

Electromagnetic conductivity of dense quark-gluon plasma

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JINR

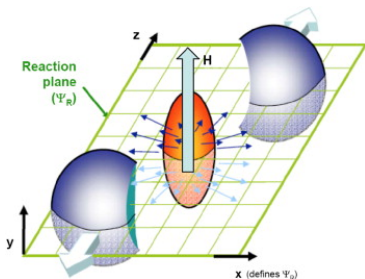
Models in Quantum Field Theory,
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In collaboration with

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Motivation



- ▶ Charge transport of QGP is important for dynamics of QGP
- ▶ QGP in heavy-ion collisions may have non-zero baryon density
- ▶ Baryon density introduces additional fermion states to QGP
- ▶ Baryon density might change σ significantly

Lattice studies of electromagnetic conductivity

- ▶ H. T. Ding, A. Francis, O. Kaczmarek, F. Karsch, E. Laermann, and W. Soeldner, Phys. Rev. D83, 034504 (2011)
- ▶ A. Amato, G. Aarts, C. Allton, P. Giudice, S. Hands, and J.-I. Skullerud, Phys. Rev. Lett. 111, 172001 (2013)
- ▶ G. Aarts, C. Allton, A. Amato, P. Giudice, S. Hands, and J.-I. Skullerud, JHEP02, 186 (2015)
- ▶ B. B. Brandt, A. Francis, B. Jager, and H. B. Meyer, Phys. Rev. D93, 054510 (2016)
- ▶ H.-T. Ding, O. Kaczmarek, and F. Meyer, Phys. Rev. D94, 034504 (2016)
- ▶ P.V. Buividovich, D. Smith, L. von Smekal, Phys.Rev.D 102 (2020) 9, 094510

Conductivity in lattice simulations

▶ $J_i = \sigma_{ij} E_j$

▶ Electromagnetic conductivity

$$\sigma_{ij} = \lim_{\omega \rightarrow 0} \frac{1}{\omega} \int_0^\infty dt \int d^3x e^{i\omega t} \langle [J_i(x), J_j(0)] \rangle$$

$$\rho_{ij} = -\frac{1}{\pi} \text{Im} G_R^{ij}(\omega, \vec{k} = 0)$$

$$\sigma_{ij} = \pi \lim_{\omega \rightarrow 0} \frac{1}{\omega} \rho_{ij}(\omega)$$

▶ Analytic continuation

$$G_E(\omega, \vec{p}) = -G_R(i\omega, \vec{p}), \quad \omega > 0$$

▶ On lattice we measure

$$C_E(\tau) = \int d^3x \langle J_i(\tau, \vec{x}) J_j(0, \vec{0}) \rangle$$

$$C_E(\tau) = \int_0^\infty d\omega \rho(\omega) \frac{\text{ch}\left(\frac{\omega}{2T} - \omega\tau\right)}{\text{sh}\left(\frac{\omega}{2T}\right)}, \quad \tau \in \left(0, \frac{1}{T}\right)$$

Conductivity with staggered fermions

- ▶ We account only connected diagrams
- ▶ Correlation function for staggered fermions

$$C_{ij}(\tau) = \frac{1}{L_s^3} \langle J_i(\tau) J_j(0) \rangle,$$

$$J_i(\tau) = \frac{1}{4} e \sum_f q_f \sum_{\vec{x}} \eta_i(x) (\bar{\Psi}_x^f U_{x,i} \Psi_{x+i}^f + \bar{\Psi}_{x+i}^f U_{x,i}^+ \Psi_x^f)$$

- ▶ Two branches of staggered correlator

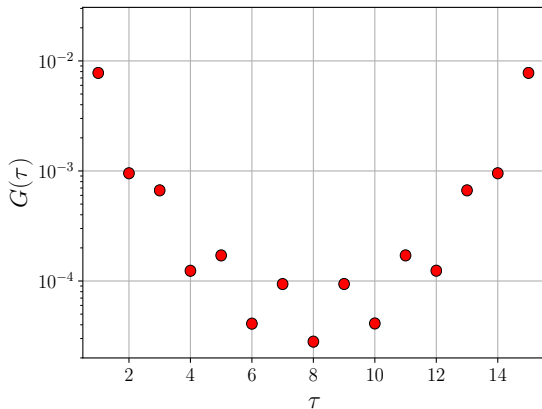
$$C_{ij}^e(\tau = 2n \times a) = \int d^3y (\langle A_i(\tau, \vec{y}) A_j(0, \vec{0}) \rangle - \langle B_i(\tau, \vec{y}) B_j(0, \vec{0}) \rangle)$$

$$C_{ij}^o(\tau = (2n+1) \times a) = \int d^3y (\langle A_i(\tau, \vec{y}) A_j(0, \vec{0}) \rangle + \langle B_i(\tau, \vec{y}) B_j(0, \vec{0}) \rangle)$$

$$A_i = e \sum_f q_f \bar{\psi}^f \gamma_i \psi^f, \quad B_i = e \sum_f q_f \bar{\psi}^f \gamma_5 \gamma_4 \gamma_i \psi^f$$

Conductivity with staggered fermions

- ▶ Typical plot for the staggered correlation function



Conductivity with staggered fermions

The strategy of the calculation

- ▶ Measure $C_E^{even,odd}(\tau)$ on two branches
- ▶ Reconstruct the $\rho^{even,odd}(\omega)$ (Backus-Gilbert method)

$$C_E^{even,odd}(t) = \int_0^\infty d\omega \rho^{even,odd}(\omega) \frac{ch(\frac{\omega}{2T} - \omega t)}{sh(\frac{\omega}{2T})}$$

- ▶ Calculate $\rho(\omega) = \frac{1}{2}(\rho^{even}(\omega) + \rho^{odd}(\omega))$
(what corresponds to the $\langle J_{el}(\tau)J_{el}(0) \rangle$)
- ▶ Calculate the conductivity $\sigma = \pi \frac{\rho(\omega)}{\omega} \Big|_{\omega \sim 0}$

Backus-Gilbert method for the spectral function

- ▶ Problem: find $\rho(\omega)$ from the integral equation

$$C(x_i) = \int_0^\infty d\omega \rho(\omega) K(x_i, \omega), \quad K(x_i, \omega) = \frac{\text{ch}\left(\frac{\omega}{2T} - \omega x_i\right)}{\text{sh}\left(\frac{\omega}{2T}\right)}$$

- ▶ Define an estimator $\tilde{\rho}(\bar{\omega})$ ($\delta(\bar{\omega}, \omega)$ - resolution function):

$$\tilde{\rho}(\bar{\omega}) = \int_0^\infty d\omega \hat{\delta}(\bar{\omega}, \omega) \rho(\omega)$$

- ▶ Let us expand $\delta(\bar{\omega}, \omega)$ as

$$\delta(\bar{\omega}, \omega) = \sum_i b_i(\bar{\omega}) K(x_i, \omega) \quad \tilde{\rho}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

- ▶ Goal: minimize the width of the resolution function

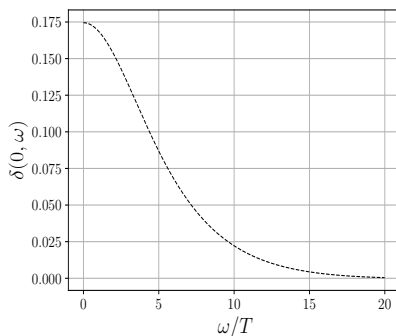
$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$

$$W_{ij} = \int d\omega K(x_i, \omega) (\omega - \bar{\omega})^2 K(x_j, \omega), \quad R_i = \int d\omega K(x_i, \omega)$$

- ▶ Regularization by the covariance matrix S_{ij} :

$$W_{ij} \rightarrow \lambda W_{ij} + (1 - \lambda) S_{ij}, \quad 0 < \lambda < 1$$

Backus-Gilbert method for the spectral function



- ▶ We calculate the estimator of the spectral function

$$\bar{\rho}(\bar{\omega}) = \int d\omega \delta(\omega, \bar{\omega}) \rho(\omega)$$

- ▶ Average spectral function (conductivity) over the width $\sim \text{few} \times T$

Backus-Gilbert method for the spectral function

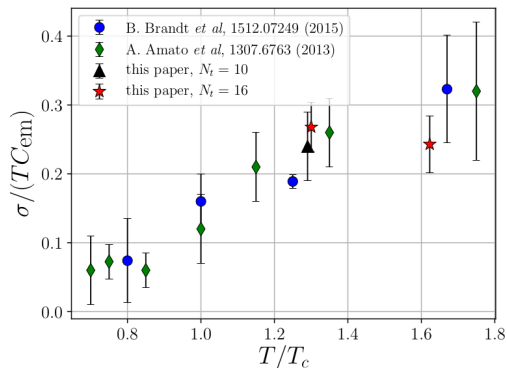
- ▶ Width of the resolution function is $\sim (3 - 4) \times T$
- ▶ For very narrow spectral density BG method underestimates conductivity
- ▶ But lattice studies give the width $\sim 4T$ or larger
 - ▶ G. Aarts et al, JHEP02, 186 (2015)
 - ▶ B. B. Brandt et al, Phys. Rev.D93, 054510 (2016)
 - ▶ H.-T. Ding, et al, Phys. Rev.D94, 034504 (2016)

Details of lattice simulations

- ▶ Stout smeared staggered 2 + 1 fermions
- ▶ Physical pion m_π and strange m_s quark masses
- ▶ $T \approx 200, 250$ MeV
- ▶ $\mu_u = \mu_d = \mu_B/3, \mu_s = 0$
- ▶ Because of the sign problem the simulations are carried out at imaginary $\mu_B = I\mu_I$
- ▶ Lattice parameters:

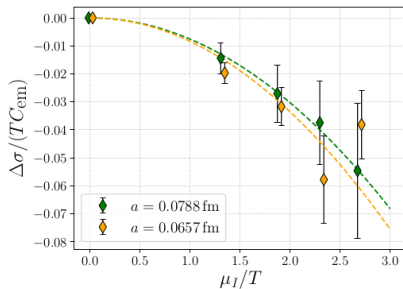
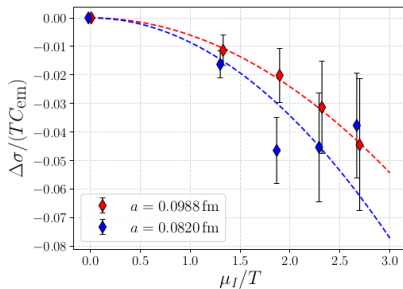
a , fm	L_s	N_t	T , fm
0.0988	48	10	200
0.0788	48	10	250
0.0820	48	12	200
0.0657	48	12	250

Conductivity at zero density



- ▶ **First calculation of the conductivity at physical pion mass**
- ▶ Agreement with previous papers
- ▶ Discretization effects are under control

E.m. conductivity at finite baryon density



- ▶ $\Delta\sigma = \sigma(\mu_I) - \sigma(\mu_I = 0)$ (to subtract UV contribution)
- ▶ Discretization effects are under control
- ▶ Our results can be well described by

$$\frac{\Delta\sigma}{TC_{em}} = -c(T) \left(\frac{\mu_I}{T} \right)^2 \Rightarrow \frac{\Delta\sigma}{TC_{em}} = c(T) \left(\frac{\mu_B}{T} \right)^2, \quad C_{em} = e^2 \sum_f q_f^2$$

- ▶ $c(T) \sim 0.007 \Rightarrow$ **BARYON DENSITY ENHANCES E.M. CONDUCTIVITY**
- ▶ $c(T)$ weakly depends on temperature
- ▶ At $\mu_q \sim T$ $\frac{\Delta\sigma}{\sigma} \sim 30\%$
reasonable agreement with Phys. Rev. C 89, 035203 (2014), Phys. Rev. C 91, 044903 (2015)

Conclusion:

- ▶ E.m. conductivity at finite baryon density was calculated
- ▶ **Baryon density enhances e.m. conductivity**

