Monopoles, instantons, and eta-prime meson in external magnetic fields

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Introduction

- I have studied magnetic monopoles, instantons, and chiral symmetry breaking with A. Di Giacomo since 2012.
- We add monopoles and anti-monopoles by applying a monopole creation operator [PRD 85 065001 (2012)].
- We have demonstrated the relations among monopoles, color confinement, instantons, and chiral symmetry breaking [ArXiv: 2203.11357].
- **In this study, the Pisa group generates the gauge field configurations with dynamical fermions applying uniform magnetic fields.**
- **We calculate eigenvalues and eigenvectors of overlap fermions and estimate hadron spectroscopy.**

Purpose and goal

The purpose is to demonstrate the effects of the strong magnetic fields on monopoles, instantons, and eta-prime meson mass.

The final goal is to show impacts of the strong magnetic fields on the color confinement and chiral symmetry breaking from the magnetic monopoles and instantons.

(1) The monopole loops, monopole density, and Polyakov loops.

(2) Fermion zero modes, topological charges, and the number of instantons and anti-instantons.

(3) Estimation of eta-prime meson mass.

Simulation parameters

- The configurations with Nf = 2 + 1 dynamical fermions in SU(3) are generated by the Pisa group [PRD 95, 074515 (2017)].
- The staggered fermion action and the Symantic tree-level improved gauge action are used.
- The uniform magnetic fields apply along the Z direction (Bz).
- The intensity of the uniform magnetic fields varies from **|e|B = 0.57** (Bz = 3) and **1.14** (Bz = 6) [GeV^2].
- The temperatures also vary from **50** to **200** [MeV].

V	T [MeV]	a [fm]	Bz	Nconf
$8^3 \times 16$	50	0.2457	Normal conf, 3, 6	60
$8^3 \times 4$	200	0.2457	Normal conf, 3, 6	60

Monopoles in external magnetic fields

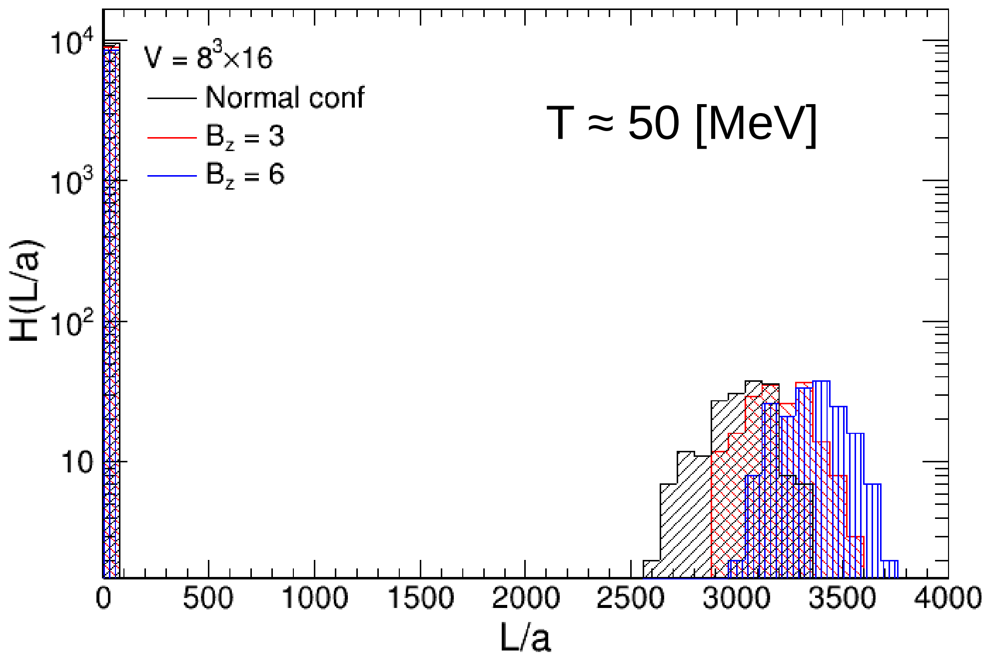
- We diagonalize the SU(3) matrix under the condition of the **maximal Abelian gauge**.
- We compute the density $\boldsymbol{\rho_m}$ of the monopole current $\boldsymbol{k_\mu}$ which satisfies the current conservation law $\nabla^*_\mu k^i_\mu(^*n) = 0$ [PRD 22 (1980) 2478].
- Monopole currents form closed loops.
- The definitions of the monopole current, density, and length:

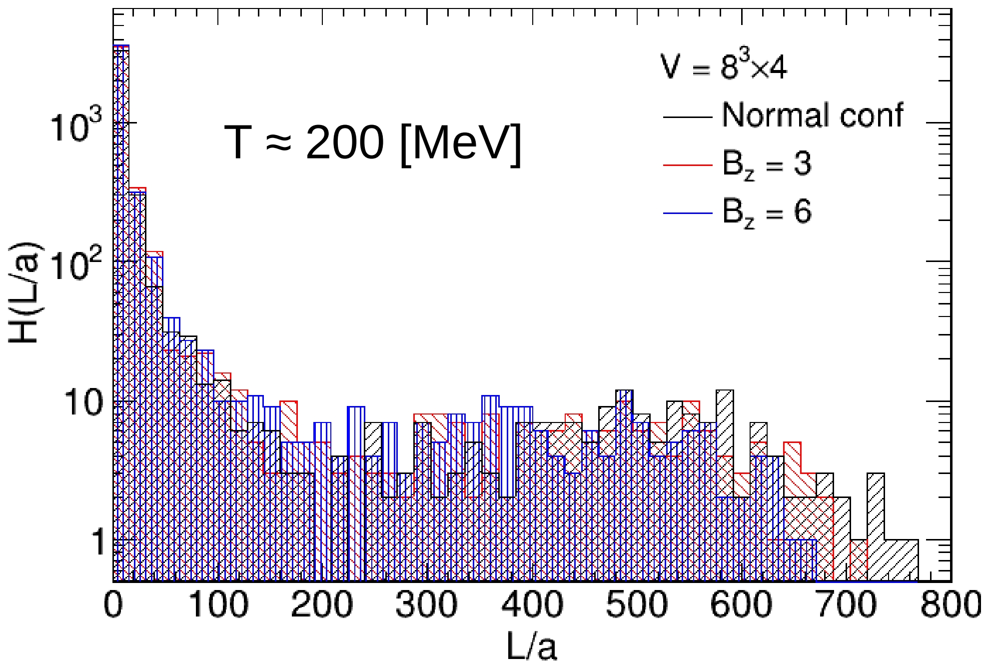
$$k^i_\mu(^*n) \equiv -\epsilon_{\mu\nu\rho\sigma}\nabla_\nu n^i_{\rho\sigma}(n+\hat{\mu}),$$

$$\rho_m = \frac{1}{12V}\sum_{i,\mu}\sum_{^*n}|k^i_\mu(^*n)|/a^3 \ [\mathrm{GeV}^3], \ L_m \equiv \frac{a}{12}\sum_{i,\mu}\sum_{^*n\in C}|k^i_\mu(^*n)| \ [\mathrm{fm}].$$

Monopoles in external magnetic fields

The histograms of monopole loops which satisfies the current conservation law [NPB PS 34 (1994) 549].





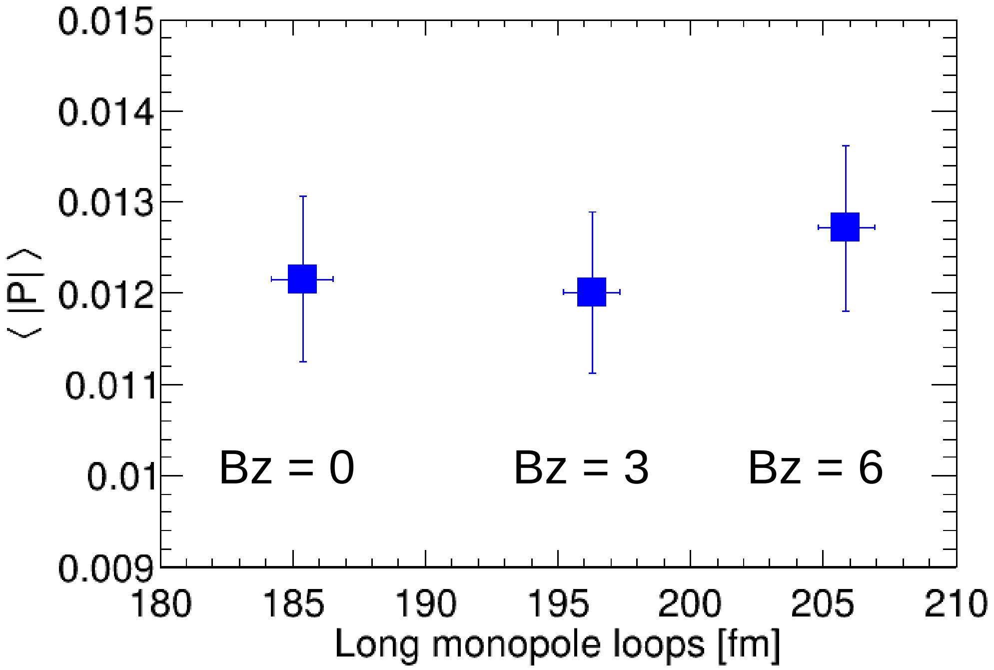
Monopoles and color confinement

We find as follows:

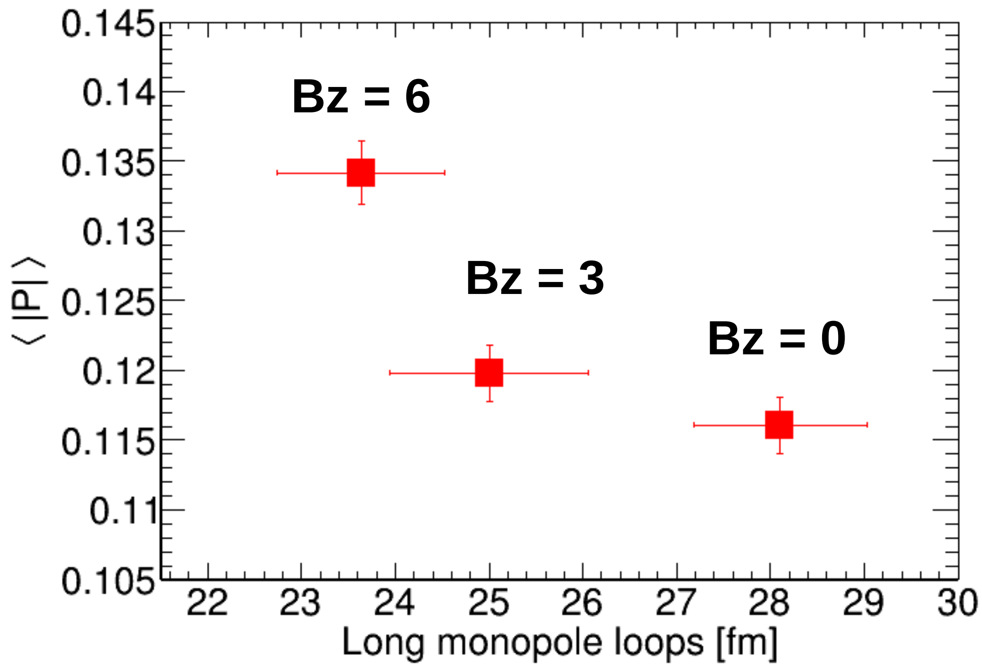
- **In low temperatures**, the long monopole loops become **longer** when the intensity of magnetic fields becomes strong.
- **In finite temperatures**, the long monopole loops become **shorter** when the intensity of magnetic fields becomes strong.
- Comparisons the average values of Polyakov loops with the length of monopole loops.

Monopoles and color confinement

- At low temperatures:



- At finite temperatures:



Similar result [ArXiv: 2203.11357]

Overlap Dirac operator

- Chiral symmetry: $\mathcal{L} = \bar{\psi} D \psi$

$$\gamma_5 D + D\gamma_5 = 0$$

- The Dirac operator ***D*** which preserves the exact chiral symmetry in the lattice gauge theory satisfies the following Ginsparg-Wilson relation [P. H. Ginsparg and K. G. Wilson PRD 25 (1982) 2649].

$$\gamma_5 \mathbf{D} + \mathbf{D}\gamma_5 = \mathbf{aDR}\gamma_5 \mathbf{D}$$

- The following **overlap Dirac operator *D*** satisfies the Ginsparg-Wilson relation [H. Neuberger, PLB 427 (1998) 353]:

$$D(\rho) = \frac{\rho}{a}\left[1 + \frac{D_W(\rho)}{\sqrt{D_W^\dagger(\rho) D_W(\rho)}}\right]$$

$$= \frac{\rho}{a}\{1 + \gamma_5 \epsilon(H_W(\rho))\}$$

- The sign function **ϵ** is approximated by the Chebyshev polynomials [Com. Phys. Comm. 153 (2003) 31].
- We solve the eigenvalue problems using by ARPACK.

$$\mathbf{D}(\rho)|\psi_\mathbf{i}\rangle = \lambda_\mathbf{i}|\psi_\mathbf{i}\rangle$$

- We compute the pairs of eigenvalues λ_i and eigenvectors $|\psi_\mathbf{i}\rangle$ from the lowest energy level to approximately 400.

D: Massless overlap Dirac operater.

ϵ: Sign function.

D_w: Massless Wilson Dirac operator.

ρ: Mass parameter **ρ = 1.4.**

H_W: Hermitian Wilson Dirac operator.

Zero modes and instantons

- **Fermion zero modes in the eigenvalues.**

  The number of **zero modes** of the **positive chirality is** n_+.

  The number of **zero modes** of the **negative chirality is** n_-.

- The exact zero modes $\boldsymbol{\lambda}_{Zero}$ are $|\lambda_{\mathbf{Zero}}| \leq \mathbf{O}(\mathbf{10}^{-8})$.
- We suppose that the Atiyah-Singer index theorem.

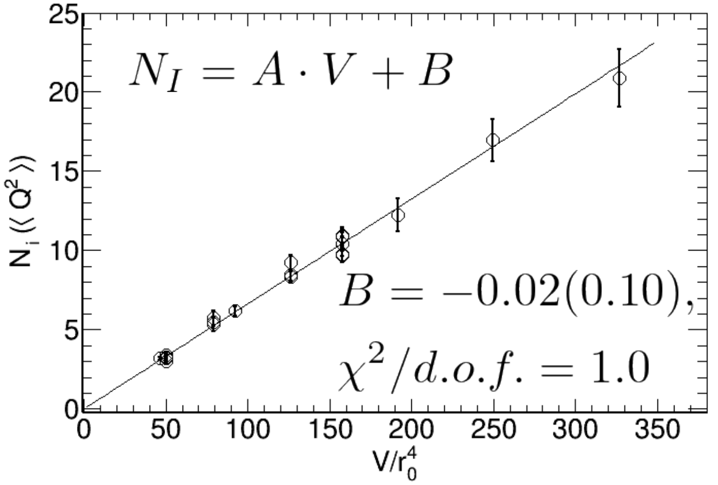
  The number of **instantons** of the **positive charge is** n_+.

  The number of **anti-instantons** of the **negative charge is** n_-.

- We have shown that the number of instantons and anti-instantons N_I can be calculated from the average square of the topological charges Q^2 [PRD 91 (2015) 054512]:

$$\mathbf{N_I = \langle Q^2 \rangle, \ \ Q = n_+ - n_-}$$

- **Topological susceptibility is** $\chi = \dfrac{\langle Q^2 \rangle}{V}$.



- **Instanton (anti-instanton) density**

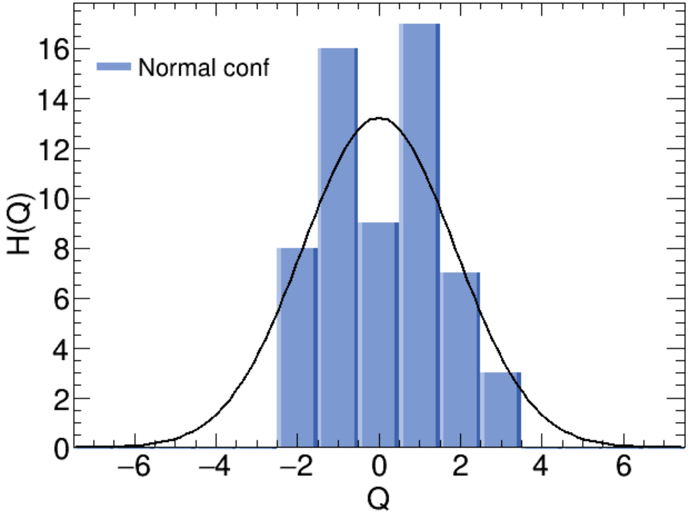
$$\frac{Ar_0^4}{2} = \frac{N_I r_0^4}{2V}$$
$$= \mathbf{8.09(17)} \times \mathbf{10^{-4}}\ [\mathrm{GeV}^4]$$

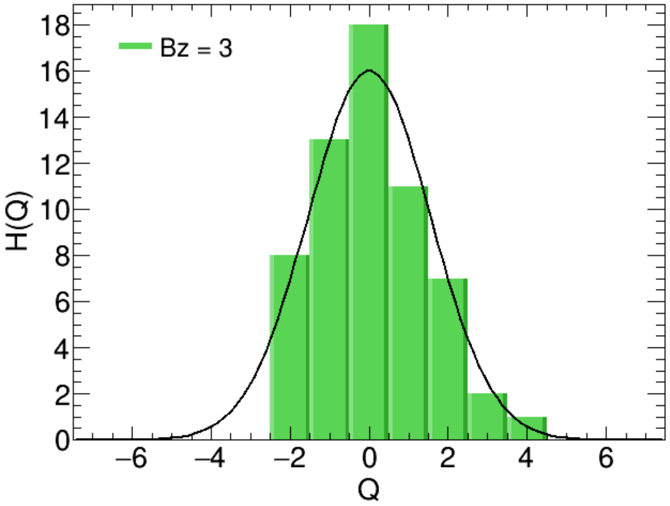
- **E. V. Shuryak [NPB 203 (1982) 93]:**

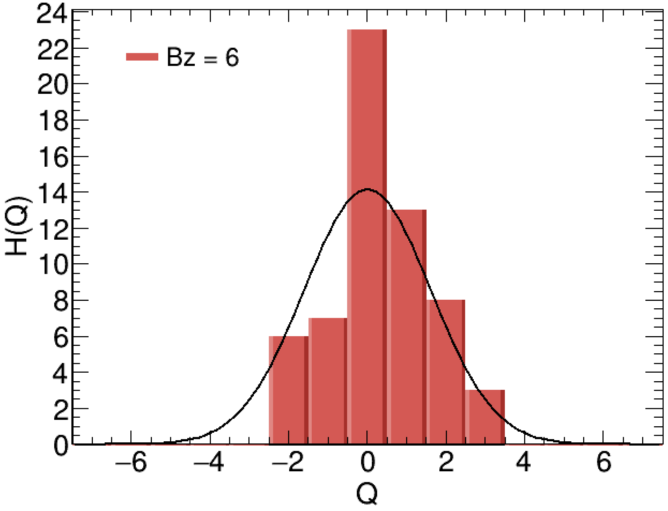
$$\mathrm{n}_c = \mathbf{8} \times \mathbf{10^{-4}}\ [\mathrm{GeV}^4]$$

Topological charges and instantons

The impact of the magnetic fields on the distributions of the topological charges and number of instantons and anti-instantons.







N_I = 3.5 (1.2), X^2/ndf = 5/4.

N_I = 2.4 (6), X^2/ndf = 1/5.

N_I = 2.6 (1.0), X^2/ndf = 7/4.

The number of instantons and anti-instantons

- **The impact of the number of instantons and anti-instantons.**

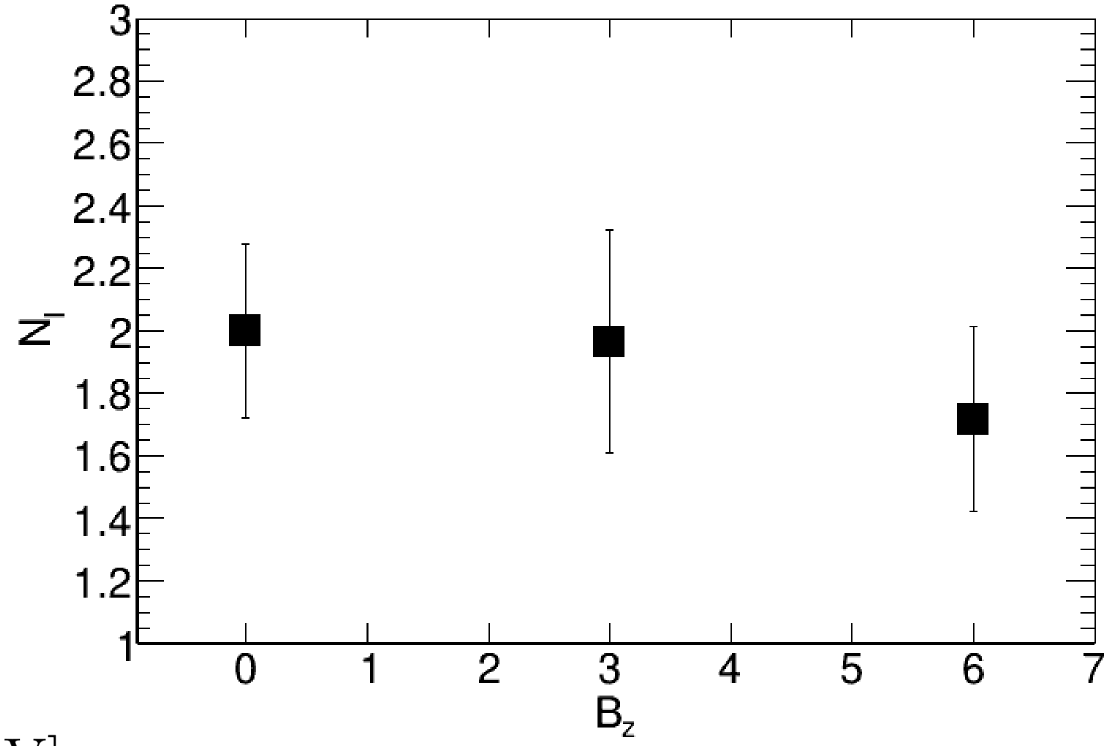
$$N_I = \langle Q^2 \rangle$$

- **Topological susceptibility is**

$$\frac{\langle Q^2 \rangle}{V} = (100(4))^4 [\mathrm{GeV}].$$

- **Quenched QCD**

$$\frac{\langle Q^2 \rangle}{V} = (196(2))^4 [\mathrm{GeV}]$$



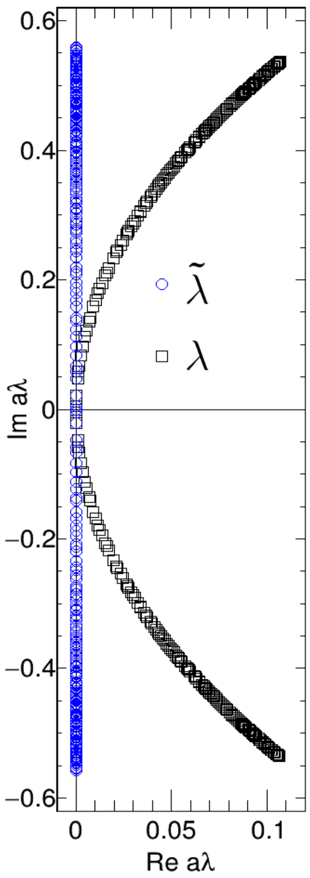
Comparisons with GRMT

To inspect the effects, we compare the eigenvalues with RMT.

- The Gaussian random matrix theory predicts the fluctuations of the eigenvalues of the Dirac operators **universally** [J. Math. Phys. 4 (1963) 701, Phys. Rept. 299 (1998) 189].

Unfolding [PRD 59 (1999) 054501]

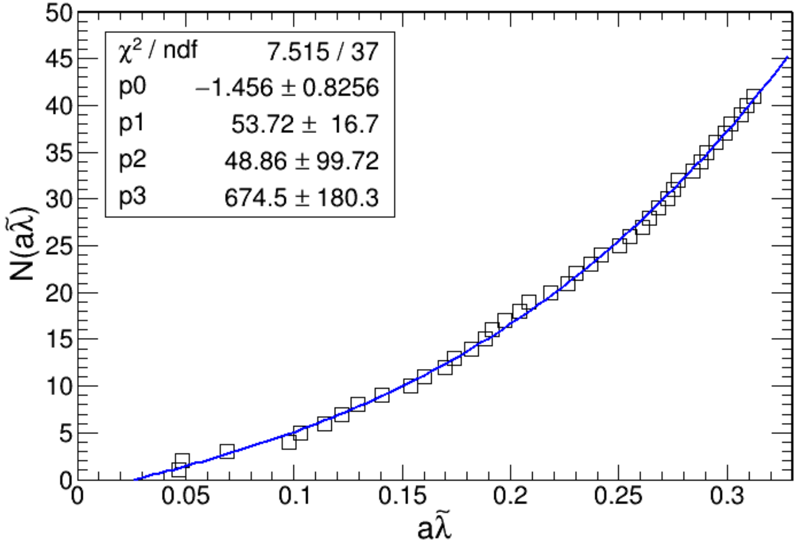
- We first calculate the improved eigenvalues $\tilde{\lambda}$ [PLB 468 (1999) 150].
- Putting the improved nonzero and positive eigenvalues in ascending order.
- Fitting the polynomial function.



- **Unfolding** [PRD 59 (1999) 054501]

Polynomial function:

$$N_{pol}(\tilde{\lambda}^n) = \sum_{d=0}^{4} a_d^n \tilde{\lambda}_d^n$$



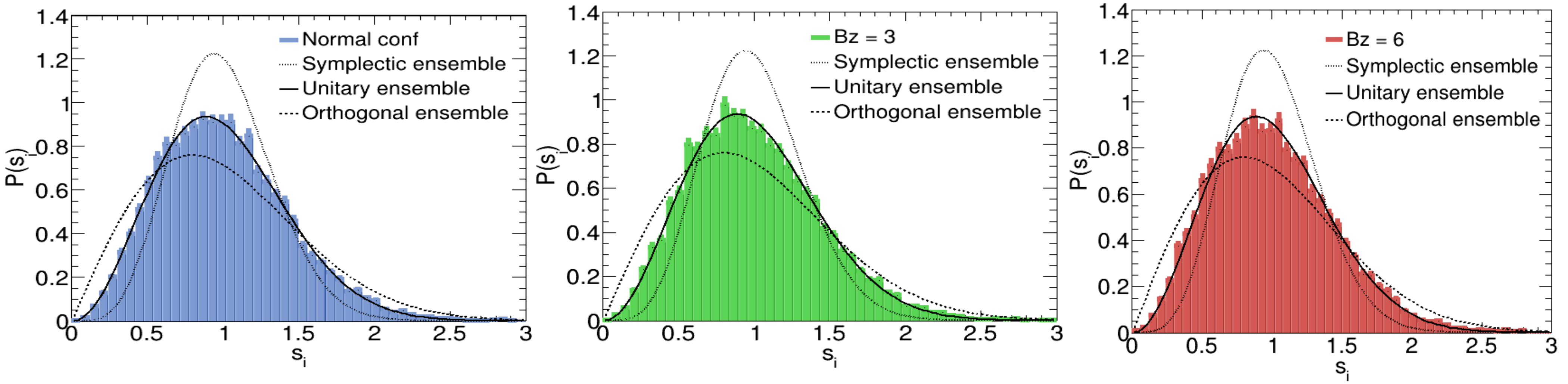
- **Unfolded eigenvalues ξ are**

$$\xi_i^n = N_{pol}(\tilde{\lambda}_i^n).$$

Comparisons with GRMT

- To inspect the effects on the short-range fluctuations of the eigenvalues, we compute the **nearest-neighbor spacing *s*,** make distributions, and compare them with the GRMT.

$$\xi_i^n = N_{pol}(a\tilde{\lambda}_i^n),\ \mathbf{s_i^n} = \xi_{\mathbf{i+1}}^{\mathbf{n}} - \xi_{\mathbf{i}}^{\mathbf{n}}$$



Predictions in GRMT [PR 299 (1998) 189].

Operators and correlation functions

- **Quark propagator:**

$$G(\vec{y},y^0;\vec{x},x^0) \equiv \sum_i \frac{\psi_i(\vec{x},x^0)\psi_i^\dagger(\vec{y},y^0)}{\lambda_i^{mass}}$$

- λ_i^{mass} **of massive Dirac operator:**

$$\lambda_i^{mass} = \left(1 - \frac{a\bar{m}_q}{2\rho}\right)\lambda_i + \bar{m}_q$$

- **Pseudoscalar:**

$$\mathcal{O}_{PS} = \bar{\psi}_1\gamma_5\left(1 - \frac{a}{2\rho}D\right)\psi_2$$

- **Connected pseudoscalar density:**

$$C_{PS}(\Delta t) =$$

$$\frac{a^3}{V}\sum_{\vec{x}_1}\sum_{\vec{x}_2,\ t}\langle \mathcal{O}_{PS}^C(\vec{x}_2,t)\mathcal{O}_{PS}(\vec{x}_1,t+\Delta t)\rangle$$

- **Dis-connected pseudoscalar density:**

$$C_{Dis-PS}(\Delta t) =$$

$$\frac{a^3}{V}\sum_t\langle\sum_{\vec{x}_2}\mathcal{O}_{PS}^C(\vec{x}_2,t)\sum_{\vec{x}_1,\ t}\mathcal{O}_{PS}(\vec{x}_1,t+\Delta t)\rangle$$

Operators and correlation functions

- **Fitting function for connected pseudoscalar density:**

$$C_{PS}(t) = \frac{a^4 \mathbf{Z_{PS}}}{am_{PS}} \exp\left(-\frac{m_{PS}}{2}T\right) \cosh\left[m_{PS}\left(\frac{T}{2} - t\right)\right].$$

- **Fitting function for dis-connected pseudoscalar density:**

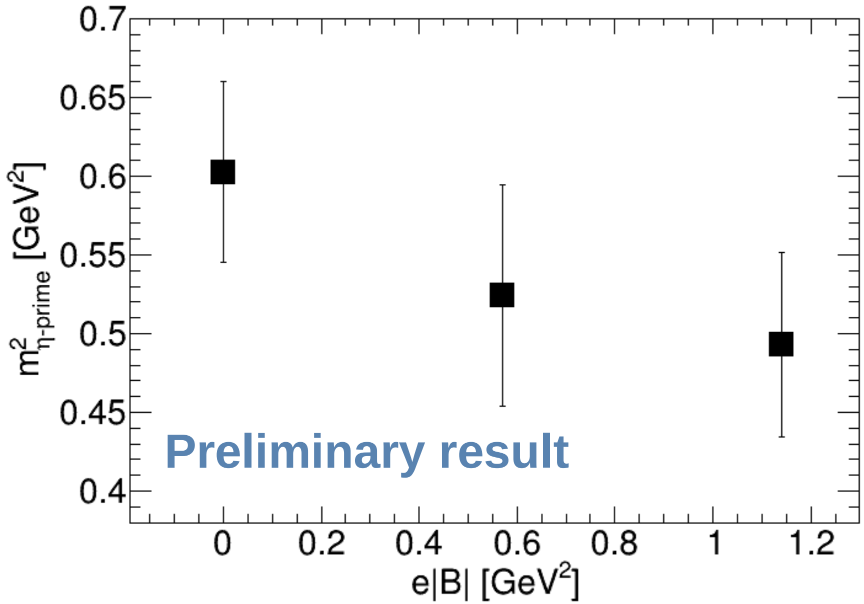
$$C_{dis-PS}(t) =$$

$$\frac{\mathbf{Z_{PS}}}{4m_{PS}} \frac{\boldsymbol{\mu_0^2}}{N_f} \left[(1 + m_{PS}t) \exp(-m_{PS}t) + \{1 + m_{PS}(T-t)\} \exp\{-m_{PS}(T-t)\}\right]$$

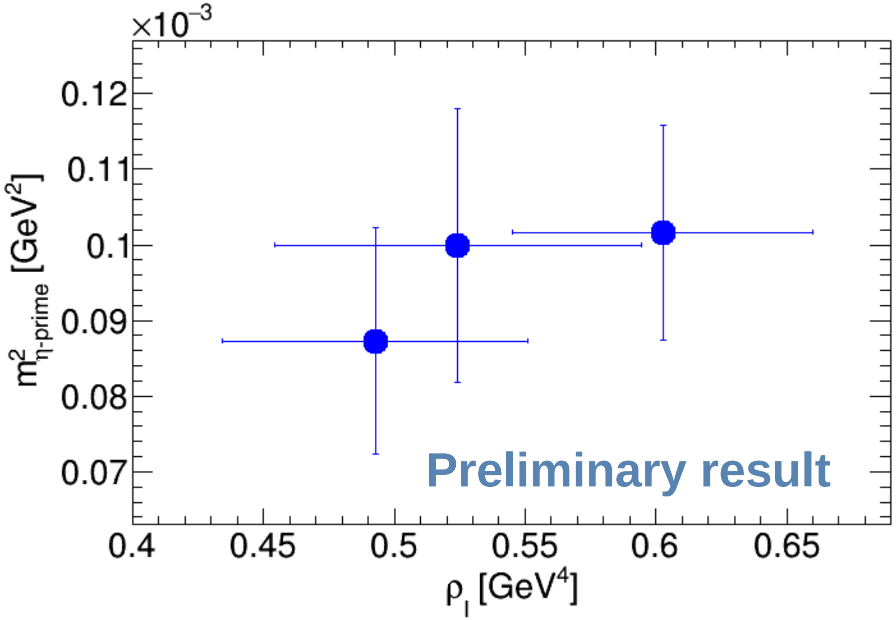
Ref [PRD 65 114501 (2002)]

Eta-prime meson in external magnetic fields

- Eta-prime mass (chiral limit) vs. magnetic fields



- Eta-prime mass and the number density of instantons and anti-instantons



Summary and Conclusions

We investigated the impacts of the magnetic fields (MF) on monopoles, instantons, and eta-prime meson mass.

- We observed the effects of MF on the long monopole loops at low and finite temperatures.
- The effects of MF on the number of instantons and anti-instantons are small.
- The distribution of topological charges are slightly affected by MF.
- MF do not affect the distribution of the nearest-neighbor spacing.
- The eta-prime mass slightly decreases with increasing the intensity of MF.

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- I really appreciate your providing the computer resources for our research.