Scaling behavior of viscosity in model A of critical dynamics

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Introduction

The description of the dynamic critical properties of a superfluid substance, that is, the effect of equilibrium fluctuations on the critical behavior.

Classical monographs Hohenberg and Halperin (1977) indicate that superfluid critical dynamics is described by stochastic dynamical models F or E. In model F, the system of stochastic equations

$$
\begin{aligned}
\partial_t \psi &= \lambda(1+ib)\left[\partial^2\psi - \frac{g_1}{3}(\psi^+\psi)\psi + g_2 m\psi\right] + i\lambda g_3 \psi\left[g_2\psi^+\psi - m\right] + f_\psi \\
\partial_t \psi^+ &= \lambda(1-ib)\left[\partial^2\psi^+ - \frac{g_1}{3}(\psi^+\psi)\psi^+ + g_2 m\psi^+\right] - i\lambda g_3 \psi^+\left[g_2\psi^+\psi - m\right] + f_{\psi^+} \\
\partial_t m &= -\lambda u \partial^2\left[g_2\psi^+\psi - m\right] + i\lambda g_3\left[\psi^+\partial^2\psi - \psi\partial^2\psi^+\right] + f_m
\end{aligned}
$$

is built from phenomenological considerations based on conservation laws. The ψ, ψ^+ are the complex fields of the order parameter, the field m is responsible for density and temperature fluctuations, g_i , b, u are coupling constants, λ is the kinetic coefficient, f_i are random forces, for which is assumed to have a Gaussian distribution of the "white noise."

- Two infrared stable fixed points of the equation of the renormalization group in model E.
- In model E nor in a more complex model F it is impossible to calculate the critical index that determines the decrease in viscosity when approaching the point of phase transition to the superfluid state.
- It was proposed to study the stability of the E and F models to the impact of the hydrodynamic velocity field, and with respect to density wave perturbations Zhavoronkov et al. (2019).
- The models are sensitive to the inclusion of hydrodynamic modes, which significantly affect the critical behavior in general and the exponents.
- Taking into account the compressibility of the medium led to a paradoxical conclusion: models E and F of critical dynamics are unstable with respect to such perturbations, and in the IR region are reduced to the model A, which has a single fixed point.
- Confirmed from microscopic point Honkonen et al. (2019)

Action Functional

Dynamical action functional for the model F with velocity field

$$
\begin{aligned}
S_{dyn}(\phi) \;=\;& b_1\psi^{+'}\psi - b_2 m'\partial^2 m' + b_3\partial_i v_j'\partial_i v_j + b_4\partial_j v_j'\partial_i v_i + \psi^{+'}[-\partial_t\psi - v_i\partial_i\psi \\
&+a_1\partial^2\psi + a_8\psi^+\psi\psi + a_9 m\psi + a_{10}\psi v^2] + \psi'[-\partial_t\psi - v_i\partial_i\psi + a_2\partial^2\psi^+ \\
&+a_{11}\psi^+\psi^+\psi + a_{12}m\psi^+ + a_{13}\psi^+ v^2] + m'[-\partial_t m - \partial_i(mv_i) + a_4\partial^2 m \\
&+a_3\partial_i v_i + a_{14}\partial^2(\psi^+\psi) + a_{15}\partial^2 v^2 + a_{16}\psi^+\partial^2\psi + a_{17}\psi\partial^2\psi^+] \\
&+v_i'[-\partial_t v - v_j\partial_j v_i + a_5\partial^2 v_i + a_6\partial_i\partial_j v_j + a_7\partial_i m + a_{18}(\partial_i\psi^+)\partial^2\psi \\
&+a_{19}\psi^+\psi^2\partial_i\psi^+ + a_{20}m\psi\partial_i\psi^+ + a_{21}(\partial_i\psi)\partial^2\psi^+ + a_{22}(\psi^+)^2\psi\partial_i\psi \\
&+a_{23}m\partial_i(\psi^+\psi) + a_{24}\partial_i(\psi^+\psi) + a_{25}\partial_i v^2 + a_{27}m\partial^2 v_i + a_{28}m\partial_i\partial_j v_i] + \ldots
\end{aligned}
$$

$\phi \in \{\psi^+, \psi, m, v, \psi^{+'}, \psi', m', \mathbf{v}'\}$

Action Functional

- The final IR-efficient dynamical action of the MSR type for describing the critical dynamics of the main fields (Zhavoronkov et al. (2019)):

$$\begin{aligned} S_{dyn,eff} \quad = \quad & b_1 \psi^{+'} \psi' + a_1 \psi^{+'} \partial^2 \psi + a_2 \psi' \partial^2 \psi^+ - \psi^{+'} \partial_t \psi - \psi' \partial_t \psi + a_8 \psi^{+'} \psi^+ \psi^2 \\ & - \frac{a_9 a_{24}}{a_7} \psi^{+'} \psi^+ \psi^2 + a_{11} \psi' (\psi^+)^2 \psi - \frac{a_{12} a_{24}}{a_7} \psi' (\psi^+)^2 \psi \end{aligned}$$

- The action differs from the dynamic action of the stochastic model A only by the expansion of fields and parameters.
- Action S_{dyn} allows to construct IR-effective action $S_{dyn,eff}$, but also defines critical dimensions of IR - irrelevant fields v, m or possible monomials constructed from them, through dimensions of composite operators of fields $\psi^{+'}, \psi^+, \psi', \psi$.

Action Functional

Dynamical action functional $\phi \in \{\psi^+, \psi, m, v, \psi^{+'}, \psi', m', \mathbf{v}'\}$

$$
\begin{aligned}
S_{dyn}(\phi) \quad = \quad & b_1\psi^{+'}\psi - b_2 m'\partial^2 m' + b_3\partial_i v_j'\partial_i v_j + b_4\partial_j v_j'\partial_i v_i + \psi^{+'}[-\partial_t\psi - v_i\partial_i\psi \\
& + a_1\partial^2\psi + a_8\psi^+\psi\psi + a_9 m\psi + a_{10}\psi v^2] + \psi'[-\partial_t\psi - v_i\partial_i\psi + a_2\partial^2\psi^+ \\
& + a_{11}\psi^+\psi^+\psi + a_{12}m\psi^+ + a_{13}\psi^+v^2] + m'[-\partial_t m - \partial_i(mv_i) + a_4\partial^2 m \\
& + a_3\partial_i v_i + a_{14}\partial^2(\psi^+\psi) + a_{15}\partial^2 v^2 + a_{16}\psi^+\partial^2\psi + a_{17}\psi\partial^2\psi^+] \\
& + v_i'[-\partial_t v - v_j\partial_j v_i + a_5\partial^2 v_i + a_6\partial_i\partial_j v_j + a_7\partial_i m + a_{18}(\partial_i\psi^+)\partial^2\psi \\
& + a_{19}\psi^+\psi^2\partial_i\psi^+ + a_{20}m\psi\partial_i\psi^+ + a_{21}(\partial_i\psi)\partial^2\psi^+ + a_{22}(\psi^+)^2\psi\partial_i\psi \\
& + a_{23}m\partial_i(\psi^+\psi) + a_{24}\partial_i(\psi^+\psi) + a_{25}\partial_i v^2 + a_{27}m\partial^2 v_i + a_{28}m\partial_i\partial_j v_i] + \ldots
\end{aligned}
$$

Canonical dimension of viscosity is IR irrelevant parameters and critical dimension is proposed to define in dimension of composite operators.

Composite operators

- Composite operators:
$f_1 = \partial_i v_j' \partial_i v_j', \qquad f_2 = \partial_j v_j' \partial_i v_i', \qquad f_3 = v_i' \partial^2 v_i, \qquad f_4 = v_i' \partial_i \partial_j v_j.$
- The critical dimension of the viscosity coefficient (source $\Delta_{a'}$) can be obtained from dimensionless condition

$$\int dx\ dt\ a_i' f_i \qquad \Delta_{a'} + \Delta_f = d + \Delta_\omega \qquad \Delta_\omega = z$$

- Canonical dimension: $d[v'] = d - 1, \qquad d[v] = 1, \qquad d[\partial] = 1$
- The composite operators mix in the process of renormalization and therefore it is necessary to investigate the full mixing matrix in the renormalization of composite operators of an one canonical dimension.
- Critical dimension of viscosity is determined by the most essential critical dimension of one of the family of composite operators of fields $\psi^{+'}, \psi^+, \psi', \psi$ of canonical dimension 8.

Model A

- Action functional

$$S_A(\varphi', \varphi) = -\alpha_0 \varphi'^2 + \varphi' \left(\partial_t \varphi - \alpha_0 \nabla^2 \varphi + \alpha_0 \tau_0 \varphi + \frac{\alpha_0 g_0}{6} \varphi^3 \right)$$

where $\tau = 0$ an $\psi^{+'} = \varphi_1'$, $\psi' = \varphi_2'$, $\psi = \varphi_1$, $\psi^+ = \varphi_2$

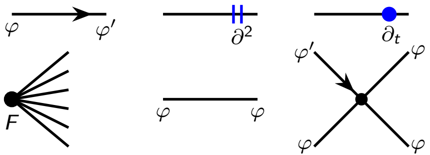
$$G(A) = c \int D\varphi' D\varphi exp[-S_A(\varphi, \varphi') + aF(\varphi, \varphi') + A\varphi + A'\varphi']$$

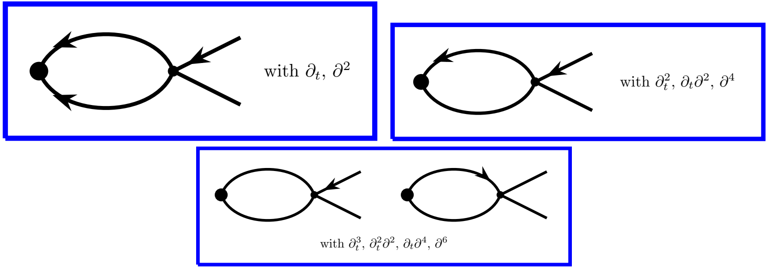
- Massles MS shceme ($d = 4 - \varepsilon$)
- Canonical dimension

$$d[\varphi'] = \frac{d}{2} + 1, \qquad d[\varphi] = \frac{d}{2} - 1, \qquad d[\partial] = 1, \qquad d[\partial_t] = 2$$

- The composite operator in the logarithmic theory, i.e. $\epsilon = 0$ and then $d[F(\varphi', \varphi)] = 8$.

Feynman rules

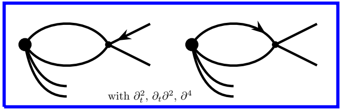


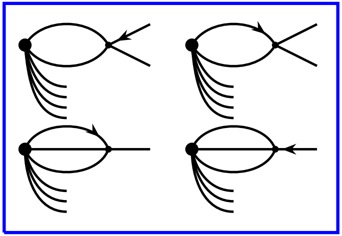


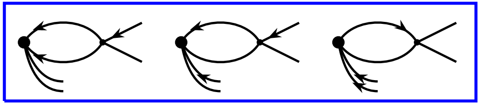
Composite operator of the form ∂F Vasil'ev (2004) : $[\partial F(\varphi', \varphi)]_R = \partial[F(\varphi', \varphi)]_R$

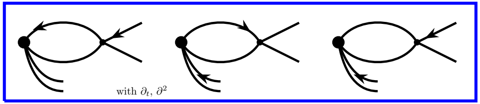
Composite operator for $d[F] = 8$

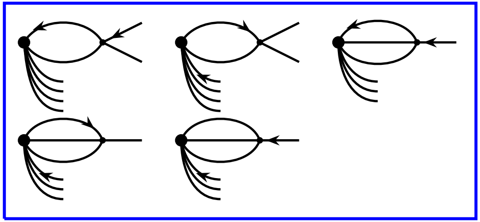
$$
\begin{array}{lll}
F_1 = (\varphi'\varphi')(\varphi\varphi) & F_2 = (\varphi'\varphi)(\varphi'\varphi) & F_3 = (\varphi\partial^2\varphi')(\varphi\varphi) \\
F_4 = (\varphi\partial\varphi')(\varphi\partial\varphi) & F_5 = (\partial\varphi'\partial\varphi)(\varphi\varphi) & F_6 = (\varphi'\partial^2\varphi)(\varphi\varphi) \\
F_7 = (\varphi'\varphi)(\varphi\partial^2\varphi) & F_8 = (\varphi'\varphi)(\partial\varphi)^2 & F_9 = (\varphi'\partial\varphi)(\varphi\partial\varphi) \\
F_{10} = (\varphi'\partial_t\varphi)(\varphi\varphi) & F_{11} = (\varphi'\varphi)(\varphi\partial_t\varphi) & F_{12} = (\varphi\partial_t\varphi')(\varphi\varphi) \\
F_{13} = (\varphi'\varphi)(\varphi\varphi)^2 & F_{14} = (\varphi\varphi)^4 & F_{15} = (\varphi\partial^2\varphi)(\varphi\varphi)^2 \\
F_{16} = (\partial\varphi\partial\varphi)(\varphi\varphi)^2 & F_{17} = (\varphi\partial\varphi)^2(\varphi\varphi) & F_{18} = (\varphi\partial^4\varphi)(\varphi\varphi) \\
F_{19} = (\partial^2\varphi\partial^2\varphi)(\varphi\varphi) & F_{20} = (\varphi\partial^2\varphi)(\varphi\partial^2\varphi) & F_{21} = (\partial\varphi\partial^3\varphi)(\varphi\varphi) \\
F_{22} = (\varphi\partial^3\varphi)(\varphi\partial\varphi) & F_{23} = (\partial\varphi\partial\varphi)(\varphi\partial^2\varphi) & F_{24} = (\partial^2\varphi\partial\varphi)(\varphi\partial\varphi) \\
F_{25} = (\partial\varphi\partial\varphi)(\partial\varphi\partial\varphi) & F_{26} = (\varphi\partial_t\varphi)(\varphi\varphi)^2 & F_{27} = (\varphi\partial_t^2\varphi)(\varphi\varphi) \\
F_{28} = (\partial_t\varphi\partial_t\varphi)(\varphi\varphi) & F_{29} = (\varphi\partial_t\varphi)(\varphi\partial_t\varphi) & F_{30} = (\varphi\partial_t\partial^2\varphi)(\varphi\varphi) \\
F_{31} = (\partial_t\varphi\partial^2\varphi)(\varphi\varphi) & F_{32} = (\varphi\partial^2\varphi)(\varphi\partial_t\varphi) & F_{33} = (\varphi\partial_t\varphi)(\partial\varphi\partial\varphi) \\
F_{34} = (\partial\varphi\partial_t\varphi)(\varphi\partial\varphi) & F_{35} = (\partial\varphi\partial_t\partial\varphi)(\varphi\varphi) & F_{36} = (\varphi\partial_t\partial\varphi)(\varphi\partial\varphi)
\end{array}
$$

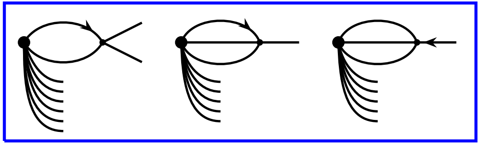

with ∂_t^2, $\partial_t\partial^2$, ∂^4






with ∂_t, ∂^2





Mixing matrix $\mathcal{Q}$

- Mixing matirx $\mathcal{Q}$

$$F_R = \mathcal{Q}(\alpha, \varepsilon, g)F$$

$$\mathcal{Q}(36\times 36) = \mathbb{1} + \begin{pmatrix} \mathcal{Q}_1(13\times 13) & 0 & 0 \\ ? & \mathcal{Q}_2(11\times 11) & 0 \\ ? & ? & \mathcal{Q}_3(12\times 12) \end{pmatrix}$$

- Matrix of renormalization constants

$$Z_a^{ij} = \mathcal{Q}_{ij}(Z_\varphi)^{-n_j}(Z_{\varphi'})^{-m_j}, \qquad Z_F = Z_a^{-1}$$

where n_j and m_j is number of fields φ, φ', rexpectively

- The RG constants fields have form for the model A

$$Z_\varphi = 1 + O(g^2), \qquad Z_{\varphi'} = 1 + O(g^2).$$

- The final form the RG constants: $Z_a = \mathcal{Q}$ $\qquad Z_F = \mathcal{Q}^{-1}$

Mixing matrix $\mathcal{Q}_1$

$$
\begin{pmatrix}
-\frac{32g}{3\varepsilon} & -\frac{32g}{3\varepsilon} & -\frac{3g}{\varepsilon} & 0 & -\frac{6g}{\varepsilon} & -\frac{3g}{\varepsilon} & 0 & 0 & 0 & -\frac{6g}{\alpha\varepsilon} & 0 & -\frac{6g}{\alpha\varepsilon} & 0 \\
-\frac{4g}{\varepsilon} & -\frac{56g}{3\varepsilon} & -\frac{9g}{4\varepsilon} & -\frac{3g}{\varepsilon} & -\frac{3g}{2\varepsilon} & -\frac{3g}{4\varepsilon} & -\frac{3g}{2\varepsilon} & 0 & 0 & -\frac{3g}{2\alpha\varepsilon} & -\frac{3g}{\alpha\varepsilon} & -\frac{9g}{2\alpha\varepsilon} & 0 \\
0 & 0 & -\frac{25g}{\varepsilon} & -\frac{4g}{\varepsilon} & -\frac{6g}{\varepsilon} & -\frac{3g}{\varepsilon} & -\frac{2g}{\varepsilon} & 0 & 0 & \frac{6g}{\alpha\varepsilon} & \frac{4g}{\alpha\varepsilon} & \frac{10g}{\alpha\varepsilon} & 0 \\
0 & 0 & -\frac{3g}{\varepsilon} & -\frac{29g}{\varepsilon} & -\frac{15g}{2\varepsilon} & -\frac{g}{\varepsilon} & -\frac{2g}{\varepsilon} & -\frac{3g}{2\varepsilon} & -\frac{9g}{\varepsilon} & -\frac{g}{\alpha\varepsilon} & -\frac{2g}{\alpha\varepsilon} & -\frac{3g}{\alpha\varepsilon} & 0 \\
0 & 0 & -\frac{4g}{\varepsilon} & -\frac{14g}{\varepsilon} & -\frac{23g}{\varepsilon} & -\frac{4g}{\varepsilon} & 0 & -\frac{3g}{\varepsilon} & -\frac{6g}{\varepsilon} & -\frac{4g}{\alpha\varepsilon} & 0 & -\frac{4g}{\alpha\varepsilon} & 0 \\
\frac{32g}{9\varepsilon} & \frac{64g}{9\varepsilon} & \frac{2g}{\varepsilon} & 0 & \frac{4g}{\varepsilon} & -\frac{10g}{\varepsilon} & -\frac{8g}{\varepsilon} & 0 & 0 & 0 & -\frac{8g}{\alpha\varepsilon} & \frac{4g}{\alpha\varepsilon} & 0 \\
\frac{16g}{9\varepsilon} & \frac{32g}{3\varepsilon} & \frac{3g}{2\varepsilon} & \frac{2g}{\varepsilon} & \frac{g}{\varepsilon} & -\frac{7g}{2\varepsilon} & -\frac{15g}{\varepsilon} & 0 & 0 & -\frac{g}{\alpha\varepsilon} & -\frac{10g}{\alpha\varepsilon} & \frac{3g}{\alpha\varepsilon} & 0 \\
0 & -\frac{64g}{9\varepsilon} & 0 & -\frac{2g}{\varepsilon} & -\frac{g}{\varepsilon} & 0 & -\frac{8g}{\varepsilon} & -\frac{21g}{\varepsilon} & -\frac{10g}{\varepsilon} & 0 & \frac{8g}{\alpha\varepsilon} & 0 & 0 \\
-\frac{16g}{9\varepsilon} & -\frac{32g}{9\varepsilon} & 0 & -\frac{3g}{\varepsilon} & -\frac{g}{2\varepsilon} & -\frac{2g}{\varepsilon} & -\frac{4g}{\varepsilon} & -\frac{13g}{2\varepsilon} & -\frac{23g}{\varepsilon} & \frac{2g}{\alpha\varepsilon} & \frac{4g}{\alpha\varepsilon} & 0 & 0 \\
0 & 0 & -\frac{2g\alpha}{\varepsilon} & 0 & -\frac{4g\alpha}{\varepsilon} & -\frac{2g\alpha}{\varepsilon} & 0 & 0 & 0 & -\frac{20g}{\varepsilon} & -\frac{16g}{\varepsilon} & -\frac{4g}{\varepsilon} & 0 \\
0 & 0 & -\frac{3g\alpha}{2\varepsilon} & -\frac{2g\alpha}{\varepsilon} & -\frac{g\alpha}{\varepsilon} & -\frac{g\alpha}{2\varepsilon} & -\frac{g\alpha}{\varepsilon} & 0 & 0 & -\frac{7g}{\varepsilon} & -\frac{30g}{\varepsilon} & -\frac{3g}{\varepsilon} & 0 \\
0 & 0 & \frac{5g\alpha}{\varepsilon} & \frac{4g\alpha}{\varepsilon} & \frac{6g\alpha}{\varepsilon} & \frac{3g\alpha}{\varepsilon} & \frac{2g\alpha}{\varepsilon} & 0 & 0 & -\frac{6g}{\varepsilon} & -\frac{4g}{\varepsilon} & -\frac{30g}{\varepsilon} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1920g}{\varepsilon}
\end{pmatrix}.
$$

Critical dimension

- The RG equations are derived from the generating functional of renormalized connected Green functions of the fields and composite operator Vasil'ev (2004):

$$(\mathcal{D}_\mu + \beta_g \partial_g - \gamma_\alpha \partial_\alpha) F_R = -\gamma_F F_R,$$

- At the fixed point $g = g^*$, in time-coordinate space, the equation of critical scaling for the closed set of renormalized operators

$$\left[-\mathcal{D}_x + \Delta_t \mathcal{D}_t\right] F_R = \Delta_F F_R, \qquad \text{where} \qquad \Delta_t = -\Delta_\omega$$

- Critical dimension of composite operator Δ_F

$$\Delta_F = d[F] + \gamma_F^*, \text{ where } d[F] = d_k[F] + (2 - \gamma_\alpha^*) d_\omega[F]$$

  where γ_F^*, γ_α^* are taken in the IR-stable fixed point.
- The coordinate of fixed point (Vasil'ev (2004), chapter 4 (4.7d))

$$g^* = \frac{3}{n+8}\varepsilon \Big|_{n=2} = \frac{3}{10}\varepsilon$$

Critical dimension

- Calculation of γ_F, Vasil'ev (2004)

$$\gamma_F \equiv Z_F^{-1} \cdot (\widetilde{\mathcal{D}}_\mu Z_F), \qquad \gamma_F = - \text{ UV-finite part} \left[\widetilde{\mathcal{D}}_\mu Z_a\right]$$

- The relation for the g and $d = 4 - \varepsilon$

$$\gamma_F = \varepsilon g \partial_g Z_a = \varepsilon g \partial_g Q$$

- The critical dimension of a composite operator

$$\Delta_F = d_p[F] + \triangle_\omega d_\omega[F] + \gamma_F^* = d[F] - \gamma_\alpha^* d_\omega[F] + \gamma_F^*,$$

- Diagonalization of matrix: $\triangle'_F = U \triangle_F U^{-1}$

$$8 - 579\varepsilon, \quad 8 - 14\varepsilon, \quad 8 - 14\varepsilon, \quad 8 - 8\varepsilon, \quad 8 - 5.534\varepsilon,$$
$$8 - 11.066\varepsilon, \quad 8 - 4.4\varepsilon, \quad 8 - 11.426\varepsilon, \quad 8 - 7.006\varepsilon, \quad 8 - 5.505\varepsilon,$$
$$8 - 4.202\varepsilon, \quad 8 - (7.731 + 1.564i)\varepsilon, \quad 8 - (7.731 - 1.564i)\varepsilon$$

- Calculation of part $\mathcal{Q}_2$, $\mathcal{Q}_3$.

Conclusion

- Construction of the composite operator of model A in $d[F] = 8$.
- Composite operators of lower dimensions $(6, 4, 2)$ are added to them, the missing powers are compensated by the powers of the control parameter responsible for the phase transition (chemical potential μ).
- Calculate the critical index that determines the decrease in viscosity when approaching the point of phase transition to the superfluid state.

Thank you for your attention.

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