

Strong coupling limit, "convergent perturbation" for QED and "Landau pole"

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Motivation — How can you deal with path integral

- RG
- Faddeev-Popov trick, ghost
- Lipatov's asymptotics
- MSR-formalism
- Ushveridze's convergent models

I. Divergent and convergent series

The action terms, $m = 0$

$$\begin{aligned} S &= i\bar{\psi}\not{\partial}\psi - \bar{\psi}m\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\bar{\psi}\gamma^\mu A_\mu\psi = \\ &= S_{\bar{\psi}\psi} + S_{FF} + S_{\bar{\psi}A\psi} \end{aligned}$$

Perturbations in $\alpha = e^2$

$$S_{\text{free}} = S_{\bar{\psi}\psi} + S_{FF},$$

$$S_{\text{int}} = S_{\bar{\psi}A\psi}$$

I. Divergent and convergent series

The action terms \pm

$$\begin{aligned} S &= i\bar{\psi}\partial\psi - \bar{\psi}m\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\bar{\psi}\gamma^\mu A_\mu\psi = \\ &= S_{\bar{\psi}\psi} + S_{FF} + S_{\bar{\psi}A\psi} \pm aS_{\bar{\psi}\psi}\sqrt{S_{FF}} \end{aligned}$$

Perturbations in ζ , $\zeta_{ph} = 1$

$$S_{free} = S_{\bar{\psi}\psi} + S_{FF} + aS_{\bar{\psi}\psi}\sqrt{S_{FF}},$$

$$S_{int} = \zeta(S_{\bar{\psi}A\psi} - aS_{\bar{\psi}\psi}\sqrt{S_{FF}}).$$

How not to recalculate diagramms

$$S = S_{\bar{\psi}\psi} + S_{FF} + aS_{\bar{\psi}\psi}\sqrt{S_{FF}} + \zeta(S_{\bar{\psi}A\psi} - aS_{\bar{\psi}\psi}\sqrt{S_{FF}}).$$

$$e^{iS} = \int dy \delta(y - S_{FF}) e^{iS_{\bar{\psi}\psi} + iS_{FF} + iaS_{\bar{\psi}\psi}\sqrt{y} + i\zeta(S_{\bar{\psi}A\psi} - aS_{\bar{\psi}\psi}\sqrt{y})} =$$

$$\int dy \int dy' e^{iy'(y - S_{FF})} e^{iS_{\bar{\psi}\psi} + iS_{FF} + iaS_{\bar{\psi}\psi}\sqrt{y} + i\zeta(S_{\bar{\psi}A\psi} - aS_{\bar{\psi}\psi}\sqrt{y})} =$$

$$\int dy \int dy' e^{iy'y} e^{i(1+a(1-\zeta)\sqrt{y})S_{\bar{\psi}\psi} + i(1-y')S_{FF} + i\zeta S_{\bar{\psi}A\psi}} =$$

$$A_\mu \rightarrow \frac{A_\mu}{\sqrt{1-y'}}, \quad \psi, \bar{\psi} \rightarrow \frac{\psi, \bar{\psi}}{\sqrt{1+a(1-\zeta)\sqrt{y}}},$$

How not to recalculate diagramms

$$\int dy \int dy' e^{iy'y} e^{i(1+a(1-\zeta)\sqrt{y})S_{\bar{\psi}\psi} + i(1-y')S_{FF} + i\zeta S_{\bar{\psi}A\psi}} =$$

$$A_\mu \rightarrow \frac{A_\mu}{\sqrt{1-y'}}, \quad \psi, \bar{\psi} \rightarrow \frac{\psi, \bar{\psi}}{\sqrt{1+a(1-\zeta)\sqrt{y}}},$$

$$e \rightarrow \tilde{e} = \frac{e\zeta}{\sqrt{(1-y')(1+a(1-\zeta)\sqrt{y})}}$$

$$\langle A^n (\psi \bar{\psi})^{m/2} \rangle = \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi} A^n (\psi \bar{\psi})^{m/2} e^{iS(e, A, \psi, \bar{\psi})}$$

$$G_{nm}^{\text{new}}(e) = \int dy \int dy' e^{iy'y} \frac{G_{nm}^{\text{standard}}(\tilde{e})}{(1-y')^{n/2} (1+a(1-\zeta)\sqrt{y})^m}$$

How not to recalculate diagramms

$$e \rightarrow \tilde{e} = \frac{e\zeta}{\sqrt{(1-y')(1+a(1-\zeta)\sqrt{y})}}$$

$$G_{nm}^{\text{new}}(e) = \int dy \int dy' e^{iy'y} \frac{G_{nm}^{\text{standard}}(\tilde{e})}{(1-y')^{n/2}(1+a(1-\zeta)\sqrt{y})^m}$$

$$(D_{nm}^{\text{new}})^{(N)} = \int dy \int dy' e^{iy'y} \frac{(D_{nm}^{\text{standard}})^{(N)} \zeta^N}{(1-y')^{(n+N)/2}(1+a(1-\zeta)\sqrt{y})^{m+N}}$$

$$U_{nm}^{(N)} = \int dy \int dy' \frac{e^{iy'y}}{(1-y'+i\epsilon)^{(N+n)/2}(1+a(1-\zeta)\sqrt{y})^{N+m}} =$$

$$= \frac{1}{\Gamma((N+n)/2)} \int_0^\infty \frac{dt \ t^{(N+n)/2-1} e^{-t}}{(1+\sqrt{ta}(1-\zeta))^{N+m}}$$

New perturbation vs the standard one

$$(D_{nm}^{\text{new}})^{(N)} = U_{nm}^{(N)} (D_{nm}^{\text{standard}})^{(N)}$$

$$U_{nm}^{(N)} = \frac{1}{\Gamma((N+n)/2)} \int_0^\infty \frac{dt}{(1 + \sqrt{t}a(1-\zeta))^{N+m}} t^{(N+n)/2-1} e^{-t}$$

Some properties

- $\zeta = \zeta_{ph} = 1$ produces the standard perturbation
- expanding in ζ generates new perturbation
- there is no Lipatov's instanton for the new case. What about the convergence?

Convergency

Expanding in ζ yields a series with the radius of convergence R_ζ :

$$R_\zeta^{-1} = \left| 1 - \sqrt{\frac{e^2}{2e_0^2 a^2}} \right|,$$

where $e_0^2 \simeq 5.9$ according to the papers

Bogomilniy E.B., Kybishin Yu.A., J. of Nucl.Phys. (rus) 34, 1535 (1981).

Bogomilniy E.B., Kybishin Yu.A., J. of Nucl.Phys. (rus) 35, 202 (1982)

Then ζ -expansion in QED is convergent at $\zeta = 1$ if $e \leq 2\sqrt{2}ae_0$,

Non-trivial RG-equation and composite operators $(\bar{\psi} A \gamma \psi)^k$

$$(D_\mu + \beta(\alpha)\partial_\alpha + \beta_2\partial_\alpha^2 + \beta_3\partial_\alpha^3 + \dots + \gamma) G_{nm}^{\text{Renorm}} = 0$$

$$\beta = \frac{\alpha^2}{U_{nm}^{(0)}} \frac{\partial}{\partial \alpha} \sum_{k=0} \alpha^k \zeta^{2k} [Z_\alpha - m(Z_{\bar{\psi}} + Z_\psi) - nZ_A]^{(k)} U_{nm}^{(2k+2)} - \alpha \gamma_{nm}$$

$$\gamma_{nm} = -\frac{\alpha}{U_{nm}^{(0)}} \frac{\partial}{\partial \alpha} \sum_{k=0} \alpha^k \zeta^{2k} [mZ_\psi + mZ_{\bar{\psi}} + nZ_A]^{(k)} U_{nm}^{(2k)},$$

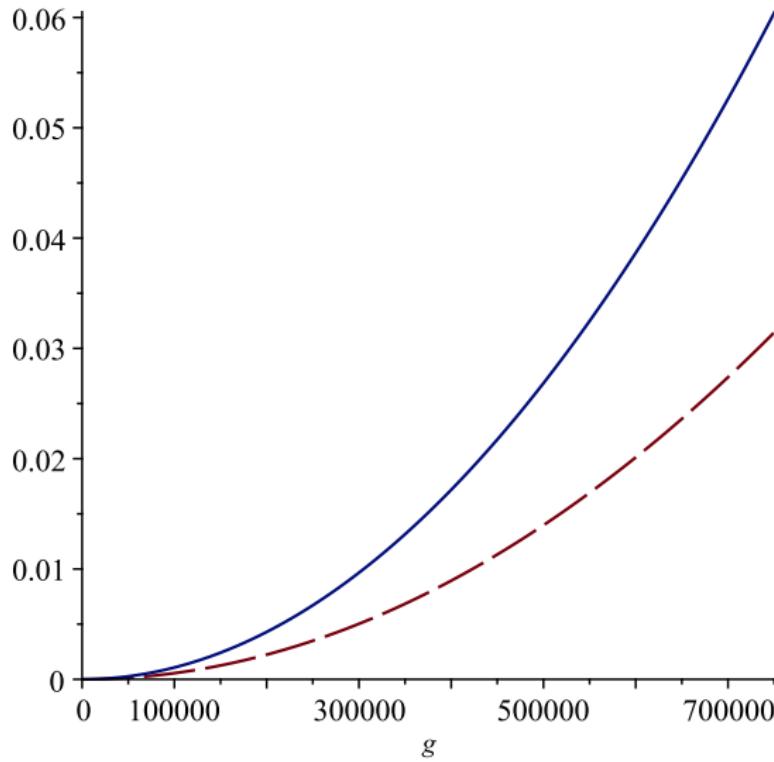
where $[\dots]^{(k)}$ denotes α^k contribution to the simple pole in ϵ ,
 $\alpha = e^2/(4\pi)$, $Z_\alpha, Z_\psi, Z_{\bar{\psi}}$ are renormalization constants.

A. L. Kataev , S. A. Larin JETP Letters volume 96, 61 (2012):

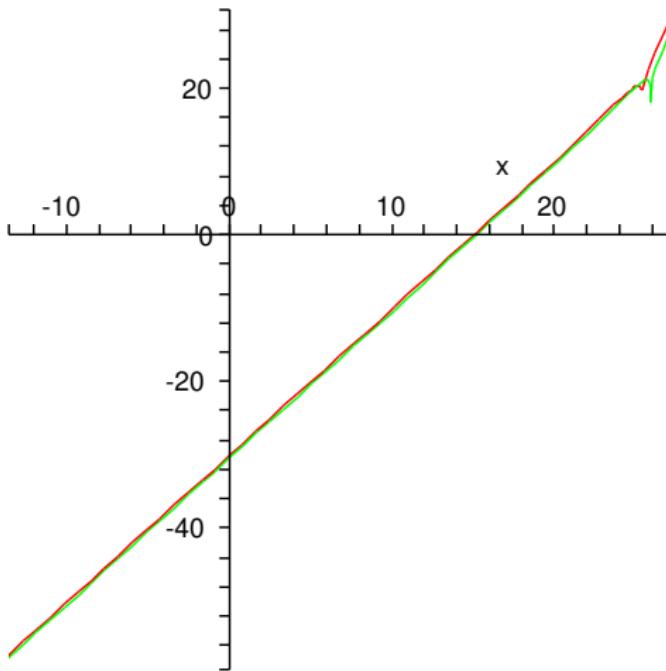
$$\beta = 0.0337\alpha^2 + 0.008062\alpha^3 - 0.04312\alpha^4 - 0.001866\alpha^5 - 0.0406\alpha^6.$$

$\beta(\alpha)$ graph. $\zeta = 1$

β -function in the circle of convergence $a = 100000$.

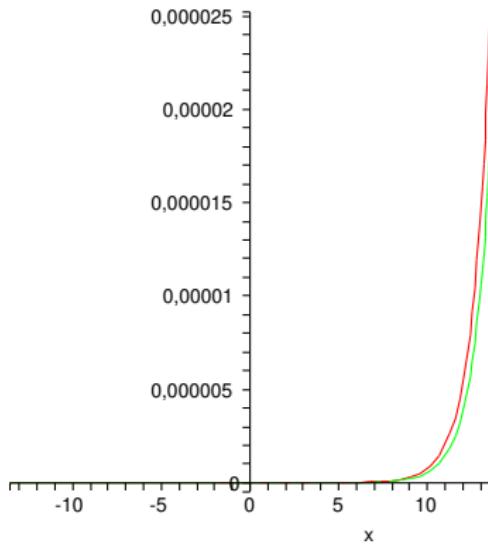
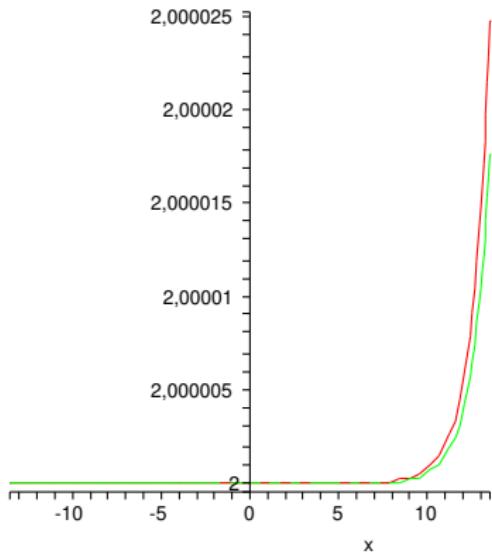


$\ln \beta(\ln \alpha)$ graph. $\ln \alpha_{max} \simeq 26$



$\partial_{\ln \alpha} \ln \beta(\ln \alpha)$ and $\partial_{\ln \alpha}^2 \ln \beta(\ln \alpha)$ graph.

$\ln \alpha_{max} \simeq 26$



$$\partial_{\ln \alpha}^2 \ln \beta \approx \partial_{\ln \alpha} \ln \beta - 2, \quad \beta \approx C_1 \alpha^2 e^{C_2 \alpha}$$

Using $\partial_\alpha \ln \beta \approx C_2$, for $\alpha = \alpha_{max}$ one can obtain $C_2 \approx 0.000003$.

Summary

Thank you!