Monopole and monopoleless components of the lattice gauge field in the maximal Abelian gauge

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The work is completed in collaboration with

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Roman Rogalyov

Publication:

Decomposition of the SU(2) gauge field in the maximal Abelian gauge.
V.G. Bornyakov, I. Kudrov, R.N. Rogalyov.   Phys.Rev.D 105 (2022) 5, 054519.

OUTLINE

- Motivation
- DS scenario of confinement and Maximal Abelian gauge
- Decomposition of the static potential in SU(2) gluodynamics, QC_2D, SU(3) gluodynamics
- Other observables
- Conclusions and perspectives

Dual superconductor scenario - one of the most popular ideas about nature of confinement t' Hooft '75, Mandelstam '76

A dual superconductor is a superconductor in which the roles of the electric and magnetic fields are exchanged.
Formation of the Abrikosov-Nilsen-Olesen string in a usual superconductor due to condensation of electric charges is dual to formation of the flux tube in QCD due to condensation of color-magnetic monopoles
Superconductor is described by Landau - Ginzburg model (Abelian Higgs model)
Dual superconductor – by dual Abelian Higgs model

It is yet unsolved task to rigorously prove that infrared QCD is dual to Abelian Higgs model

Abelian dominance hypothesis

Ezawa, Iwazaki '82

Physical observables, related to the infrared properties of the theory, can be computed with the help of the Abelian variables i.e.

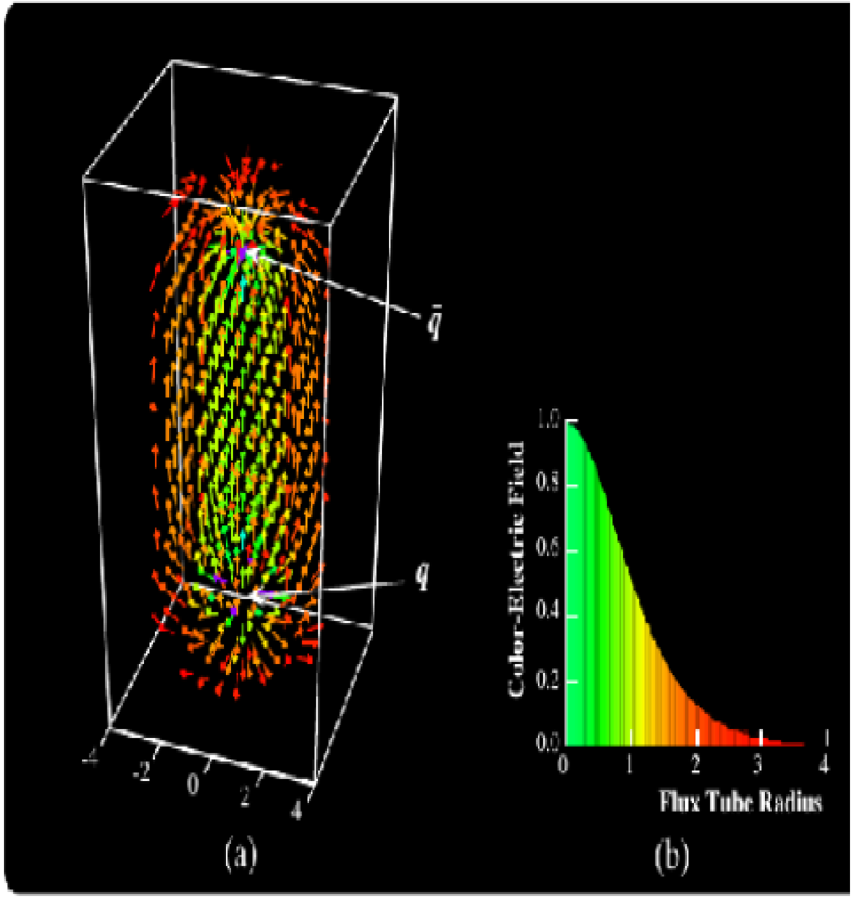
$$<\mathcal{O}>=\frac{1}{Z}\int e^{-S}\mathcal{O}(U_\mu)\mathcal{D}U_\mu$$

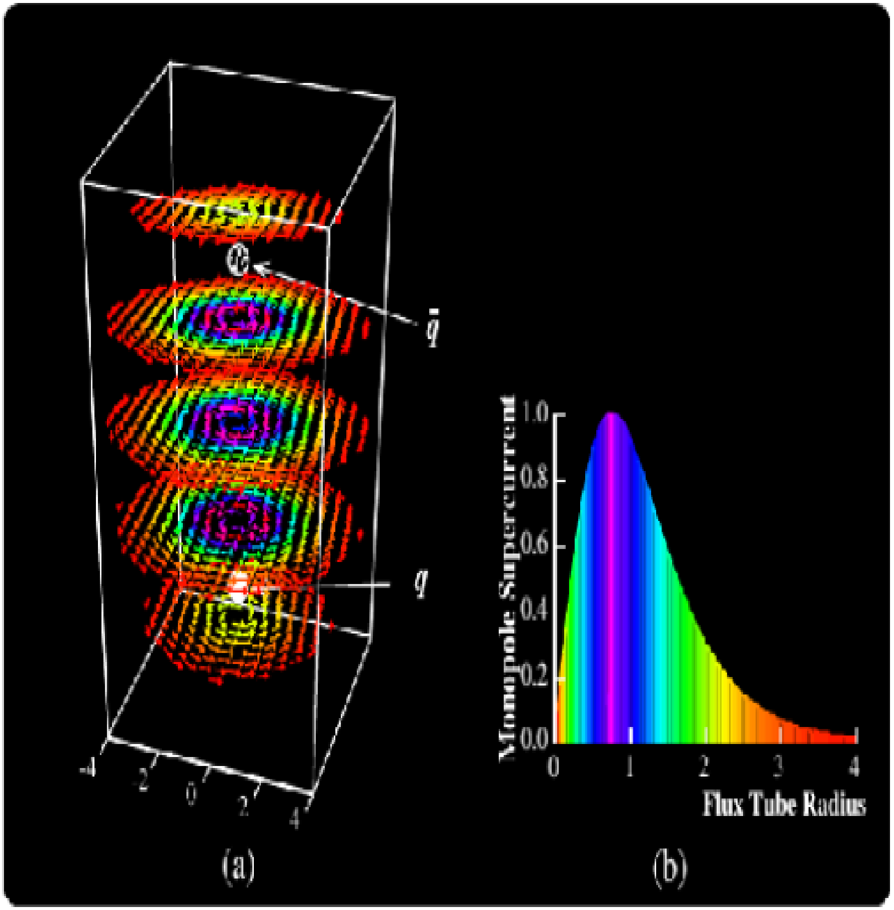
and

$$<\mathcal{O}>^{Ab}=\frac{1}{Z}\int e^{-S}\mathcal{O}(u_\mu)\mathcal{D}U_\mu$$

give approximately equal values of the infrared physical quantities.

Example: $O = W(r,t)$; static potential is derived from the Wilson loop: $V(r) = \alpha/r + \sigma r$.
Abelian projection gives very good approximation for σ but not for α
Suzuki and Yotsuyanagi, 1990





profile of the color-electric field(left) and profile of the magnetic currents (right) in DLG .

Koma, 2001

Maximal Abelian gauge

('t Hooft, 1981)

MA gauge condition

$$\Big(\partial_\mu \delta_{kl} + \epsilon_{k3l} A^3_\mu(x)\Big) A^l_\mu(x) = 0\,, \quad k = 1,2$$

solutions: extremums over gauge transformations of the functional

$$F[A] = \int d^4x \;\{(A^1_\mu)^2 + (A^2_\mu)^2\}$$

Abelian projection:

$$A^a_\mu T^a \to A^3_\mu T^3$$ (in observables)

Lattice formulation - by Kronfeld, Laursen, Schierholz, Wiese,1989

Bonati, D'Elia and Di Giacomo, 2010

It was argued that MAG is a proper Abelian gauge to find gauge invariant monopoles since monopoles can be identified in this gauge by the Abelian flux, but this is not possible in other Abelian gauges.

In other words, the efficiency of the method to detect monopoles (DeGrand-Toussaint) depends on the choice of the gauge.

It was demonstrated for a class of gauges which interpolate between the Maximal Abelian gauge and the Landau gauge, how monopoles gradually escape detection.

One can decompose the Abelian vector potential into monopole and photon parts

$$A_\mu^{mon}(x) = 2\pi \sum_{y,\nu} D(x-y)\partial_\nu m_{\mu\nu}(x)$$

$$A_\mu^{phot}(x) = A_\mu(x) - A_\mu^{mon}(x)$$

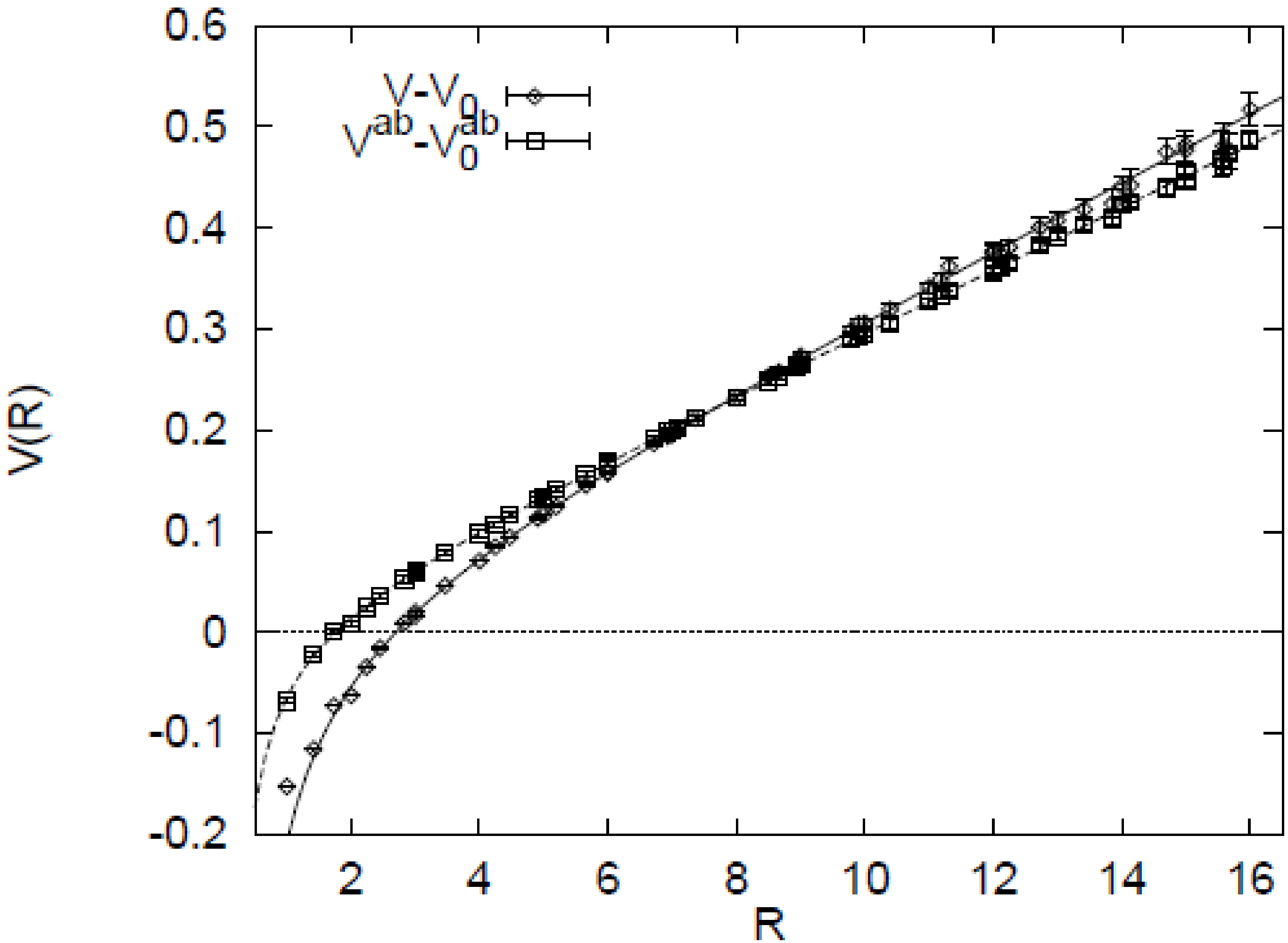
$$u_\mu^{mon}(x) = exp(iA_\mu^{mon}(x))$$

$$u_\mu^{ph}(x) = exp(iA_\mu^{ph}(x))$$

$$U_\mu^{mod}(x) = U_\mu(x) u_\mu^{mon,\dagger}(x)$$

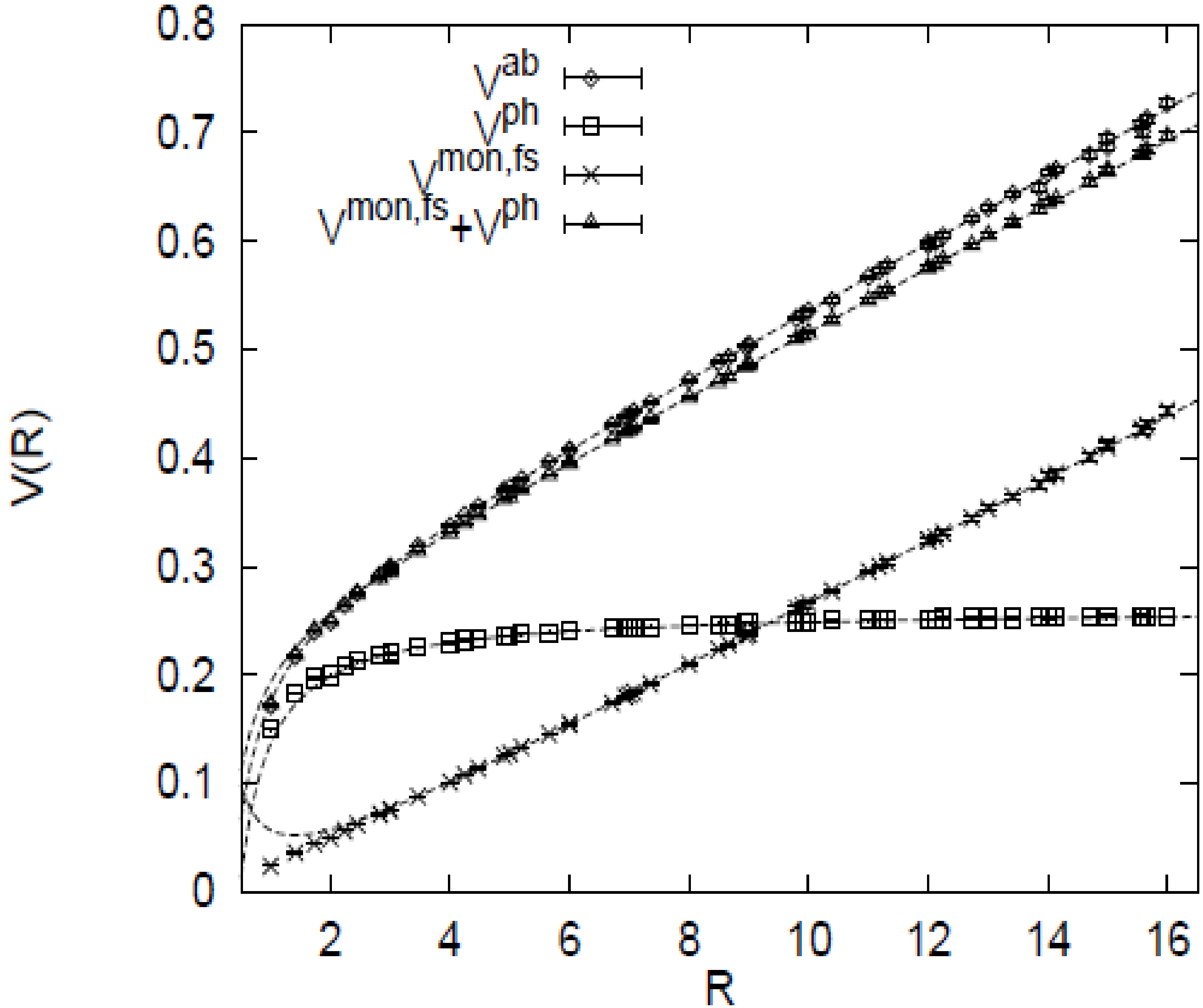
U_μ^{mod} - nonabelian gauge field with monopoles removed (modified)

Abelian dominance (first results by Suzuki and Yotsuyanagi, 1990)



Abelian and nonabelian static potentials. Bali, VB, Mueller-Preussker, Schilling, 1996

Monopole dominance



Abelian static potential in comparison with 'monopole' and 'photon' static potentials

Results in $SU(2)$:

$$\sigma^{ab}/\sigma = 0.92(4)$$

$$\sigma^{mon}/\sigma^{ab} = 0.95(2)$$

$$\sigma^{ab,2}/\sigma^{ab} = 2.23(5)$$

(it is 8/3 in SU(2)

σ^{ab}/σ was computed in the limit of infinite cutoff
σ^{ab}/σ was computed for improved lattice action and universality of the Abelian dominance had been demonstrated
VB, Ilgenfritz, Mueller-Preussker, 2005

Recent results for SU(3) gluodynamics from Hideo Suganuma and co-authors:

'Perfect Abelian dominance for the string tension'

Phys.Rev.D 102 (2020), 014512

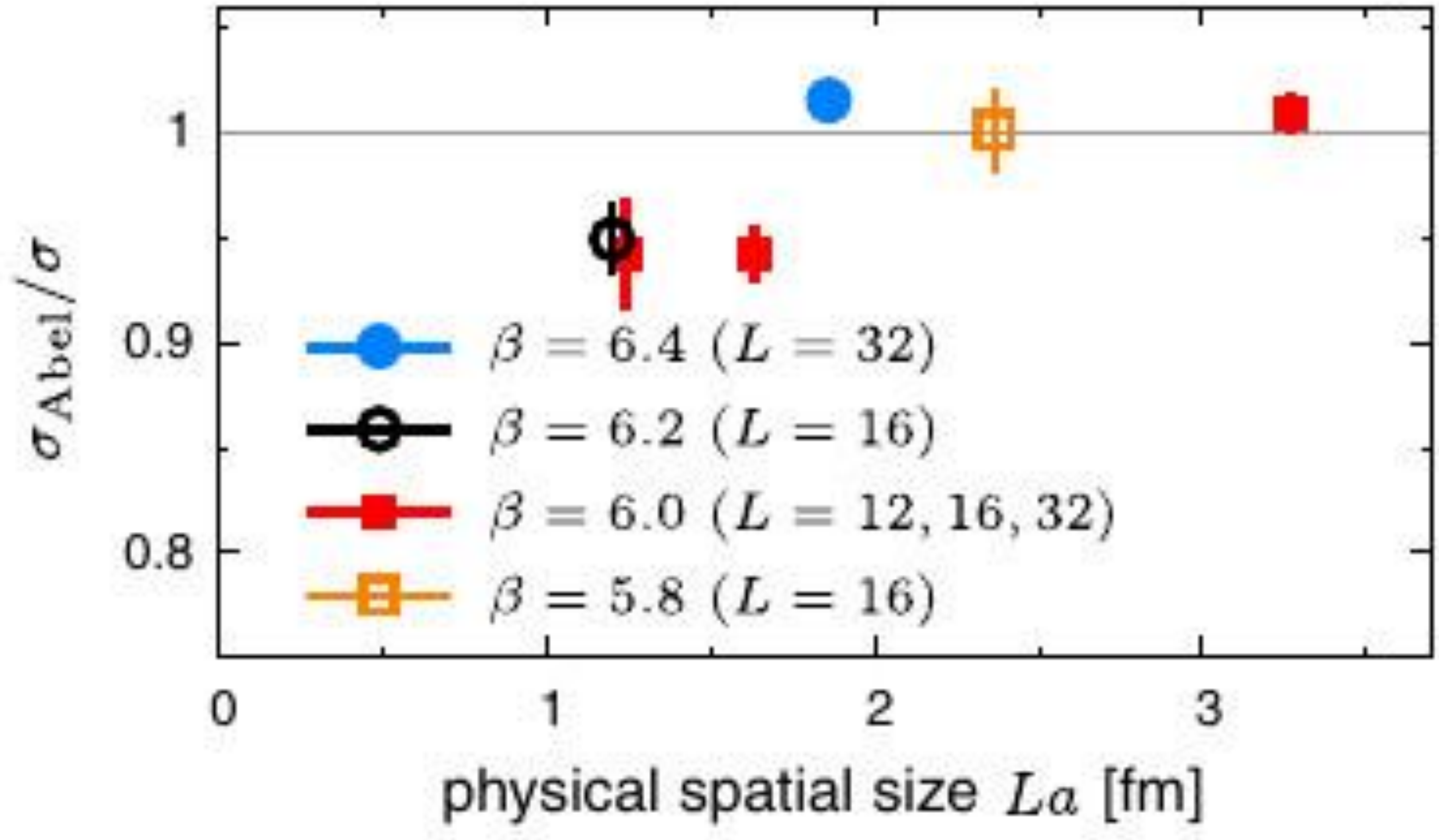
EPJ Web Conf. 126 (2016) 04042

PoS LATTICE2015 (2016) 323

Phys.Rev.D 92 (2015) 3, 034511

AIP Conf.Proc. 1701 (2016) 1, 020016

Phys.Rev.D 90 (2014) 11, 111501



Suganuma and Sakumichi, 2014

Decomposition of the static potential

usual representation:

$$U_\mu(x) = C_\mu(x) u_\mu(x)$$

$$u_\mu(x) = u_\mu^{mon}(x) u_\mu^{ph}(x)$$

We suggest:

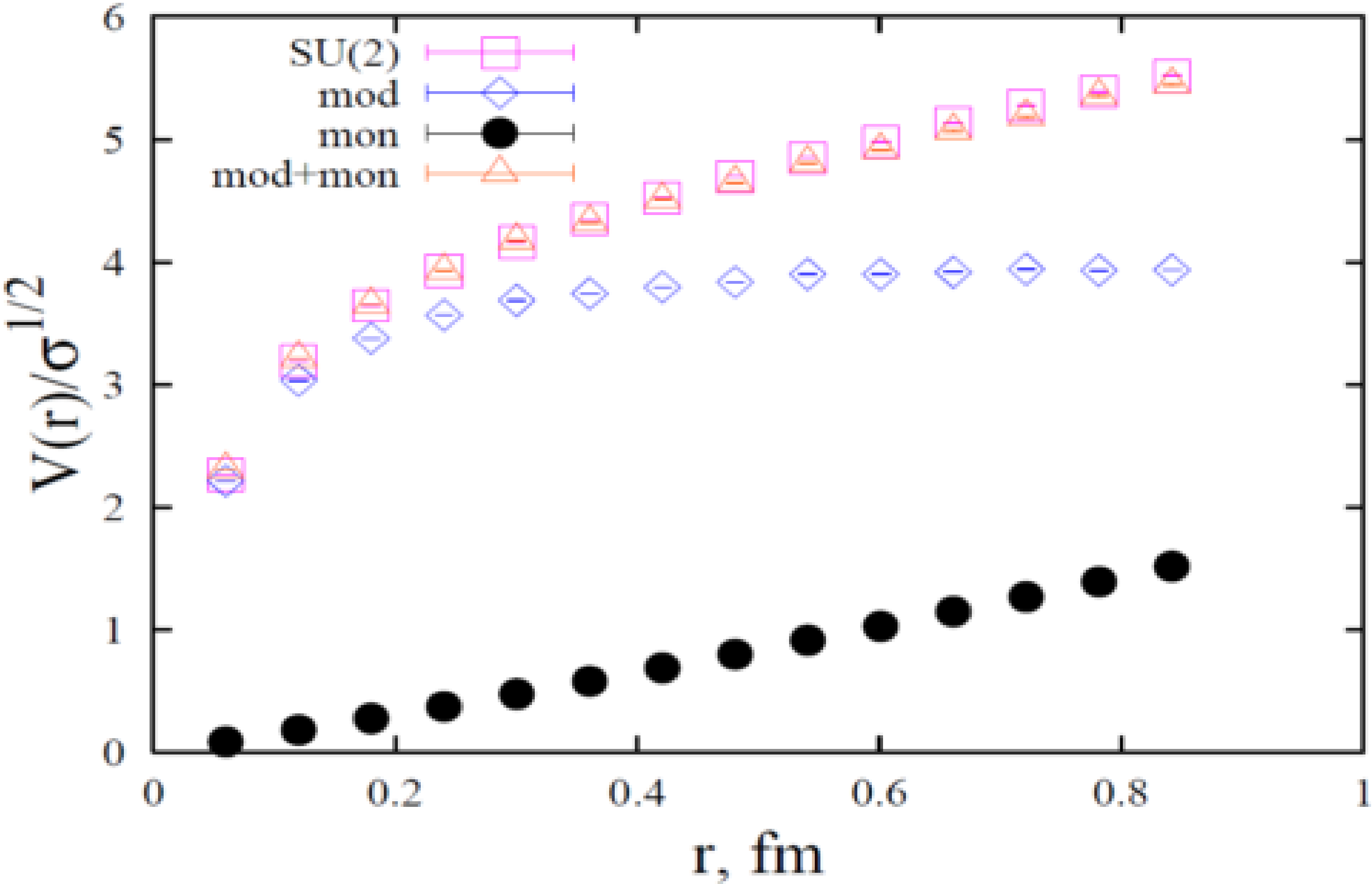
$$U_\mu(x) = U_\mu^{mod}(x) U_\mu^{mon}(x)$$

Then:

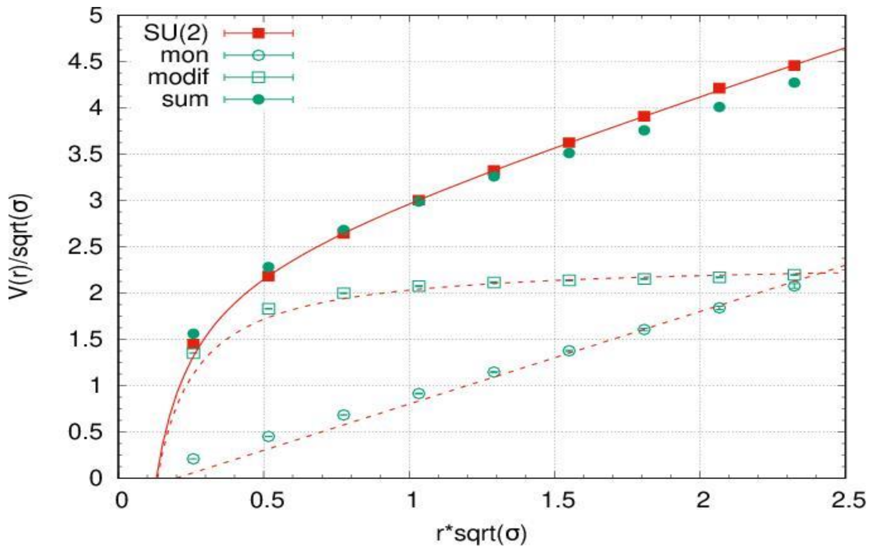
$$V(r) \approx V_{mon}(r) + V_{mod}(r)$$

$V^{mon} + V^{mod}$ approximates the nonabelian static potential with high accuracy at all distances.
SU(2) gluodanamics, $24^4, a = 0.08$fm

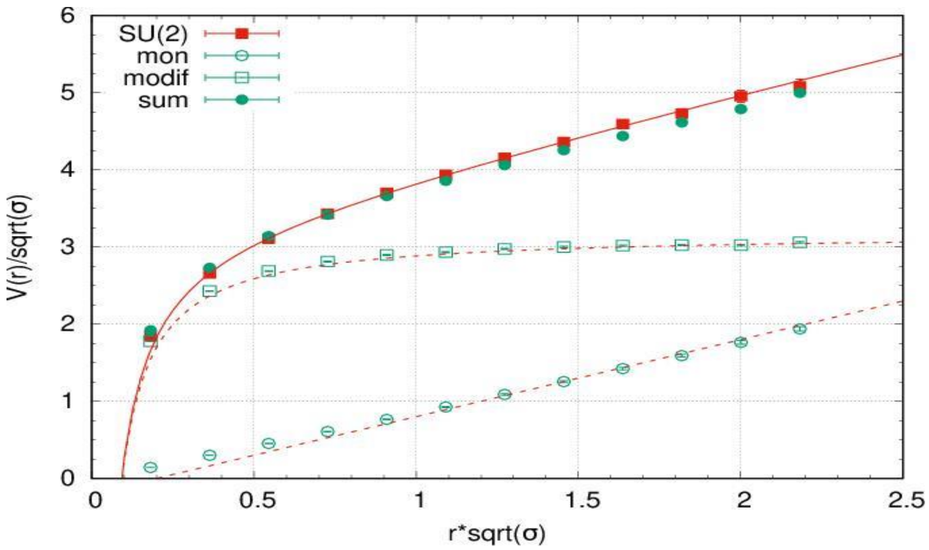
VB, Polikarpov, Schierholz, Suzuki, Syritsyn 2005



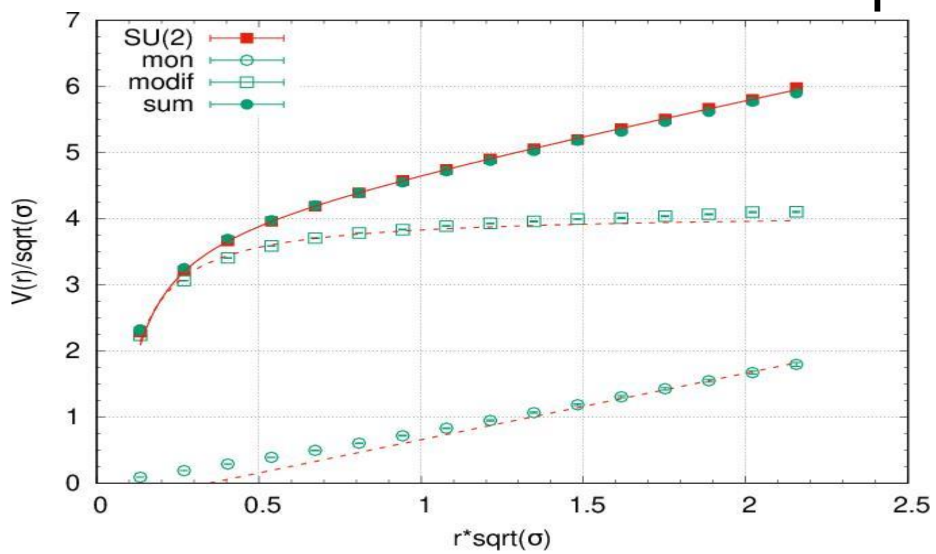
SU(2) gluodynamics with Wilson action (2202.04196[hep-lat])



β = 2.4



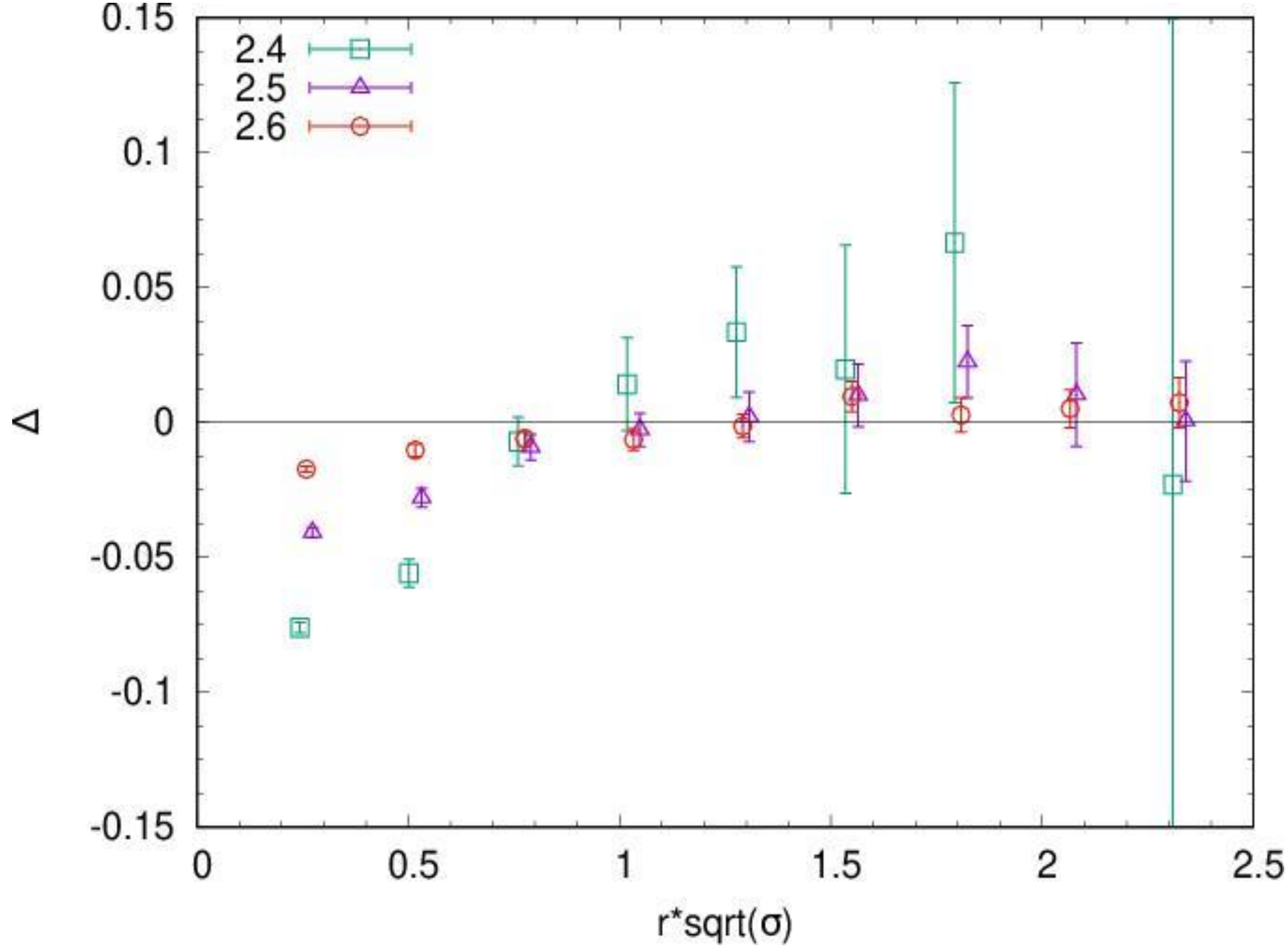
β = 2.5



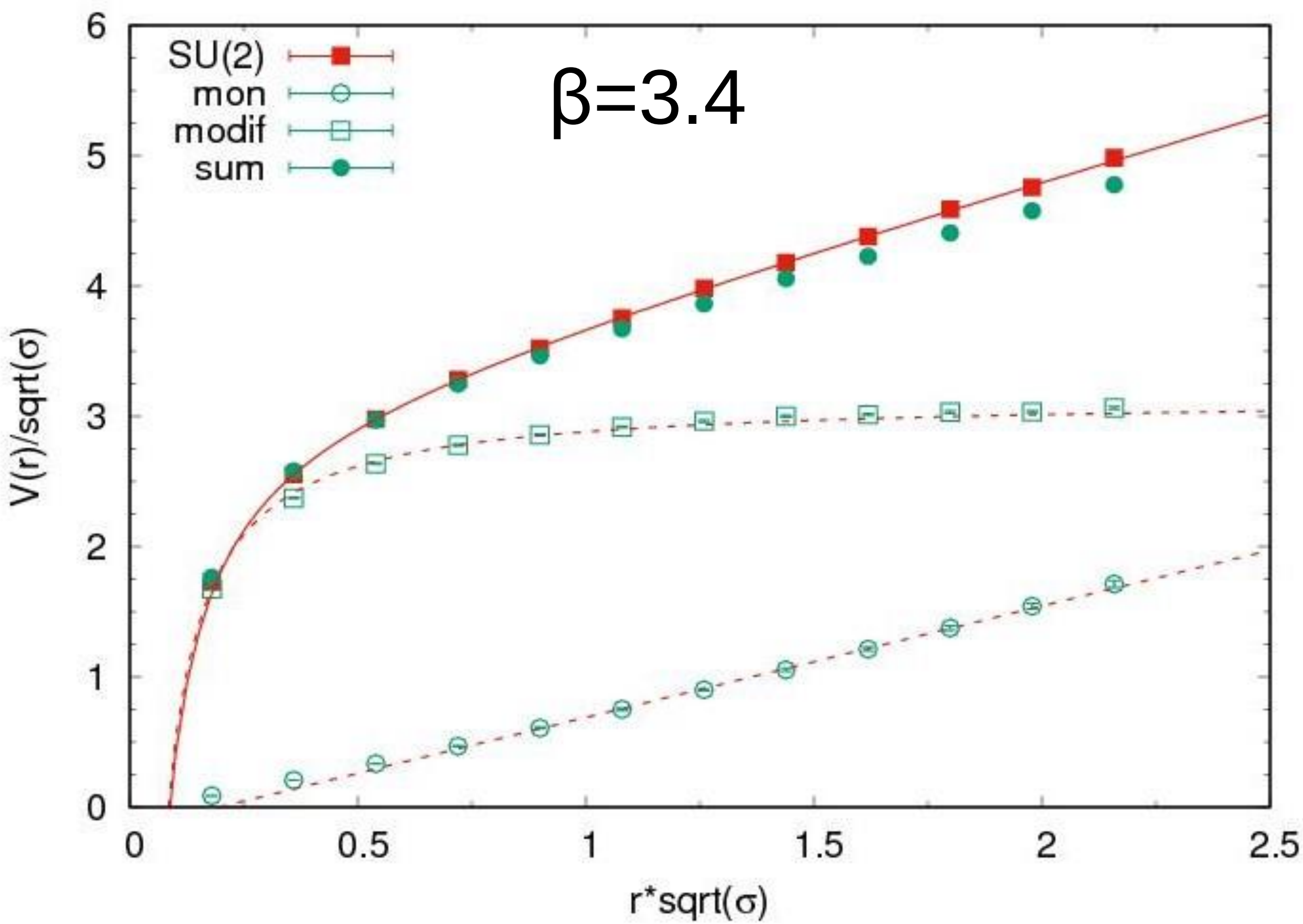
β = 2.6

Relative deviation

$$\Delta(r) = \frac{V(r) - (V_{mon}(r) + V_{mod}(r))}{V(r)}$$

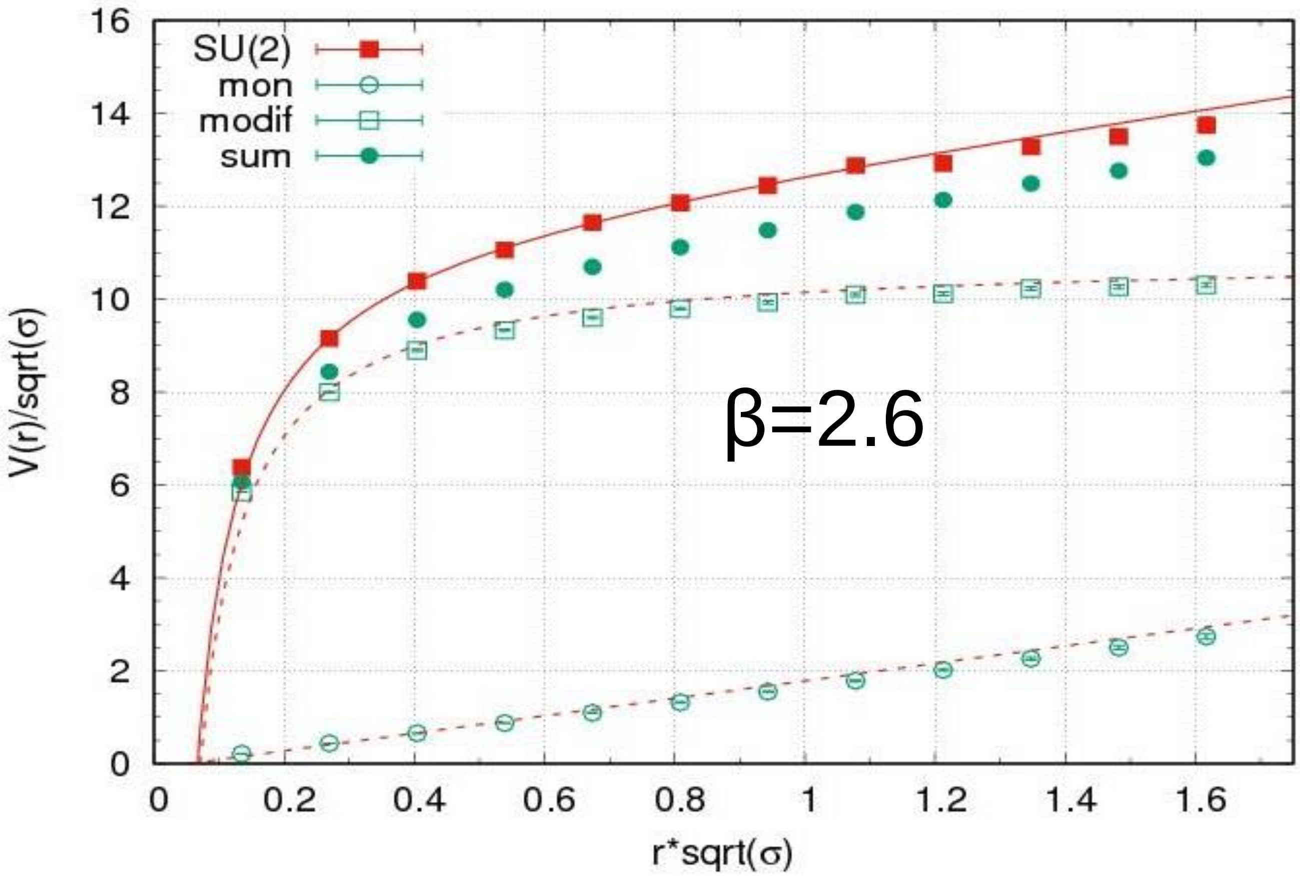


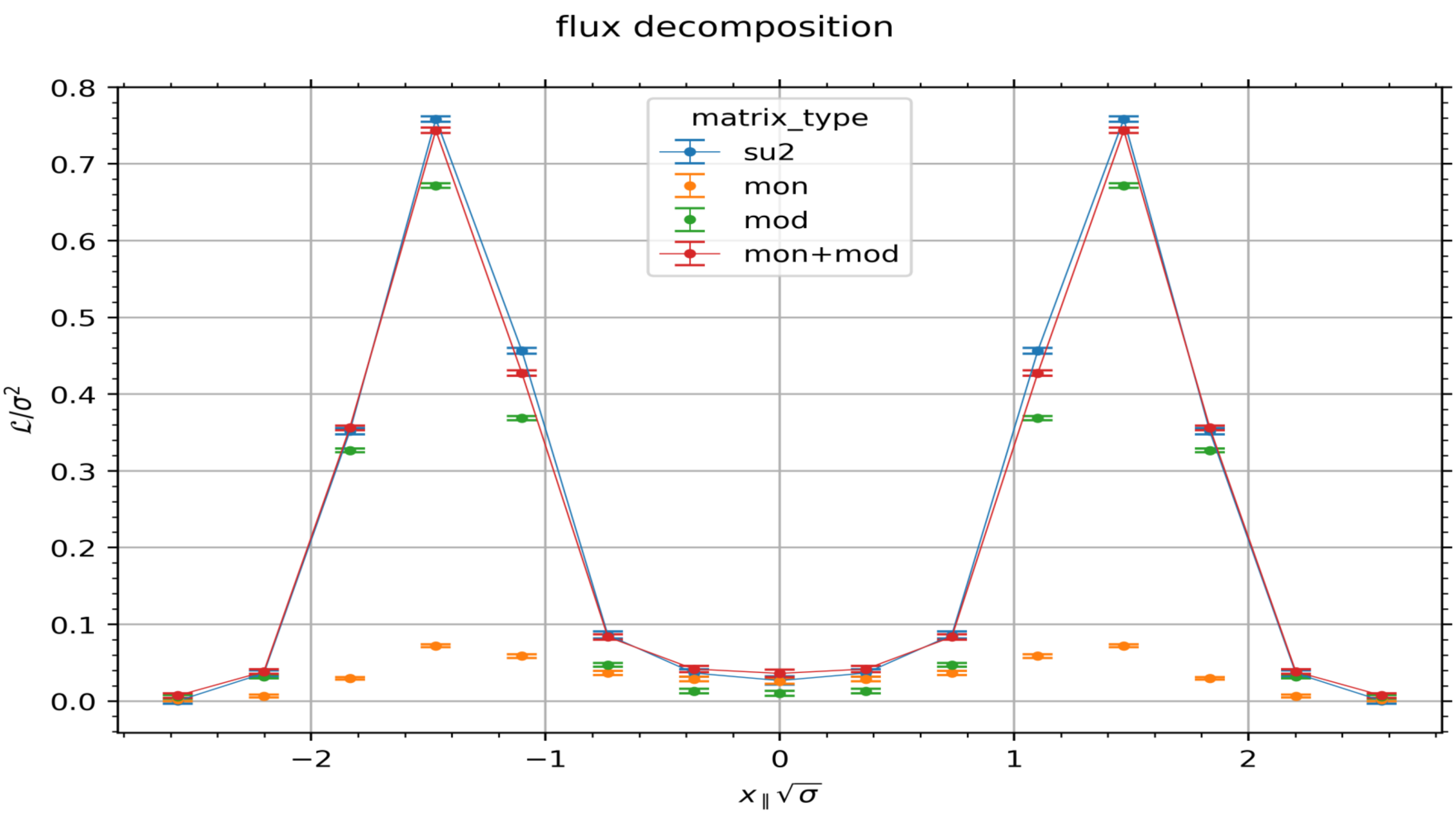
SU(2) gluodynamics with tadpole improved action
Universality



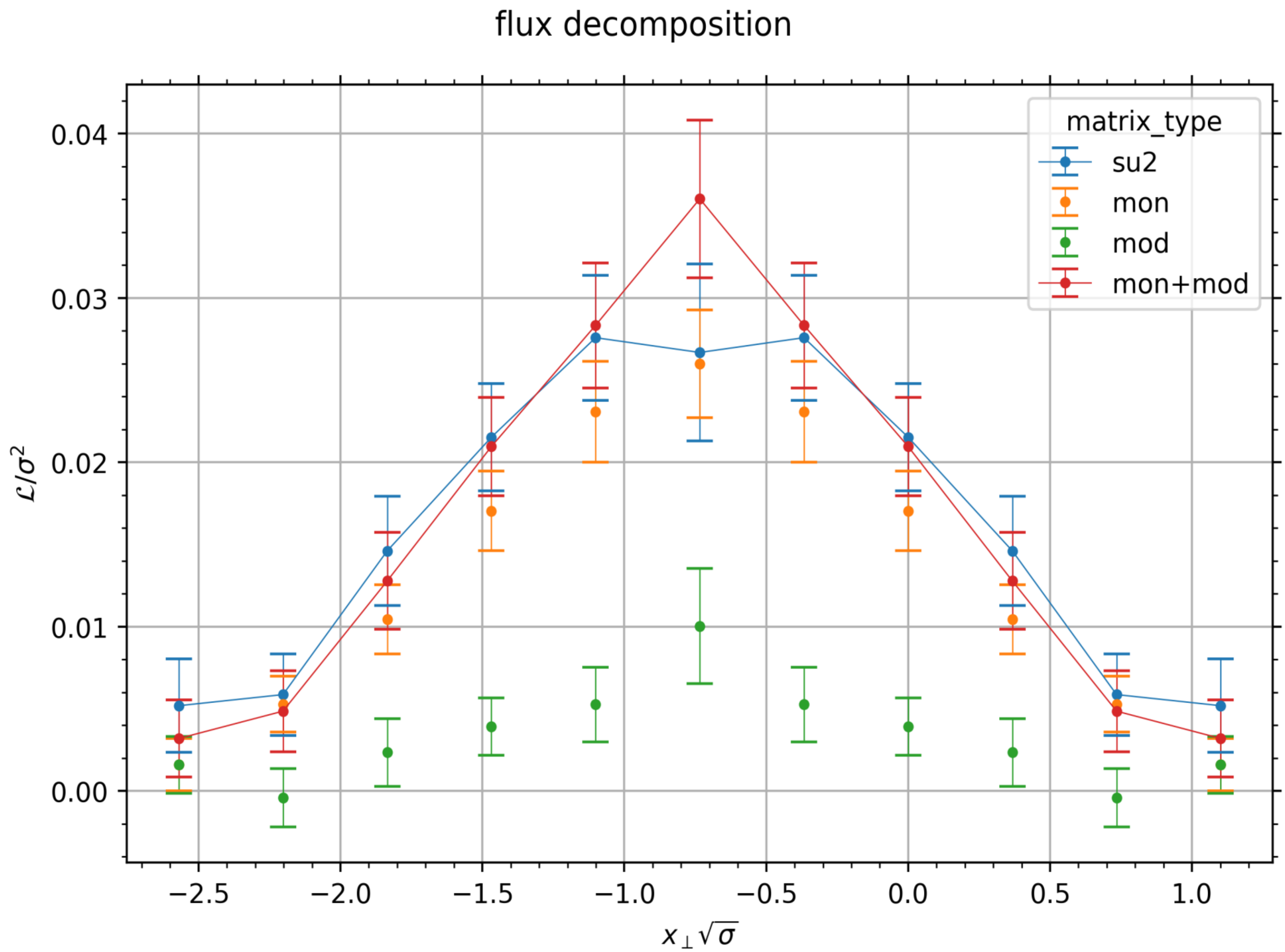
Adjoint representation

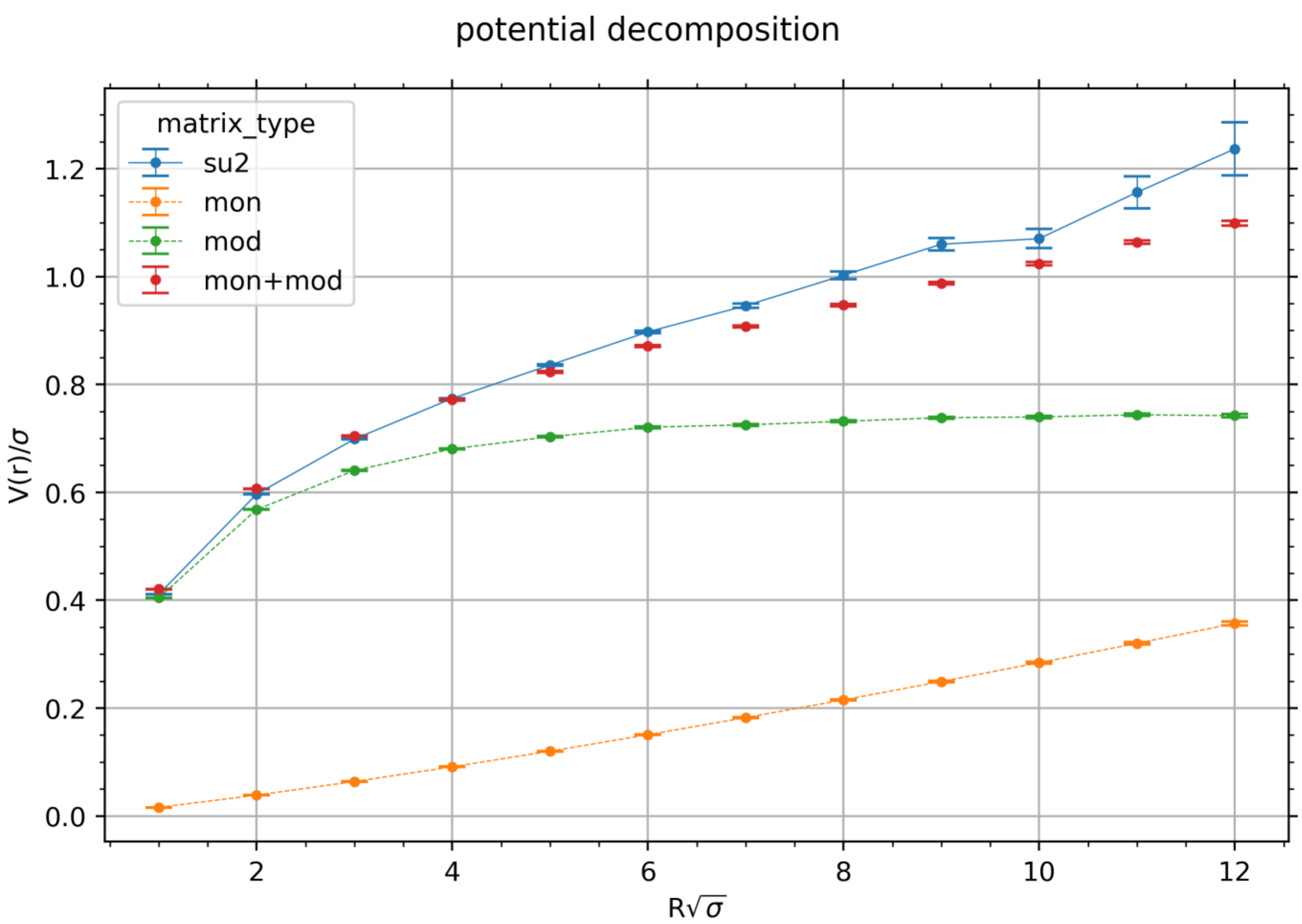
$$V_{adj}(r) \approx V_{adj,mod}(r) + V_{mon,q2}(r)$$

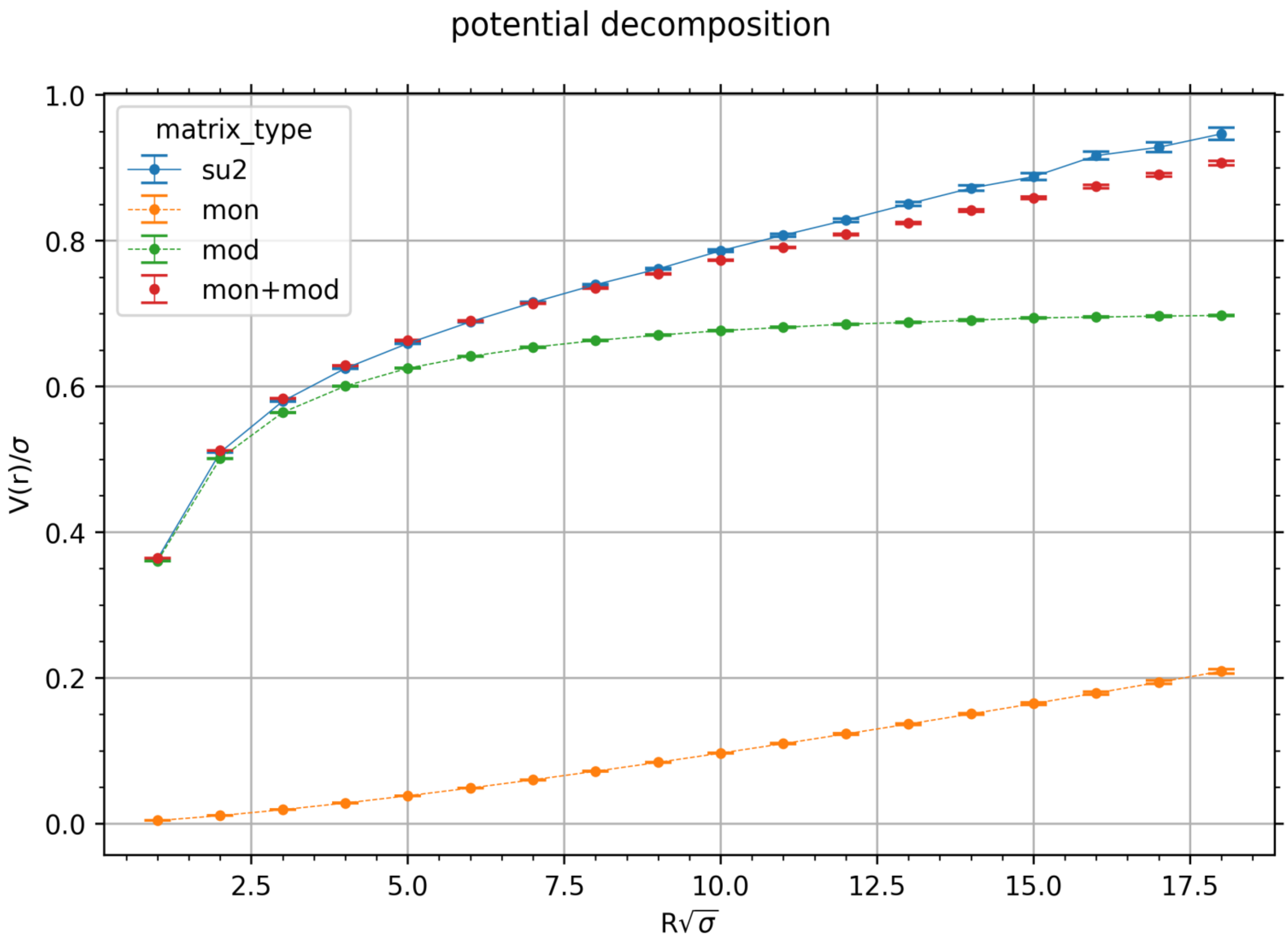




Action density β=2.6, R/a = 8


flux decomposition
matrix_type
su2
mon
mod
mon+mod
$\mathcal{L}/\sigma^2$
0.04
0.03
0.02
0.01
0.00
−2.5
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
$x_\perp\sqrt{\sigma}$


potential decomposition
matrix_type
su2
mon
mod
mon+mod
V(r)/σ
R√σ
0.0
0.2
0.4
0.6
0.8
1.0
1.2
2
4
6
8
10
12



potential decomposition
matrix_type
su2
mon
mod
mon+mod
V(r)/σ
R√σ
0.0
0.2
0.4
0.6
0.8
1.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5

in collaboration with V. Goy, A. Begun, N. Gerasimenyuk

- SU(2) lattice QCD with $N_f = 2$ staggered Dirac operator
- Lattice size 32^4
- Lattice spacing $a = 0.044$ fm
- Pion mass $m_\pi = 740(40)$ MeV
- Range of μ values: $0 \le a\mu \le 0.5$

or

$0 \le \mu \lesssim 2000$ MeV

Simulation settings

Lattice fermion action:

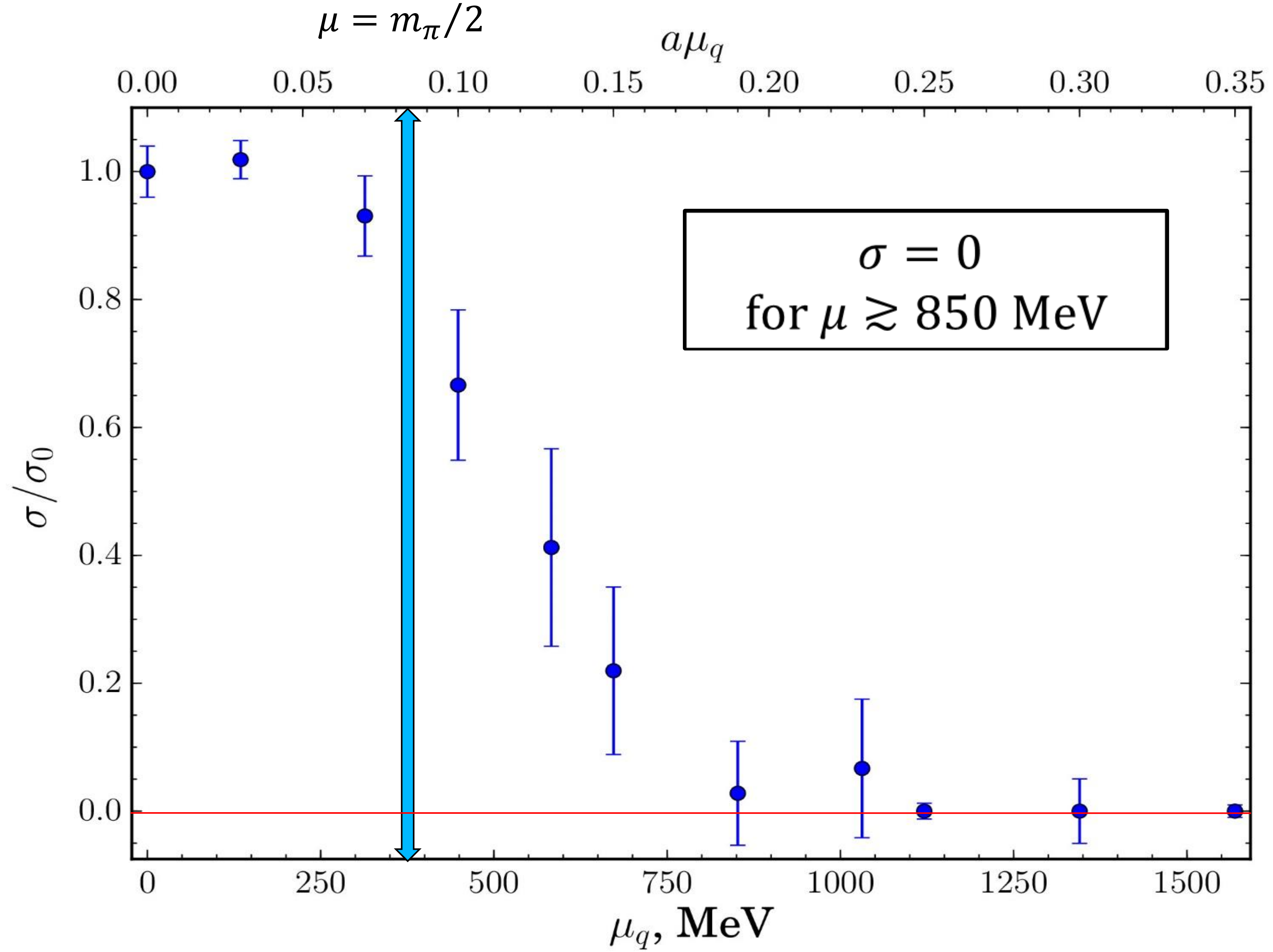
$$S_F = \sum_{x,y} \bar{\psi}_x M(\mu, m)_{x,y} \psi_y + \frac{\lambda}{2} \sum_x \left(\psi_x^T \tau_2 \psi_x + \bar{\psi}_x \tau_2 \bar{\psi}_x^T \right)$$

M is the staggered lattice Dirac operator,

λ- term is needed to make the di-quark condensate nonzero

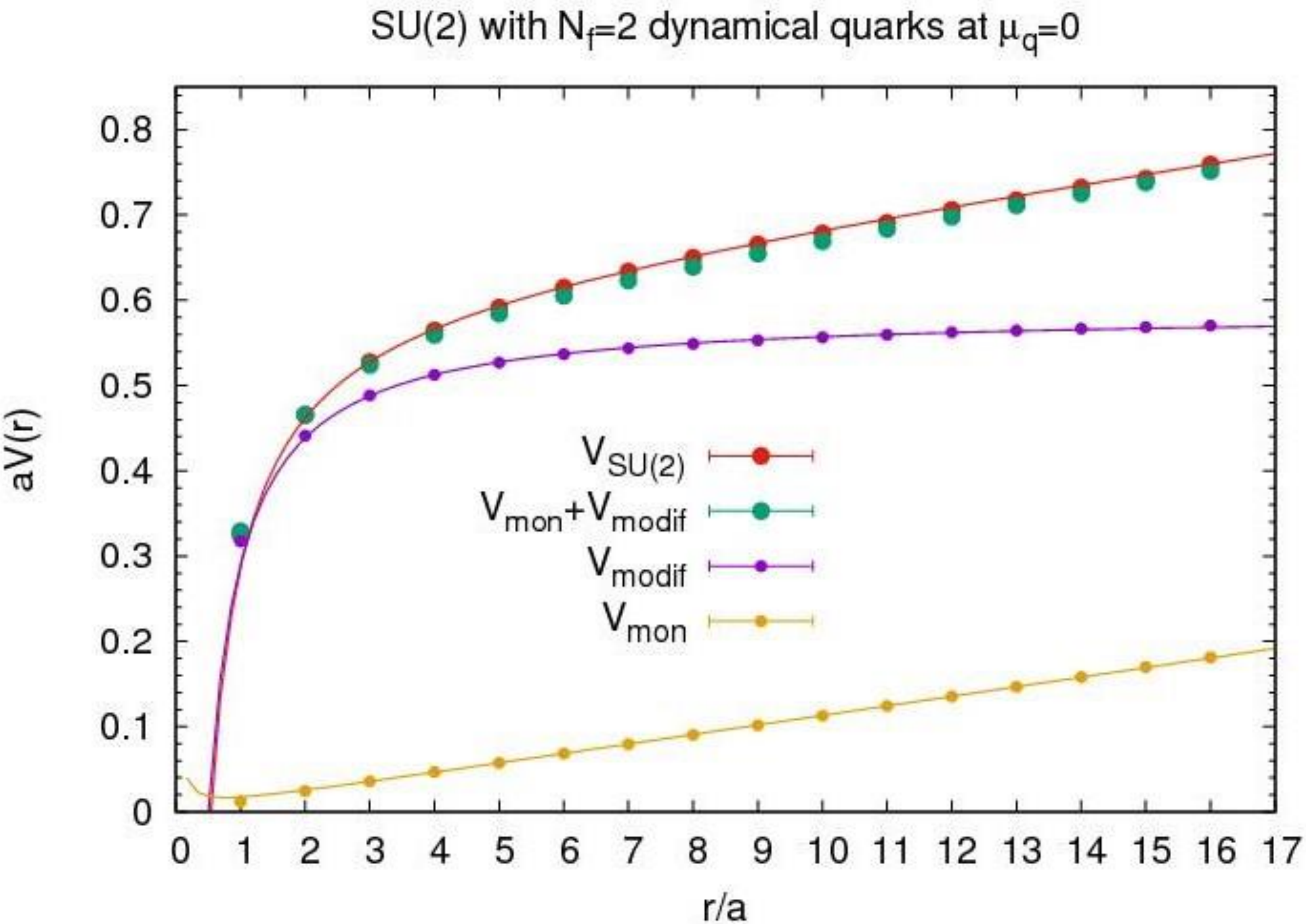
Partition function:

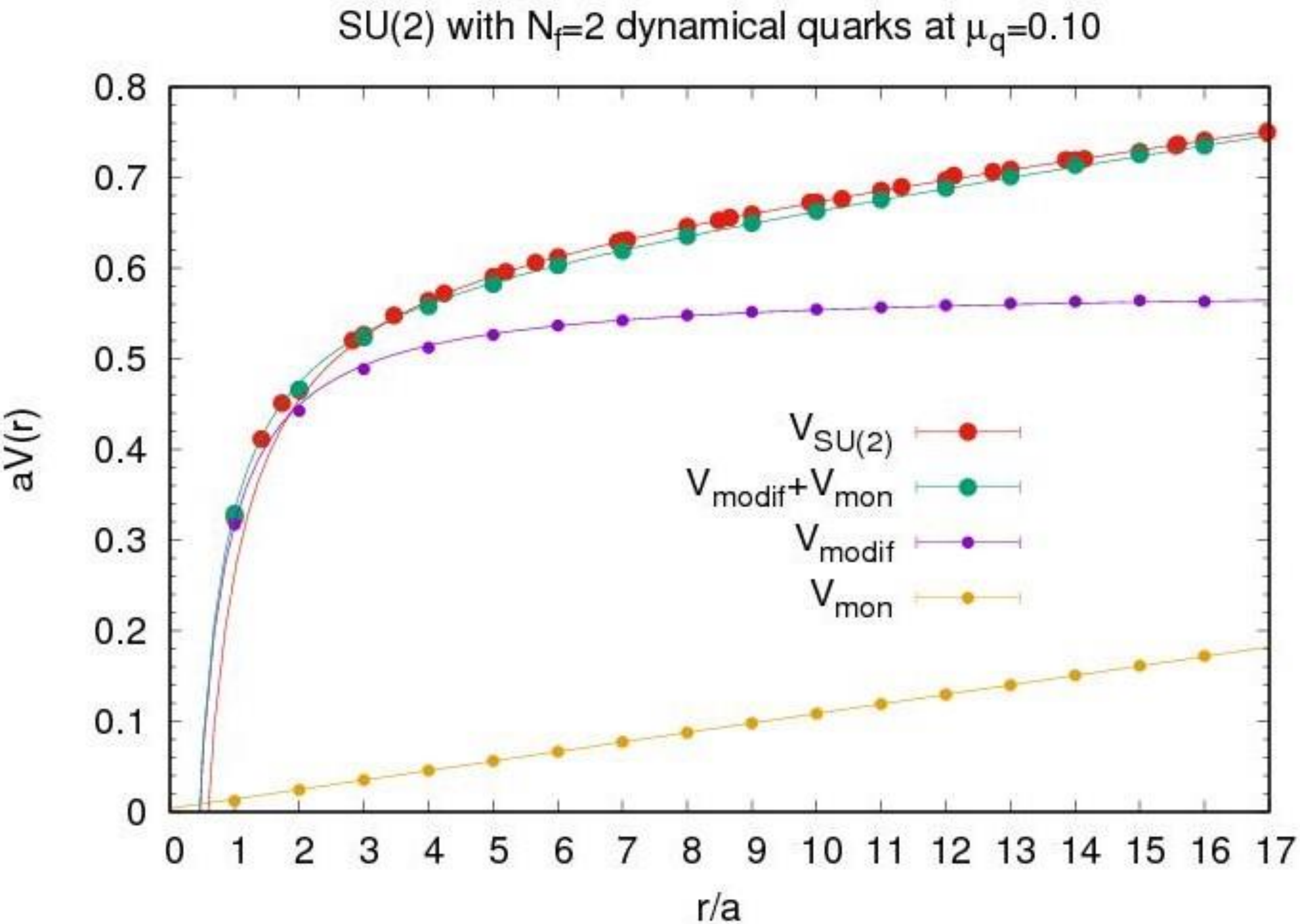
$$Z = \int DU e^{-S_G} \cdot \left(\det(M^\dagger M + \lambda^2)\right)^{\frac{1}{4}}$$



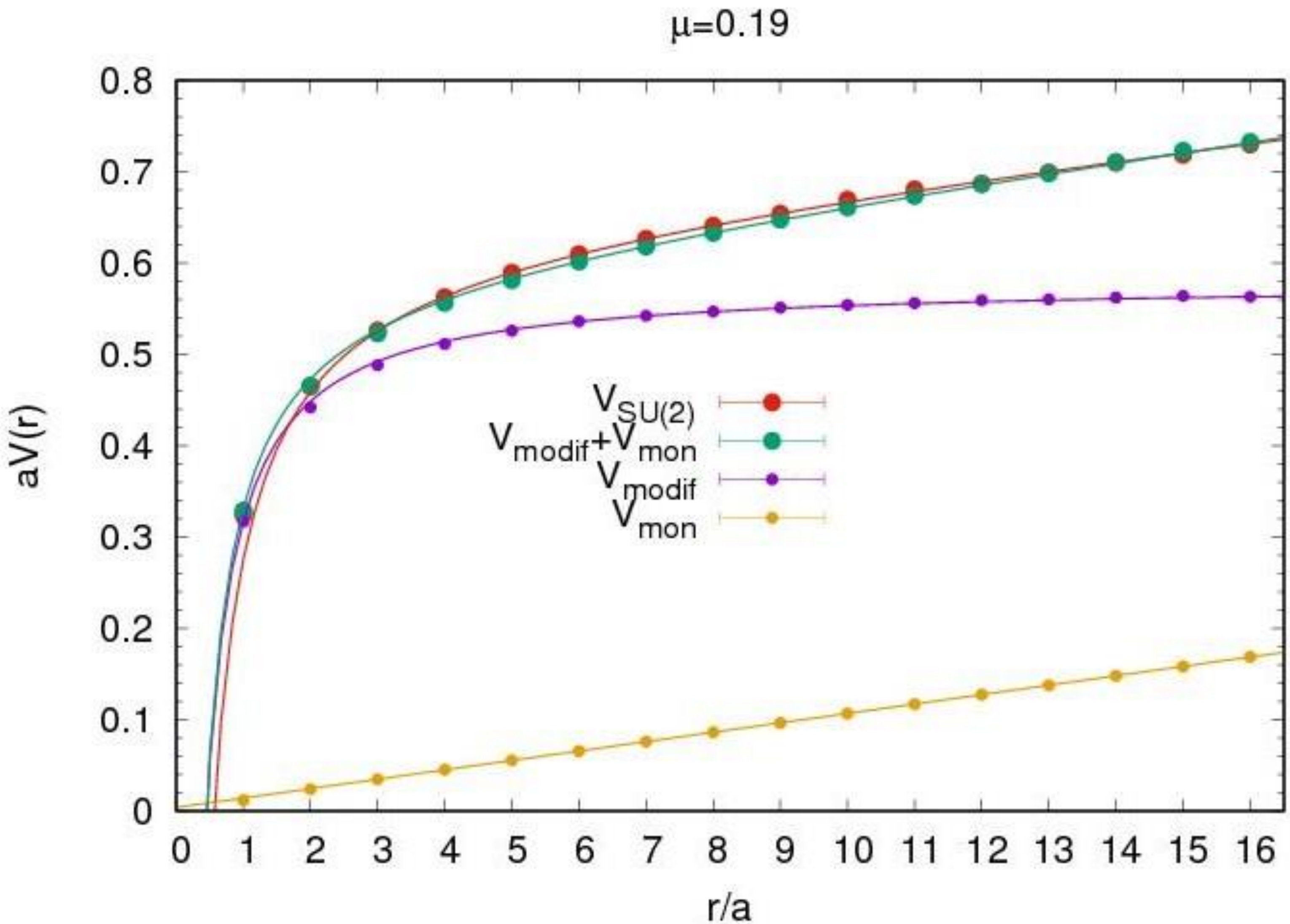
JHEP 03 (2018) 161 • e-Print: 1711.01869 [hep-lat]

V. G. Bornyakov, V. V. Braguta, E.-M. Ilgenfritz, A. Yu. Kotov, A. V. Molochkov, and A. A. Nikolaev


SU(2) with N_f=2 dynamical quarks at μ_q=0
aV(r)
r/a
$V_{SU(2)}$
$V_{mon}+V_{modif}$
V_{modif}
V_{mon}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17



SU(2) with N_f=2 dynamical quarks at μ_q=0.10
aV(r)
r/a
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
$V_{SU(2)}$
$V_{modif}+V_{mon}$
V_{modif}
V_{mon}



μ=0.19
aV(r)
r/a
V_SU(2)
V_modif+V_mon
V_modif
V_mon
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

Implications

Independence of U_{mon} and U_{mod} (to be demonstrated via studies of field correlators)

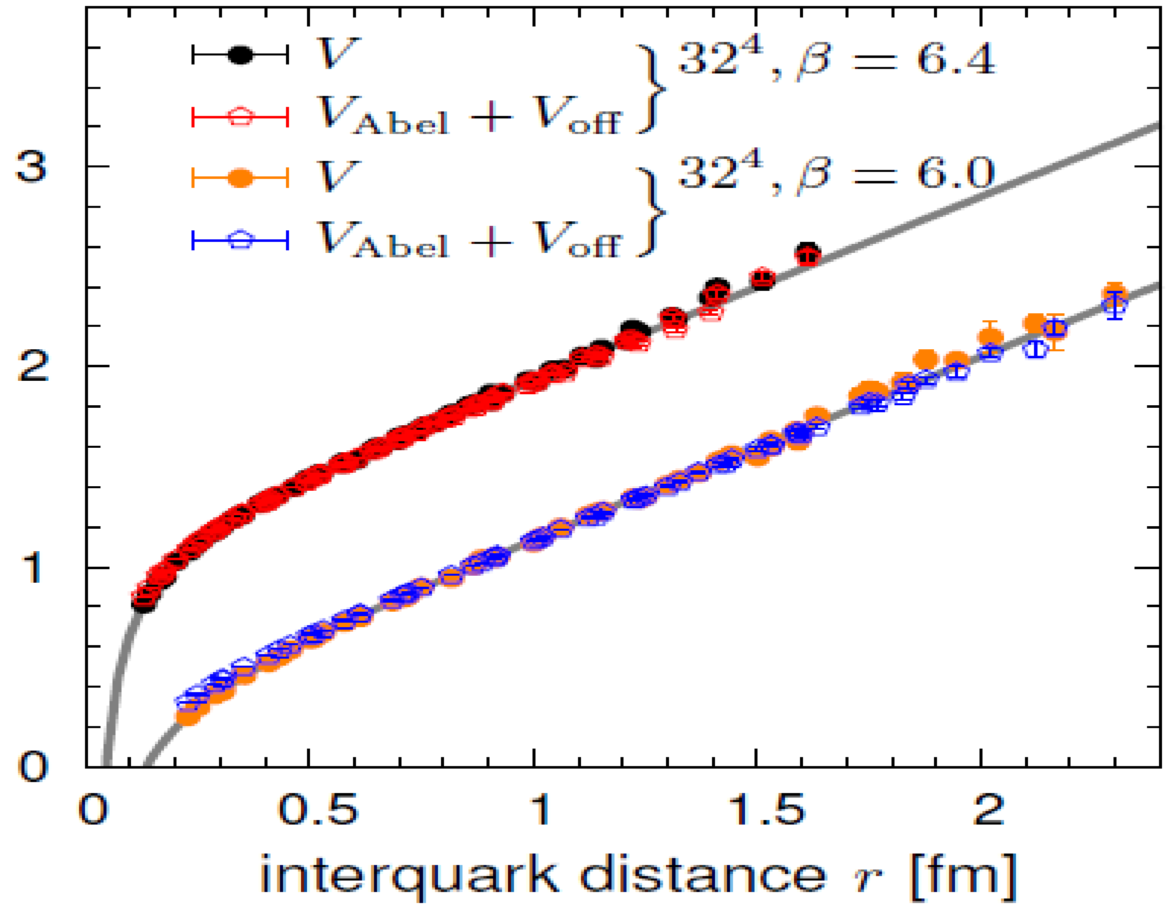
Monopole gauge field U_{mon} describes classical part of the hadron string energy, while monopoleless gauge field U_{mod} describes string fluctuations

Short strings? (V.I. Zakharov, F. Gubarev, 2005)

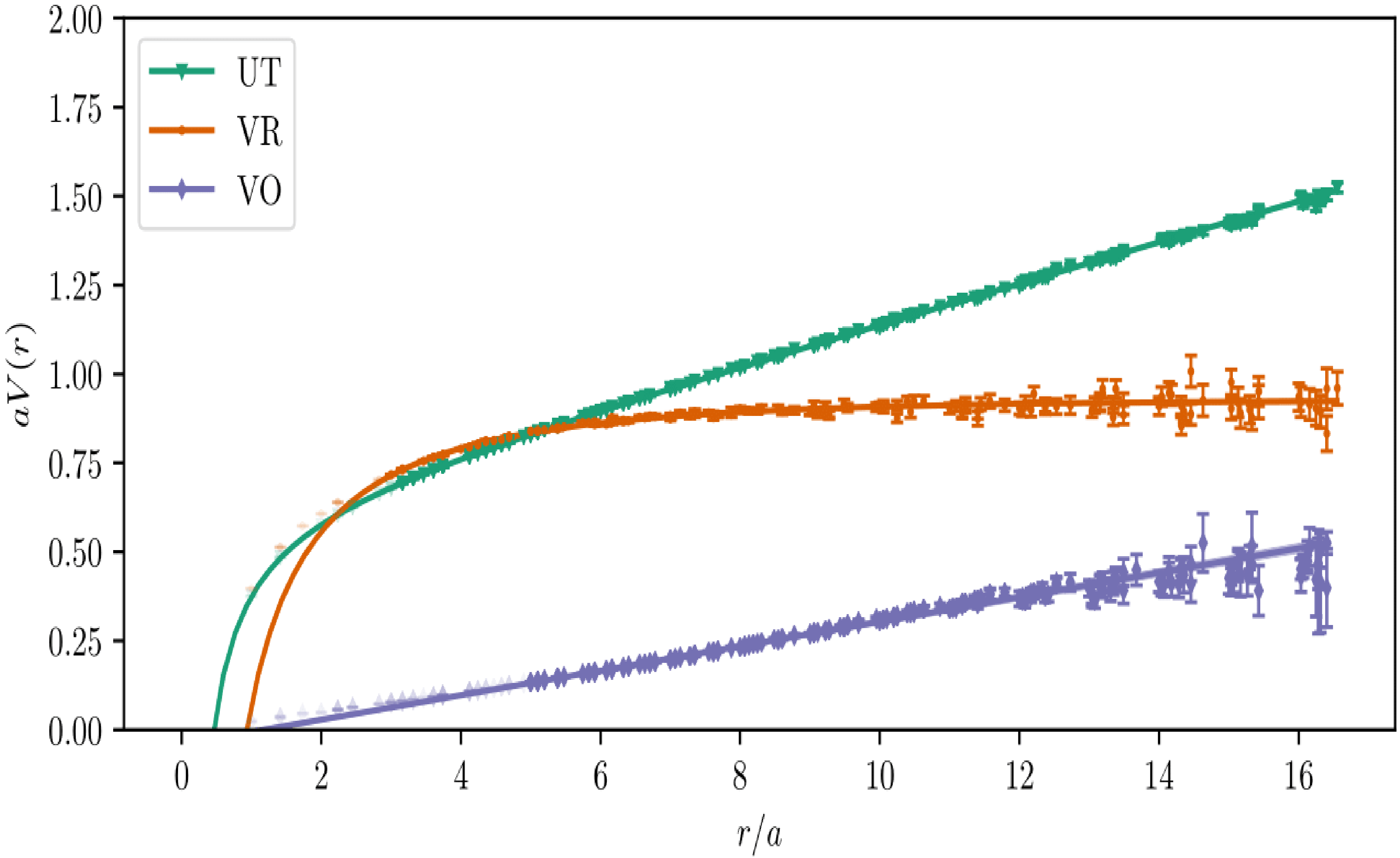
Another decomposition

Suganuma and Sakumichi, 2014

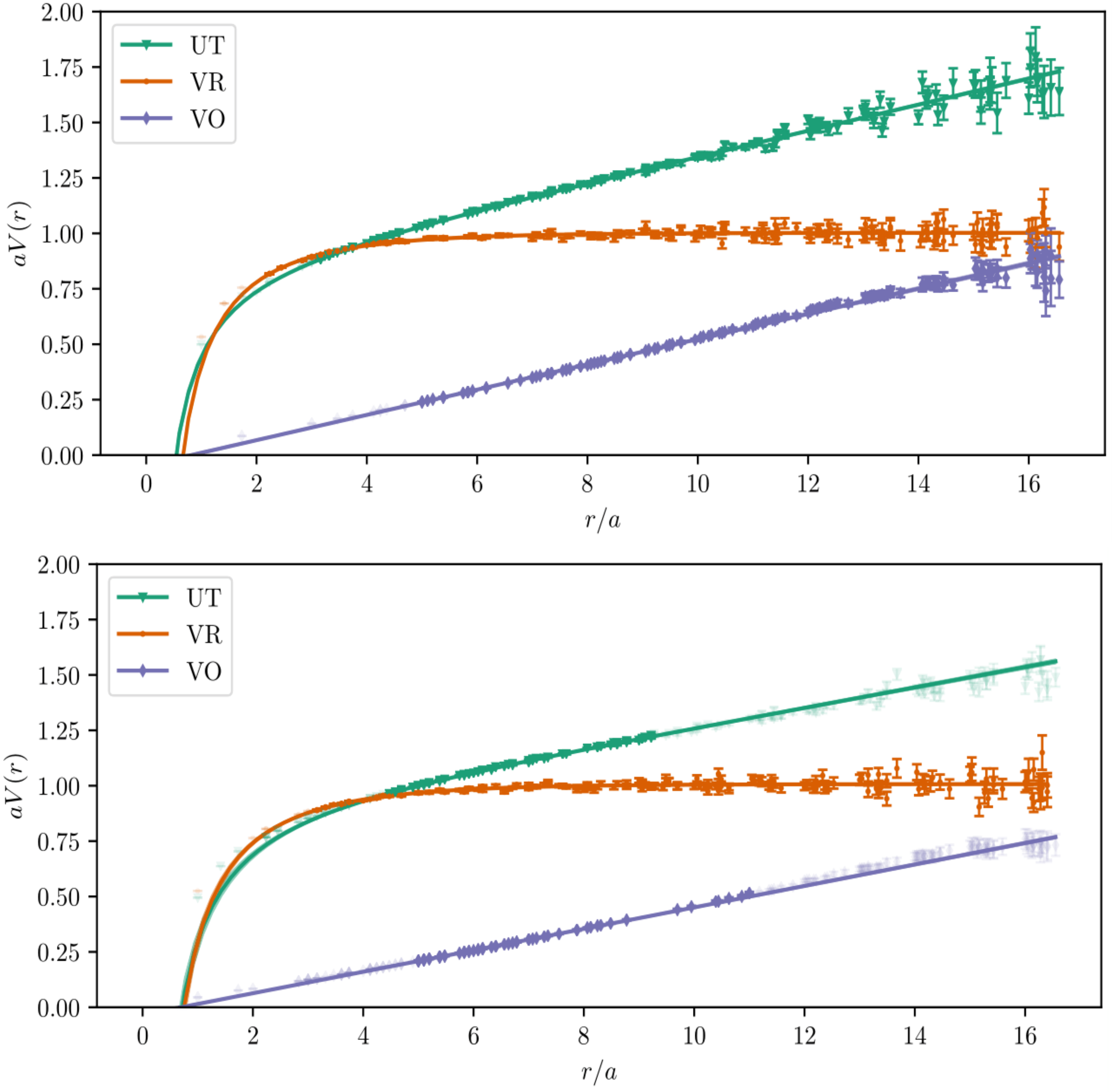
$$V(r) \simeq V_{\text{Abel}}(r) + V_{\text{off}}(r)$$



Static quark potential from centre vortices in the presence of dynamical fermions
James C. Biddle, Waseem Kamleh, and Derek B. Leinweber
Phys.Rev.D 106 (2022) 5, 054505

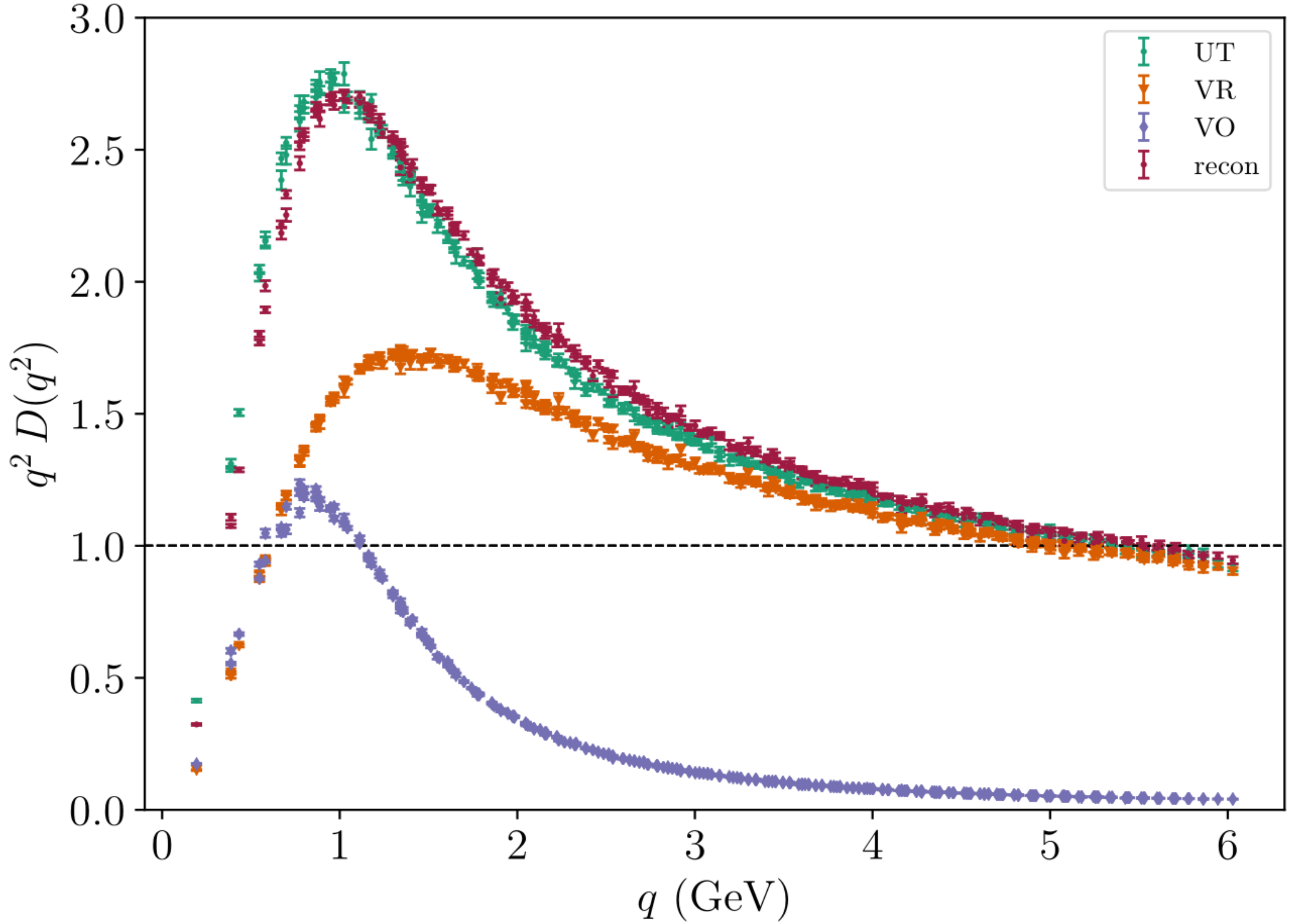


- SU(3) gluodynamics, improved Iwasaki action, a=0.1 fm, 32^3 x 64 lattice



- $N_{f=2}$ QCD for m_π = 700MeV (top), m_π =156MeV (bottom)

Impact of Dynamical Fermions on the Centre Vortex Gluon Propagator
Phys.Rev.D 106 (2022) 1, 014506   SU(3) gluodynamics



Conclusions

- In MA gauge the static potential can be decomposed

$$V(r) = V_{mon}(r) + V_{modif}(r)$$

- This was demonstrated in SU(2) gluodynamics and in SU(2) QCD
- This suggests that the classical part of the hadron string action is described by $A_{\mu}^{mon}(x)$ while its vibrations (Luescher term) are described by $A_{\mu}^{modif}(x)$
- These two components of $A_{\mu}(x)$ are not correlated – this should be demonstrated

- Two methods of decomposition should be studied