Level densities of heavy and superheavy nuclei

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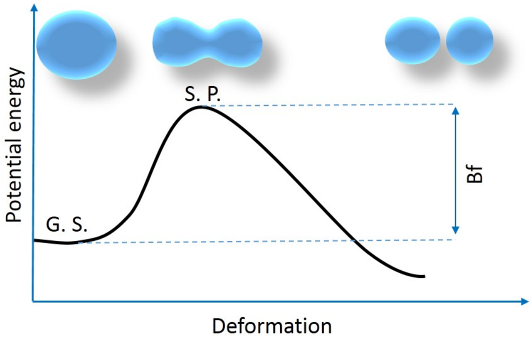
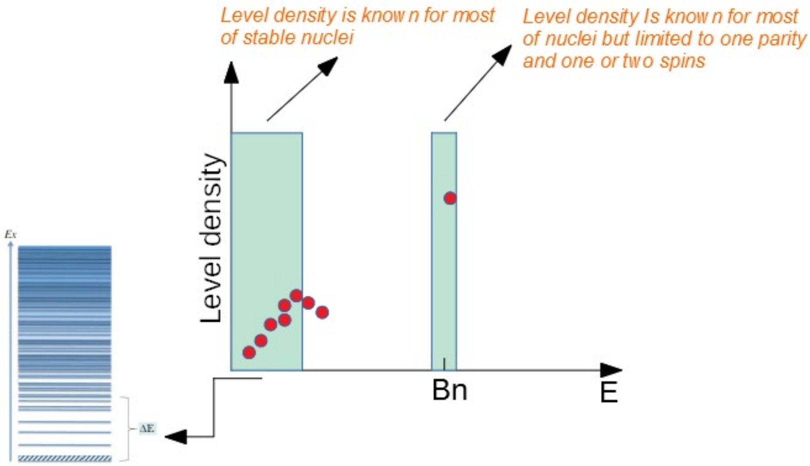
Outline

Why level density is important in superheavy region?!

Level density is the number of levels per energy unit (MeV).



$$\frac{\Gamma_f}{\Gamma_n} \propto \frac{\rho_f}{\rho_n}$$



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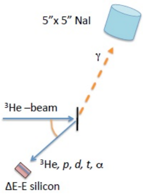
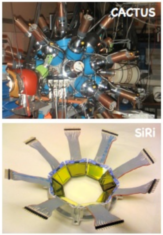
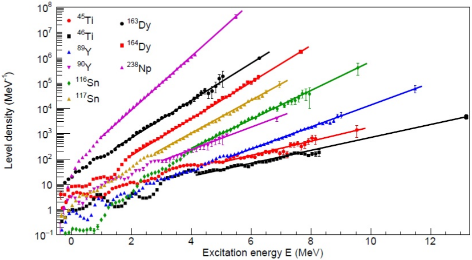






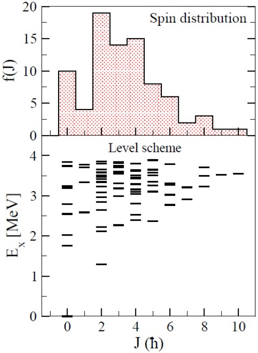
T. Renstrøm *et al.*, Phys. Rev. C **98**, 054310 (2018).

R. Chankova *et al.*, Phys. Rev. C **73**, 034311 (2006).

H. Utsunomiya *et al.*, Phys. Rev. C **88**, 015805 (2013).

M. Guttormsen *et al.*, Phys. Rev. C **68**, 064306 (2003).

H.T. Nyhus *et al.*, Phys. Rev. C **85**, 014323 (2012).



$$\rho(U, J, \pi) = F_\pi(U) F_J(U, J) \rho(U)$$

$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{12 a^{1/4} U^{5/4}} \exp\left(2\sqrt{aU}\right); \; U = aT^2$$

$$F_\pi = \frac{1}{2}$$

$$F_\pi(U) = \frac{1}{2} \tanh(\alpha \frac{U}{2})$$

$$F_J(U, J) = e^{-\frac{J^2}{2\sigma^2}} - e^{-\frac{(J+1)^2}{2\sigma^2}} \approx \frac{2J+1}{2\sigma^2} e^{\frac{-J(J+\frac{1}{2})}{2\sigma^2}}$$

$$\rho_{def}(U, J, \pi) = \rho_{spher}(U, J, \pi) K_{coll}; \; K_{coll} \propto \sigma_\perp^2$$

S. I. Al-Quraishi, S. M. Grimes, T. N. Massey, and D. A. Resler, *Phys. Rev. C* **67**, 015803 (2003).

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Aims

- To obtain survival probabilities for superheavy nuclei.
- To examine calculation formalism for lighter mass region where experimental data on nuclear level density is available.
- To study spin and parity distributions and collective effects.

Nucleus is considered as a system of independent quasiparticles.

The thermal equilibrium is assumed between neutron and proton subsystems.

$$\Omega = -\beta \sum_{\tau=p,n} \sum_{k} (\varepsilon_{\tau k} - \lambda_\tau - E_{\tau k}) + 2\sum_{k} \log[1 + \exp(-\beta E_{\tau k})] - \beta \frac{\Delta_\tau^2}{G_\tau}$$

The BCS equations, which determine the temperature dependence of Δ_τ and λ_τ , are derived from Ω.

$$N_\tau = \sum_k \left(1 - \frac{\varepsilon_{\tau k} - \lambda_\tau}{E_{\tau k}} \tanh \frac{\beta E_{\tau k}}{2}\right), \frac{2}{G_\tau} = \sum_k \frac{\tanh(\beta E_{\tau k})/2}{E_{\tau k}}$$

λ_τ, Δ_τ: chemical potential, pairing gap.

$E_{\tau k} = \sqrt{(\varepsilon_k - \lambda)^2 + \Delta^2}$: quasiparticle energies.

G_τ: The constant of the pairing interaction.

Data

Mass, shell correction and deformations of nuclei with $Z = 112 - 120$ at the ground state and at the saddle point.

Ground state								
Z	N	A	Mass (MeV)	E_{tot} (MeV)	ELD (MeV)	δE_{sh} (MeV)	β_2	β_3
112	165	277	151.79	-4.99	3.20	-8.18	0.2076	0.0000
112	166	278	152.43	-5.38	2.26	-7.64	0.2040	0.0000
112	167	279	154.31	-4.67	3.66	-8.32	0.2025	0.0000
112	168	280	155.25	-5.00	2.60	-7.60	0.1935	0.0018
112	169	281	157.40	-4.25	4.28	-8.53	0.1966	0.0000
112	170	282	158.31	-4.86	0.86	-5.71	0.1453	0.0000
112	171	283	160.24	-4.56	1.67	-6.23	0.1308	0.0006
112	172	284	161.41	-5.12	0.85	-5.97	0.1308	0.0006

Data

Saddle point								
Z	N	A	Mass (MeV)	E_{tot} (MeV)	ELD (MeV)	δE_{sh} (MeV)	β_2	β_3
112	165	277	156.25	-0.53	2.13	-2.65	0.34	0.00
112	166	278	156.44	-1.37	1.70	-3.08	0.33	0.00
112	167	279	158.30	-0.68	2.16	-2.84	0.33	0.00
112	168	280	159.03	-1.22	0.86	-2.08	0.31	0.00
112	169	281	161.28	-0.38	2.33	-2.70	0.28	0.00
112	170	282	162.05	-1.12	1.51	-2.63	0.28	0.00
112	171	283	164.81	0.02	2.31	-2.29	0.28	0.00
112	172	284	165.76	-0.78	1.19	-1.97	0.28	0.00
112	173	285	168.73	0.33	2.91	-2.58	0.28	0.00
112	174	286	169.90	-0.47	1.45	-1.91	0.28	0.00

$$E_\tau(T) = \sum_k \varepsilon_{k,\tau} \left(1 - \frac{\varepsilon_{k,\tau} - \lambda_\tau}{E_{k,\tau}} tanh \frac{\beta E_{k,\tau}}{2}\right) - \frac{\Delta_\tau^2}{G_\tau},$$

$$U(T) = \sum_\tau E_\tau(T) - E_\tau(0).$$

$$S(T) = \sum_\tau \sum_k \{ln[1 + exp(-\beta E_{k,\tau})] + \frac{\beta E_{k,\tau}}{1 + exp(\beta E_{k,\tau})}\}.$$

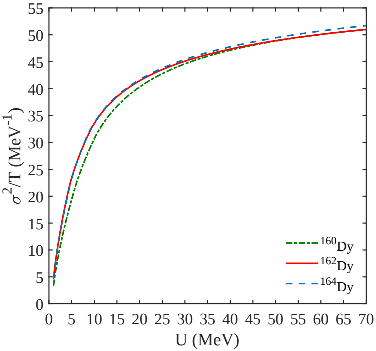
$$\rho_i(U) = \frac{\exp(S)}{(2\pi)^{\frac{3}{2}} \sqrt{D}}$$

$$\rho(U) = \frac{\rho_i(U)}{\sqrt{2\pi\sigma^2}}$$

$$\sigma^2 = \frac{1}{2}\sum_{\tau=p,n}\sum_{k} m_{\tau k}^2 \cosh^{-2}(1/2\beta E_{\tau k}),$$

$m_{\tau k}$: The single-particle spin projections.

$$\Im = \frac{\hbar^2\sigma^2}{T}$$



Nucleus	$\Im_{r.b.}$ ($\hbar^2$/MeV)
^{160}Dy	65.46
^{162}Dy	66.83
^{164}Dy	68.21

$$\Im_{r.b.} = 0.4MR^2$$

$$\rho_{tot}(U) = \sum_c \rho_i(U - U_c)\tau_c(U_c)$$

$$\tau_c(U_c) = 2I_c + 1$$

$$U_c = \hbar\omega_\beta(n_\beta + 1/2) + \hbar\omega_\gamma(2n_\gamma + |K|/2 + 1) + \frac{\hbar^2}{2\Im}\left[I_c(I_c + 1) - K^2\right]$$

n_β, n_γ: the quantum numbers of harmonic oscillator energies.

K: the projection of I_c on the symmetry axis.

$$\rho_{tot}(U) = \rho(U)K_{coll}$$

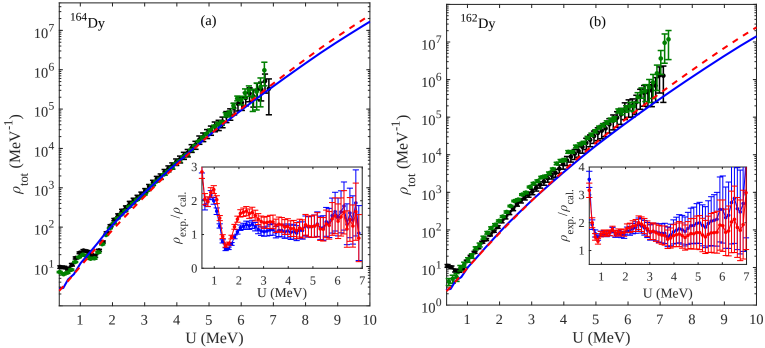
$$K_{coll} = K_{rot}K_{vib}$$

$$K_{vib} = \exp\left(0.0555A^{2/3}T^{4/3}\right)$$

$$K_{rot} = \begin{cases} 1, & \text{for spherical nuclei} \\ \Im_\perp T, & \text{for deformed nuclei,} \end{cases}$$

$$\Im_\perp = \Im_{r.b} f(\beta_2, \beta_4)$$

$$f(\beta_2, \beta_4) = 1 + \sqrt{5/16\pi}\beta_2 + (45/28\pi)\beta_2^2 + (15/7\pi\sqrt{5}\beta_2\beta_4)$$



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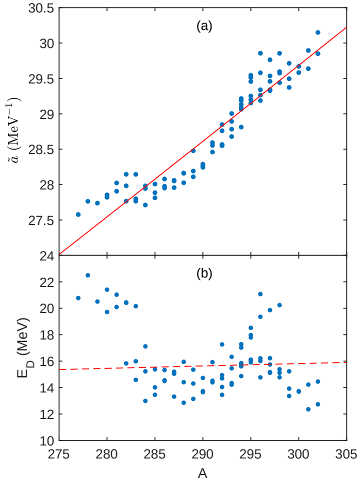
Green symbols: M. Guttormsen *et al.*, Phys. Rev. C **68**, 064306 (2003), and

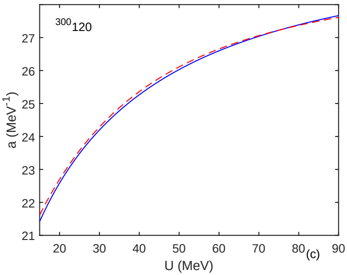
H.T. Nyhus *et al.*, Phys. Rev. C **85**, 014323 (2012).

Black symbols: T. Renstrøm *et al.*, Phys. Rev. C **98**, 054310 (2018).

$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{12 a[U]^{\frac{1}{4}} U^{\frac{5}{4}}} \exp(2\sqrt{a[U]U})$$

$$a(A, U) = \tilde{a}(A) \left[1 + \frac{1 - \exp(-\frac{U}{E_D})}{U} \delta E_{sh} \right]$$





$$\tilde{a} = a_1 A + a_2 A^2$$

$a_1 = 0.09$ MeV^{-1},
$a_2 = 2.89 \times 10^{-4}$ MeV^{-1},
$E_D \approx 15$ MeV.

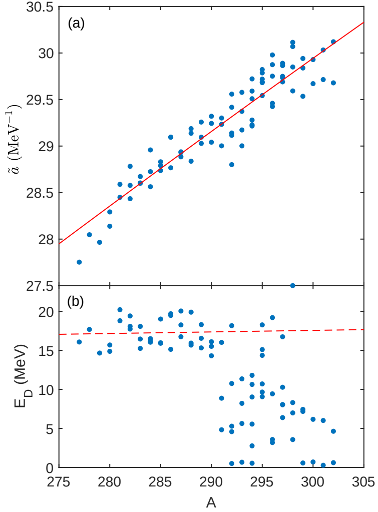
Saddle point

$|\delta E_{sh}| \leq 1.7$ MeV

$$\delta E_{sh} \rightarrow (\delta E_{sh} - \Delta)$$

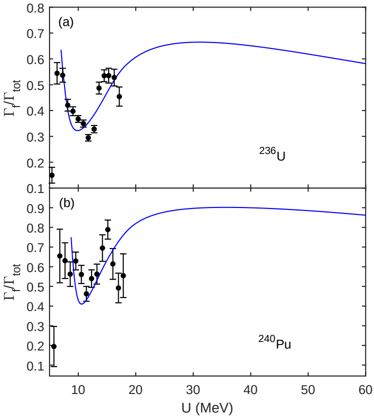
$$\tilde{a} = a_1 A + a_2 A^2$$

$a_1 = 0.1217$ MeV^{-1},
$a_2 = -7.3 \times 10^{-5}$ MeV^{-1},
$E_D \approx 17$ MeV.



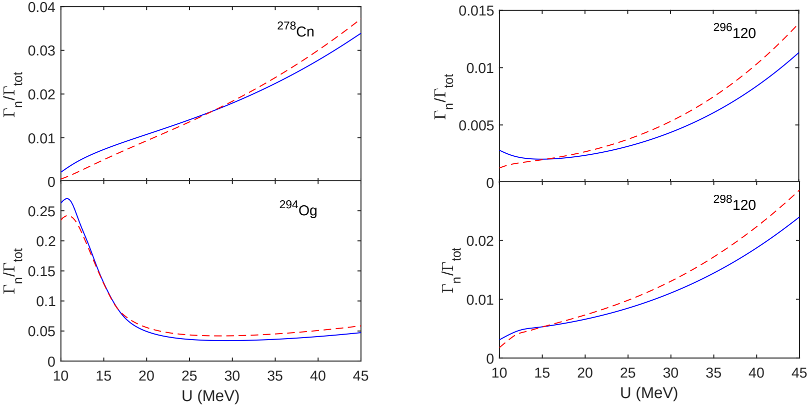
$$\frac{\Gamma_n}{\Gamma_f} = \frac{gA^{2/3}}{K_0} \frac{\int_0^{U-B_n} \varepsilon \rho_{GS} \left(U - B_n - \varepsilon\right) d\varepsilon}{\int_0^{U-B_f} \rho_{SP} \left(U - B_f - \varepsilon\right) d\varepsilon}$$

$$\frac{\Gamma_f}{\Gamma_{tot}} = \frac{1}{1 + \Gamma_n/\Gamma_f}$$



E. Cheifetz, H. C. Britt, and J. B.Wilhelmy, Phys. Rev. C**24**, 519 (1981).

$$\frac{\Gamma_n}{\Gamma_f} = \frac{4A^{2/3} a_f \,(U - B_n - \Delta_n)}{K_0 a_n \left[2 a_f^{1/2} (U - B_f - \Delta_f)^{1/2} - 1\right]} \times \exp\left[2 a_n^{1/2} (U - B_n - \Delta_n)^{1/2} - 2 a_f^{1/2} (U - B_f - \Delta_f)^{1/2}\right]$$



π^g: The same parity as the whole nucleus at the ground state

π^s: The parity opposite to the ground state

$$P(n) = \frac{f^n}{n!} e^{-f}$$

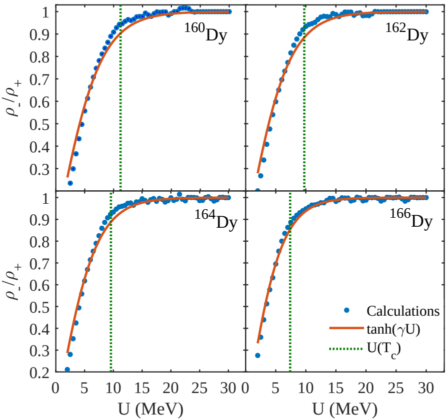
$$f = \sum_{k \in \pi_s} \frac{1}{1 + \exp(\beta E_k)}$$

$$\frac{\rho^s}{\rho^g} = \frac{\beta^s}{\beta^g} \frac{Z^s}{Z^g} \sqrt{\frac{C^g}{C^s}} e^{(\beta^s - \beta^g)U}$$

D. Mocelj *et al.*, *Phys. Rev. C* **75**, 045805 (2007).

$$\frac{\rho_-}{\rho_+} = tan\gamma U$$

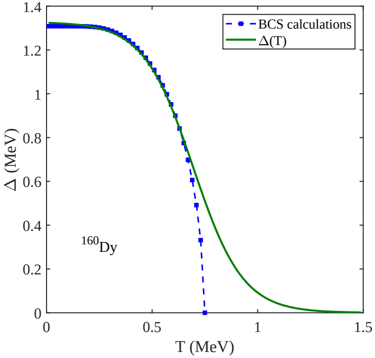
Nucleus	γ (MeV^{-1})
^{160}Dy	0.133
^{162}Dy	0.143
^{164}Dy	0.147
^{166}Dy	0.172



S. I. Al-Quraishi, S. M. Grimes, T. N. Massey, and D. A. Resler, *Phys. Rev. C* **67**, 015803 (2003).

Thank you for your attention!

$$\Delta = \frac{\Delta_0}{1 + \exp\left(\frac{T - T_m}{f_m}\right)}$$



$$P(n) = \frac{f^n}{n!} e^{-f}$$

$$P^+ = \sum_n^{even} \frac{f^n}{n!} e^{-f} = e^{-f} \cosh f$$

$$P^- = \sum_n^{odd} \frac{f^n}{n!} e^{-f} = e^{-f} \sinh f$$

$$\frac{P^-}{P^+} = \tanh f$$

$$\frac{P^s}{P^g} = \tanh f', \; f' = f_p + f_n$$

$$P^{s(g)} = \frac{Z^{s(g)}}{Z}$$

$$Z^s = \frac{Z}{1 + \frac{1}{\tanh f'}}$$

$$Z^g = \frac{Z}{1 + \tanh f'}$$

$E_\tau(T) = \sum_k \varepsilon_{k,\tau} \left(1 - \frac{\varepsilon_{k,\tau} - \lambda_\tau}{E_{k,\tau}} tanh \frac{\beta E_{k,\tau}}{2}\right) - \frac{\Delta_\tau^2}{G_\tau},$

$U(T) = \sum_\tau E_\tau(T) - E_\tau(0).$

$S(T) = \sum_\tau \sum_k \{ln[1 + exp(-\beta E_{k,\tau})] + \frac{\beta E_{k,\tau}}{1+exp(\beta E_{k,\tau})}\}.$

$C_\tau(T) = \frac{1}{2} \sum_k \cosh^{-2} \left(\frac{\beta E_{k,\tau}}{2}\right) \left[\beta^2 E_{k,\tau}^2 - \beta \Delta_\tau \frac{d\Delta_\tau}{dT}\right].$

$U^{s(g)} = -\frac{\partial \ln Z^{s(g)}}{\partial \beta}$

$$U^g = U + \left[(1 - \tanh(f))\frac{\partial f}{\partial \beta}\right]$$

$$U^s = U + \left[(1 - \coth(f))\frac{\partial f}{\partial \beta}\right]$$

$S^{s(g)} = \beta U^{s(g)} + \ln Z^{s(g)}$

$$S^g = \beta U^g + S - \log[(1 + \tanh(f))]$$

$$S^s = \beta U^s + S - \log[(1 + \coth(f))]$$

$C^{s(g)} = -\beta^2 \frac{\partial^2 \ln Z^{s(g)}}{\partial \beta^2}$

$$C^g = C - \beta^2(1 - \tanh(f)) \left[(1 + \tanh(f)) \left(\frac{\partial f}{\partial \beta}\right)^2 - \left(\frac{\partial^2 f}{\partial \beta^2}\right)\right]$$

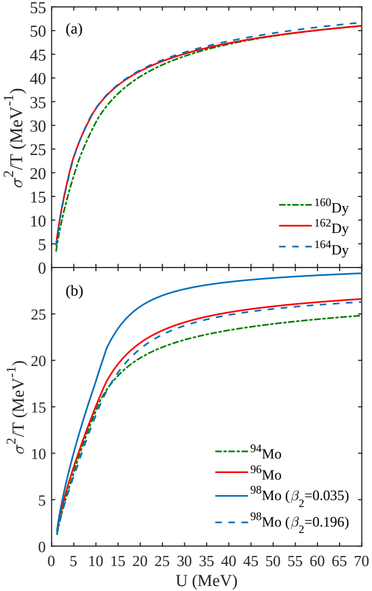
$$C^s = C - \beta^2(1 - \coth(f)) \left[(1 + \coth(f)) \left(\frac{\partial f}{\partial \beta}\right)^2 - \left(\frac{\partial^2 f}{\partial \beta^2}\right)\right]$$

The spin cut-off factor is related to the effective moment of inertia

$$\Im = \frac{\hbar^2 \sigma^2}{T}$$

Nucleus	$\Im_{r.b.}$ ($\hbar^2$/MeV)	$\Im_{exp}$ ($\hbar^2$/MeV)
^{160}Dy	65.46	34.56
^{162}Dy	66.83	37.19
^{164}Dy	68.21	40.87
^{94}Mo	26.98	3.44
^{96}Mo	27.94	3.85
^{98}Mo	28.92	4.08

$\Im_{r.b.} = 0.4MR^2$, $\Im_{exp} = 3/E_{2^+}$.



Assumption of a decoupling between intrinsic and collective degrees of freedom

$$U = U_i + U_c$$

$$\rho_{tot}(U) = \int \rho_i(U_i)\rho_{coll}(U - U_i)dU_i$$

$$\rho_{coll}(U - U_i) = \sum_c \delta(U - U_i - U_c)\tau_c(U_c).$$

$$\tau_c(U_c) = 2I_c + 1$$

$$\rho_{tot}(U) = \sum_c \rho_i(U - U_c)\tau_c(U_c)$$

$$\rho_{tot}(U) \simeq \sum_c \left[\rho_i(U) - U_c \frac{\partial \rho_i(U)}{\partial U}\right] \tau_c(U_c)$$

$$= \sum_c \left[\rho_i(U) - \frac{U_c}{T}\rho_i(U)\right] \tau_c(U_c).$$

$$\rho_{tot}(U) \simeq \rho_i(U) \sum_c \exp(-\frac{U_c}{T})\tau_c(U_c)$$

$$K_{coll} = \sum_c \exp(-\frac{U_c}{T})\tau_c(U_c)$$

$$U_c = \hbar\omega_\beta(n_\beta + 1/2) + \hbar\omega_\gamma(2n_\gamma + |K|/2 + 1) + \frac{\hbar^2}{2\Im}\left[I_c(I_c+1) - K^2\right]$$

n_β, n_γ: the quantum numbers of harmonic oscillator energies.

K: the projection of I_c on the symmetry axis.

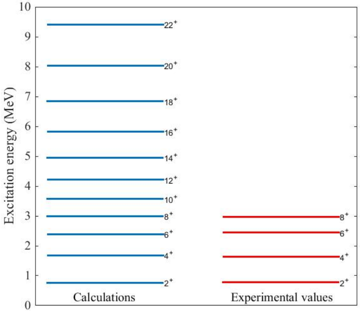
Yrast band

Quantum numbers:
$K = 0,\ I = 0, 2, 4, \ldots$
$n_\beta = n_\gamma = 0$
$U_c = \frac{\hbar^2}{2\Im}\left[I(I+1)\right].$

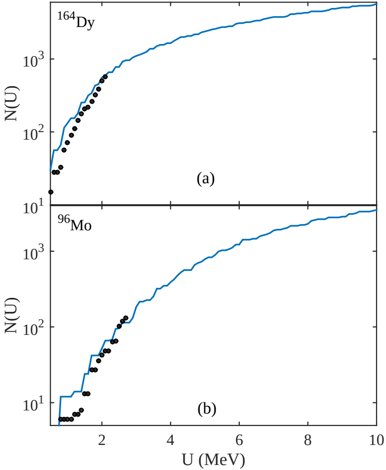
$E(2^+) = \frac{2\times3}{2\Im}$

$\Im_{exp} = \frac{3}{E(2^+)}.$

$\Im = \Im_{r.b}\left(1 - a_1 e^{-a_2 I(I+1)}\right)$
$a_1 = 0.89,\ a_2 = 0.006$



D. Abriola, and A. A. Sonzogni, Nuclear Data Sheets **107**, 2423 (2006).



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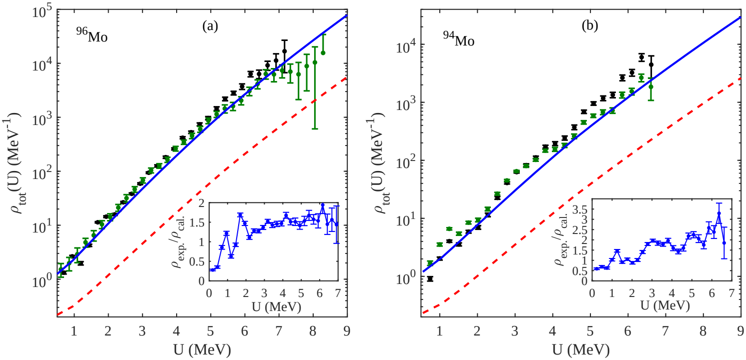

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