Solitonic Vortex in 4d SQCD vs Critical Superstring

The XXIV International Scientific Conference of Young Scientists and Specialists (AYSS-2020)

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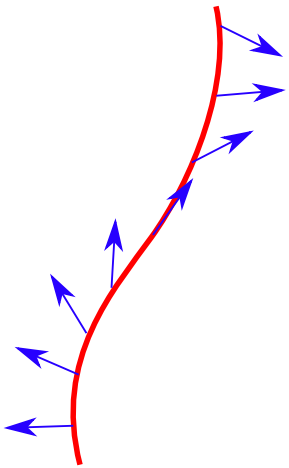
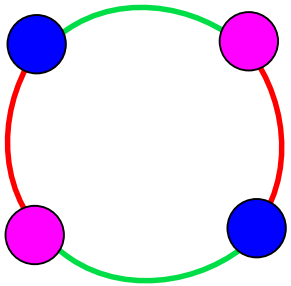
Joint work with Mikhail Shifman and Alexei Yung

Confinement checklist

Understanding of strong coupling dynamics in QCD:

1. Understand nature of confinement and confining string
   1. Abelian confinement: Seiberg-Witten
   2. Non-Abelian vortices: "instead-of-confinement" (will explain here)
2. Quantize the string and study the hadron spectrum
   1. Generally the vortices are thick, unknown higher-derivative corrections $\rightarrow$ hard to deal with
   2. But there is (at least) one suitable example in $\mathcal{N} = 2$ realm
3. Study effective hadron interactions (not there just yet)





Based on 2006.12054. Stringy review: 1611.03111

So far without chiral symmetry

4D $\mathcal{N} = 2$ supersymmetric QCD

Quark vacuum

Bulk theory: $\mathcal{N} = 2$ supersymmetric QCD
Scalar quarks condense $\Rightarrow$ broken symmetry.

$$m_G \sim g\sqrt{\xi}$$

In the limit of equal quark masses, a *global* subgroup survives:

$$\mathrm{U}(N_c)_{\mathrm{gauge}} \times \mathrm{SU}(N_f)_{\mathrm{flavor}} \to \mathrm{SU}(N_c)_{C+F} \times \mathrm{SU}(\widetilde{N})_F \times \mathrm{U}(1)$$

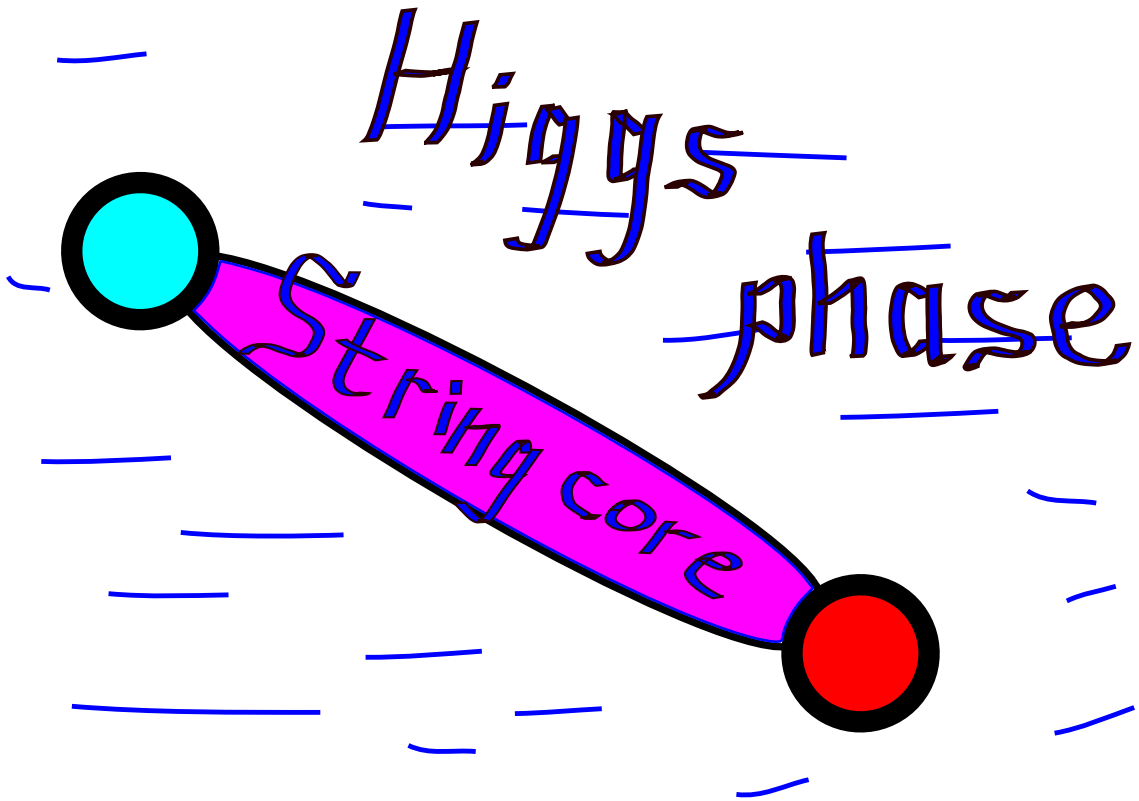
$\widetilde{N} = N_f - N$
Abrikosov-Nielsen-Olesen strings emerge, which then are promoted to non-Abelian strings.

Confining string

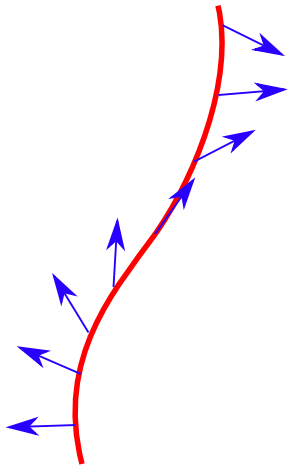
Meißner effect

(Scalar) quarks condense $\rightarrow$ monopoles are confined

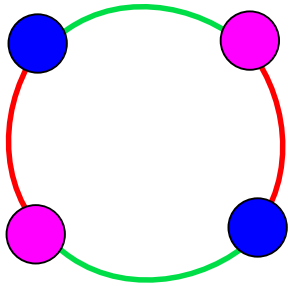
String tension is $T = 2\pi\xi$



Non-Abelian vortices in 4d



1. Non-Abelian strings were found in $\mathcal{N} = 2$ $U(N)$ SQCD '03 [Hanany, Tong; Auzzi, Bolognesi, Evslin, Konishi, Yung]
2. $N_f = 2N \to$ (almost) superconformal
   At strong coupling $\to$ thin string regime (hypothesis)
   '15 [Shifman, Yung]
3. World sheet theory is $\mathcal{N} = (2,2)$ weighted $\mathbb{CP}$ sigma model
4. At $N = 2$, $N_f = 4$ the string becomes critical



2d $\mathcal{N} = (2,2)$ $\mathbb{WCP}(2,2)$ sigma model

2d sigma model action (bosonic part):

$$S_{\text{bos}} = \int d^2x \left\{ \left|\nabla_\alpha n^P\right|^2 + \left|\tilde{\nabla}_\alpha \rho^K\right|^2 + 2\left|\sigma + \frac{m_P}{\sqrt{2}}\right|^2 \left|n^P\right|^2 + 2\left|\sigma + \frac{m_K}{\sqrt{2}}\right|^2 \left|\rho^K\right|^2 + \frac{e^2}{2}\left(|n^P|^2 - |\rho^K|^2 - \beta\right)^2 \right\}$$

$n^P \leftarrow$ internal $\mathrm{SU}(N_c)_{C+F}$ non-Abelian moduli

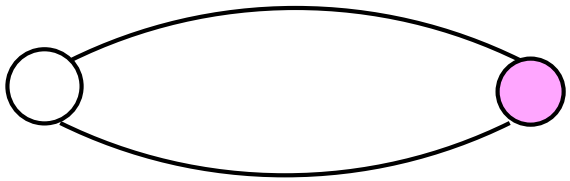
$\rho^K \leftarrow$ correspond to massless fields living on the string

Tool: 2d-4d correspondence

World sheet theory is 2d $\mathcal{N} = (2,2)$ $\mathbb{WCP}(2,2)$ sigma model

BPS spectra coincide on the world sheet and in the bulk

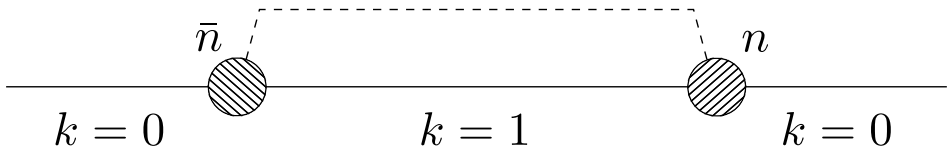
(noted [Dorey'98, Dorey Hollowood Tong '99], explained [Hanany Tong '04, Shifman Yung '04])



4d monopoles attached to two non-Abelian strings

$\Updownarrow$

2d kinks interpolating between two vacua



2d spectrum

Start with $\beta \gg 1$

If masses m_3, m_4 are "large", the fields ρ^3, ρ^4 are heavy and decouple $\rightarrow$ we are left with the "ordinary" $\mathbb{CP}(1)$ sigma model

The spectrum includes:

- BPS perturbative states with mass $|i(m_1 - m_2)|$
- Non-perturbative states (BPS kinks) with mass $M = |Z|$, where Z is the central charge of the $\mathcal{N} = 2$ algebra (a complex number).
  Analogy: ordinary Bogomol'ny inequalities for the soliton.

Central charge is a linear function of the "ordinary" charges:

$$Z_{\text{kink}} \approx |-(\beta_{CP(1)}\, T - i\, q)\,(m_1 - m_2)|$$

T is a topological charge, q is the global flavor charge

Wall-crossing

From weak to strong coupling

Consider a "decay" of some BPS particles

$$1 \to 2 + 3$$

Conservation laws:

$$\text{charge: } Z_1 = Z_2 + Z_3$$

$$\text{energy: } |Z_1| = |Z_2| + |Z_3|$$

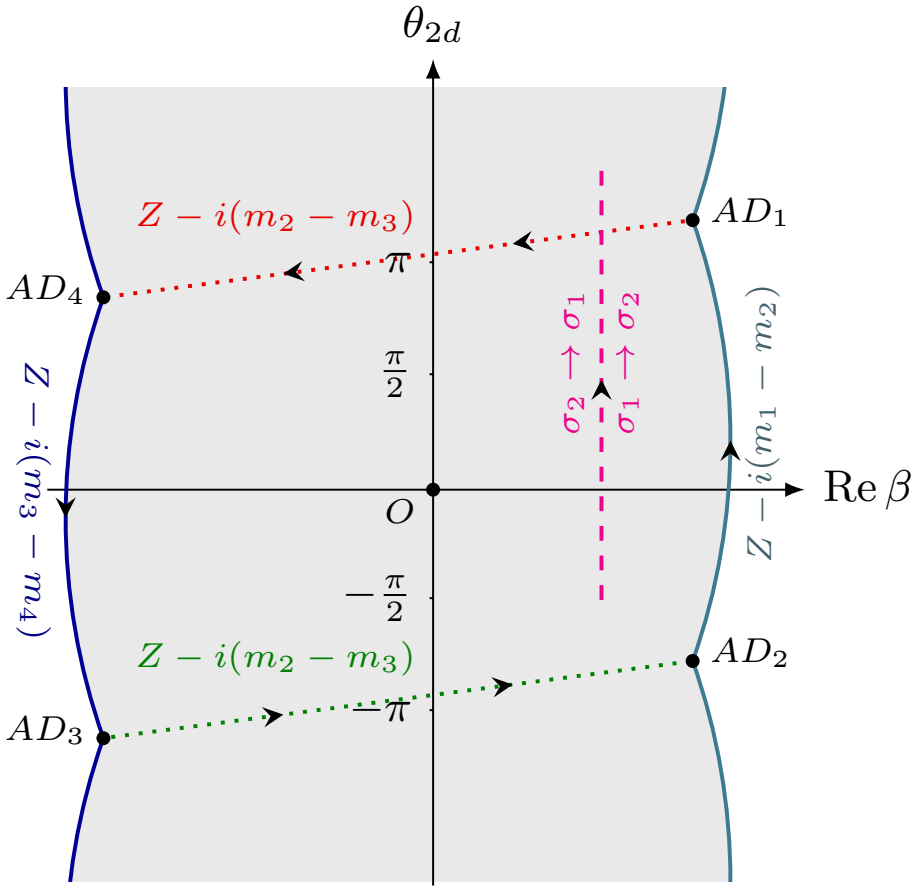
$\to$ decay is possible only when the three complex numbers Z_1, Z_2, Z_3 are collinear $\to$ equation for the so-called walls of marginal stability

2d theory is conformal $\to$ can consider the central charge as a function of the coupling β

Curves of marginal stability

At weak coupling, there are perturbative states
When crossing into the strong coupling domain, these decay into kink-antikink pairs

In 4d, this corresponds to the screened quarks and Higgsed gluons decaying to *mesonic* monopole-antimonopole pairs
$\rightarrow$ instead-of-confinement phase



CMS on the plane of complexified 2d coupling β

Stringy baryon from field theory

Baryon charge = 2, gauge-invariant $\rightarrow$ a four-kink state

$$n^P \rho^K n^{P'} \rho^{K'} = w^{PK} w^{P'K'},$$

Triplet representation (**3**, **3**, 2) $\rightarrow$ need to modify.

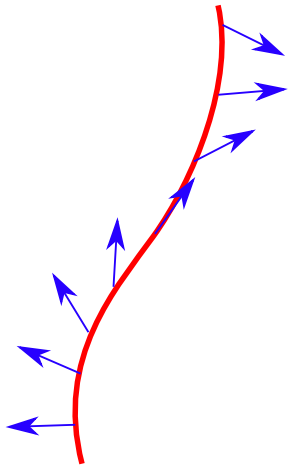
The singlet representation (**1**, **1**, 2) we are looking for would correspond to $\det(w)$. Classically it is zero, but at the point $\beta = 0$ there is a deformation:

$$\det(w) = b$$

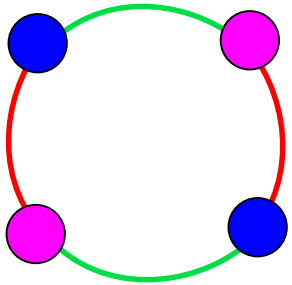
This deformation corresponds to a massless field (b = VEV of this field). By 2D-4D correspondence we conclude that there is a similar state in the bulk.

Last slide

1. **Setup: Yang-Mills in four dimensions**
   1. 4d $\mathcal{N} = 2$ SQCD, gauge group $U(2)$, $N_f = 4$ flavors
   2. Squark VEVs $\to$ Higgs branch $\to$ BPS solitonic vortex
   3. Strong coupling $\to$ Thin string regime
      4 translational + 6 orientational moduli



2. **String theory viewpoint:**
   1. The vortex $\sim$ 10d critical superstring on $\mathbb{R}^4 \times$ *conifold*
   2. LST, without gravity *(began with YM!)*
   3. The only massless hypermultiplet = "baryon"
3. **Field theory viewpoint:**
   1. Vortex world sheet = 2d $\mathbb{WCP}(2,2)$ sigma model
   2. See "instead-of-confinement" in action
   3. Strong coupling $\to$ 4-kink massless composite state
      = stringy "baryon"!



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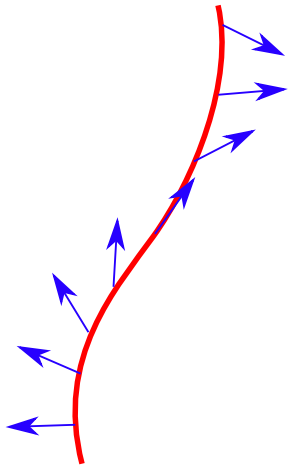

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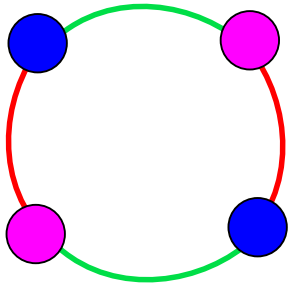

Thank you for your attention!

Acknowledgments

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Talk by E. Ievlev

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