Effective QG, Cosmological Constant and the Standard Model of Particle Physics

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Based on: *Breno Giacchini, Tibério P. Netto, I.Sh, 2112.06314*

Previous works: *2006.04217, 2009.04122*

- **Mathematically, the cosmological constant term comes to our mind first when we want formulating covariant action for gravity**

$$S_{grav} = - \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, (R + 2\Lambda)\,, \qquad \rho_\Lambda = \frac{\Lambda}{8\pi G}\,.$$

The greatest legend about CC: The CC term can be calculated in the framework of Quantum Field Theory (QFT). And it has a strange value, about 120 orders of magnitude greater than ρ_Λ^{obs}.

Real deal: In QFT we can not derive any independent, massive or massless, parameter from the first principles.

CC is not an exception. Naive calculation typically gives an infinite value for a massive parameter, with both potential and logarithmic-type divergences. After infinity is subtracted, one has to fix the finite value, that requires a measurement.

General structure of renormalization in curved space.

Starting from the paper *R. Utiyama & B. DeWitt, J.M.Phys. (1962)*, **we know that curved-space counterterms satisfy two conditions:**

- **They are covariant using appropriate regularization.**
- **They are local functionals of the metric.**

Renormalizable theory of matter fields on classical curved background requires classical action of vacuum

$$S_{vac} = S_{HE} + S_{HD}\,, \qquad S_{HE} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R+2\Lambda)\,.$$

The theory without independent vacuum parameter $\Lambda = \Lambda_{vac}$ is inconsistent as loops of massive fields give CC-type divs.

If $\Lambda_{vac} \equiv 0$, this kind of divergences cannot be removed by renormalization and we have a kind of theoretical disaster.

Of course, the same is true for all other terms in S_{vac}, including the Einstein-Hilbert and the fourth-derivative terms forming S_{HD}.

What is the required magnitude of the vacuum CC?

Within the renormalizable theory of matter fields on classical curved background, one can expect running of the essential physical parameters. Consider the RG running for CC and G:

$$(4\pi)^2\,\mu\frac{d\rho_\Lambda^{vac}}{d\mu} = \frac{N_s m_s^4}{2} - 2N_f m_f^4\,, \qquad \rho_\Lambda = \rho_\Lambda^{vac} = \frac{\Lambda_{vac}}{8\pi G_{vac}}$$

$$(4\pi)^2\,\mu\,\frac{d}{d\mu}\Big(\frac{1}{16\pi G_{vac}}\Big) = \frac{N_s m_s^2}{2}\,\Big(\xi-\frac{1}{6}\Big) + \frac{N_f m_f^2}{3}\,.$$

It is unclear how these equations can be used in cosmology, where the energy scale is much smaller than the masses.

However, even the UV running means that ρ_Λ^{vac} cannot be smaller then the fourth power of a typical mass of the theory.

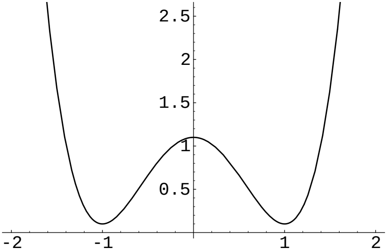
Thus, the natural value from the MSM perspective is

$$\rho_\Lambda^{vac} \;\propto\; M_F^4 \;\propto\; 10^8\,GeV^4\,.$$

Induced CC from SSB in the Standard Model.

In the stable point of the Higgs potential $V = -m^2\phi^2 + f\phi^4$, we meet $\rho_\Lambda^{ind} = \langle V \rangle \approx 10^8\, GeV^4$ – of the same magnitude as ρ_Λ^{vac} !

This is induced CC, similar to the one found by Zeldovich (1967).



The observed CC is a sum $\rho_\Lambda^{obs} = \rho_\Lambda^{vac} + \rho_\Lambda^{ind}$. Since ρ_Λ^{vac} is an independent parameter, the renormalization condition is

$$\rho_\Lambda^{vac}(\mu_c) = \rho_\Lambda^{obs} - \rho_\Lambda^{ind}(\mu_c)\,.$$

Here μ_c is the energy scale where ρ_Λ^{obs} is “measured”.

Finally, the main CC relation is $\rho_\Lambda^{obs} = \rho_\Lambda^{vac}(\mu_c) + \rho_\Lambda^{ind}(\mu_c)\,.$

The ρ_Λ^{obs} **which is likely observed in SN-Ia, LSS and CMB is**

$$\rho_\Lambda^{obs}(\mu_c) \;\approx\; 0.7\,\rho_c^0 \;\propto\; 10^{-47}\,GeV^4.$$

The CC Problem is that the magnitudes of $\rho_\Lambda^{vac}(\mu_c)$ **and** $\rho_\Lambda^{ind}(\mu_c)$ **are a huge 55 - 56 orders of magnitude greater than the sum!**

Obviously, these two huge terms do cancel. "Why they cancel so nicely" is the main (great) CC Problem (Weinberg, 1989). The origin of the problem is the huge difference between the M_F **scale of** ρ_Λ^{ind} **and** ρ_Λ^{vac} **vs the** μ_c **scale of** ρ_Λ^{obs}.

CC Problem is a sort of (the most difficult) hierarchy problem.

There were many attempts to solve this problem. The general impression is that it is impossible without moving the fine-tuning from CC to another sector of the theory.

- **In what follows, we accept that the CC fine tuning takes place and will just recognize this as a fact.**

It is remarkable that the theoretical predictions for the two ingredients have the same order of magnitude.

$$\rho_\Lambda^{obs} \;=\; \rho_\Lambda^{vac}(\mu_c) + \rho_\Lambda^{ind}(\mu_c).$$

In the particle physics units, the value $\rho_\Lambda^{obs} \approx 10^{-47}\,\mathrm{GeV}^4$ **.**
In MSM, we need about 55-56 **orders fine-tuning for** $\rho_\Lambda(\mu_0)$**.**

Assuming a symmetry breaking at energy scales beyond MSM, the amount of fine tuning may significantly increase.

The CC problem is a real mystery, as the fine tuning can be violated even by very small changes, e.g., in Yukawa couplings via one-loop or higher-loop corrections (up to 21 loops!).

Even a small mismatch in $\rho_\Lambda(\mu_0)$ **may lead to either a negative, zero or too big (100 times greater) positive value of** ρ_Λ^{obs}**.**
All these options contradict our own existence:

S. Weinberg, Anthropic bound on the cosmological constant, (1987).

And there is still a chance of the renormalization group running of CC, even in the “deep IR”.

Brief review of renormalization group

The renormalization group is a useful and economic way to describe quantum corrections. Thus, it is quite natural trying to use it in quantum gravity (QG).

As an example, consider a fermion loop effect in QED,

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}\big[i\gamma^\mu(\partial_\mu - A_\mu) - m\big]\psi.$$

With the one-loop correction, we get, approximately,

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu}\left[1 - \beta \ln\left(\frac{\Box + m^2}{\mu^2}\right)\right] F^{\mu\nu}.$$

In the IR, when (Euclidean) momentum is $k^2 \ll m^2$, **this becomes an irrelevant redefinition of** e.

However, in the UV, when $k^2 \gg m^2$, **there is an effective running:**

$$e^2(k) = \frac{e_0^2}{1 - \beta\, \ln(k^2/\mu^2)}. \qquad \beta = \frac{e^2}{6\pi^2}.$$

Renormalization group running for ρ_Λ^{vac} or ρ_Λ^{ind}

may break down the fine tuning and produce significant effect, even if the running is very weak compared to basic values.

In semiclassical theory, such a running may take place only because of the effects of massive particles, and it is supposed to be weakened by AC-like decoupling (*Gorbar & Sh. 2003 ...***).**

This interesting subject was discussed in many papers, e.g.
I.Sh., J. Solà, JHEP **02** *(2002) 006; Phys. Lett.* **B682** *(2009) 105.*

- **Another possibility is the running in QG, i.e., in a quantum theory of the metric, which is the main subject of this talk.**

There are a few serious challenging problems on this way, namely 1) Maintain covariance; 2) Apply renormalization group in the nonrenormalizable theory such as quantum gravity; 3) Extract the unambiguous beta functions.

Let us discuss these issues in the effective QG framework.

- **Gauge invariant renormalizability in QG**

Covariant action of gravity is $S = \int d^4x \sqrt{-g}\, \mathcal{L}(g_{\mu\nu})\,.$

where $\mathcal{L}(g_{\mu\nu})$ **can be** $\mathcal{L}(g_{\mu\nu}) = -\kappa^{-2}(R + 2\Lambda)$ **or another.**

The gauge transformation $\delta g_{\mu\nu} = R_{\mu\nu,\alpha}\xi^\alpha$**. The gauge invariance:**

$$\frac{\delta S}{\delta g_{\mu\nu}}\, R_{\mu\nu,\alpha} = 0\,.$$

Renormalizability in fourth derivative QG:

K. Stelle, Phys. Rev. D (1977).

General proof in QG, using Batalin-Vilkovisky formalism:

P.M. Lavrov, I.Sh., Gauge invariant renormalizability of quantum gravity, arXiv:1902.04687; PRD.

Textbook-level introduction: *I.L. Buchbinder and I.Sh., Introduction to Quantum Field Theory with Applications to Quantum Gravity (Oxford Univ. Press, 2021).*

Gauge invariant renormalizability in QG/GR

The Faddeev-Popov approach, with $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$

$$S_{tot} = S(h) + \frac{1}{2}\chi^{\mu}Y_{\mu\nu}\chi^{\nu} + \bar{C}^{\mu}M^{\nu}_{\ \mu}C_{\nu}, \qquad M^{\nu}_{\ \mu} = \frac{\delta\chi^{\nu}}{\delta h_{\lambda\sigma}}R_{\lambda\sigma,\mu}\,.$$

The useful choices of gauge fixing conditions and the weight function depend on the theory, e.g., the background gauge

$$\chi_{\mu} = \nabla^{\nu}h_{\mu\nu} - \beta\nabla_{\mu}h\,, \qquad Y_{\mu\nu} = \alpha g_{\mu\nu},$$

Independent on the parametrization and gauge fixing, e.g., α **and** β, **one can prove that the divergent part of effective action,** $\Gamma_{div} = \Gamma_{div}(g_{\mu\nu})$, **in all orders of loop expansion, is local and satisfies the gauge identity**

$$\frac{\delta\Gamma_{div}}{\delta g_{\mu\nu}}\,R_{\mu\nu,\alpha} = 0\,.$$

Multiplicative renormalizability depends on a power counting.

Power counting in QG

The power counting of a diagram with an arbitrary number of external lines $h_{\mu\nu}$ and number of their derivatives $d(G)$ is defined by the superficial degree of divergence $\omega(G)$,

$$\omega(G) + d(G) = \sum_{l_{int}} (4 - r_l) - 4V + 4 + \sum_V K_V .$$

The first sum is over internal lines of the diagram, r_l is the inverse power of momentum in the propagator of the given internal line, and V is the number of vertices. K_V is the number of derivatives acting on the given vertex.

In addition, there is the topological relation $L = I - V + 1$.

As important example, consider quantum GR.

$$S_{EH} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g}\,(R + 2\Lambda) .$$

In this case we get $\omega(G) + d(G) = 2 + 2L - 2K_\Lambda$.

What we know (almost) for sure

is that higher derivatives bring massive degrees of freedom. Those can be normal fields, ghosts, tachyons, and/or tachyonic ghosts.

We can assume that the contributions of these massive degrees of freedom decouple in the IR.

The natural (and probably correct) assumption is that below the typical mass scale of the massive degrees of freedom there remain only quantum effects of the massless graviton, i.e., the effective QG should be based on Einstein's gravity.

J.F. Donoghue, Phys. Rev. D 50 (1994) 3874.

From this perspective, the running which takes place in renormalizable or superrenormalizable models of QG, may take place only above the greater mass in the spectrum of QG.

This means, typically, above the Planck scale.

Gauge-fixing dependence

One can analyse the situation with gauge and parametrization dependencies without explicit calculations, using the general QFT theorems, see, e.g.,

I.Y. Aref'eva, A.A. Slavnov, L.D. Faddeev, Theor. Math. Phys. (1974).

G. Costa and M. Tonin, Rivista Nuovo Cim. ***5*** *(1975).*

B.L. Voronov, P.M. Lavrov, I.V. Tyutin, Sov. J. Nucl. Phys. (1982).

W. Kummer, Eur. Phys. J. ***C21*** *175 (2001), hep-th/0104123.*

This formalism was applied to quantum gravity in

E.S. Fradkin and A.A. Tseytlin, NPB 201 (1982).

I.Sh, A. Jacksenaev, PLB 324 (1994) 284.

Also, it was confirmed by explicit calculations, e.g., in

R.E. Kallosh, O.V. Tarasov and I.V. Tyutin, NPB 137 (1978).

M. Kalmykov, Class. Quant. Grav. 12 (1995) 1401.

J. Gonçalves, T. de Paula Netto, I.Sh., PRD 97 (2018), 1712.03338.

How can we get an invariant definition of RG in QG?

The best solution is based on the Vilkovisky–DeWitt (VdW) scheme for constructing effective action in quantum gravity.

*G.A. Vilkovisky, Nucl. Phys. B***234** *(1984).*

A.O. Barvinsky and G.A. Vilkovisky, Phys. Repts. 119 (1985).

B.S. DeWitt, The effective action, (Essays in honor of the sixtieth birthday of E.S. Fradkin, 1987).

We need that (at least) one-loop divergences do not depend on the gauge-fixing and parametrization of the quantum metric.

Then, we can achieve the universal running of the coefficients of $\rho_\Lambda,\ R,\ R^2$ **and, in fact, of all other terms of the action.**

And the VdW approach makes it possible, and even gives more

T. Taylor and G. Veneziano, Nucl. Phys. B ***345*** *(1990).*

B. Giacchini, T. de Paula Netto, I.Sh., JHEP (2020), 2009.04122.

For the effective QG based on Einstein's GR with the CC, this prescription gives

$$\bar{\Gamma}^{(1)}_{\rm div} = -\frac{1}{\epsilon}\int d^4x\sqrt{-g}\,\left\{\frac{121}{60}C^2 - \frac{151}{180}E + \frac{31}{36}R^2 + 8\Lambda R + 12\Lambda^2\right\}.$$

This, completely invariant, result enables us to construct the renormalization group equations

$$\mu\frac{d}{d\mu}\left(\frac{1}{16\pi G}\right) = \frac{8\Lambda}{(4\pi)^2}\,, \qquad \mu\frac{d\Lambda}{d\mu} = -\frac{2(16\pi G)\Lambda^2}{(4\pi)^2}\,. \qquad (*)$$

The solutions can be easily found in the form ($\gamma_0 = 16\pi G_0\Lambda_0^2$).

$$G(\mu) = \frac{G_0}{\left[1 + \frac{10}{(4\pi)^2}\,\gamma_0 \ln\frac{\mu}{\mu_0}\right]^{4/5}}\,, \qquad \Lambda(\mu) = \frac{\Lambda_0}{\left[1 + \frac{10}{(4\pi)^2}\,\gamma_0 \ln\frac{\mu}{\mu_0}\right]^{1/5}}.$$

In the V-DW approach, we get a well-defined renormalization group running of the Newton and cosmological constants between the Planck and Hubble scales!

This result is because of effective QG, as we assume that below the Planck scale all extra degrees of freedom get inactive.

The renormalization group (RG) equations in effective QG:

$$\mu \frac{d}{d\mu}\left(\frac{1}{16\pi G}\right) = \frac{8\Lambda}{(4\pi)^2}\,, \qquad \mu \frac{d\Lambda}{d\mu} = -\frac{2(16\pi G)\Lambda^2}{(4\pi)^2}\,. \qquad (*)$$

The two most remarkable properties of these equations and their solutions are as follows:

i) Universality. Eqs. (*) do not depend on the gauge fixing, parametrization of quantum fields or any kind of hypothesis and assumptions, except using the VdW effective action.

ii) Can be regarded exact, i.e., not restricted to one-loop order. All higher-loop corrections are suppressed by the powers of

$$\frac{\Lambda}{M_P^2} = \frac{\rho_\Lambda}{M_P^4}.$$

In the present-day Universe this quantity is of the order of 10^{-120}, **but even in the inflationary epoch it is at least** 10^{-12}.

Thus, RG equations (2) **describe an effectively exact running.**

For the cosmological constant

(vacuum energy) density, is there a chance that the running of vacuum CC breaks down the fine tuning between $\rho_\Lambda^{vac} = \rho_\Lambda$ and ρ_Λ^{ind}, making an extra trouble related to the CC problem?

Consider the strongest option - the SUSY GUT case. Then $M_X \sim 10^{16}$ GeV **and hence**

$$\big|\rho_\Lambda\big| = \big|\rho_\Lambda^{vac}\big| \approx \rho_\Lambda^{ind} \propto M_X^4 \qquad \Longrightarrow \qquad \frac{\gamma_0}{(4\pi)^2} \sim 8\Big(\frac{M_X}{M_P}\Big)^4 \approx 10^{-11}.$$

Accordingly, we get, as a very good approximation,

$$G(\mu) = G_0\Big[1 - \frac{8\,\gamma_0}{(4\pi)^2}\ln\frac{\mu}{\mu_0}\Big] \quad \textbf{and} \quad \rho_\Lambda(\mu) = \rho_\Lambda^0\Big[1 + \frac{6\,\gamma_0}{(4\pi)^2}\ln\frac{\mu}{\mu_0}\Big].$$

The main point is that the effective quantum gravity running depends on the vacuum part ρ_Λ only.

But what happens with the observed sum $\rho_\Lambda^{obs} = \rho_\Lambda + \rho_\Lambda^{ind}$??

The answer depends of the magnitude of the running, i.e., of

$$\delta\rho_\Lambda^{obs} \;=\; \rho_\Lambda^{obs}(UV) - \rho_\Lambda^{obs}(IR) \;=\; \frac{6\gamma_0}{(4\pi)^2}\,\rho_\Lambda^0\,\ln\left(\frac{\mu_{\rm UV}}{\mu_{\rm IR}}\right).$$

In cosmology, standard identification is $\mu \sim H$ **(Hubble const).**

This running produces a discrepancy with the cosmological observations much more than 10^{50} **for a change of just about an order of magnitude in the parameter** H**.**

As we already know, for a typical SUSY GUT model

$$\frac{\gamma_0}{(4\pi)^2} \propto 10^{-11}, \qquad \textbf{also} \qquad \ln\left(\frac{\mu_{\rm UV}}{\mu_{\rm IR}}\right) = \ln\left(\frac{H_{\rm inflation}}{H_0}\right) \approx 130.$$

Consequently, if we “believe” in the VDW effective action in QG, then SUSY GUT’s are ruled out.

What about lower energy physics?

What if we have to "rule out" the Minimal Standard Model of particle physics (MSM)??

Assuming that the MSM is valid up to the Planck scale, we get

$$\frac{6}{(4\pi)^2}\,\gamma_0 \,\sim\, 48\Big(\frac{M_F}{M_P}\Big)^4 \,\approx\, 10^{-65}.$$

Multiplying by $\rho_\Lambda^0 \sim M_F^4$, the variation is

$$\delta\rho_\Lambda^{obs} \approx 10^{-55}\ln\Big(\frac{H}{H_0}\Big)\,\mathrm{GeV}^4 \approx 10^{-53}\,\mathrm{GeV}^4 \approx 10^{-6}\,\rho_\Lambda^{obs}.$$

Thus, the model of effective QG running is lucky enough to pass the test related to MSM. The opposite output would mean the disproval of the Vilkovisky and DeWitt approach in QG.

However, since the result is $\mathcal{O}(M_F^8)$, the existence of any kind of "new physics" based on the symmetry breaking beyond the scale $10M_F$, contradicts the CC running in effective QG.

Conclusions

- **The great CC problem looks impossible to solve, as it comes from summing up completely independent contributions: induced and vacuum, the last has to be fine-tuned.**

- **Even accepting the fine-tuning, this does not make our life completely free of the CC problem.**

- **In the IR region, i.e., below the mass spectrum of the fundamental higher derivative gravity, we meet an effective QG, which is remarkable in several respects.**

- **Assuming the Vilkovisky-DeWitt unique effective action, we arrive at the well-defined RG equations, which turn out exact, in the sense they are free from the higher-loop corrections.**

- **It is remarkable that the effective CC running, derived in this framework, provides a relation between the cosmological constant problem and the particle physics.**