Various corners of QCD and 2 color QCD phase diagrams

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Russian Science Foundation

БАЗИС
Фонд развития теоретической физики и математики

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RSF | Russian Science Foundation

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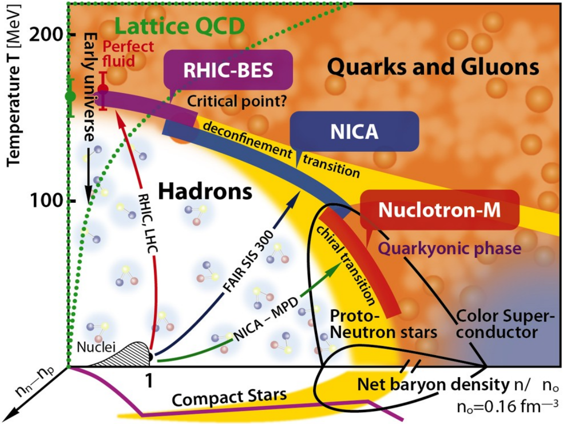
БАЗИС

Фонд развития теоретической физики и математики

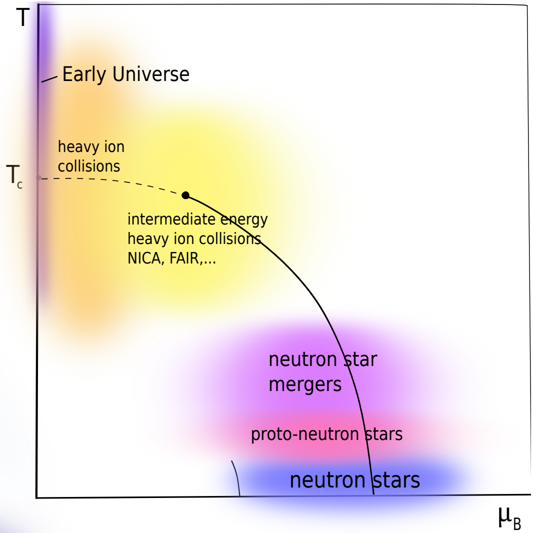
QCD Phase Diagram



QCD Dhase Diagram

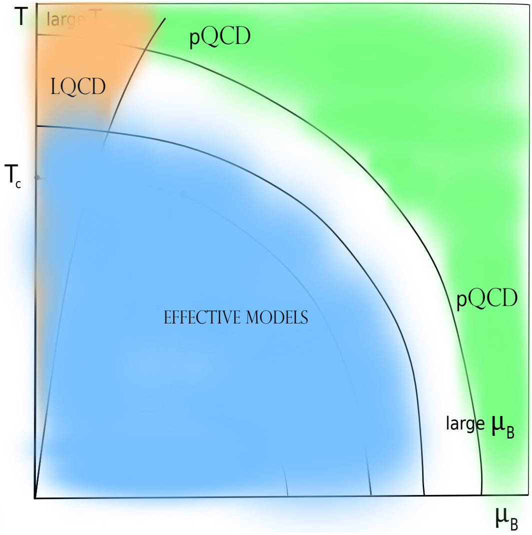
QCD at T and μ
(QCD at extreme conditions)

- Early Universe
- heavy ion collisions
- neutron stars
- proto- neutron stars
- neutron star mergers



Methods of dealing with QCD

- Perturbative QCD
- First principle calculation – lattice QCD
- Effective models
- DSE, FRG
- .....



Effective models

NJL model can be considered as **effective model for QCD**.

the model is **nonrenormalizable**
Valid up to $E < \Lambda \approx 1$ GeV

$$\mu, T < 600\,\mathrm{MeV}$$

Parameters G, Λ, m_0

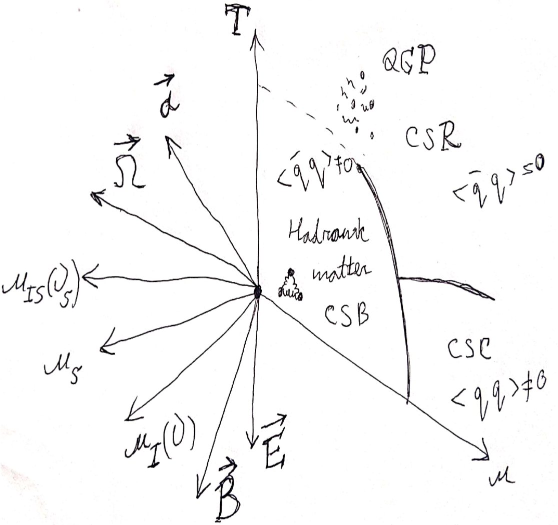
chiral limit $m_0 = 0$

in many cases chiral limit is a very good approximation

dof– **quarks**
no gluons only **four-fermion interaction**
attractive feature — dynamical CSB
the main drawback – lack of confinement (PNJL)

More than just QCD at (μ, T)

- more chemical potentials μ_i
- magnetic fields
- rotation of the system $\vec{\Omega}$
- acceleration $\vec{a}$
- finite size effects (finite volume and boundary conditions)



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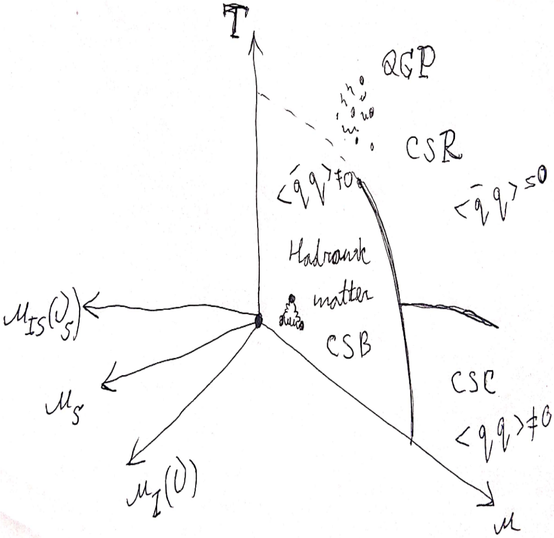

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad n_B = \frac{1}{3}(n_u + n_d)$$

Baryon chemical potential μ_B

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$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad n_B = \frac{1}{3}(n_u + n_d)$$

Isotopic chemical potential μ_I

Allow to consider systems with isospin imbalance ($n_n \neq n_p$).

$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\,(\bar{q}\gamma^0\tau_3 q)$$

$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

chiral (axial) chemical potential

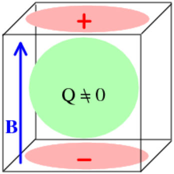
Allow to consider systems with chiral imbalance (difference between densities of left-handed and right-handed quarks).

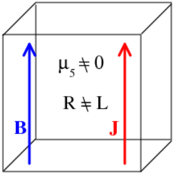
$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

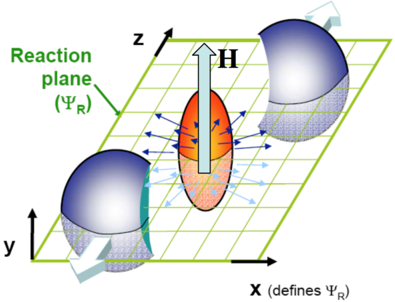
The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

Chiral magnetic effect



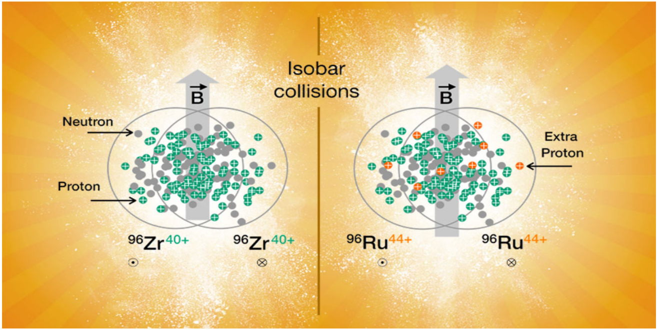




$$\vec{J} \sim \mu_5 \vec{B},$$

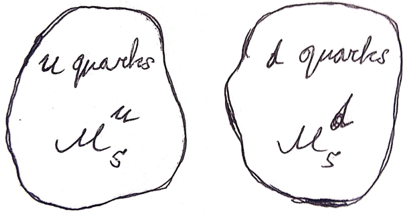
A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033

Chiral magnetic effect



The first blind analysis results isoba run have been recently released by the STAR Collaboration. Under the pre-defined assumption of identical background in RuRu and ZrZr, the results are **inconsistent with the presence of CME**, as well as with all existing theoretical models (whether including CME or not). However **the observed difference of backgrounds** must be taken into account **before any physical conclusion is drawn**.

Chiral isospin imbalance



$$\mu_5^u \neq \mu_5^d \qquad \text{and} \qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$

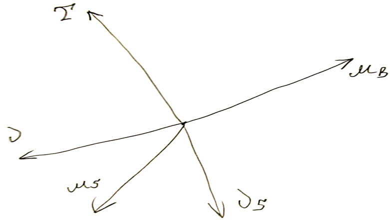
$$n_{I5} = n_{u5} - n_{d5}, \qquad n_{I5} \longleftrightarrow \nu_5$$

- Chiral isospin imbalance and chiral imbalance μ_{I5} and μ_5 can be generated in parallel magnetic and electric fileds $\vec{E} \,||\, \vec{B}$

- Chiral imbalance could appear in dense matter
  - Chiral separation effect
    (Thanks for the idea to Igor Shovkovy)
  - Chiral vortical effect

Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$



- **QC_2D phase diagram** and **diquark condensation** phenomenon

  with different chemical potentials, including μ_5

- **QCD phase diagram** and **color superconductivity phenomenon**

  with **different chemical potentials**

  and matter content including **chiral imbalance**

Recall that in NJL model **without color superconductivity phenomenon** there have been found **dualities**

It is not related to holography or gauge/gravity duality

it is the dualities of the phase structures of different systems

Dualities

Chiral symmetry breaking $\Longleftrightarrow$ pion condensation

Isospin imbalance $\Longleftrightarrow$ Chiral imbalance

The TDP

$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...)$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP (phase daigram) is invariant under

Interchange of - condensates - matter content

$$\Omega(M, \pi, \nu, \nu_5)$$

$$M \longleftrightarrow \pi, \qquad \nu \longleftrightarrow \nu_5$$

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

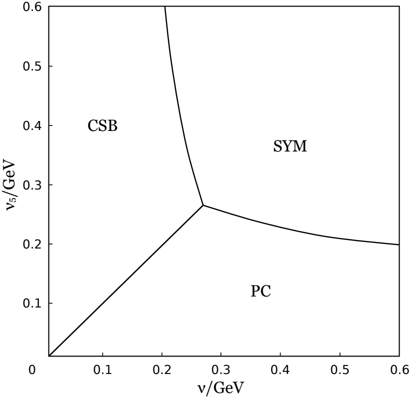


Figure: NJL model results

$$\mathcal{D}: \; M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

Duality between chiral symmetry breaking and pion condensation

$$\mathrm{PC} \longleftrightarrow \mathrm{CSB} \quad \nu \longleftrightarrow \nu_5$$

Two colour QCD case

QC_2D

There are a lot similarities:

- similar phase transitions:

  confinement/deconfinement, chiral symmetry breaking/restoration at large T and μ

- A lot of physical quantities coincide with some accuracy

  Critical temperature, shear viscosity etc.

There is **no sign problem** in SU(2) case

$$(Det(D(\mu)))^{\dagger} = Det(D(\mu))$$

and lattice simulations at non-zero baryon density are possible

It is a great playground for studying dense matter

Phase diagram of QC_2D

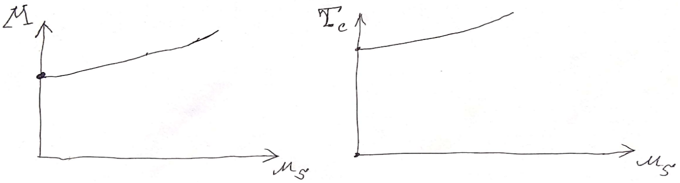
Condensates and phases

$$M = \langle \sigma(x) \rangle \sim \langle \bar{q}q \rangle, \qquad \text{CSB phase:} \quad M \neq 0,$$

$$\pi_1 = \langle \pi_1(x) \rangle = \langle \bar{q}\gamma^5 \tau_1 q \rangle, \qquad \text{PC phase:} \quad \pi_1 \neq 0,$$

$$\Delta = \langle \Delta(x) \rangle = \langle qq \rangle = \langle q^T C \gamma^5 \sigma_2 \tau_2 q \rangle, \qquad \text{BSF phase:} \quad \Delta \neq 0.$$

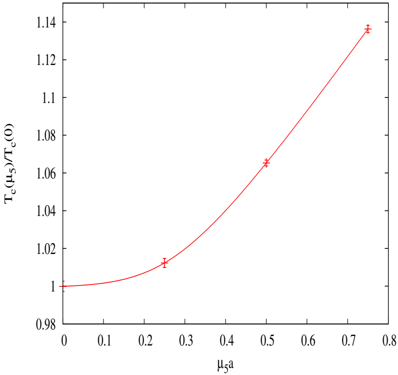
QCD at non-zero μ_5



catalysis of CSB by chiral imbalance:

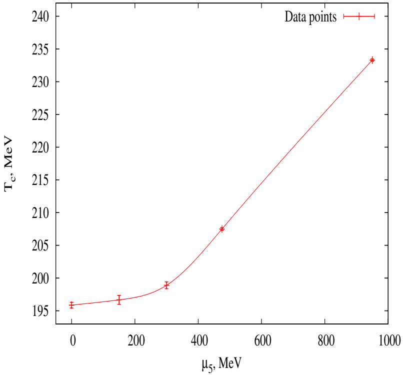
- increase of $\langle\bar{q}q\rangle$ as μ_5 increases
- increase of critical temperature T_c of chiral phase transition (crossover) as μ_5 increases

SU(2)

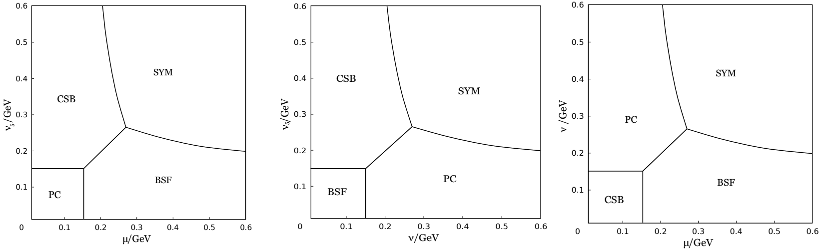


V. Braguta, A. Kotov et al, JHEP 1506, 094 (2015),PoS LATTICE 2014, 235 (2015)

SU(3)



V. Braguta, A. Kotov et al, Phys. Rev. D 93, 034509 (2016), arXiv:1512.05873 [hep-lat]



$$(a) \qquad \mathcal{D}_1 : \quad \mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow |\Delta|, \qquad \text{PC} \longleftrightarrow \text{BSF}$$

J. Andersen, T. Brauner, D. T. Son, M. Stephanov, J. Kogut, ...

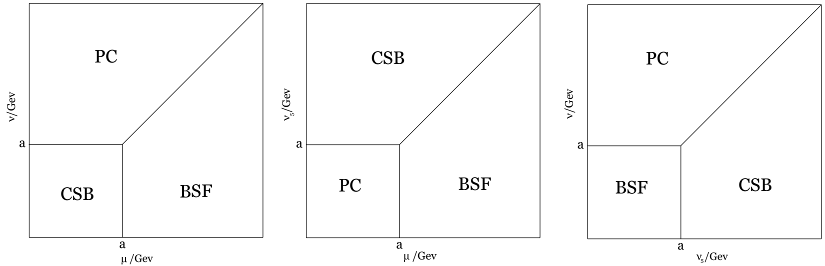
$$(b) \qquad \mathcal{D}_3 : \quad \nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1, \qquad \text{PC} \longleftrightarrow \text{CSB}$$

$$(c) \qquad \mathcal{D}_2 : \quad \mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow |\Delta|, \qquad \text{CSB} \longleftrightarrow \text{BSF}$$

Each chemical potential is connected in one-to-one correspondence with some phenomenon (condensation)

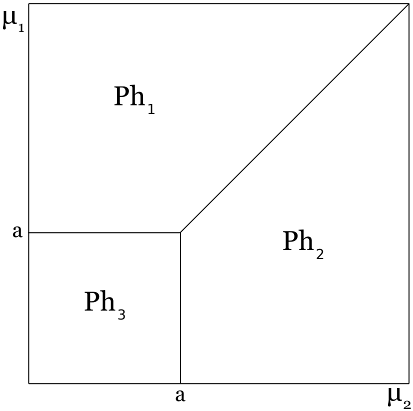
- Baryon density $\mu \iff$ diquark condensation
- Isospin imbalance $\nu \iff$ pion condensation
- Chiral imbalance $\nu_5 \iff$ chiral symmetry breaking

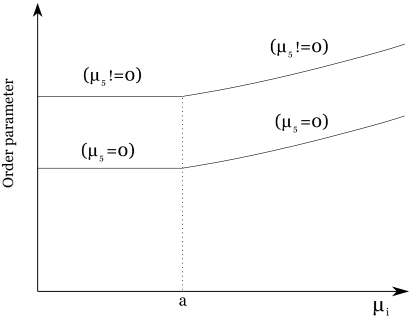
One to one correspondence



$$\mu \longrightarrow \text{BSF}, \qquad \nu \longrightarrow \text{PC}, \qquad \nu_5 \longrightarrow \text{CSB}$$

Universal catalysis effect of chiral imbalance





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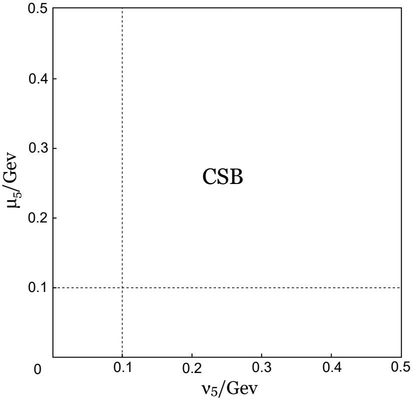

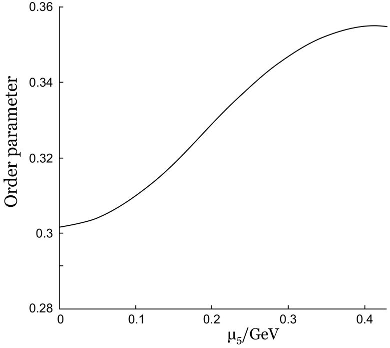

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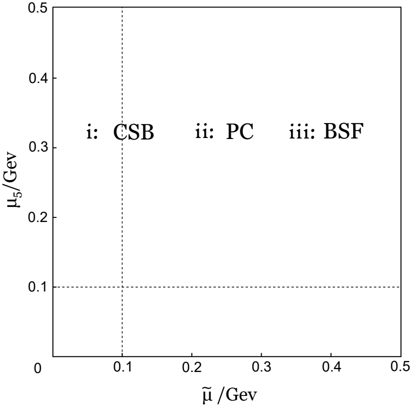
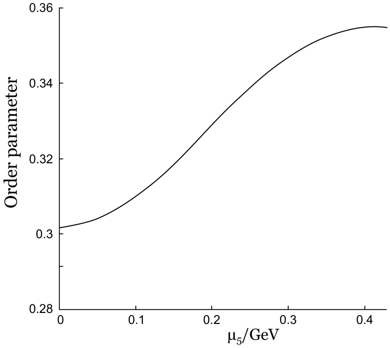
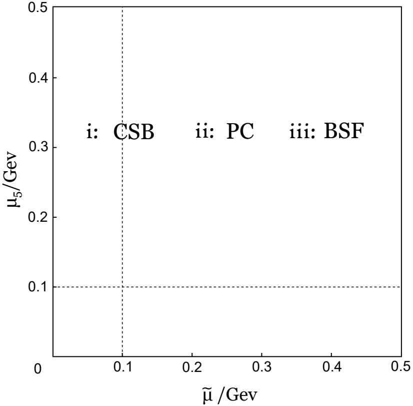
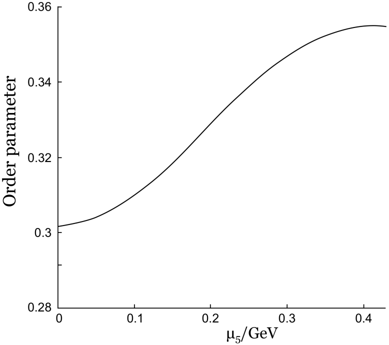




Chameleon property (mimicry) of μ_5



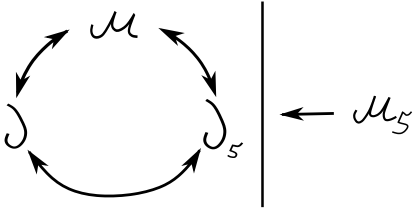


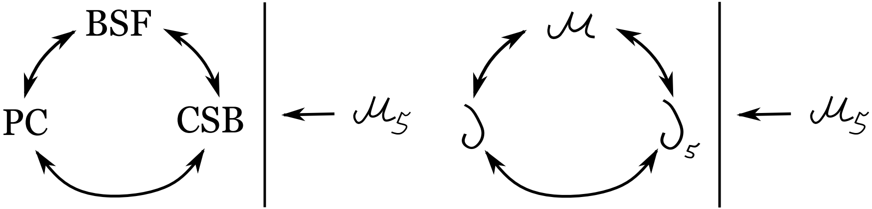
Chameleon nature of chiral imbalance μ_5

μ_5 mimics other chemical potentials μ, ν, ν_5

Universality of chirality

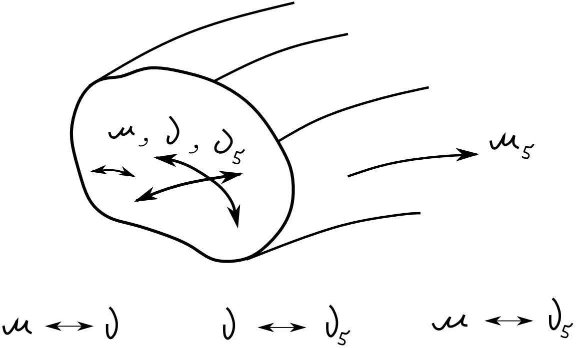
Chiral imbalance μ_5 does not participate in dual transformations

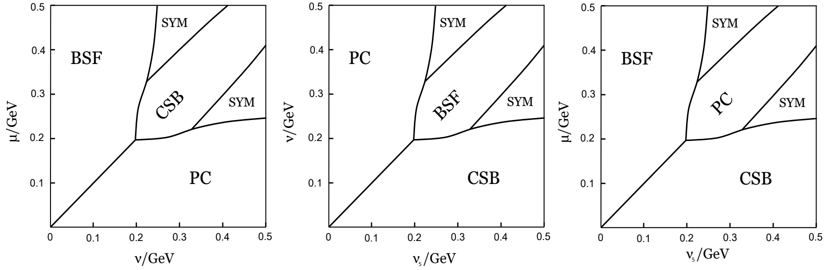




Chiral imbalance μ_5 does not participate in dual transformations

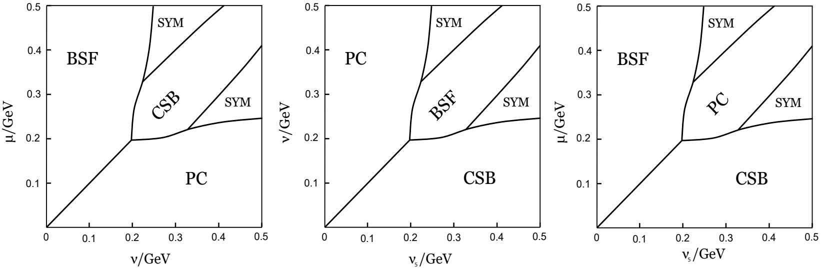
The phase diagram is foliation of dually connected cross-section of (μ, ν, ν_5) along the μ_5 direction





All phase diagrams are dually connected

Phase structure in the large values regime

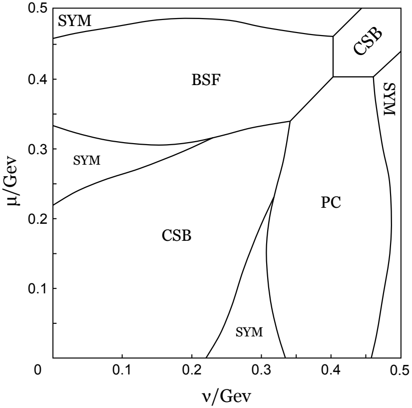


Chiral imbalance μ_5 could universally trigger all the phenomena

Diquark condensation at $\mu = 0$

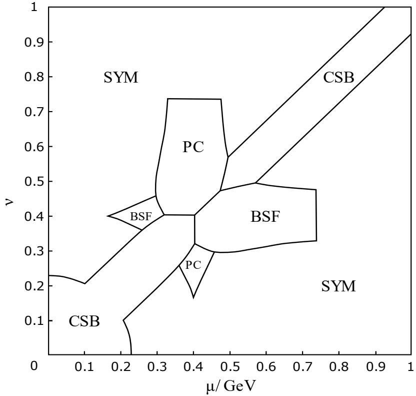
Chiral imbalance μ_5 leads to several rather peculiar phases in the system, e. g. the **diquark condensation** in the region of the phase diagram at $\mu = 0$

It was known that μ_5 leads to pion condensation in dense quark matter with zero $\nu = 0$ in SU(3) case and in SU(2) as well



PC_d phase and diquark condensation

- PC_d phase has been predicted without possibility of diquark condensation
- Diquark condensation can take over the PC_d phase
- In two colour case diquark condensation is in a sense even stronger than in three colour case and starts from $\mu > 0$



PC_d phase is unaffected by BSF phase in two color case. Maybe one can infer that it is the case also for 3 color QCD

Phase diagram of QCD and color superconductivity at finite μ_5

The Lagrangian of three color NJL model

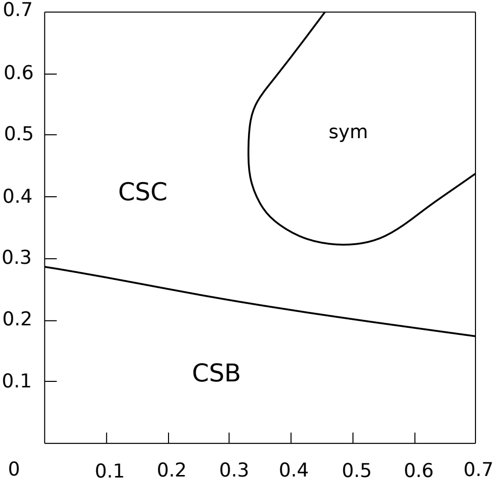
$$L = \bar{q}\Big[\gamma^{\nu} i\partial_{\nu} - m\Big]q + G\Big[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2\Big] +$$

$$+H \sum_{A=2,5,7} [\overline{q^c} i\gamma^5\tau_2\lambda_A q][\bar{q}i\gamma^5\tau_2\lambda_A q^c]$$

$$m_\pi,\ f_\pi,\ \langle\overline{q}q\rangle \longrightarrow \text{quark-antiquark coupling } G$$

H is not precisely determined

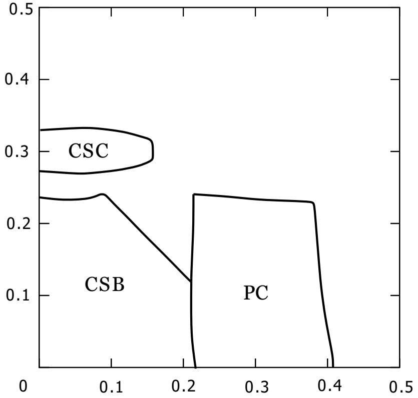
If the quark-antiquark interaction has been constrained empirically, the most natural solution is to determine the quark-quark coupling constants empirically, too. Unfortunately, the analog to the meson spectrum would be a diquark spectrum, which of course does not exist in nature

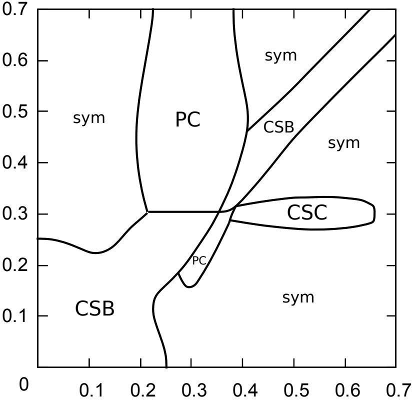


Chiral imbalance μ_5 facilitates the generation of color superconductivity

Diquark condensation at $\mu = 0$ in SU(3)

Chiral imbalance μ_5 leads to the **diquark condensation** in the region of the phase diagram at $\mu = 0$ in three color case

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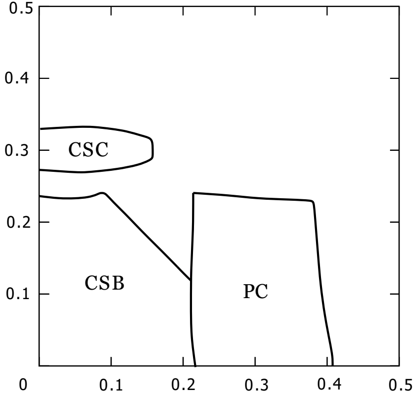
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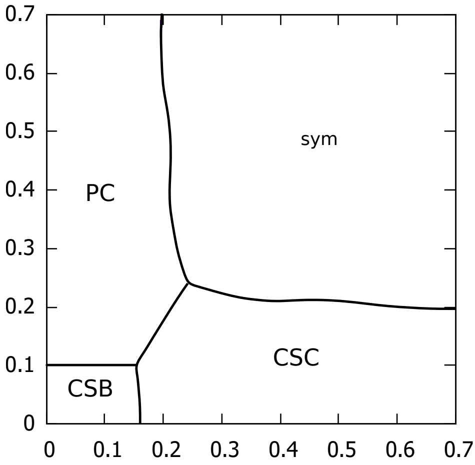
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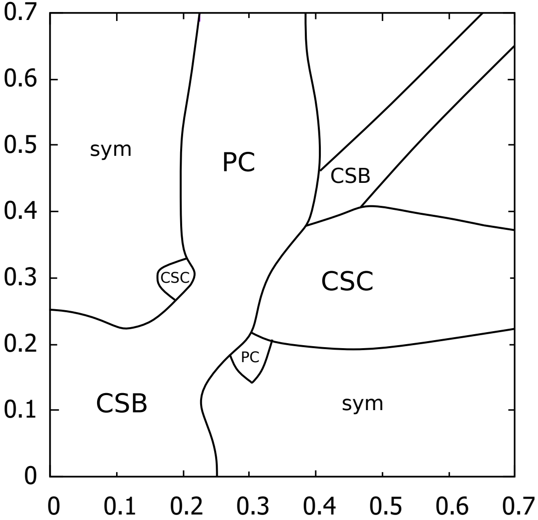
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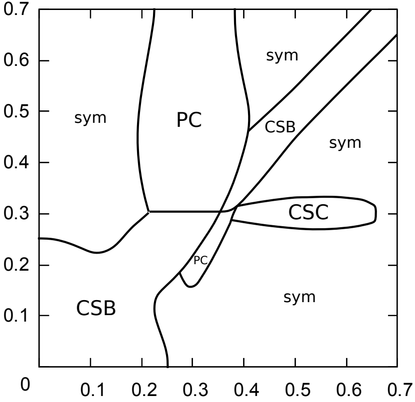

PC_d phase is unaffected by BSF phase in three color case.

Dualities $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$ were found in

- In the framework of NJL model

- In the mean field approximation

Dualities are connected with Pauli-Gursey group

Dualities were found in

- In the framework of NJL model beyond mean field

- In QC_2D non-pertubartively (at the level of Lagrangian)

Duality $\mathcal{D}$ is a remnant of chiral symmetry

Duality was found in

- In the framework of NJL model beyond mean field or at all orders of N_c approximation
- In QCD non-pertubartively (at the level of Lagrangian)

- $(\mu_B, \mu_I, \nu_5, \mu_5)$ phase diagram was studied in two color color case
- It was shown that there exist dualities in QCD and QC_2D

  Richer structure of Dualities in the two colour case
- There have been shown ideas how dualities can be used

  Duality is not just entertaining mathematical property but an instrument with very high predictivity power
- Dualities have been shown non-perturbetively in the two colour case
- Duality has been shown non-perturbarively in QCD