_2^\mu(x)$  is a solution to the homogeneous problem

$$\square A_2^\mu = 0 \quad , \quad A_2^\mu(u=0, \mathbf{x}) = \hat{A}_2^\mu(\mathbf{x}) \quad , \quad \hat{A}_{2,\pm}^\mu(\mathbf{x}) = \hat{A}_\pm^\mu(\mathbf{x}) - \hat{A}_{1,\pm}^\mu(\mathbf{x}) \quad .$$

# Field of a single charge

How does a null string change the electric field of a charged particle?

An electric charge is at rest below the horizon as a point with coordinates

$$x_e = 0, \quad y_e = a, \quad z_e = 0, \quad a > 0.$$

The corresponding current is  $\bar{j}^\mu(x) = e\delta^{(3)}(\vec{x} - \vec{x}_e)u^\mu$ , with  $u^\mu = \delta_0^\mu$ .

The field of the particle below  $\mathcal{H}$  is

$$\bar{A}^\mu = \frac{e}{4\pi} \frac{\delta_0^\mu}{\sqrt{x^2 + (y - a)^2 + z^2}}.$$

The considered particle moves freely and crosses  $\mathcal{H}_+$  part of the horizon. In the  $R$ -chart nothing happens with the 4-velocity of the particle at  $u = 0$ . Therefore,

$$j^\mu(x) = \bar{j}^\mu(x), \quad u > 0,$$

and the inhomogeneous part of the solution can be taken as

$$A_1^\mu(x) = \bar{A}^\mu(x).$$

# Field of a single charge

The homogeneous part solves the wave equation with the following Cauchy data:

$$\hat{A}_{2,+}^{\mu}(\mathbf{x}) = 0 \quad , \quad \hat{A}_{2,-}^{\mu}(\mathbf{x}) = \hat{A}_{-}^{\mu}(\mathbf{x}) - \hat{A}_{1,-}^{\mu}(\mathbf{x}) = [M_{\nu}^{\mu}(\omega) \bar{A}^{\nu}(\bar{x}) - \bar{A}^{\mu}(x)]|_{\mathcal{H}_{-}} \quad .$$

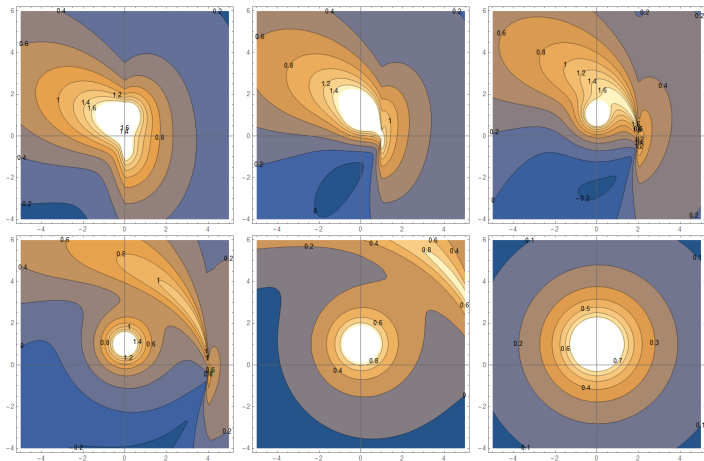
The homogeneous solution is

$$A_{2,-}^{\mu}(u, \mathbf{x}) = -\frac{1}{\pi u} \int_{-\infty}^0 dy' \int_{-\infty}^{\infty} dz' a^{\mu}(u, \mathbf{x}, y', z') \quad ,$$

$$\begin{aligned} a^u(u, \mathbf{x}, y', z') &= \partial_v [f(v - \Delta - 2\omega y', y', z') - f(v - \Delta, y', z')] \quad , \\ a^v(u, \mathbf{x}, y', z') &= \partial_v [(1 + \omega^2)f(v - \Delta - 2\omega y', y', z') - f(v - \Delta, y', z')] \\ a^y(u, \mathbf{x}, y', z') &= \omega \partial_v f(v - \Delta - 2\omega y', y', z') \\ a^z(u, \mathbf{x}, y', z') &= 0, \end{aligned}$$

$$f(v, y, z) = \frac{e}{4\pi} \frac{1}{\sqrt{v^2/4 + (y - a)^2 + z^2}} \quad , \quad \Delta = \frac{1}{u} ((y - y')^2 + (z - z')^2) \quad ,$$

# Scalar potential $A^0$ of a point charge



Scalar potential  $A^0$  of a point charge is disturbed by a null string passing in  $x$  direction,  $\mu = 1$ . The time variable changes as  $t/a = 0.01, 1, 2, 4, 6, 20$ . The last picture corresponds to undisturbed Coulomb potential of a charge placed in the point (0, 1, 0).

# Observable effects

- The near field ( $u \rightarrow 0$ )

The effects are complicated because of the form-factor which appears in the solution due to the string. After scattering on the string a monochromatic wave is not monochromatic anymore.

- The far field ( $u \rightarrow \infty$ )

$$\begin{aligned} A &= A_+ + A_-, \\ A_{\pm} &= \theta(\pm f_{\pm}) e^{ik_{\mu}^{\pm} x^{\mu}} + \text{"tail"}, \quad \text{"tail"} \sim \mathcal{O}(u^{-1/2}). \end{aligned}$$

A polarized wave  $\hat{E} e^{ikx}$  transforms to

$$\theta(\pm f_+) \hat{E}_+ e^{ik_+^{\mu} x_{\mu}} + \theta(-f_-) \hat{E}_- e^{ik_-^{\mu} x_{\mu}} + \text{"tail"},$$

where  $k_+ = k$ ,  $\hat{E}_+ = \hat{E}$  for the right observer, and

$$k_- = M(\omega)k, \quad \hat{E}_- = S(\omega)\hat{E}.$$

# Observable effects in far field region

From an observational point of view, the interaction of a null string with electromagnetic and gravitational waves, is of interest. When they are scattered by a null string the following effects emerge:

- **refraction** of the part of the wave that propagates behind the passing string with respect to the observer. All "left" vectors turn in the same way due to holonomy.
- **diffraction** on the string, when the "right" part of the wave partially overlaps the "left" region. Diffraction is accompanied by **interference**, and depends on the string energy and wavelength (conditions  $f_{\pm} = 0$ );
- perturbation of the electric field of a charged particle
- self force similar to the self-force acting on a point charge in the spacetime of a massive static cosmic string

*B. Linet, PRD 33. 1833, 1986*

# Conclusion

- We have developed the holonomy method for fields. It allows to study **different fields** (fiber bundles) in the null string background.
- We predict physical effects related to wave propagating in the gravitational field of a **straight null string** in Minkowsky spacetime  $R^{1,3}$ .
- Gravitational effects in the the null string background is work in progress.

Thank you!