Dissipation in the formalism of time-dependent Green's functions at finite temperature

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«Models in Quantum Field Theory»

The known methods

The main questions:

- How to describe dissipation in quantum systems?
- How to calculate the kinetic coefficients from first principles?

The known methods:

- Non-Hamiltonian disspative formalism (Lindbland equation,. . .).
- Projection operators, $\hat{\rho} = \hat{\rho}_{relevant} + \hat{\rho}_{irrelevant}$.
- Linear response theory (Green – Kubo formulas,. . .).
- Phenomenological models (stochastic differential equations),

$$\partial_t \varphi_a = (\alpha_{ab} + \beta_{ab}) \frac{\delta S^{st}}{\delta \varphi_b} + \zeta_a. \tag{1}$$

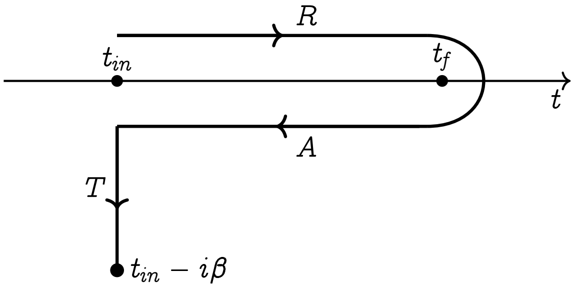
Introduction

- Motivation: to provide the method of calculation of kinetic coefficients and probe it on a simple model.
- Model systems: fermions or bosons with weak local interactions,

$$\hat{H} = \int d^3\mathrm{x}\, \hat{\psi}^+(\mathrm{x}, t)\Big(-\frac{\Delta}{2m} - \mu\Big)\hat{\psi}(\mathrm{x}, t) + g\int d^3\mathrm{x}\, \hat{\psi}^+(\mathrm{x}, t)\hat{\psi}(\mathrm{x}, t)\hat{\psi}^+(\mathrm{x}, t)\hat{\psi}(\mathrm{x}, t). \quad (2)$$

- We want to describe the dynamics of equilibrium fluctuations on large scales and calculate the kinetic coefficient.

The main object



The 2-point Green's function:

$$\left\langle \hat{\psi}(x_1, t_1)\hat{\psi}^+(x_2, t_2) \right\rangle = \mathrm{Sp}\Big(\mathrm{T}\{\hat{\psi}(x_1, t_1)\hat{\psi}^+(x_2, t_2)\}\hat{\rho}\Big). \tag{3}$$

Action:

$$\mathcal{S} = \int\limits_C dt \left[\psi^+(\mathrm{x}, t)\Big(i\partial_t + \frac{\Delta}{2m} + \mu\Big)\psi(\mathrm{x}, t) - g\psi^+(\mathrm{x}, t)\psi(\mathrm{x}, t)\psi^+(\mathrm{x}, t)\psi(\mathrm{x}, t) \right]. \tag{4}$$

Keldysh variables

$$\xi = \frac{\psi_R + \psi_A}{\sqrt{2}},\ \eta = \frac{\psi_R - \psi_A}{\sqrt{2}},\ \xi^+ = \frac{\psi_R^+ + \psi_A^+}{\sqrt{2}},\ \eta^+ = \frac{\psi_R^+ - \psi_A^+}{\sqrt{2}}. \tag{5}$$

Propagators in Keldysh variables:

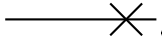
$$G_{\eta\xi^+} = -e^{-i\varepsilon(t-t')}\theta(t'-t), \tag{6}$$

$$G_{\xi\eta^+} = e^{-i\varepsilon(t-t')}\theta(t-t'), \tag{7}$$

$$G_{\xi\xi^+} = e^{-i\varepsilon(t-t')}\big(1 \pm 2n(\varepsilon)\big), \tag{8}$$

$$G_{\eta\eta^+} = 0. \tag{9}$$

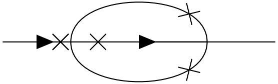
Notations:

$G_{\eta\xi^+} =$ , $G_{\xi\eta^+} =$ , $G_{\xi\xi^+} =$ . (10)

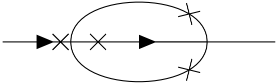
Interactions:

$$S_{int} = g\big(\xi^+\xi\xi^+\eta + \xi^+\xi\eta^+\xi + \xi^+\eta\eta^+\eta + \eta^+\xi\eta^+\eta\big). \tag{11}$$

«Pinch» singularities



$$\Big|_{p=0,\ \omega=0} = \int_{-\infty}^{\infty} d(t-t')$$

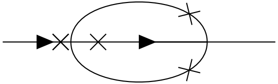
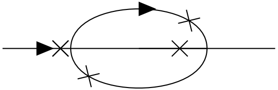
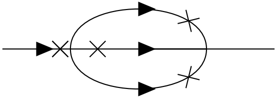


$$\Big|_{p=0} = \tag{12}$$

$$= \frac{\mp\zeta g^2}{(2\pi)^6}\int d^3\mathrm{k}\, d^3\mathrm{q}\left(\pi\delta\left(\varepsilon(\mathrm{k}+\mathrm{q})-\varepsilon(\mathrm{k})-\varepsilon(\mathrm{q})\right) + \underbrace{\frac{i}{\varepsilon(\mathrm{k}+\mathrm{q})-\varepsilon(\mathrm{k})-\varepsilon(\mathrm{q})}}_{\text{Singular}}\right)\times$$

$$\times\ \big(1 \pm 2n(\mathrm{k})\big)\big(1 \pm 2n(\mathrm{q})\big).$$

Singular diagrams:

 ,  ,  .

Regularization:

$$e^{-i\varepsilon(t-t')} \to e^{-i\varepsilon(t-t')-\gamma|t-t'|}, \quad \gamma = \alpha\frac{\mathrm{p}^2}{2m}. \tag{13}$$

The origin of dissipation

Dyson equation:

$$D^{-1} = K - \Sigma \;\Rightarrow\; D^{-1}_{\eta^+\xi} = i\omega + i\varepsilon + \underline{\gamma - \Sigma_{\eta^+\xi}}. \tag{14}$$

The sum of sunset diagrams in hydrodynamic limit:

$$I(\mathrm{p},\ \omega) \approx I(\mathrm{p},\ \omega)\Big|_{\substack{\mathrm{p}=0\\ \omega=0}} + \omega \frac{\partial I(\mathrm{p}=0,\ \omega)}{\partial \omega}\Big|_{\omega=0} + \frac{p_i p_j}{2!} \frac{\partial^2 I(\mathrm{p},\ \omega=0)}{\partial p_i \partial p_j}\Big|_{\mathrm{p}=0}. \tag{15}$$

Result

- Propagators and one loop:

$$\left\langle \hat{\psi}(t_1)\hat{\psi}^+(t_2) \right\rangle \sim \exp\left(-i\left(\frac{p^2}{2m} - \mu\right)(t_1 - t_2)\right). \tag{16}$$

- <u>Two loops:</u>

$$\left\langle \hat{\psi}(t_1)\hat{\psi}^+(t_2) \right\rangle \sim \exp\left(-i\left(\frac{p^2}{2m} - \mu\right)(t_1 - t_2) \underbrace{-\tilde{\alpha}\frac{p^2}{2m}|t_1 - t_2|}_{\text{dissipation}}\right). \tag{17}$$

- By analogy with phenomenological models, $\tilde{\alpha}$ is the kinetic coefficient.

Asymptotic of some integrals

The contributions to $\tilde{\alpha}$ are given by:

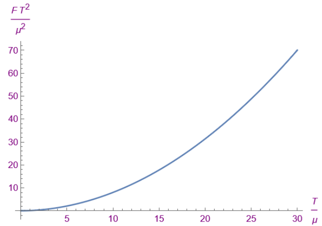
$$\lim_{\alpha\to 0}\frac{\mp 2\zeta g^2 m^2}{(2\pi)^6}\cdot \mathrm{Re}\int d^3\mathrm{k}\,d^3\mathrm{q}\;\frac{f(\mathrm{k},\mathrm{q})}{\left[i(\mathrm{k}\cdot\mathrm{q})-\alpha(\mathrm{k}^2+\mathrm{k}\cdot\mathrm{q}+\mathrm{q}^2)\right]^2}, \tag{18}$$

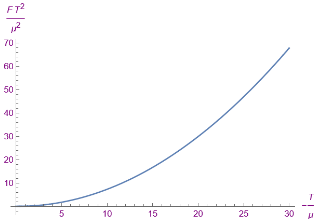
$$\begin{aligned}&\lim_{\alpha\to 0}\frac{\mp 2\zeta g^2 m}{(2\pi)^6}\cdot \mathrm{Re}\int d^3\mathrm{k}\,d^3\mathrm{q}\Bigg(-\frac{if(\mathrm{k},\mathrm{q})}{\left[i(\mathrm{k}\cdot\mathrm{q})-\alpha(\mathrm{k}^2+\mathrm{k}\cdot\mathrm{q}+\mathrm{q}^2)\right]^2}+\\&+\frac{2}{3}\frac{f(\mathrm{k},\mathrm{q})\cdot(\mathrm{k}+\mathrm{q})^2}{\left[i(\mathrm{k}\cdot\mathrm{q})-\alpha(\mathrm{k}^2+\mathrm{k}\cdot\mathrm{q}+\mathrm{q}^2)\right]^3}+\frac{f_1(\mathrm{k},\mathrm{q})}{i(\mathrm{k}\cdot\mathrm{q})-\alpha(\mathrm{k}^2+\mathrm{k}\cdot\mathrm{q}+\mathrm{q}^2)}+\\&+\frac{2}{3}\frac{if_2(\mathrm{k},\mathrm{q})\cdot(\mathrm{k}+\mathrm{q})^2}{\left[i(\mathrm{k}\cdot\mathrm{q})-\alpha(\mathrm{k}^2+\mathrm{k}\cdot\mathrm{q}+\mathrm{q}^2)\right]^2}\Bigg),\end{aligned} \tag{19}$$

$$f(\mathrm{p},\mathrm{k},\mathrm{q})=4n(\mathrm{k})n(\mathrm{q})\mp 4n(\mathrm{p}-\mathrm{k}-\mathrm{q})-8n(\mathrm{k})n(\mathrm{p}-\mathrm{k}-\mathrm{q}). \tag{20}$$

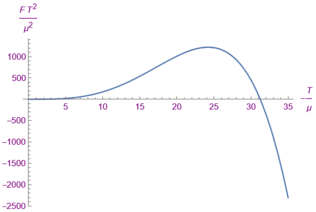
After calculating the integrals

$$\tilde{\alpha} = g^2 m^2 T^2 F(T/\mu).$$





Fermions with positive (left) or negative (right) chemical potentials.



Bosons with negative chemical potentials.

Conclusions

- Existing methods for calculating the kinetic coefficients are difficult to carry out real calculations.
- Based on the formalism of the time-dependent Green's time functions at finite temperature the new method for calculating the kinetic coefficients in quantum systems has been developed.
- To calculate the kinetic coefficient, it is necessary to carry out a rather complicated asymptotic analysis of a certain class of integrals.