A criterion for infinite positron feedback in the dynamics of runaway electron avalanches

Egor Stadnichuk

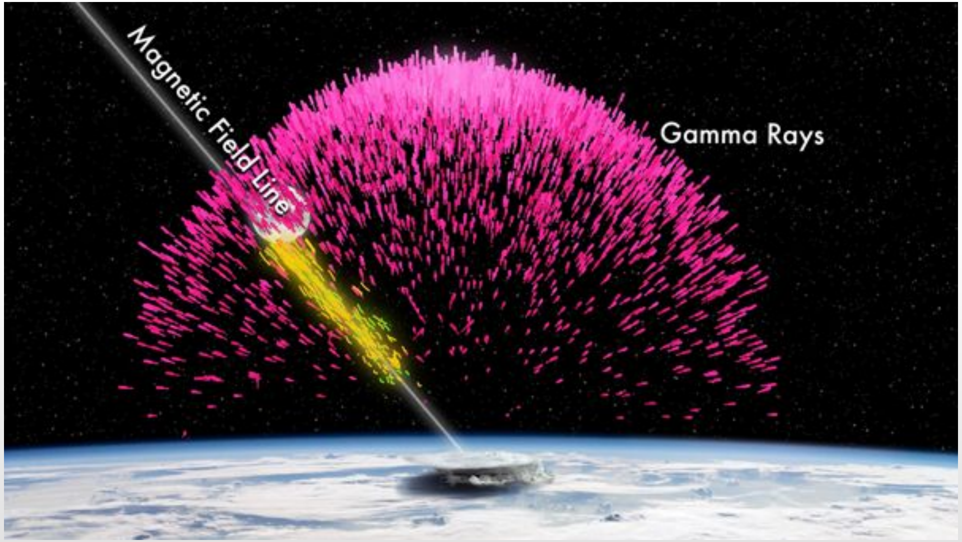
MIPT, HSE

Terrestrial Gamma-ray Flashes

Space gamma telescopes observe intensive gamma-ray flashes from the Earth.

TGF – intensive and short bursts of gamma-rays radiating from the Earth into space.

The source of TGFs are thunderstorms, mostly on equatorial latitudes.

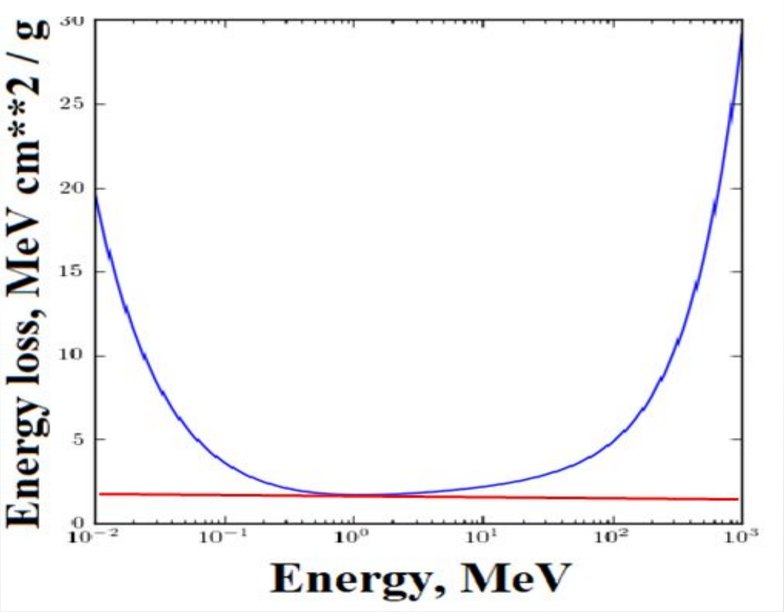


TGFs according to NASA.

The underlying basic physics

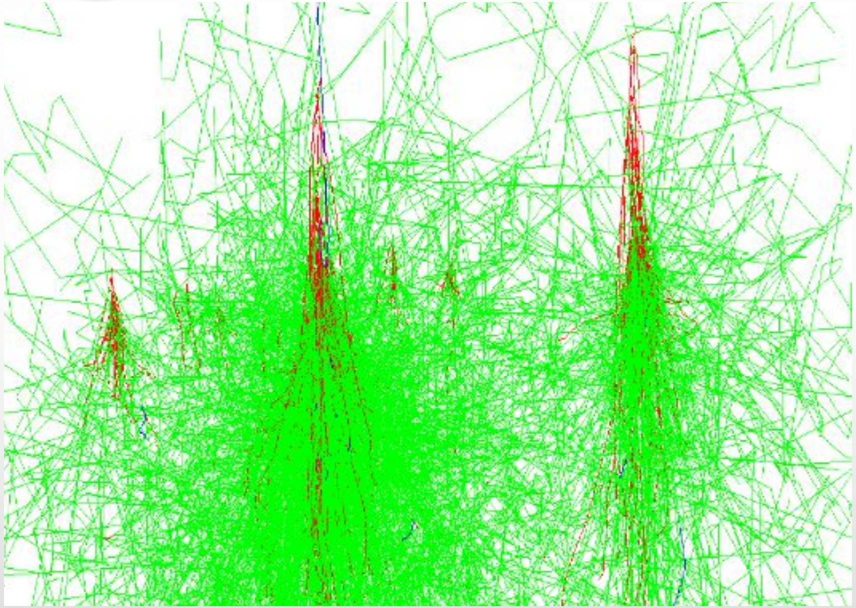
Atmospheric electric field might give to a relativistic electron more energy than it wastes on interaction with air.

Such an electric field is called the critical electric field, electrons accelerated by it are called runaway electrons.



Blue curve - electron stopping power in the air, red line - electron acceleration by the critical electric field.

Relativistic Runaway Electron Avalanches



RREA simulation on Geant4. Red lines - electron tracks, green lines - gamma-ray tracks, blue - positron tracks.

Runaway electrons produce new runaway electrons by collision with air molecules' electrons. This leads to a formation of a Relativistic Runaway Electron Avalanche (RREA).

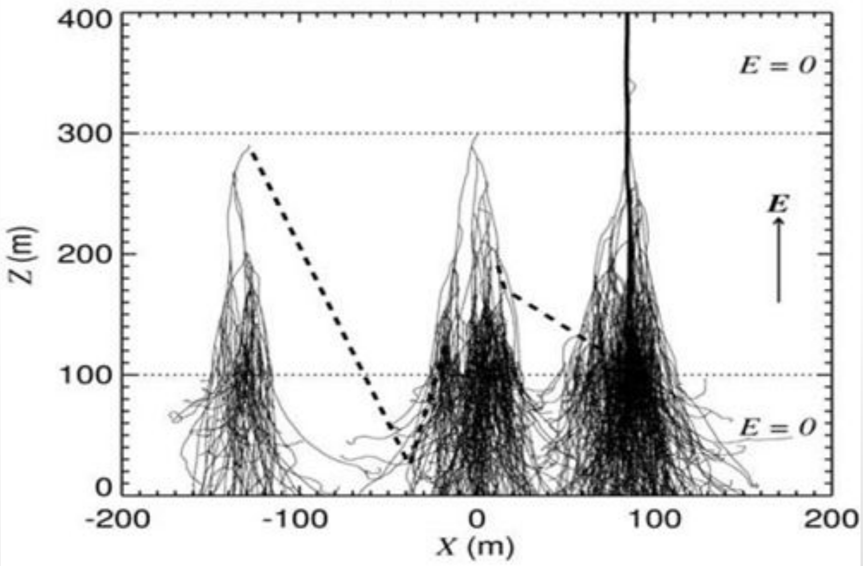
The law of the RREA growth:

$$N_{RREA}(z) = N_0 \cdot e^{\frac{z}{\lambda_{RREA}}}$$

Relativistic Feedback Discharge Model

RREA reproduce themselves by positron and gamma feedback mechanisms.

Positrons are created by RREA's bremsstrahlung and propagate in direction opposite to RREA propagation. In this way, positrons create secondary avalanches at the beginning of the cell.



Infinite feedback within Dwyer's simulation. Continuous lines - electrons, dashed lines - gamma-rays, thick lines - positrons.

Kinetic calculation assumptions

- All values are averaged.
- All runaway electrons fly strictly along the z-axis (against the electric field).
- All bremsstrahlung gamma-ray fly strictly along the mean azimuthal angle α relative to the z-axis.
- All positrons move strictly against the field.
- For gamma-rays, only the process of electron-positron pair production is taken into account.
- Positrons are runaway.
- Positrons give rise to runaway electrons with equal probability at any part of their trajectory inside the cloud. But positrons annihilate.
- The cell length is much longer than the RREA growth length.

Gamma-ray production

Let the initial RREA be starting at the point z = z_0. Then number of runaway electrons within this RREA grows as follows:

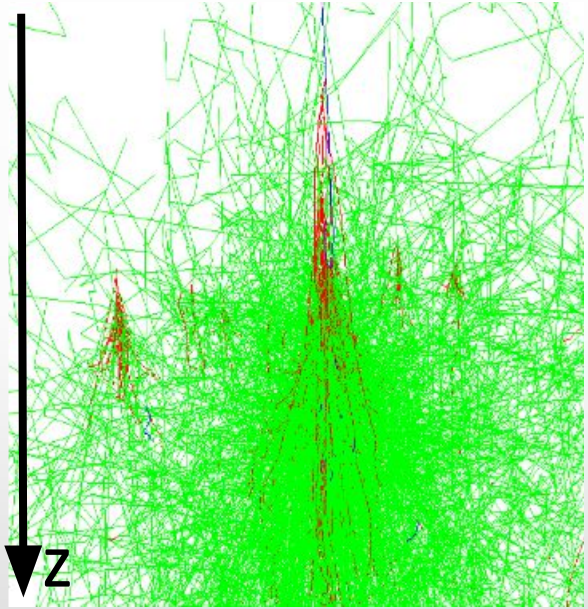
$$N_{RREA}(z,\ z_0) = e^{\frac{z\ -\ z_0}{\lambda_{RREA}}}$$

Therefore, number of gamma-rays produced within an interval (z, z + dz) is

$$df_\gamma(z,\ z_0) = N_{RREA}(z,\ z_0)\frac{dz}{\lambda_\gamma}$$

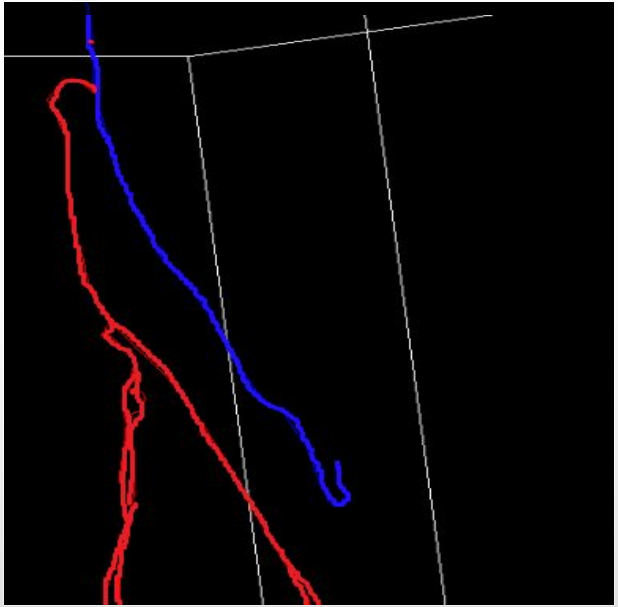
That leads to the following correspondence:

$$f_\gamma(z, z_0) = \frac{\lambda_{RREA}}{\lambda_\gamma} \cdot \left(e^{\frac{z-z_0}{\lambda_{RREA}}} - 1\right)$$



RREA simulation on Geant4. Red lines - electron tracks, green lines - gamma-ray tracks, blue - positron tracks.

Positrons



A positron reversing and generating a secondary RREA. Geant4.

In average, gamma-rays flow with a non-zero azimuthal angle. Moreover, positrons are created by gamma-rays moving against the electric field. Therefore, they have to reverse to be accelerated to the beginning of the cell. If P^1 is the reversal probability, then number of generated positrons is:

$$df_+(z, z_0) = P^1 f_\gamma(z, z_0) \frac{dz}{\lambda_+ \cos\theta}$$

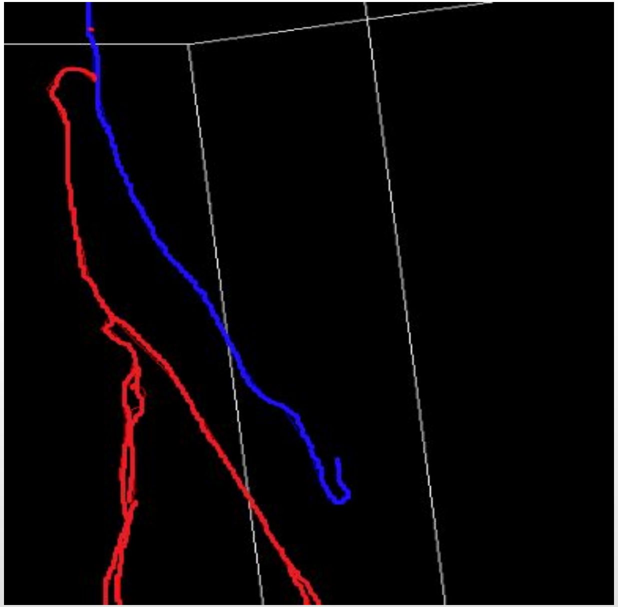
Secondary avalanches

Positrons decay moving to the beginning of the cell by the annihilation:

$$\frac{dN_+}{dz} = -\frac{N_+}{\lambda_x}$$

They generate secondary runaway electrons. Let the probability of a secondary RREA formation by a secondary runaway electron be P^2. Therefore, secondary avalanches within (z, z+dz) are generated in the following way:

$$df_2(z, z_0) = dz \cdot \frac{P^2}{\lambda_2} \cdot \int_z^L d\zeta \frac{\partial f_+(\zeta, z_0)}{\partial \zeta} e^{-\frac{\zeta - z}{\lambda_x}}$$



A positron reversing and generating a secondary RREA. Geant4.

Secondary RREAs production

Thus, number of secondary RREAs generated at interval (z, z+dz) by one primary RREA, started from the point z_0 is the following:

$$\frac{df_2(z,z_0)}{dz} = \beta \cdot e^{-\frac{z_0}{\lambda_{RREA}}} e^{\frac{z}{\lambda_x}} \left(e^{\frac{L(\lambda_x - \lambda_{RREA})}{\lambda_x \lambda_{RREA}}} - e^{\frac{z(\lambda_x - \lambda_{RREA})}{\lambda_x \lambda_{RREA}}} \right)$$

β does not depend on z and z_0: $\beta = \frac{P\lambda_{RREA}}{\lambda_2 \lambda_+ \lambda_\gamma} \frac{\lambda_{RREA}\lambda_x}{\lambda_x - \lambda_{RREA}}$

Generation of the tertiary RREAs

Let the primary RREA start from the point $z_0 = 0$.

Every single RREA from the second generation has the same physics as the primary RREA. Consequently, the distribution of tertiary RREAs should be found from the following convolutions:

$$f_3(z,0) = \int_0^z d\zeta \frac{\partial f_2(\zeta,0)}{\partial \zeta} f_2(z,\zeta)$$

$$\frac{df_3(z,0)}{dz} = \int_0^z d\zeta \frac{\partial f_2(\zeta,0)}{\partial \zeta} \frac{\partial f_2(z,\zeta)}{\partial z}$$

Tertiary RREAs

Integration results in the following formulas:

$$f_3(z,0) = \beta^2 \int_0^z d\zeta e^{-\frac{\zeta}{\lambda_{RREA}}} \left(e^{\frac{L(\lambda_x - \lambda_{RREA})}{\lambda_x \lambda_{RREA}}} \lambda_x (e^{\frac{z}{\lambda_x}} - 1) - \lambda_{RREA} (e^{\frac{z}{\lambda_{RREA}}} - 1) \right) \cdot$$
$$\cdot e^{\frac{\zeta}{\lambda_x}} \left(e^{\frac{L(\lambda_x - \lambda_{RREA})}{\lambda_x \lambda_{RREA}}} - e^{\frac{\zeta(\lambda_x - \lambda_{RREA})}{\lambda_x \lambda_{RREA}}} \right)$$

$$\frac{df_3(z,0)}{dz} = \beta^2 \frac{\lambda_{RREA}\lambda_x}{\lambda_x - \lambda_{RREA}} \left(e^{\frac{L(\lambda_x - \lambda_{RREA})}{\lambda_x \lambda_{RREA}}} - 1 - \frac{L(\lambda_x - \lambda_{RREA})}{\lambda_x \lambda_{RREA}} \right) \cdot$$
$$\cdot e^{\frac{z}{\lambda_x}} \left(e^{\frac{L(\lambda_x - \lambda_{RREA})}{\lambda_x \lambda_{RREA}}} - e^{\frac{z(\lambda_x - \lambda_{RREA})}{\lambda_x \lambda_{RREA}}} \right)$$

The fourth generation of RREAs

For the fourth generation, similarly, we have:

$$f_4(z,0) = \int_0^z d\zeta f_2(z,\zeta)\frac{\partial f_3(\zeta,0)}{\partial \zeta}$$

This results in the following correspondence:

$$f_4(z,0) = \beta^3 \frac{\lambda_{RREA}\lambda_x}{\lambda_x - \lambda_{RREA}}\left(e^{\frac{L(\lambda_x-\lambda_{RREA})}{\lambda_x\lambda_{RREA}}} - 1 - \frac{L(\lambda_x-\lambda_{RREA})}{\lambda_x\lambda_{RREA}}\right)\cdot$$

$$\cdot\int_0^z d\zeta e^{-\frac{\zeta}{\lambda_{RREA}}}\left(e^{\frac{L(\lambda_x-\lambda_{RREA})}{\lambda_x\lambda_{RREA}}}\lambda_x(e^{\frac{z}{\lambda_x}}-1) - \lambda_{RREA}(e^{\frac{z}{\lambda_{RREA}}}-1)\right)e^{\frac{\zeta}{\lambda_x}}\left(e^{\frac{L(\lambda_x-\lambda_{RREA})}{\lambda_x\lambda_{RREA}}} - e^{\frac{\zeta(\lambda_x-\lambda_{RREA})}{\lambda_x\lambda_{RREA}}}\right)$$

The connection between generations

It is easy to spot that:

$$f_4(z,0) = \beta \frac{\lambda_{RREA}\lambda_x}{\lambda_x - \lambda_{RREA}} \left(e^{\frac{L(\lambda_x - \lambda_{RREA})}{\lambda_x \lambda_{RREA}}} - 1 - \frac{L(\lambda_x - \lambda_{RREA})}{\lambda_x \lambda_{RREA}} \right) \cdot f_3(z,0) = \Gamma \cdot f_3(z,0)$$

Due to the same physics of each generation we have:

$$f_{i+1}(z,0) = \int_0^z d\zeta f_2(z,\zeta) \frac{\partial f_i(\zeta,0)}{\partial \zeta}$$

Γ does not depend on z, consequently:

$$f_i(L,0) = \Gamma^{i-3} \cdot f_3(L,0), \; \forall i \geq 4$$

The criterion of the infinite feedback

As it was shown, number of particles in the next generation of RREAs differs from the previous one by the factor Γ. Consequently, if $\Gamma > 1$, then number of particles grows with number of generation and we have infinite feedback. Thus, the criterion of the infinite feedback is

$$\frac{P\lambda_{RREA}}{\lambda_2\lambda_\gamma\lambda_{\gamma\to e^-e^+}cos\alpha}\left(\frac{\lambda_{RREA}\lambda_x}{\lambda_x-\lambda_{RREA}}\right)^2\left(e^{\frac{L(\lambda_x-\lambda_{RREA})}{\lambda_x\lambda_{RREA}}}-1-\frac{L(\lambda_x-\lambda_{RREA})}{\lambda_x\lambda_{RREA}}\right)>1$$

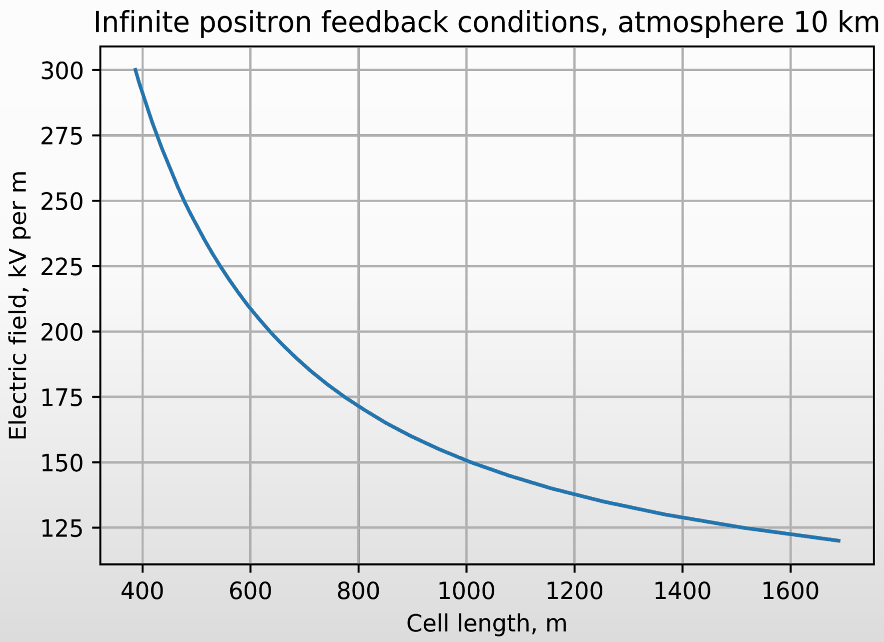
Infinite feedback conditions

Calculations were performed for 10 km atmosphere altitude.

RREA growth length was calculated according to Dwyer's formula:

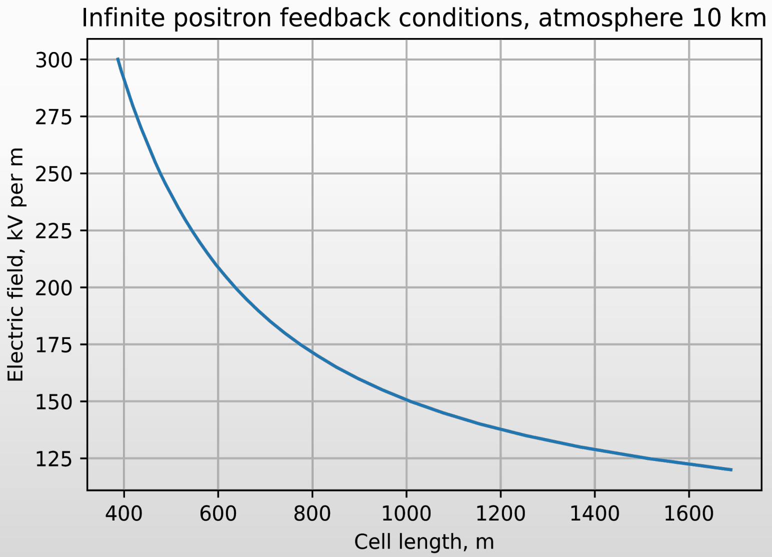
$$\lambda_{RREA} = \frac{7300\ keV}{E - \frac{\rho}{\rho_0} \cdot 276\ \frac{kV}{m}}$$

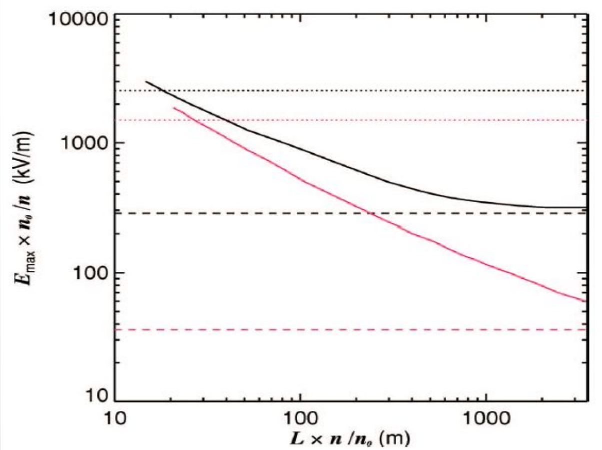
Other parameters were taken constant and were obtained from Geant4 for electric field 200 kV per m.



Results

The criterion of infinite positron feedback is consistent with Geant4 simulations. Moreover, it concurs with Dwyer’s calculations.





Black curve - infinite feedback conditions in the Earth atmosphere according to Dwyer. n - air concentration, n_0 - normal conditions.

Conclusions

- Particle spatial distributions for each generations were obtained.
- Derived infinite feedback criterion is consistent with Geant4 simulations and Dwyer simulations.
- Infinite feedback requires extended electric fields with a large magnitude, e.g. 200 kV per m with length ~650 m for 10 km altitude (critical electric field ~90 kV per m).

  Such electric fields have never been experimentally observed.

  This makes RFDM controversial for explaining TGFs.

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Characteristic length values

For 10 km atmosphere altitude and 200 kV per m electric field:

$$P = P_{e^+ reversal} \cdot P_{e^- reversal} = 0.8^2 = 0.64$$

$$\lambda_{RREA} \approx 85\, m$$

$$\lambda_x \approx 500\, m$$

$$\lambda_\gamma \approx 500\, m$$

$$\lambda_{\gamma \to e^+ e^-} \approx 14000\, m$$

$$cos\alpha \approx 0.94$$

$$\lambda_2 \approx 106.6\, m$$

Values obtained with Geant4 (G4StandartEmPhysics_option4).

Connection with Dwyer feedback coefficient

Within his simulations, Dwyer defined feedback coefficient as a ratio between number of runaway electrons in the middle of the cell in the second generation to number in the first generation:

$$\gamma = \frac{N_2(\frac{L}{2})}{N_1(\frac{L}{2})}$$

A calculation of Dwyer's feedback coefficient gives the following result:

$$\gamma \approx \Gamma \cdot \left(1 - e^{\frac{-L(\lambda_x - \lambda_{RREA})}{2\lambda_{RREA}\lambda_x}}\right)$$

Therefore, these values are almost the same.

Secondary RREAs distribution

Integration of the formula above results in the following correspondence ($P = P^1 P^2$):

$$f_2(z,z_0) = \frac{P\lambda_{RREA}}{\lambda_2\lambda_+\lambda_\gamma}\frac{\lambda_{RREA}\lambda_x}{\lambda_x-\lambda_{RREA}}e^{-\frac{z_0}{\lambda_{RREA}}}\left(e^{\frac{L(\lambda_x-\lambda_{RREA})}{\lambda_x\lambda_{RREA}}}\lambda_x(e^{\frac{z}{\lambda_x}}-1)-\lambda_{RREA}(e^{\frac{z}{\lambda_{RREA}}}-1)\right)- \\ -\frac{P\lambda_{RREA}}{\lambda_2\lambda_+\lambda_\gamma}\lambda_x(z-\lambda_x(e^{\frac{z-L}{\lambda_x}}-e^{-\frac{L}{\lambda_x}}))$$

$L \gg \lambda_{RREA}$ and $\lambda_x \gg \lambda_{RREA}$, consequently:

$$f_2(z,z_0) \approx \beta\cdot e^{-\frac{z_0}{\lambda_{RREA}}}\left(e^{\frac{L(\lambda_x-\lambda_{RREA})}{\lambda_x\lambda_{RREA}}}\lambda_x(e^{\frac{z}{\lambda_x}}-1)-\lambda_{RREA}(e^{\frac{z}{\lambda_{RREA}}}-1)\right)$$

The criterion with gamma-ray absorption

Let gamma-rays be absorbed in the air with characteristic length: λ_- .

Thus, number of gamma-rays, produced by a RREA:

$$f_\gamma(z, z_0) = \frac{\lambda_{RREA}\lambda_-}{\lambda_\gamma(\lambda_- + \lambda_{RREA})} \cdot e^{\frac{z-z_0}{\lambda_{RREA}}}$$

This leads to the following feedback coefficient:

$$\Gamma = \frac{P\lambda_{RREA}\lambda_-}{(\lambda_- + \lambda_{RREA})\lambda_2\lambda_\gamma\lambda_{\gamma\to e^- e^+} cos\alpha} \left(\frac{\lambda_{RREA}\lambda_x}{\lambda_x - \lambda_{RREA}}\right)^2 \left(e^{\frac{L(\lambda_x - \lambda_{RREA})}{\lambda_x\lambda_{RREA}}} - 1 - \frac{L(\lambda_x - \lambda_{RREA})}{\lambda_x\lambda_{RREA}}\right)$$

Total number of particles in RFDM

If feedback coefficient > 1 then the number of particles is infinite in total. Otherwise it depends on number of particles in the first generation in the following way:

$$N_{\Sigma}^{particles} = \frac{N_1^{particles}}{1-\Gamma}$$