VII International Conference “Models in Quantum Field Theory” (MQFT-2022)

October 10-14, 2022, Saint Petersburg

A constituent multiquark approach to meson resonances as a generalization of Quark Model

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Based on: S.S. Afonin, arXiv:1705.01899, 1808.07363 [hep-ph]

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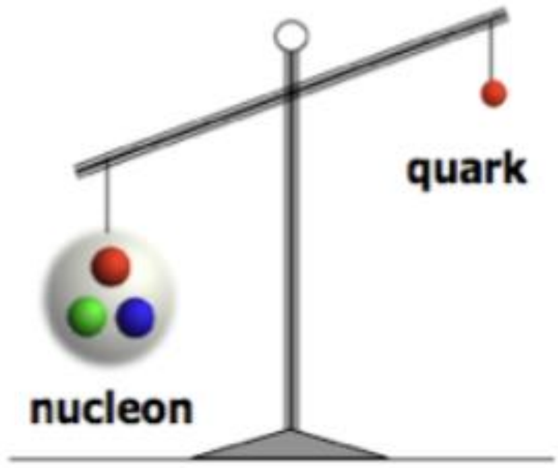
Strong interactions: almost 99% of mass of visible universe



Analytical description - ?

Mass generation mechanism - ? — Encoded in the hadron spectrum (hundreds of resonances!)

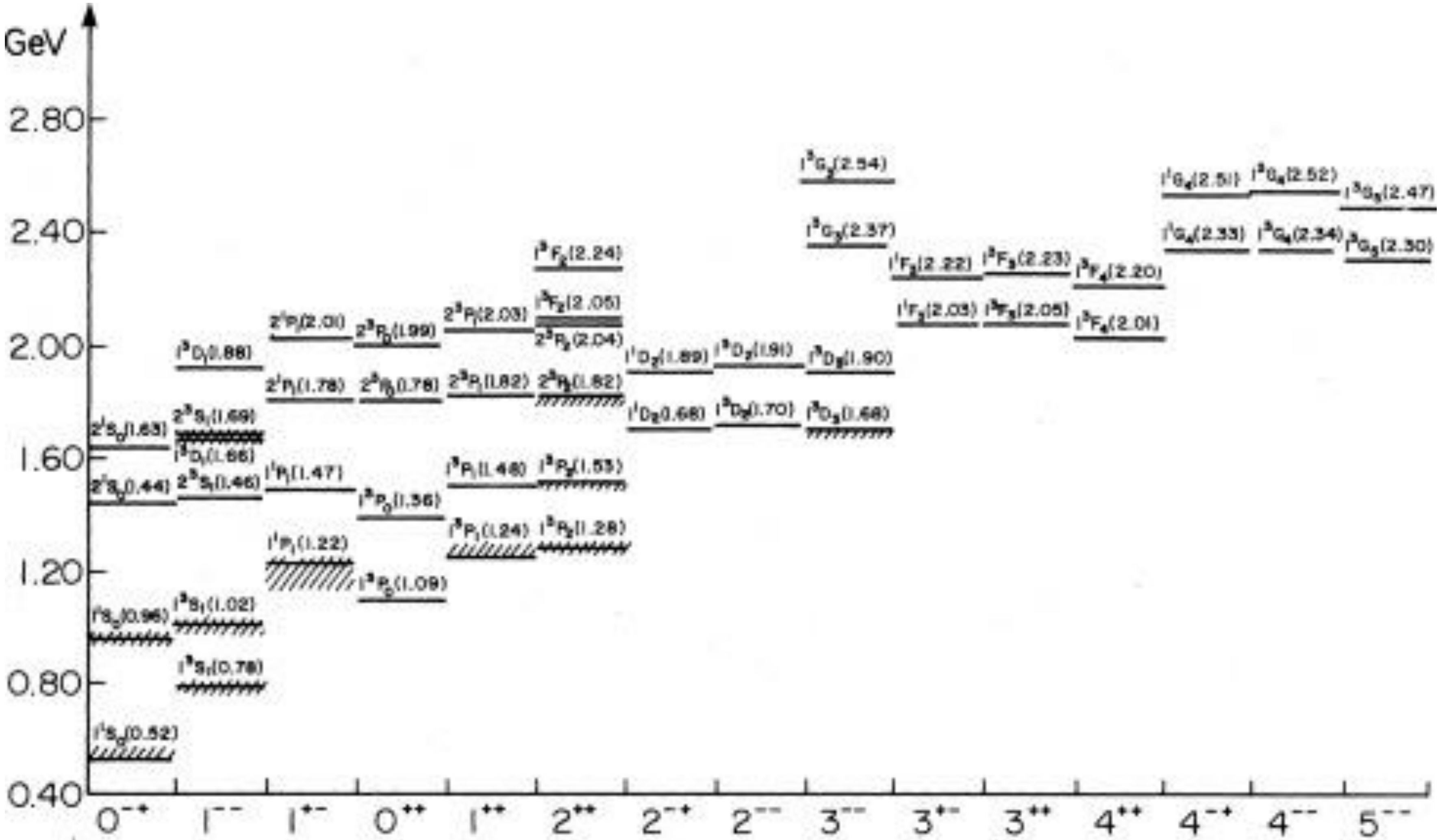
Many models in the last 60 years — success was partial

Lattice: O'k (partly) for numerical predictions
but no physical picture of resonance formation

Non-relativistic potential Quark Model – O'k for classification

Many relativistic dynamical models – a real progress is not seen

The spectrum of light non-strange mesons: An example of prediction

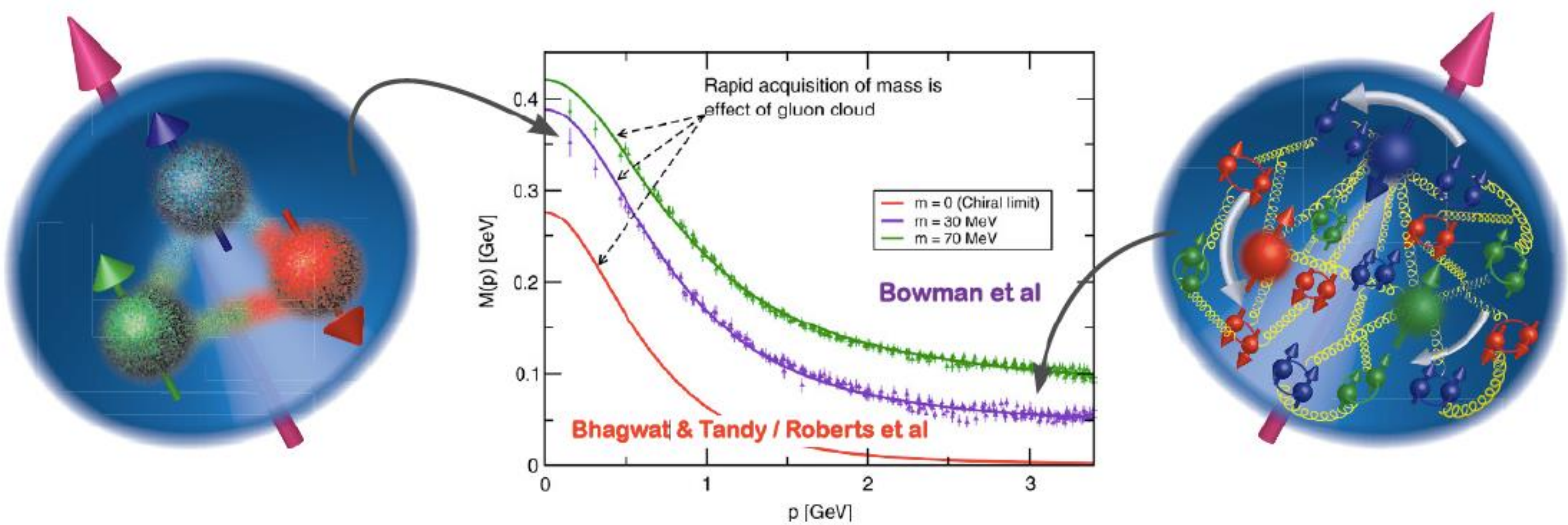


Agreement for radial excitations usually is poor

New ideas - ?

Hadron Mass

- **Three-pronged approach to explore the origin of hadron mass**
  - **Lattice QCD**
  - **Mass decomposition – roles of the constituents**
  - **Model calculation – approximated analytical approach**



$$M_p = 2m_u^{\mathrm{eff}} + m_d^{\mathrm{eff}}$$

$$H_{\mathrm{QCD}} = H_q + H_m + H_g + H_a$$

HADRON MASS: THE CENTRAL PROBLEM

Mass of resonance → ***Re*-part of *S*-matrix pole**
→ **gravitational mass**

related with

Energy-momentum tensor in QCD:

$$T^{\mu\nu} = \frac{1}{2}\,\overline{\psi}\, i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi + \frac{1}{4}\, g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}{}_{\alpha}$$

Its trace:

QCD trace anomaly

anomalous dimension of quark mass

$$T^{\alpha}_{\alpha} = \frac{\beta(g)}{2g} F^{\mu\nu,a} F^{a}_{\mu\nu} + \sum_{q=u,d,s} m_q (1+\gamma_m)\overline{\psi}_q \psi_q$$

$$\beta(g) = -(11 - 2n_f/3)\, g^3/(4\pi)^2 + \ldots$$

$$M = \frac{\langle P|H_{\rm QCD}|P\rangle}{\langle P|P\rangle}|_{\rm rest\ frame} \qquad H_{\rm QCD} = \int d^3\vec{x}\, T^{00}(0,\vec{x})$$

$\langle P|P\rangle = 1$ — non-relativistic normalization

$\langle P|P\rangle = 2E$ — relativistic normalization

Hadron mass in relativistic case: $\boxed{2m_h^2 = \langle h|T_\alpha^\alpha|h\rangle}$ renorminvariant!

A consequence of the Ward identity $2p_\mu p_\nu = \langle h|T_{\mu\nu}|h\rangle$

Our ansatz for light mesons: $\boxed{m_h^2 = \Lambda(E_h + 2m_q) = \Lambda E_h + m_\pi^2}$

where $E_h \sim \langle h|G_{\mu\nu}^2|h\rangle \neq 0$ $\qquad m_u = m_d \doteq m_q$

Λ is universal for light hadrons and fixed by GOR relation

$$m_\pi^2 = -\frac{\langle\bar{q}q\rangle}{f_\pi^2}(m_u + m_d) = \Lambda \cdot 2m_q \qquad \boxed{\Lambda \doteq -\frac{\langle\bar{q}q\rangle}{f_\pi^2}}$$

"sudden" excitations of pion?

One must specify E_h - interpretations?

$$m_h^2 = \Lambda E_h + m_\pi^2$$

By assumption $\quad E_h \sim \langle G_{\mu\nu}^2 \rangle$

ΛE_h renorminvariant!

Let us fix $\quad \langle \bar{q} q \rangle = -(250\,\mathrm{MeV})^3,\; m_u + m_d = 11\ \mathrm{MeV},\; f_\pi = 92.4\ \mathrm{MeV}$

This yields $\quad \Lambda = 1830\ \mathrm{MeV}$

Consider the rho-meson

$\pi \qquad\qquad \rho$

$E_\rho \approx 310\ \mathrm{MeV}$ - the energy cost for the given spin flip (looks like a constituent mass!)

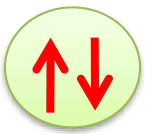
The non-renorminvariant logic does not work! $\quad m_\rho \neq E_\rho + m_\pi$

$$m_h^2 = \Lambda E_h + m_\pi^2$$

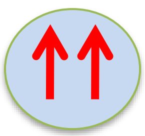
Higher spin and radial excitations with correct quantum numbers?

<u>A proposal</u>: Let us assume that gluodynamics leads to formation of gluon analogues of positronium inside hadrons

"CONSTITUENT GLUONS"?



$$J^{PC} = 0^{-+} \quad — \quad A_0$$



$$J^{PC} = 1^{--} \quad — \quad A_1$$

notations

$$\boxed{{}^J\pi_{A_0^n A_1^l}} \quad (P,C) = \left((-1)^{n+l+1},(-1)^l\right) \quad \boxed{m^2_{\pi_{A_0^n A_1^l}} = \Lambda(nE_0 + lE_1) + m_\pi^2}$$

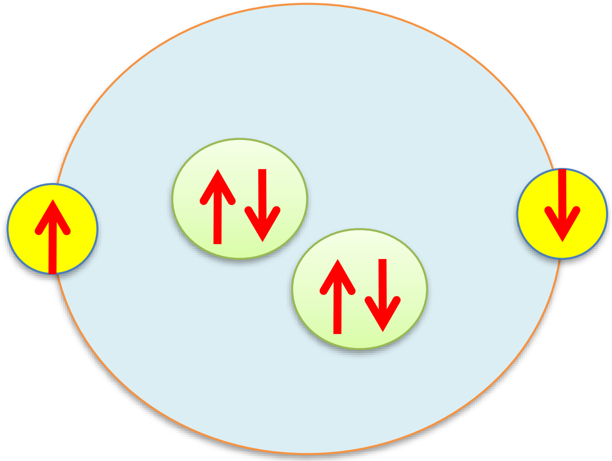
Light non-strange mesons

(some analogy with protons and neutrons inside atomic nucleus)

$$\boxed{{}^J\rho_{A_0^n A_1^l}} \quad (P,C) = \left((-1)^{n+l+1},(-1)^{l+1}\right) \quad \boxed{m^2_{\rho_{A_0^n A_1^l}} = \Lambda(E_\rho + nE_0 + lE_1) + m_\pi^2}$$

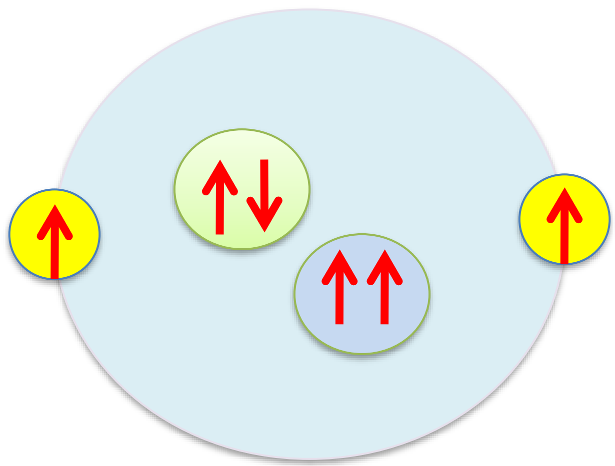
Examples

$^{0}\pi_{A_0^2} \Longrightarrow$



$\pi(1300)$

$^{J}\rho_{A_0 A_1} \Longrightarrow$



$\pi,\ \pi_1$ and π_2

Not exotic!

Phenomenologically: $E_0 \approx 450 \quad E_1 \approx 570 \quad \mathrm{MeV}$

(typical for “constituent” gluons!)

The relations $$\boxed{m^2_{\pi_{A_0^n A_1^l}} = \Lambda(nE_0 + lE_1) + m_\pi^2} \quad \textbf{(1)}$$

$$\boxed{m^2_{\rho_{A_0^n A_1^l}} = \Lambda(E_\rho + nE_0 + lE_1) + m_\pi^2} \quad \textbf{(2)}$$

lead naturally to Regge trajectories

$$\boxed{m^2_{J,n} = aJ + a'n + b, \qquad J, n = 0, 1, 2, \ldots .}$$

(1) and (2) result in some old known relations in various limits

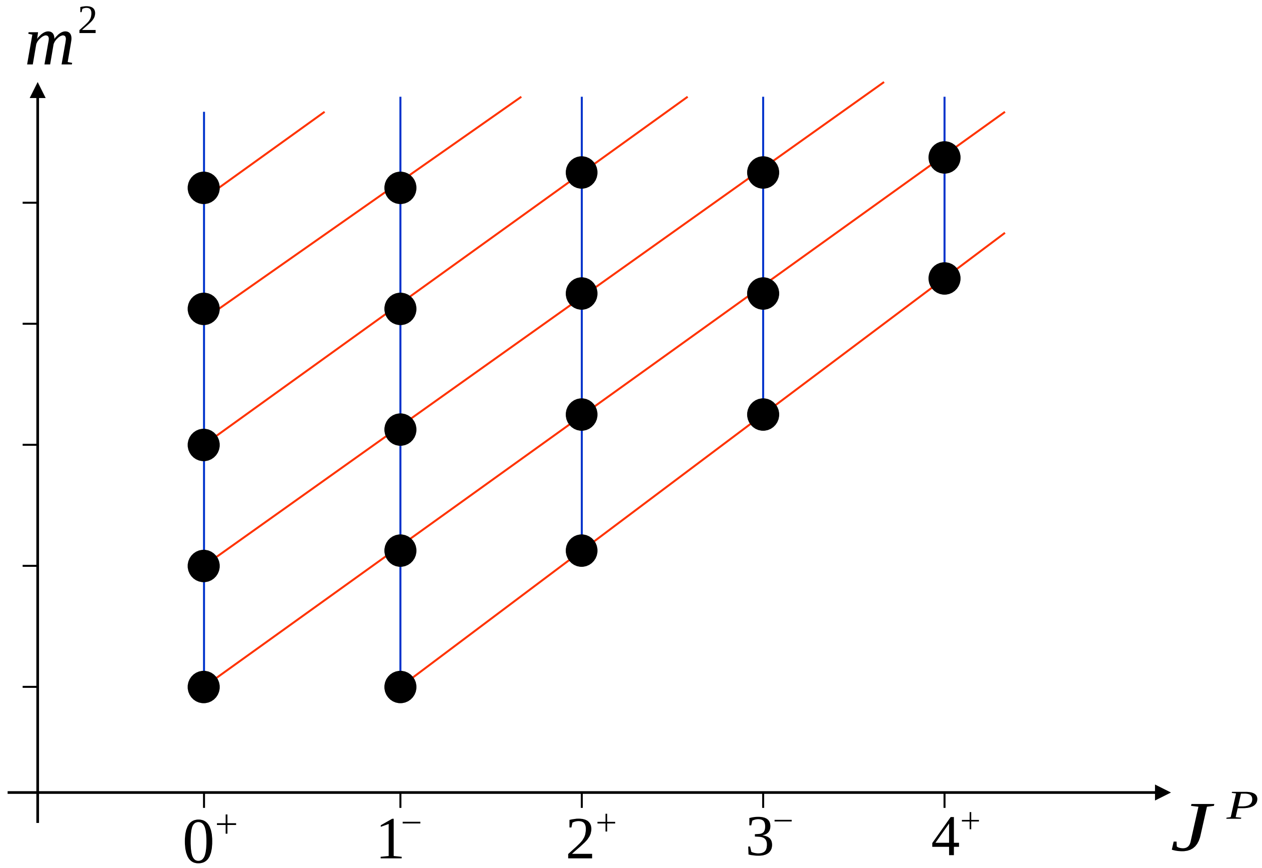
$E_0 = E_1$ - Hadron strings (*a=a'*)

$E_\rho = E_1$ - E.g., the vector and axial radial trajectories are related

$$\boxed{m^2_{a_1}(n) = m^2_\rho(n) + m^2_\rho - m^2_\pi} \qquad \begin{matrix} m_\pi = 0 \\ n = 0 \end{matrix} \; \boxed{m^2_{a_1} = 2m^2_\rho}$$

$$E_\rho = E_0 = E_1 \Rightarrow \boxed{m^2_\rho(n) = 2m^2_\rho\left(n + \frac{1}{2}\right), \quad m^2_{a_1}(n) = 2m^2_\rho\,(n+1)}$$

Regge and radial Regge linear trajectories



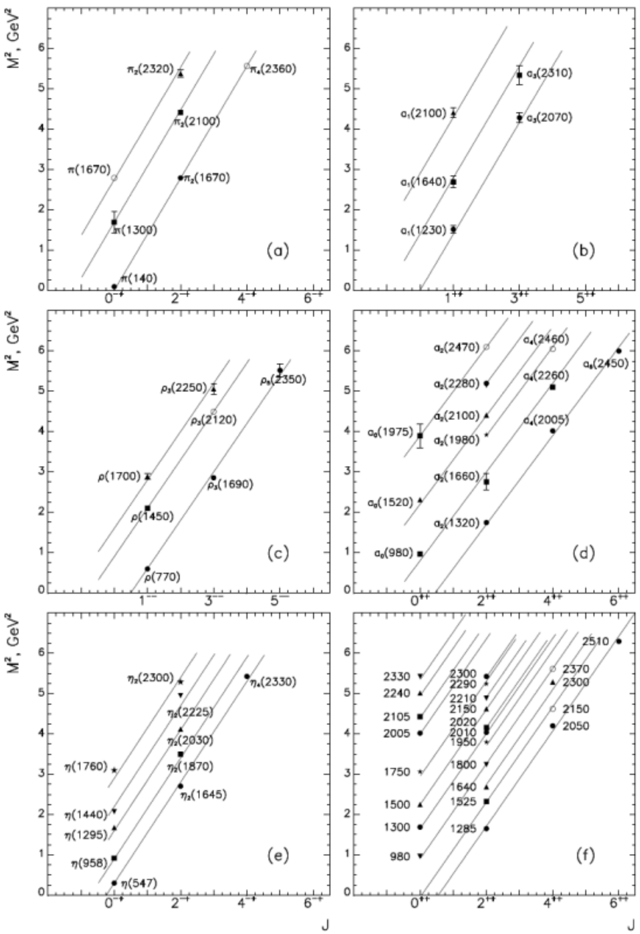
$$m^2(J) = m_0^2 + \alpha' J$$ – Regge trajectories

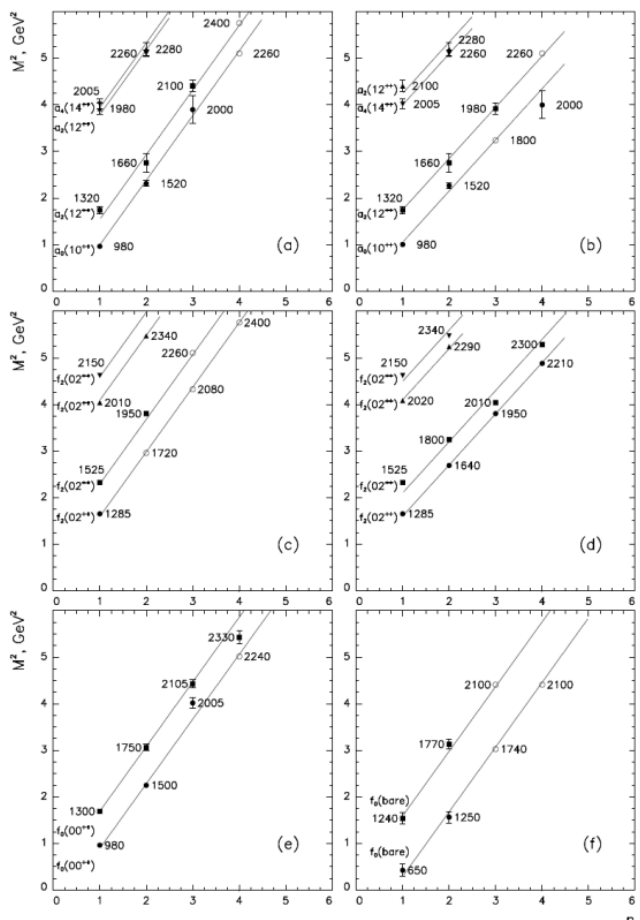
$$m^2(n) = \mu_0^2 + \alpha n$$ – Radial Regge trajectories

Linear Regge and radial trajectories: Experiment

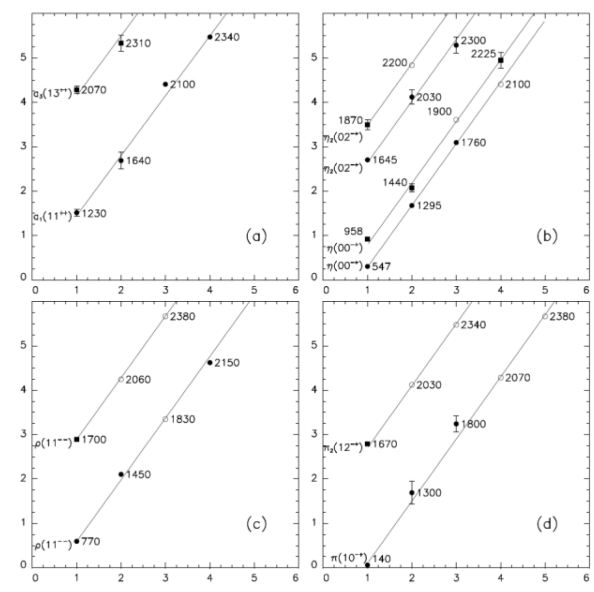
Rich source of spectral data on the light mesons – proton-antiproton annihilation

(A.V. Anisovich, V.V. Anisovich and A.V. Sarantsev, PRD (2000); D.V. Bugg, Phys. Rept. (2004))



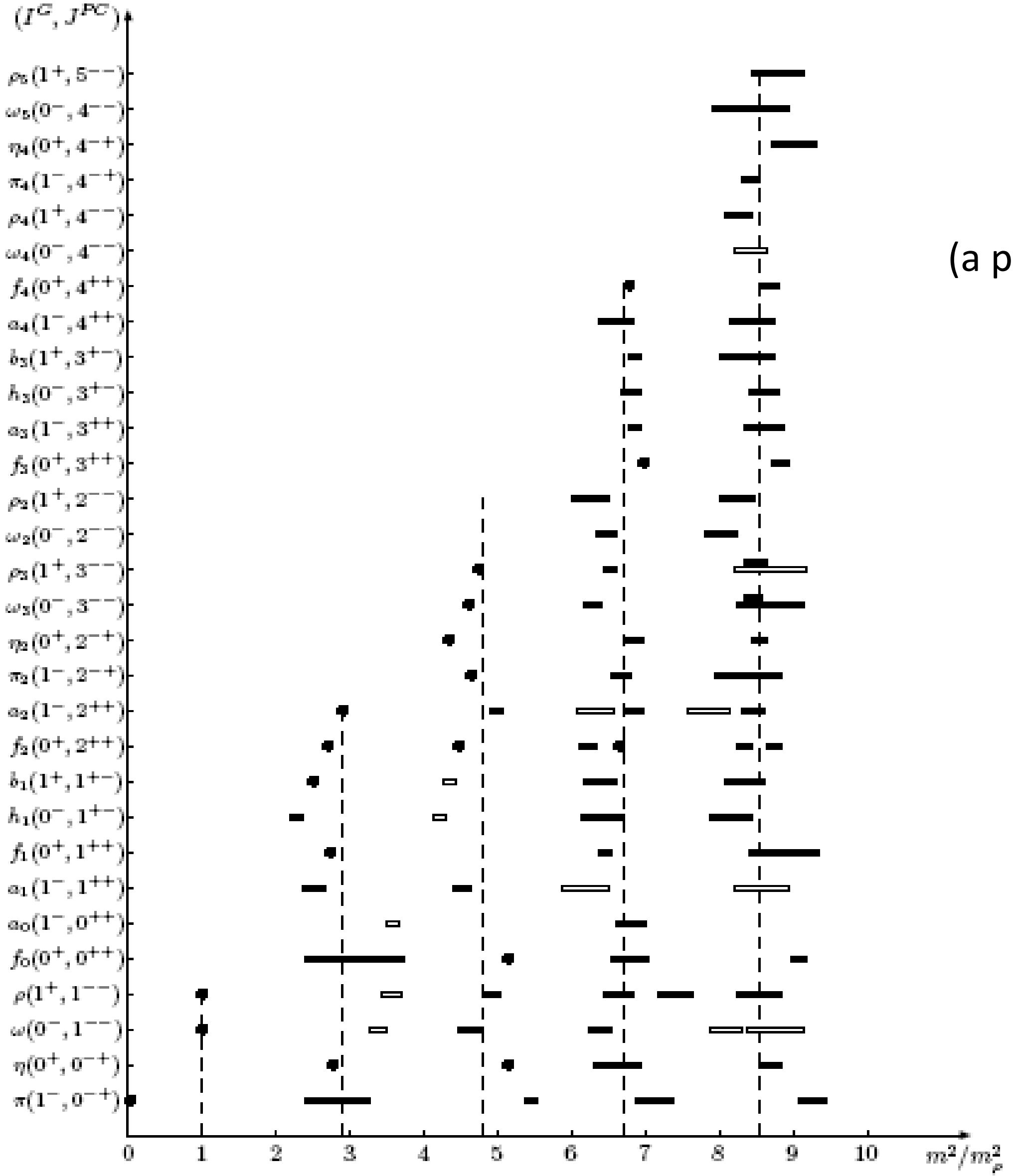


Examples



Experimental spectrum of light non-strange mesons

(a plot from S.S. Afonin, Eur. Phys. J. A 29 (2006) 327)



The major feature: Spin-parity clustering

The spectrum of light nonstrange mesons in units of M_ρ^2. Experimental errors are indicated. Circles stay when errors are negligible. The dashed lines mark the mean mass squared in each cluster of states and the open strips and circles denote the one-star states. The arrows indicate the *J > 0* mesons which have no chiral partners (the hypothetical chiral singlets).

In average (in GeV^2)

$$\bar{M}^2 \approx 1.1(N+0.6),\ N = 0,1,\ldots$$

More exactly, ***<u>N= L + n</u>***

$$\bar{M}^2(L,n) \approx 1.1(L+n+0.6)$$

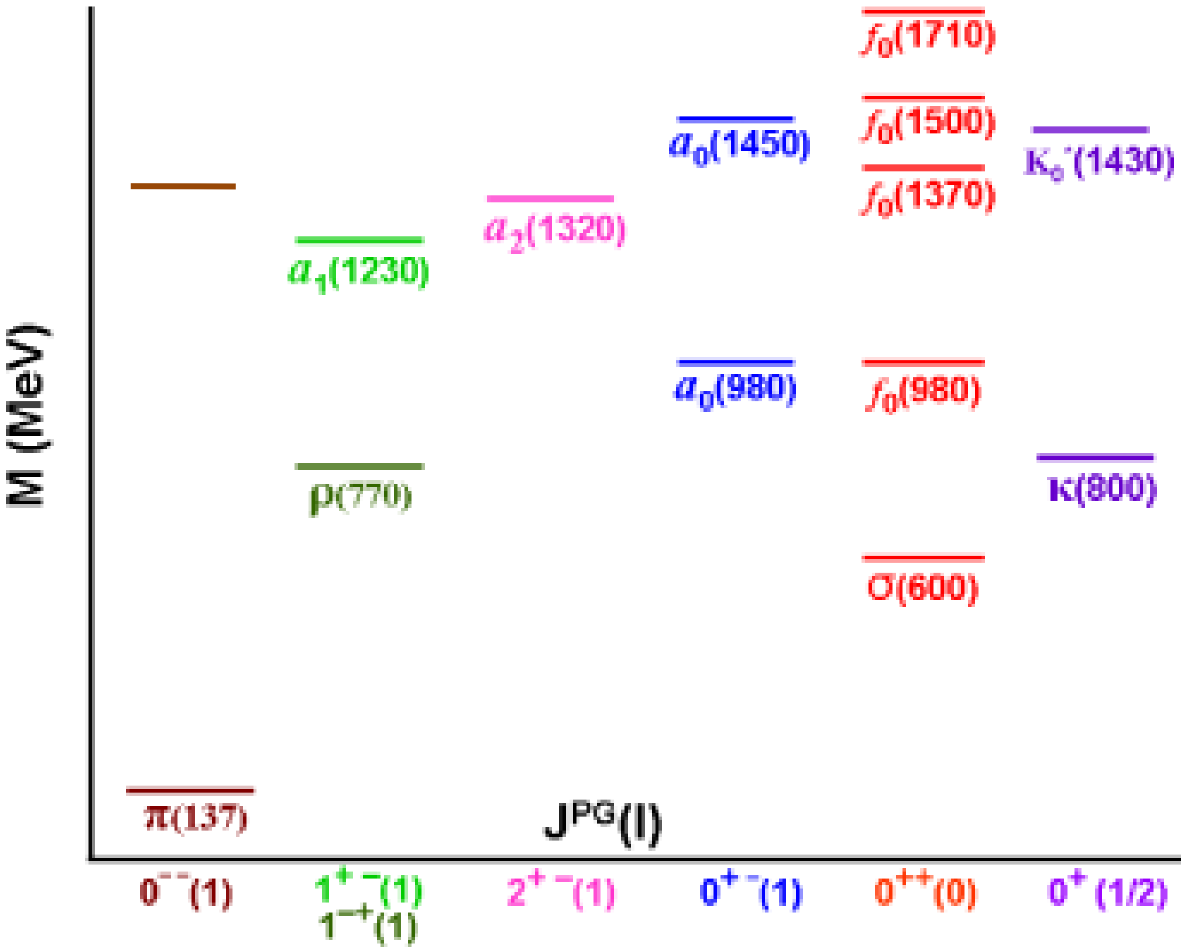
$\Longrightarrow$ The law $\boxed{M^2(L,n) \sim L+n}$ works!

Like in non-relativistic hydrogen atom:

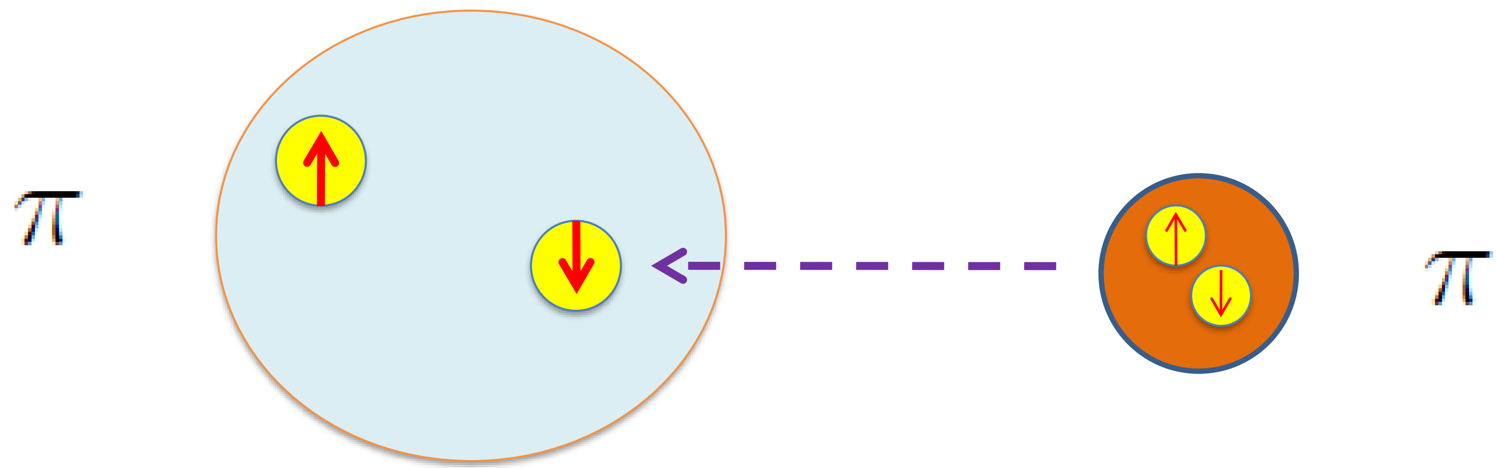
$$E(L,n) \sim \frac{1}{N^2}, \qquad N = L+n+1$$ - principal quantum number

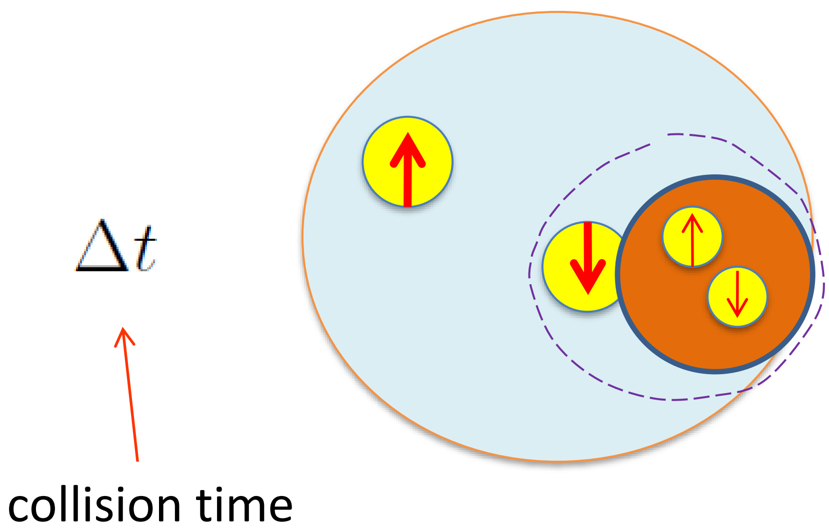
The symmetry of the spectrum is larger than O(3), it is O(4)
(V.A. Fock, Z. Phys. 98 (1935) 145)

The scalar mesons – the most enigmatic sector



Collisional excitations





$$m_h^2 = \Lambda(E_h + 2m_q) = \Lambda E_h + m_\pi^2$$

$$m_q \;\rightarrow\; m_q + m_\pi$$

$$\boxed{m_\sigma^2 = \Lambda m_\pi + m_\pi^2}$$

For our inputs: $m_\sigma \approx 525 \text{ MeV}$

General principle: $$m_{\pi_h}^2 = \Lambda m_h + m_\pi^2$$

Examples

$\boxed{\pi_\rho}$	$m_{h_1} \approx 1190\ \mathrm{MeV}$	$h_1 \to \rho\pi$	$\boxed{h_1(1170)}$
$\boxed{\pi_\eta}$	$m_{\pi_\eta} \approx 1010\ \mathrm{MeV}$	$\pi_\eta \to \eta\pi$	$\boxed{a_0(980)}$
$\boxed{\pi_K}$	$m_{\pi_K} \approx 970\ \mathrm{MeV}$	$\pi_K \to \pi\pi$	$\boxed{f_0(980)}$
$\boxed{K_\pi}$	$m_{K_\pi} \approx 710\ \mathrm{MeV}$		$\boxed{K_0^*(800)}$

PDG: $682 \pm 29\ \mathrm{MeV}$

In conclusion…

The proposed approach is broader than "just another one model" as it gives a new language for discussion of hadron resonances, for interpretation of data in the hadroproduction and formation experiments, and a possible starting point for construction of essentially new dynamical models.