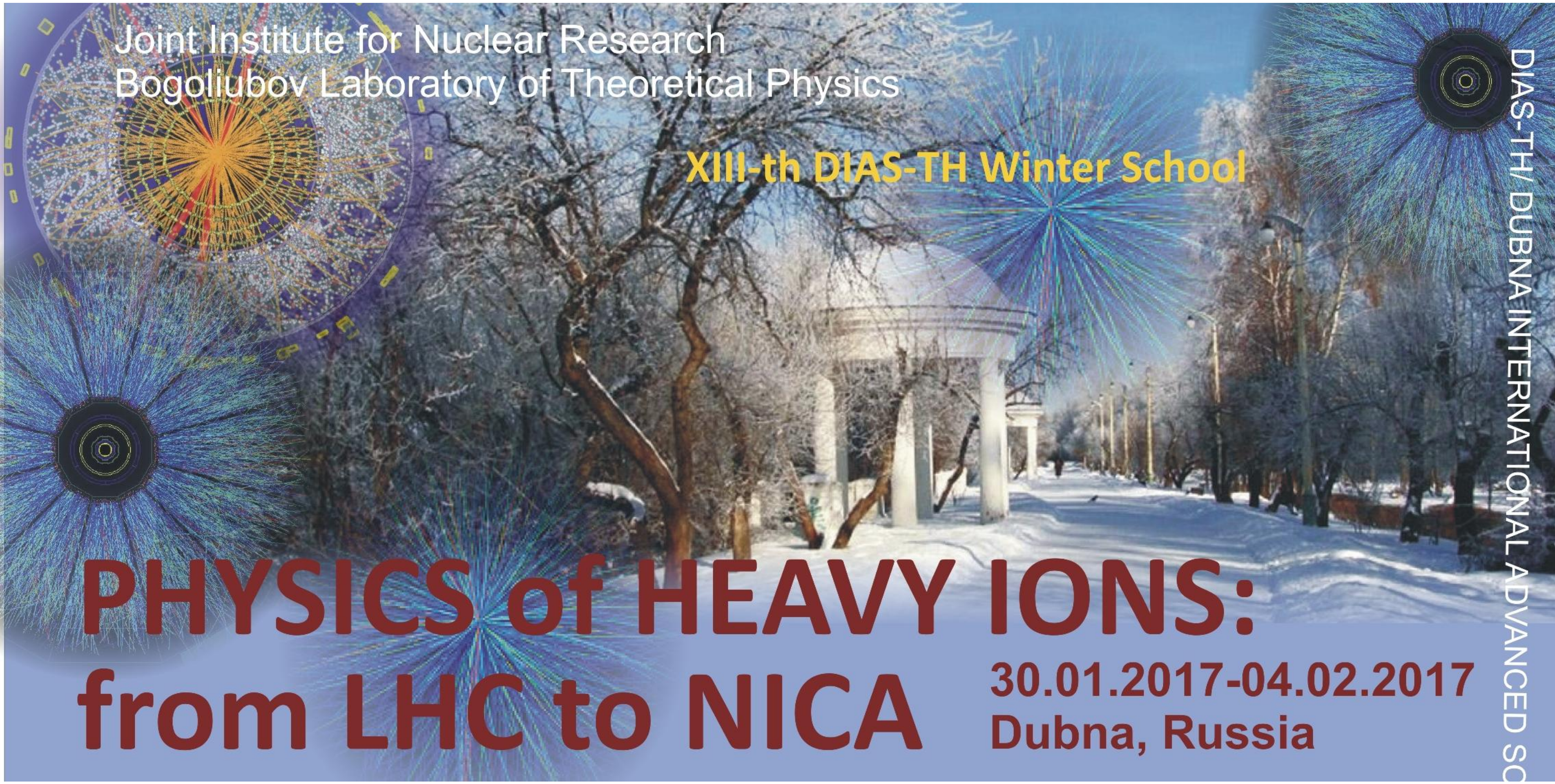
Holography for NICA

Irina Aref'eva
Steklov Mathematical Institute, RAS



Outlook

- Physical picture of Quark-Gluon Plasma in heavy-ions collisions
- Why holography?
- Results from holography:

  Fit experimental data via holography;
  top-down (top=string theory)
  bottom-up (bottom=5-dim GR+matter)

  Predict new data

- What is special for NICA

И. Я. Арефьева, “Голографическое описание кварк-глюонной плазмы, образующейся при столкновениях тяжёлых ионов”, УФН, 184:6 (2014), 569–598

I.A,``Holography for Heavy Ions Collisions at LHC and NICA, arXiv:1612.08928

From observations in HIC

- QGP strong interacting fluid
- Measurement of energy lost (jet quenching, R_{AA}-factor, J/Psi suppressions
- Transport coefficients, extremely small eta/s
- Phase transition (still near small mu)
- Energy dependence of the total multiplicity $s^{0.155}$
- Thermalization time
- Direct photons (electric conductivity)

Holography:

Connection between

a strongly coupled quantum field theory in a D-dimensional spacetime

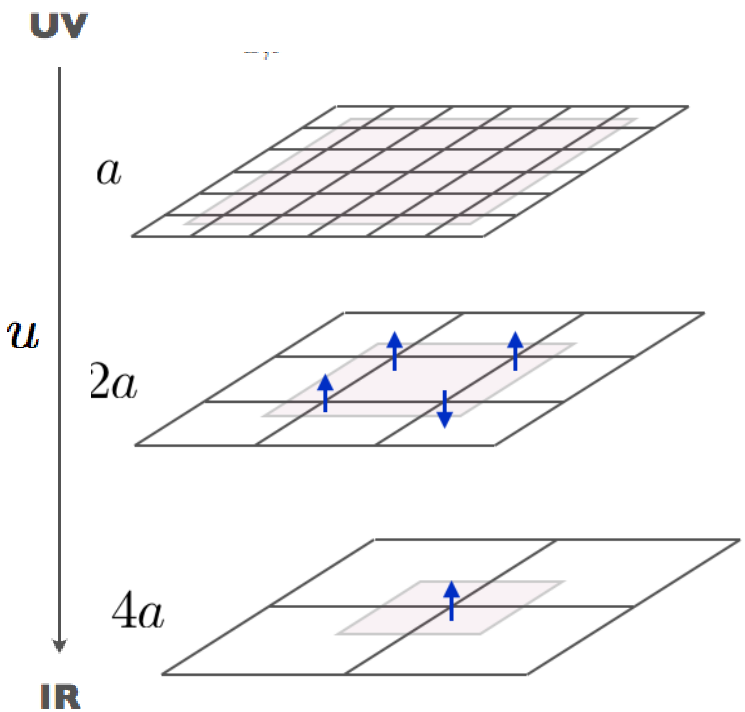
and a D+1-dimensional classical gravity in a special background

Holographic Duality & RG flow a la Kadanoff and Wilson

RG scale -> an extra spatial dimension

Currents -> dynamical filed

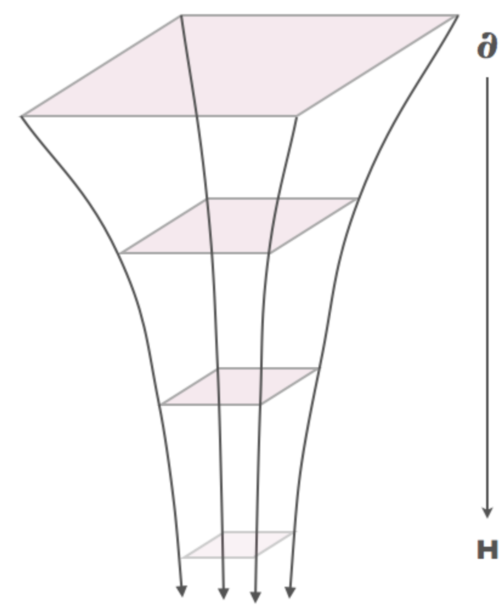
$$J_i|_{UV} \qquad = \qquad \Phi_i|_\partial$$



$$H = \sum_i J_i(x,a)\,\mathcal{O}^i(x)$$

$$H = \sum_i J_i(x,2a)\,\mathcal{O}^i(x)$$

$$H = \sum_i J_i(x,4a)\,\mathcal{O}^i(x)$$



$$u\frac{\partial}{\partial u} J_i(x,u) = \beta_i(J_j(x,u),u)$$

From: **arXiv:1205.5180**

Few mathematical background

Conformal group acting in $\mathbb{R}^{1,d-1}$

Definition.The conformal group of Minkowski space $\mathbb{R}^{1,d-1}$ *is generated by the Poincare transformations, the scale transformation*

$$x_\mu \to \lambda x_\mu \tag{A.1}$$

and the special conformal transformations

$$x_\mu \to \frac{x_\mu + a_\mu}{x^2 + 2x^\mu a_\mu + a^2} \tag{A.2}$$

---

the conformal algebra $(d > 2)$

$$\begin{aligned} P_\mu &= -i\partial_\mu \\ D &= -ix^\mu \partial_\mu \\ L_{\mu\nu} &= i\left(x_\mu \partial_\nu - x_\nu \partial_\mu\right) \\ K_\mu &= -i\left(2x_\mu x^\nu \partial_\nu - x^2 \partial_\mu\right) \end{aligned}$$

$$\begin{aligned} [M_{\mu\nu}, P_\rho] = i(\eta_{\mu\rho} P_\nu - \eta_{\nu\rho} P_\mu), \;\; [M_{\mu\nu}, K_\rho] &= i(\eta_{\mu\rho} K_\nu - \eta_{\nu\rho} K_\mu) \\ [M_{\mu\nu}, D] = 0, \;\; [P_\mu, K_\nu] &= -2(\eta_{\mu\nu} D + iM_{\mu\nu}), \\ [D, P_\mu] = P_\mu, \;\; [D, K_\mu] &= -K_\mu. \end{aligned}$$

AdS $_{d+1}$

Definition of AdS_{d+1}. *We take* $\mathbb{R}^{2,d}$ *which has the metric*

$$ds^2 = -dX_0^2 - dX_{d+1}^2 + \sum_{\mu=1}^{d} dX_\mu^2,$$

and define AdS_{d+1} *is a quadric*

$$X_M X^M = -1.$$

SO(2,d) -isometry

$$J_{MN} = -i(X_M \partial_N - X_N \partial_M)$$

$$[J_{MN}, J_{M'N'}] = i\Big(\eta_{MN'} J_{NM'} + \eta_{NM'} J_{MN'} - \eta_{MM'} J_{NN'} - \eta_{NN'} J_{MM'}\Big)$$

Conformal algebra as the isometry algebra of AdS

The conformal algebra is correctly reproduced from the AdS$_{d+1 \text{isometry}}$ if we identify

$$P_\mu = J_{\mu,d+1} - iJ_{\mu,0} \qquad K_\mu = J_{\mu,d+1} + iJ_{\mu,0} \qquad D = -J_{0,d+1} \qquad M_{\mu\nu} = J_{\mu\nu}$$

$$\begin{aligned} P_\mu &= -i\partial_\mu \\ D &= -ix^\mu\partial_\mu \\ L_{\mu\nu} &= i\left(x_\mu\partial_\nu - x_\nu\partial_\mu\right) \\ K_\mu &= -i\left(2x_\mu x^\nu\partial_\nu - x^2\partial_\mu\right) \end{aligned}$$

AdS $_{d+1.}$ Parametrization

$$X_0 = \frac{\cos t}{\cos\rho}, \qquad X_{d+1} = \frac{\sin t}{\cos\rho}, \cdots \qquad X_\mu = \tan\rho\,\Omega_\mu. \qquad \vec{\Omega}^2 = 1 \qquad \mu = 1, \ldots d$$

$$ds^2 = \frac{1}{\cos^2\rho}\left(-dt^2 + d\rho^2 + \sin^2\rho\, d\Omega^2\right)$$

The center of AdS lies at $\rho = 0$ and the boundary at $\rho = \pi/2$.

The boundary manifold is $\mathbb{R} \times S^{d-1}$

$$\boxed{\tanh\chi = \sin\rho}$$

$$D = i\frac{\partial}{\partial t}, \qquad P_\mu = -ie^{-it}\left[\Omega_\mu\left(\partial_\chi - i\tanh\chi\,\partial_t\right) + \frac{1}{\tanh\chi}\nabla_\mu\right]$$

$$M_{\mu\nu} = -i\left(\Omega_\mu\frac{\partial}{\partial\Omega^\nu} - \Omega_\nu\frac{\partial}{\partial\Omega^\mu}\right) \qquad K_\mu = ie^{it}\left[\Omega_\mu\left(-\partial_\chi - i\tanh\chi\,\partial_t\right) - \frac{1}{\tanh\chi}\nabla_\mu\right]$$

$$\nabla_\mu = \frac{\partial}{\partial\Omega^\mu} - \Omega_\mu\Omega^\nu\frac{\partial}{\partial\Omega^\nu}$$

Klein-Gordon-Fock equation on AdS:

$$\frac{1}{2} J_{AB} J^{BA} \phi = \Delta(\Delta - d)\phi \qquad m^2 = \Delta(\Delta - d)$$

Fundamental solution

$$G_N^{\Delta}(X, X') = \frac{e^{-\Delta \mathcal{L}(X,X')}}{e^{-2\mathcal{L}(X,X')} - 1}$$

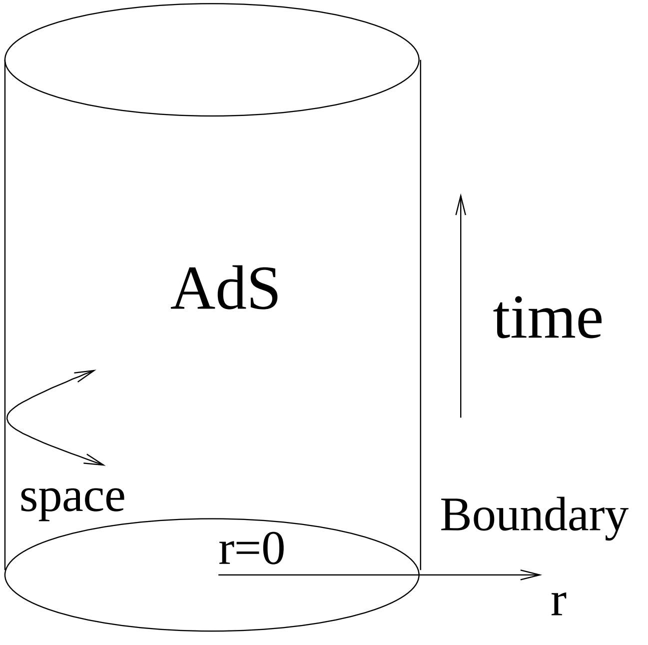
$$\cosh \mathcal{L} = -\eta_{MN} X^M X'^N$$

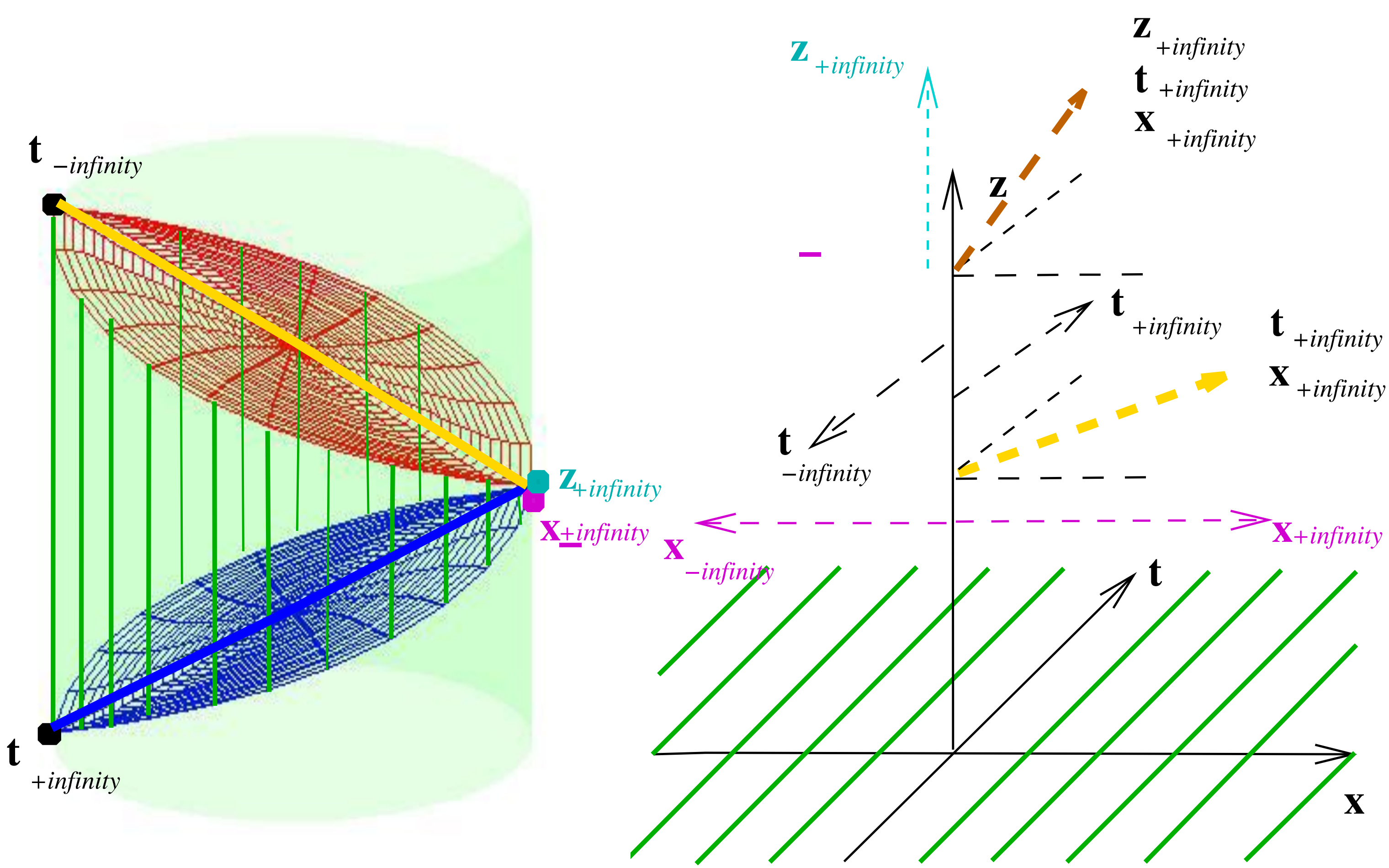
Euclidean AdS $i\epsilon$

Klein-Gordon-Fock equation on AdS:

$$G_N(t,\chi,\vec{\Omega}|t',\chi',\vec{\Omega}') \sim \frac{1}{\chi'^{\Delta}\chi^{\Delta}}\, G_{\partial\partial}(t,\vec{\Omega}|t',\vec{\Omega}')$$

$$G_{NN}(t,\chi,\vec{\Omega}|t',\chi',\vec{\Omega}') \sim \frac{1}{\chi'^{d-\Delta}}\, K(t,\chi,\vec{\Omega}|t',\vec{\Omega}')$$


AdS
time
space
Boundary
r=0
r

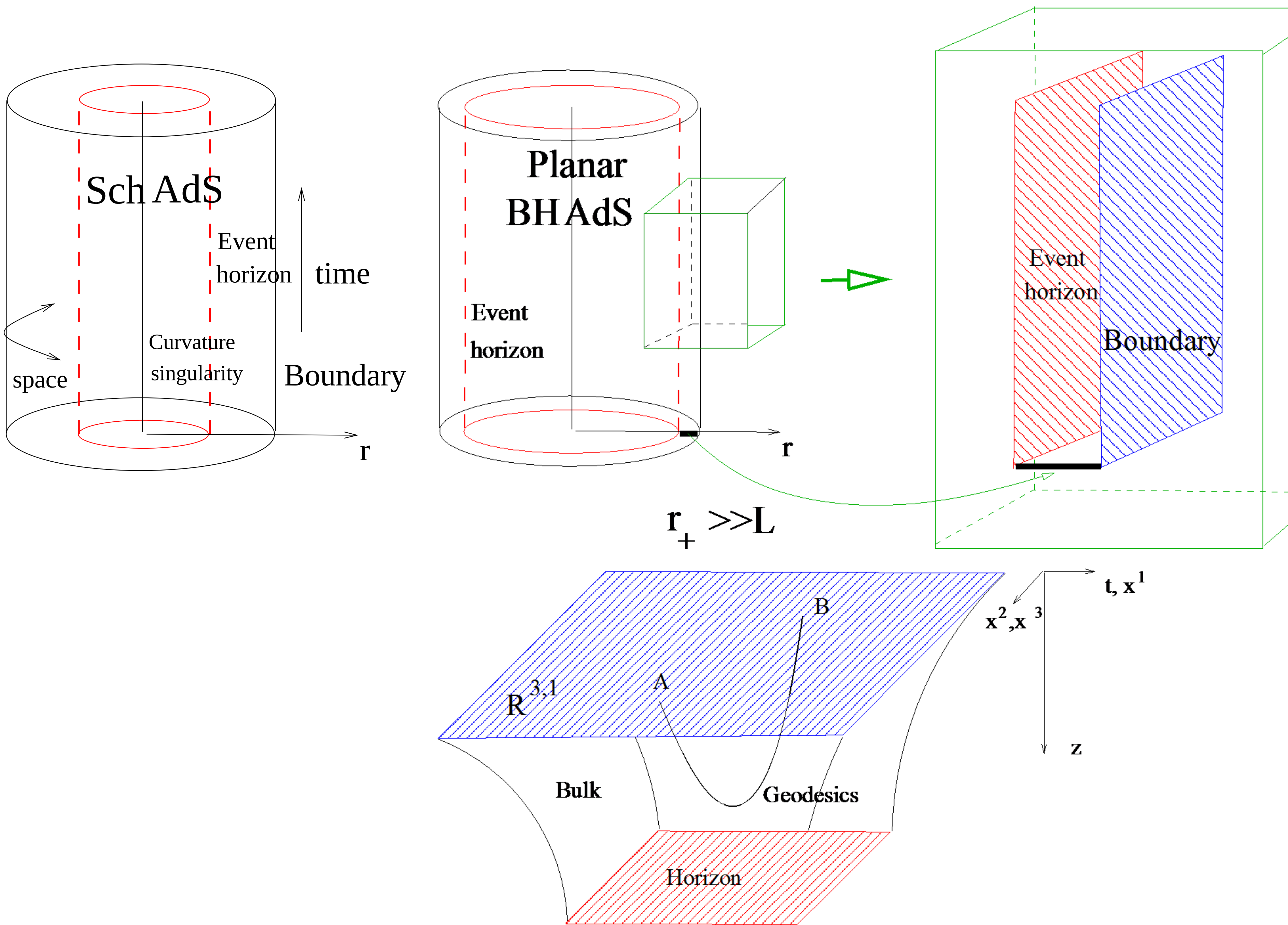


$$z = \frac{(1 - r^2)}{(1 + r^2)\cos\tau - 2r\cos\varphi},$$

$$t = \frac{(1 + r^2)\sin\tau}{(1 + r^2)\cos\tau - 2r\cos\varphi}$$

$$x = \frac{-2r\sin\varphi}{(1 + r^2)\cos\tau - 2r\cos\varphi}$$

Global -> Poincare



Sch AdS
Event horizon
time
space
Curvature singularity
Boundary
r
Planar BH AdS
Event horizon
r
Event horizon
Boundary
r+ >>L
t, x1
x2,x3
z
B
A
R3,1
Bulk
Geodesics
Horizon

Let me remind: QGP as a strongly coupled fluid

- **Conclusion from the RHIC and LHC experiments: appearance of QGP (not a weakly coupled gas of quarks and gluons, but a strongly coupled fluid).**

  Based on modification of particle spectra (compared to p+p); jet quenching; high p_T-suppression of hadrons; elliptic flow;suppression of quarkonium production

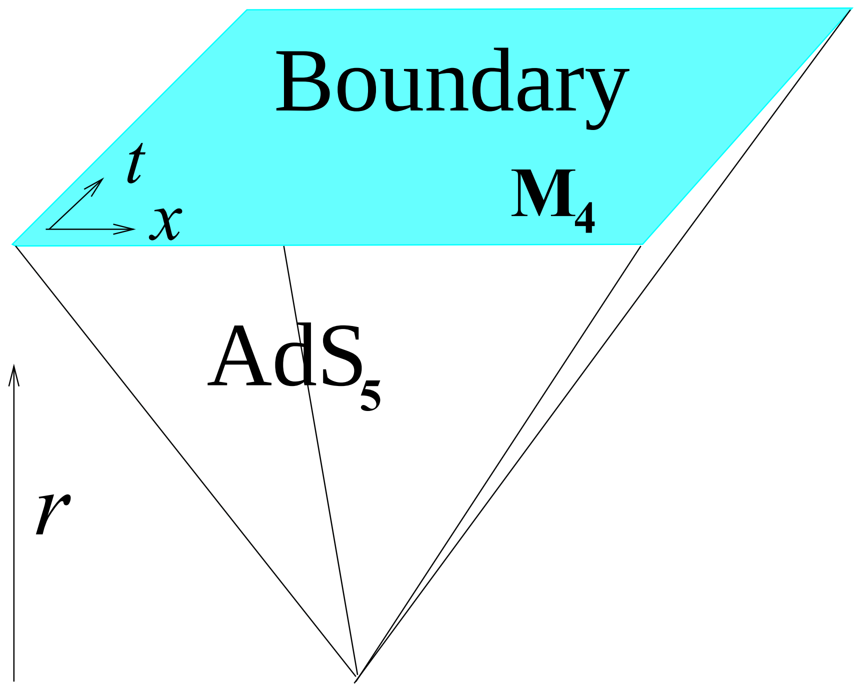
- This makes perturbative methods inapplicable

- The lattice formulation of QCD does not work for description of QGP formation, since we have to study real-time phenomena.

- This has provided a motivation to try to understand the dynamics of QGP through the **gauge/string duality**

Holography

Relation between 4-dim QFT theory and 5-dim classical solutions in gravity



Conformal symmetry

Maldacena,1998
……
O.Aharony et al.
Phys.Rep.323(2000)183

A kind of a “phenomenology”

Dual description of QGP as a part of Gauge/string duality

- There is an approximate gravity dual construction for QCD.

- Lattice calculations show that QCD exhibits a quasi-conformal behavior at temperatures T >300 MeV and the equation of state can be approximated by $E = 3P$ (a traceless conformal energy-momentum tensor).

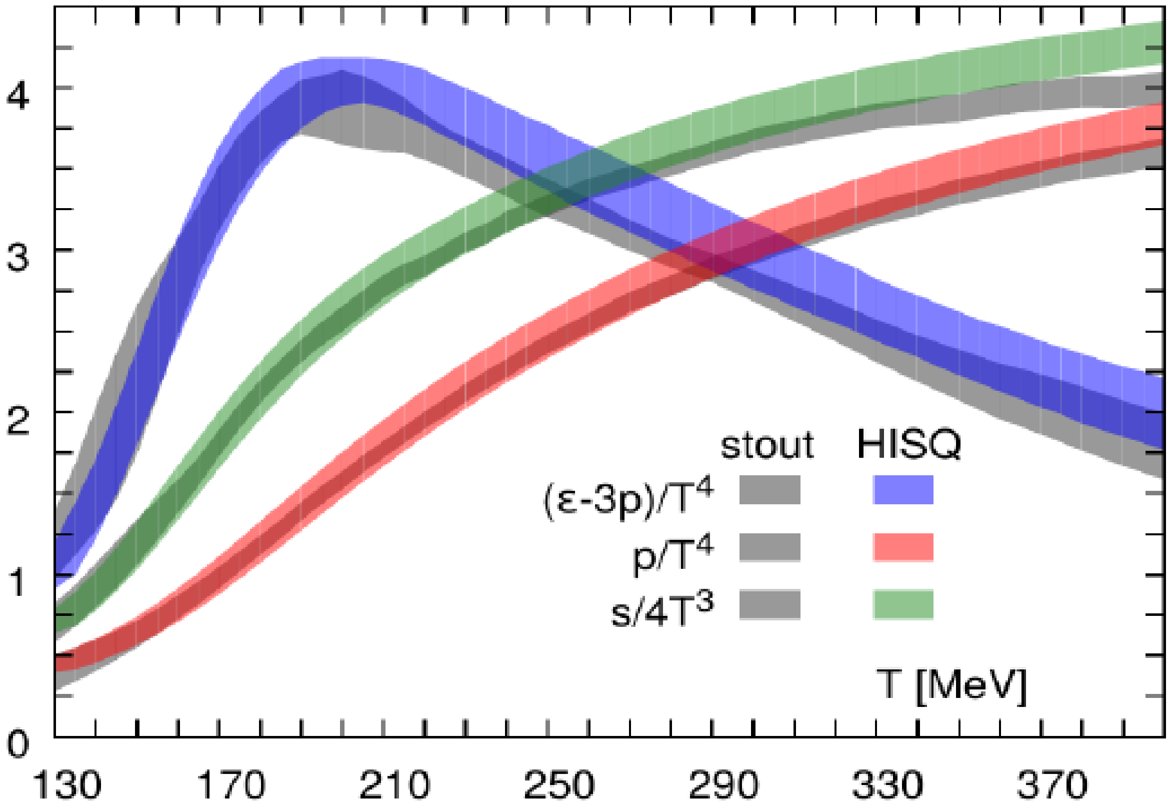
- Use the AdS/CFT correspondence as a tool to get non-perturbative dynamics of QGP.

Reviews: Solana, Liu, Mateos, Rajagopal, Wiedemann, 1101.0618

I.A., Holographic approach for QGP in HIC, UFN, 184, 2014;

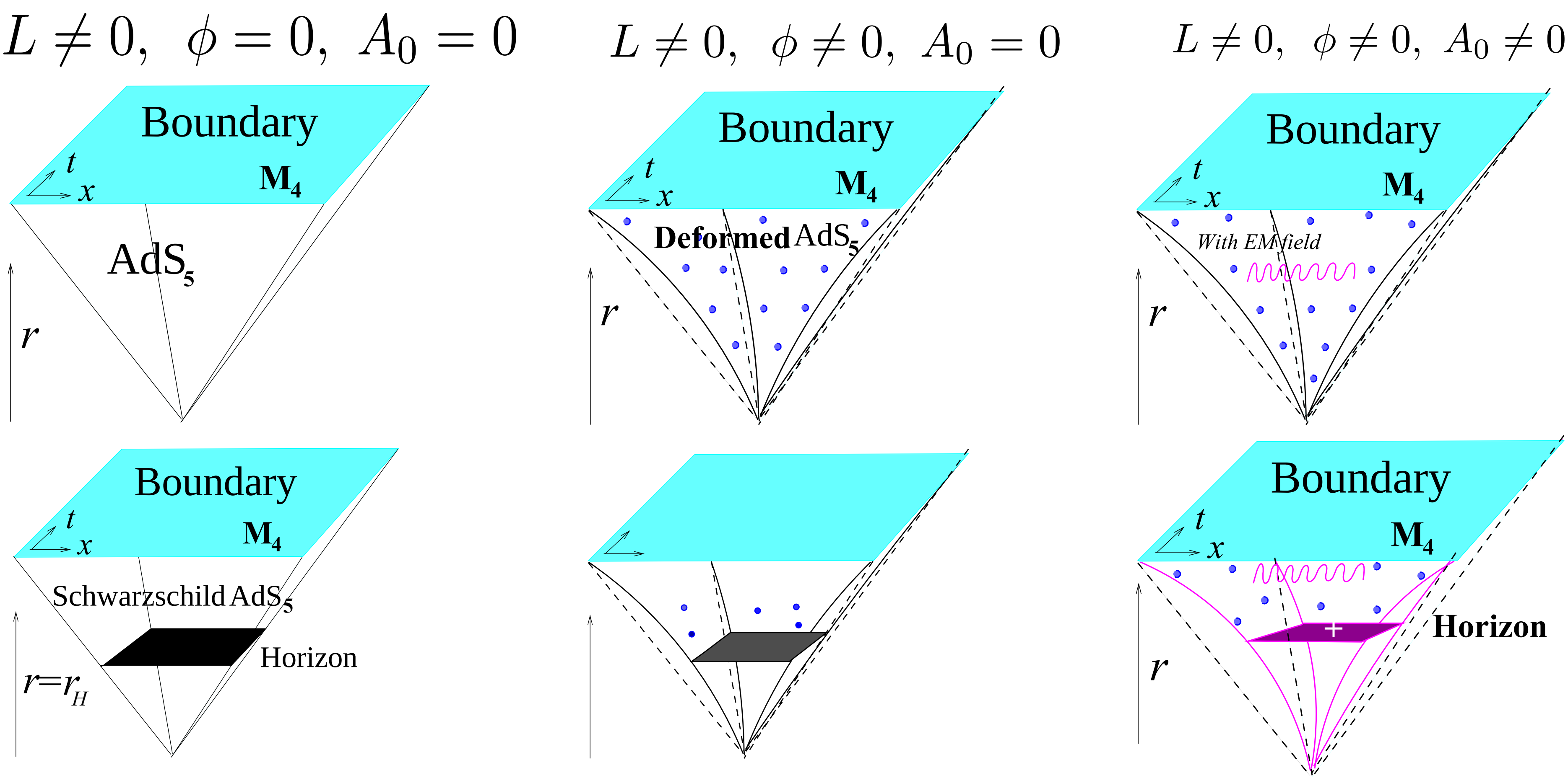
DeWolfe, Gubser, Rosen,Teaney, HI and string theory, Prog. Part.Nucl.Phys., 75, 2014
P.M.Chesler, W. van der Schee, Early thermalization, 1501.04952 [nucl-th]

Deviation from conformal symmetry



From:

Choice of the background = choice of phenomenological model



Background

$$ds^2 = \frac{b(z)}{z^2}\left((-f(z)dt^2 + dx^2) + z^{2-2/\nu}(dy_1^2 + dy_2^2) + \frac{dz^2}{f(z)}\right)$$

$$b(z) = \exp(\frac{cz^2}{2}) \qquad f = 1 - (\frac{1}{z_h^{2/\nu+2}} + q^2 z_h^2) z^{2/\nu+2} + q^2 z^{2/\nu+4}$$

Generalization of

1) b(z): **O. Andreev, V. Zakharov,Phys.Rev, D749 (2006);JHEP 0704(2007)**

Alternative: Gubser et al 0804.0434, 1108.229…
Kiritsis et al, 0903.2859, Evans et al 1002.1885

2) Charge, **P.Colangelo, F.Giannuzzi, S.Nicotri, 1008.3116**

Y.Yang, P.H.Yuan, 1506.05930

3) Anizotropy: **IA, A. Golubtsova JHEP 1504 (2015) 011**

(this anisotropy reproduces the energy dependence of multiplicity)

Thermodynamics (of 5 -dim)

$$T = \frac{1}{4\pi}\left|\frac{df}{dz}\right|_{z=z_h} = \frac{1}{2\pi}\,\frac{\frac{1}{\nu}+1}{z_h}\left(1-\frac{1}{\frac{1}{\nu}+1}q^2 z_h^{\frac{2}{\nu}+4}\right)$$

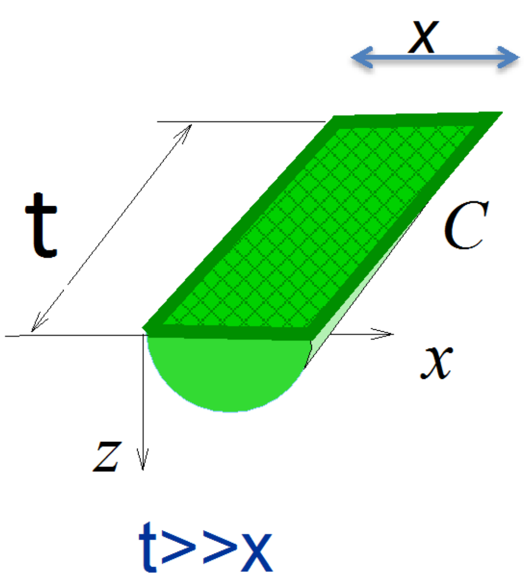
The quark chemical potential $A_0(0) = i\mu$

$$A_0(z) = i\left(\mu - \frac{4\sqrt{3}\,q}{c}\left(1-e^{-cz^2/4}\right)\right)$$

$$T = \frac{\left(\frac{1}{\nu}+1\right)}{2\pi z_h}\left(1-\frac{c^2\mu^2 z_h^{\frac{2}{\nu}+4}}{3\left(\frac{1}{\nu}+1\right)\left(1-e^{-\frac{cz_h^2}{2}}\right)^2}\right)$$

Temporal Wilson loop in the charged quark confinement background

Energy between quarks located along x-direction



$$W(T,X) = \langle \mathrm{Tr}_F\, e^{i\oint_{T\times X} dx_\mu A_\mu} \rangle \sim e^{-V(X)T},$$

Holography for a probe

$$S_{xt} = \frac{1}{2\pi\alpha'}\int d\sigma^1 d\sigma^2 \sqrt{-\det(h_{\alpha\beta})}$$

$$S_{xt} = \frac{T}{2\pi\alpha'}\int \frac{b(z)}{z^2}\sqrt{f(z)+z'^2}\,dx.$$

Temporal Wilson loop in the charged quark confinement background

Energy between quarks located along x-direction

$$S_{xt} = \frac{T}{2\pi\alpha'}\int \frac{b(z)}{z^2}\sqrt{f(z)+z'^2}\,dx.$$

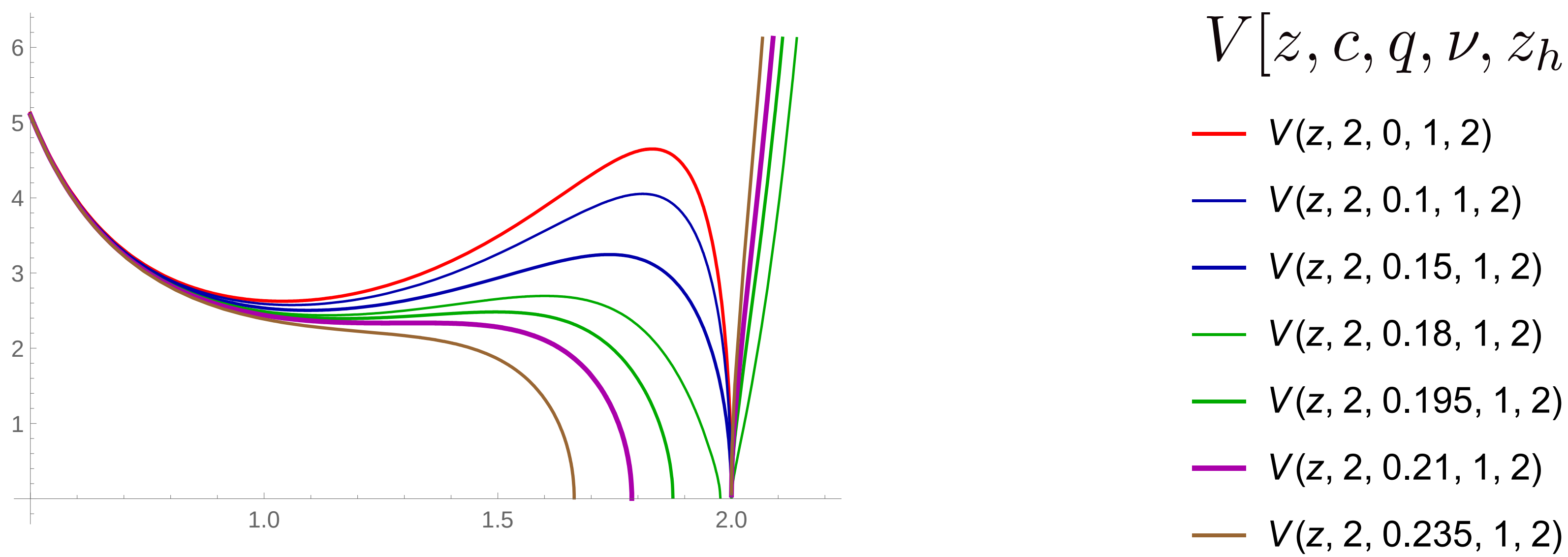
"Potential" $$V_x(z) = \frac{b(z)}{z^2}\sqrt{f(z)}$$

Symmetric parameterization $z(\pm\ell) = 0 \quad z(0) = z_* \quad z'(0) = 0$

Distance between endpoints of the string

$$L_x = 2\int_{\infty}^{z_*}\frac{dz}{z'} = 2\int_0^{z_*}\frac{dz}{\sqrt{f(z)}\sqrt{\left(\frac{V_x^2(z)}{V_x^2(z_*)}-1\right)}}$$

$z_* < z_{h_{i_0}}$ the smallest of the horizons $f(z) > 0 \;\; \text{for} \;\; 0 < z < z_*$



Temporal Wilson loop in the charged quark confinement background

Energy between quarks located along x-direction

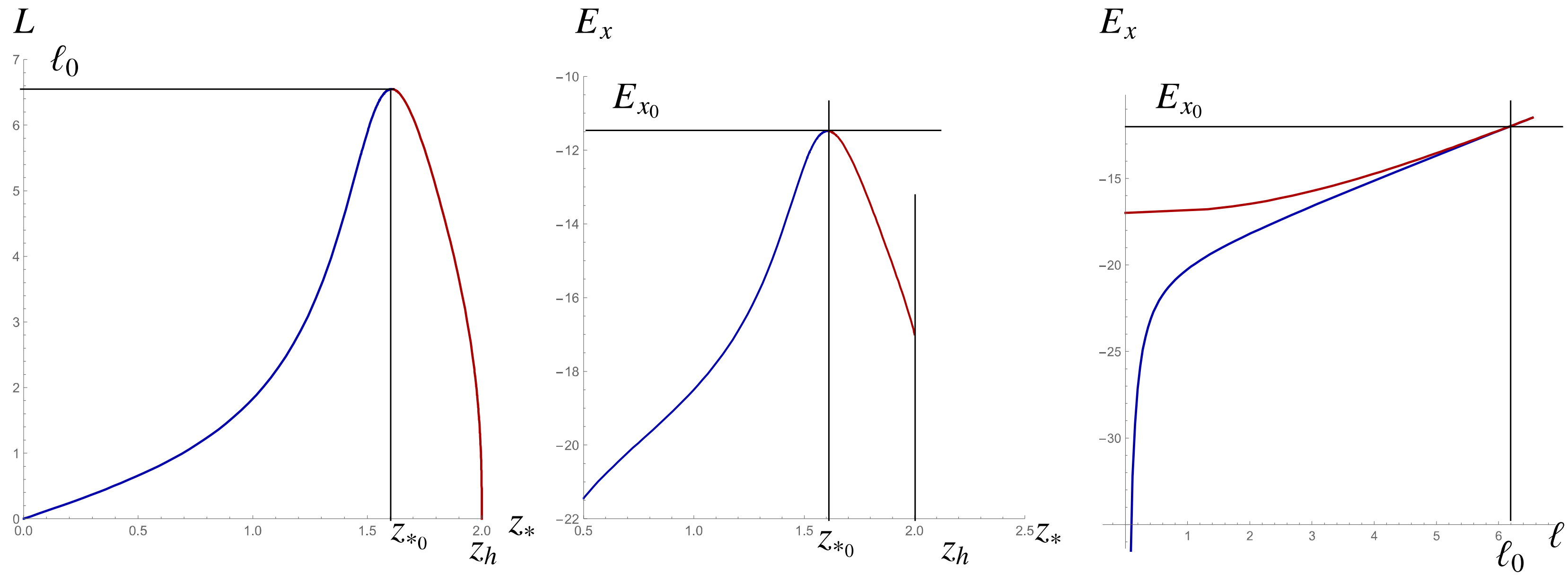
$$E_x = \int_0^{z_*} \frac{dz}{z^2}\left[\frac{b(z)V(z)}{\sqrt{V^2(z)-V^2(z_*)}}-1\right] - \frac{1}{z_*} + m^{\frac{\nu}{2\nu+2}} e^{c\, m^{-\frac{2\nu}{2\nu+2}}} - \sqrt{\pi c}\,\mathrm{erfi}\left(\sqrt{c}\, m^{-\frac{\nu}{2\nu+2}}\right)$$

$$z_h = \left(\frac{1}{m}\right)^{\frac{\nu}{2\nu+2}}$$

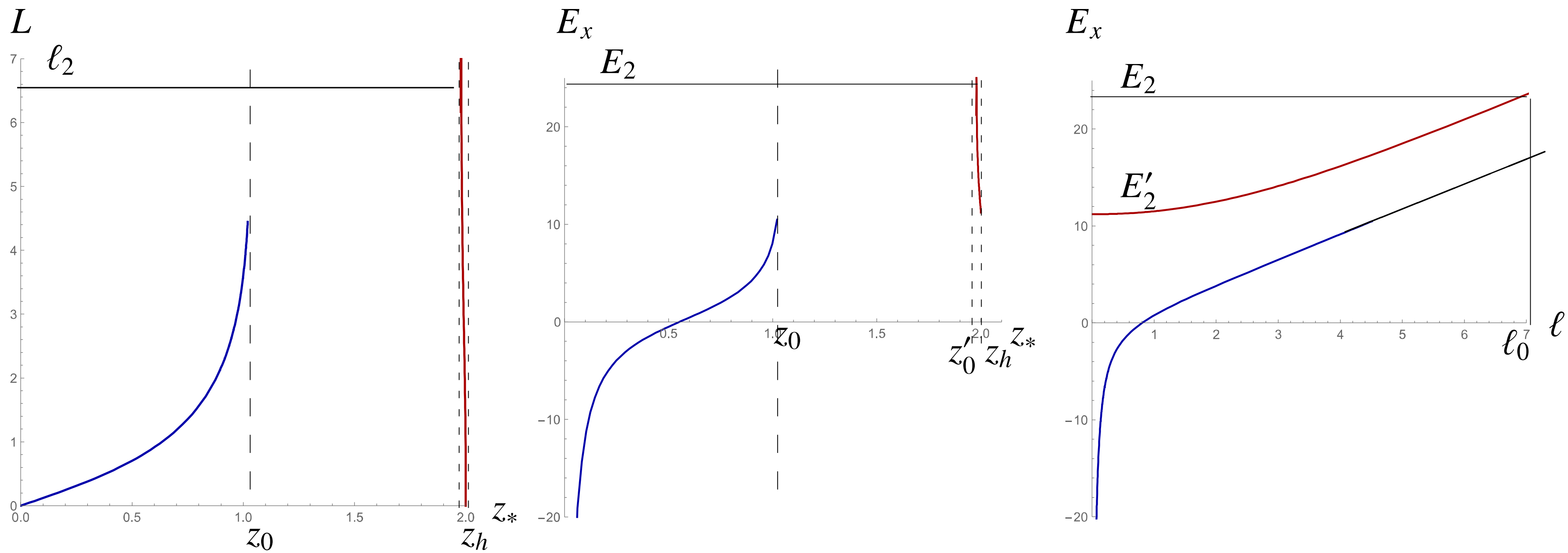
$$L_x = 2\int_0^{z_*} \frac{dz}{\sqrt{f(z)}\sqrt{\left(\frac{V_x^2(z)}{V_x^2(z_*)}-1\right)}}$$

Above the critical point (deconfinment)

There is no extremal point for the "potential" in the interval $0 < z < z_{h_{i_0}}$



Below the critical point (confinement)



The "potential" is a decreasing function only on the intervals

$$0 < z < z_{min} \quad \text{and} \quad z_{max} < z < z_{h_{i_0}}$$

$$V'(z)|_{z=z_0} = 0, \quad V''(z_0) > 0$$

Similar to analysis performed for the isotropic case

O. Andreev, V. Zakharov, JHEP 0704(2007)

M.Mia et al, Phys.Lett. B694 (2011)460 (2011)

P.Colangelo, F.Giannuzzi, S.Nicotri, Phys.Rev. D83 (2011) 035015

Few calculations

$$\sqrt{\frac{V(z)}{V(z_0)}^2 - 1} = \sqrt{\frac{V''(z_0)}{V(z_0)}}(z - z_0) + \mathcal{O}(z - z_0)^2$$

$$L_1 \underset{z_* \sim z_0}{\sim} -2\sqrt{\frac{V(z_0)}{f(z_0)V''(z_0)}} \ln(z_0 - z_*)$$

$$S_{xt} \underset{z_* \sim z_0}{\sim} -\frac{\mathcal{T}}{2\pi\alpha'} \frac{b(z_0)}{z_0^2} \frac{1}{\sqrt{\frac{V''(z_0)}{V(z_0)}}} \ln(z_0 - z_*)$$

$$S_{xt} \underset{L \to \infty}{\sim} \frac{\mathcal{T}}{\pi\alpha'} \frac{L_1}{2} \frac{b(z_0)\sqrt{f(z_0)}}{z_0^2}$$

$$\sigma_x = \frac{V_x(z_0)}{2\pi\alpha'}$$

Temporal Wilson loop in the charged quark confinement background

Energy between quarks located along transversal y-direction

$$S_{yt} = \frac{\mathcal{T}}{2\pi\alpha'} \int \frac{b(z)}{z^2} \sqrt{z^{2-2/\nu} f(z) + z'^2}\, dx.$$

The "potential"

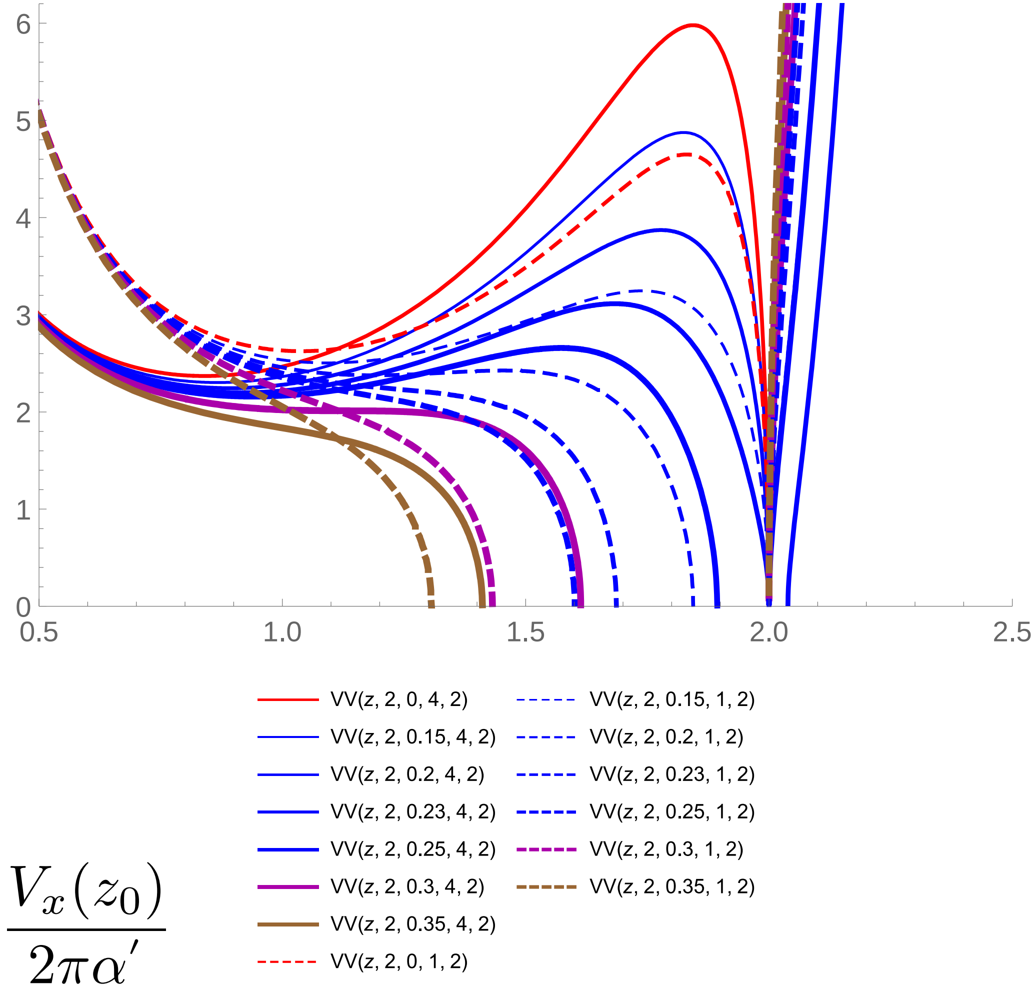
$$V_y(z) = \frac{b(z)\sqrt{f(z)}}{z^{1/\nu+1}}$$

$$\sigma_y = \frac{V_y(z_0)}{2\pi\alpha'}$$

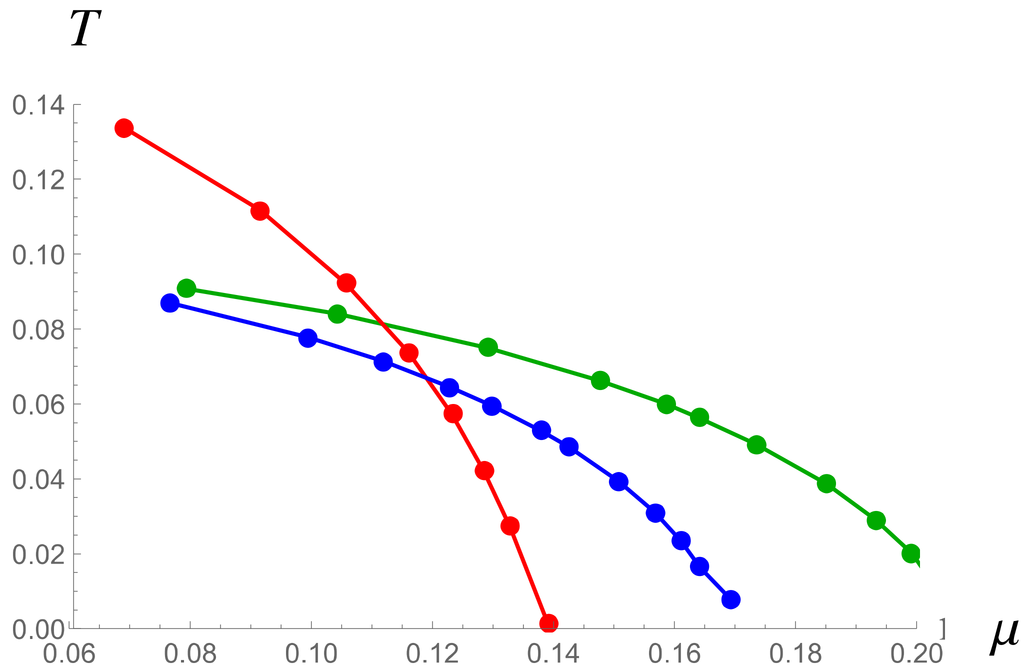
$$V_x(z) = \frac{b(z)}{z^2}\sqrt{f(z)}$$

(dashed lines)

$$\sigma_x = \frac{V_x(z_0)}{2\pi\alpha'}$$



Holographic anisotropic QCD phase diagram



Phase transitions lines dividing the plane in two regions, a hadron phase near the origin, and a deconfined phase beyond the curve.
The red line corresponds to the isotropic case.
Anisotropic case nu=4:

the blue line corresponds to **quarks located along the longitudinal x-direction** ,
the green line corresponds to **quarks located along the transversal y-direction**

Direct photons and electric conductivity

The thermal-photon production from the QGP plays an essential role, since photons after they are produced in HIC almost do not interact with the QGP and, therefore, they give us the local information in heavy ion collisions.

The photon-emission rate is related to the retarded correlator of currents in momentum space

$$G^R_{\mu\nu}(k) = i\int d^4(x-y)e^{ik\cdot(x-y)}\theta(x^0)\langle[J^a_\mu(x), J^b_\nu(0)]\rangle,$$

$$d\Gamma = -\frac{d^3k}{(2\pi)^3}\frac{e^2 n_b(|\mathbf{k}|)}{|\mathbf{k}|}\mathrm{Im}\left[\mathrm{tr}\left(\eta^{\mu\nu}G^{ab\,R}_{\mu\nu}\right)\right]_{k^0=|\mathbf{k}|},$$

S.I.Finazzo and R.Rougemont, Phys.Rev.D 93, (2016) 034017
I.Iatrakis, E.Kiritsis, C.Shen and D.L.Yang, arXiv:1609.07208 [hep-ph]

Direct photons and electric conductivity

$$S_M = -\mathcal{N}\int d^5x\sqrt{-g}\frac{V_{dil}(\phi)}{4}F^{MN}F_{MN}$$

$$ds^2 = \frac{b^2(z)}{z^2}\left(-f(z)dt^2 + dx^2 + p(z)(dy_1^2 + dy_2^2) + \frac{dz^2}{f(z)}\right)$$

$$\int d^4x dz \mathcal{V}(z)\left(-\frac{F_{01}^2 + F_{02}^2}{fp} - \frac{F_{03}^2}{f} + f\left(\frac{F_{1z}^2 + F_{2z}^2}{p} + F_{3r}^2\right) - F_{0z}^2 + \frac{F_{12}^2}{p^2} + \frac{F_{13}^2 + F_{23}^2}{p}\right)$$

$$\mathcal{V}(z) = V_{dil}(\phi(z))\frac{b(z)}{z}$$

Boundary conditions

$$\lim_{z\to 0} A_\mu(z,\omega,k) = 1, \qquad \lim_{z\to z_h} A_\mu(z,\omega,k) = 0$$

$$A_\mu(t,\mathbf{x},z) = \int\frac{d^4k}{(2\pi)^4}e^{-i\omega t + i\mathbf{k}\mathbf{x}}\mathcal{A}_\mu(z,\omega,k), \quad \mathcal{A}_\mu(z,\omega,k) = A_\mu(z,\omega,k)\,a_\mu(\omega,k)$$

$$k = k_x, \qquad k_{y_1} = k_{y_2} = 0$$

Eqs to solve:

$$E_L'' + \left(\frac{f'}{f}\frac{w^2}{w^2 - fk^2} + \frac{V'}{V}\right) E_L' + \frac{w^2 - fk^2}{f^2} E_L = 0$$

$$E_{\perp,i}'' + \left(-\frac{p'}{p} + \frac{V'}{V} + \frac{f'}{f}\right) E_{\perp,i}' + \frac{w^2 - k^2}{f} E_{\perp,i} = 0, \quad i = 1,2$$

$$E_L = kA_0 + wA_3, \quad E_{\perp,i} = wA_{\perp,i}, \;\; i = 1,2.$$

Substitute to

$$S_{boundary} = \int d^4x \left[\frac{f}{w^2 - k^2 f} E_L E_L' + \frac{f}{p\, w^2} E_\perp E_\perp' \right],$$

$$E_\perp E_\perp' \equiv \sum_{i=1,2} E_{\perp,i} E_{\perp,i}'$$

As in isotropic case

A la membrane paradigm N.Iqbal, H.Liu, Phys. Rev.D 79 (2009) 025023

$$\zeta_\perp = -\frac{V\,f}{pw}\frac{\partial_r E_i^\perp}{E_i^\perp} \qquad \zeta_L = -\frac{V\,f}{\omega}\frac{\partial_r E_L}{E_L},$$

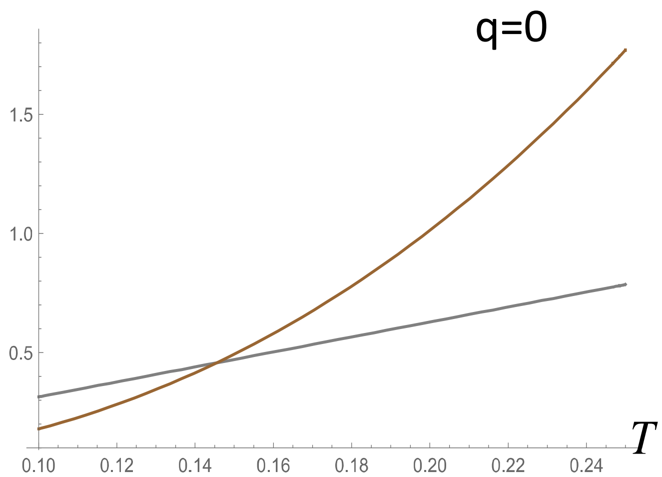
$$\zeta_\perp' - \frac{w}{f}\left[\frac{p}{V}\,\zeta_\perp^2 + \frac{V}{p}\left(1 - f\,\frac{k^2}{w^2}\right)\right] = 0.$$

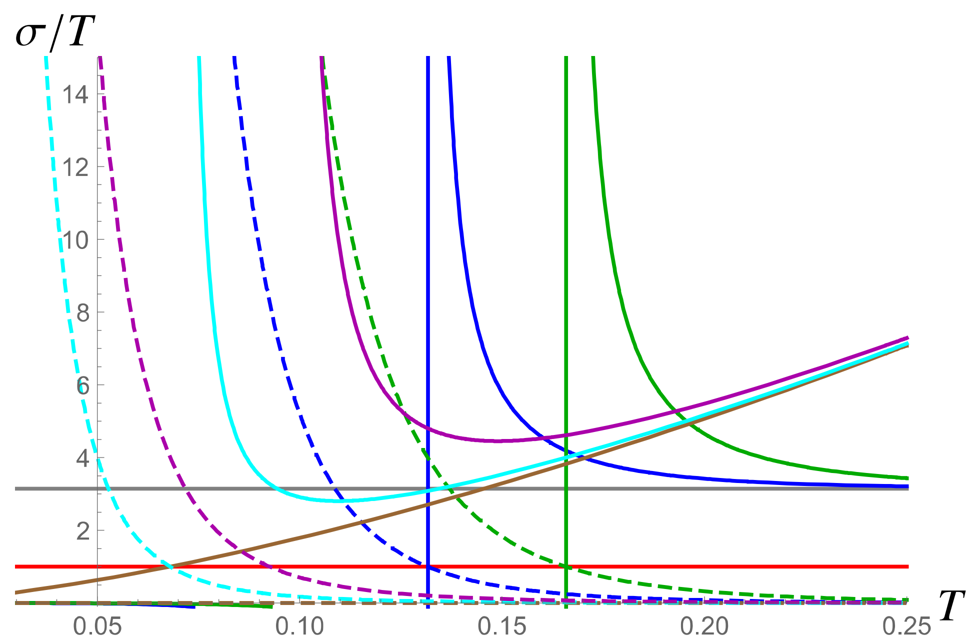
$$\zeta_L' + \frac{k^2 f'}{w^2 - f k^2}\zeta_L - \frac{w}{f}\left[\frac{\zeta_L^2}{V} + V\left(1 - f\,\frac{k^2}{w^2}\right)\right] = 0$$

$$\zeta_\perp|_{z=0} = \zeta_\perp|_{z=z_h} = i\frac{V}{p}|_{z=z_h}$$

Electric conductivity

$$|\zeta_\perp(T,\nu,q)| \approx \left(\frac{2\pi\nu}{1+\nu}\right)^{3-2/\nu} \frac{T^{3-2/\nu}}{\left(1-\left(\frac{\nu+1}{\nu}\right)^{\frac{2+3\nu}{\nu}} q^2 \left(\frac{1}{2\pi T}\right)^{\frac{2+4\nu}{\nu}}\right)^{3-2/\nu}}$$





$\nu = 1$ and $\nu = 4$

Isotropic: q=0,0.1,0.2 gray, blue and green lines
Anisotropic: q=0,0.1,0.2 are shown by brown, darker cyan and darker green.
Dashed: validity of the approximation (below the red line)

Conclusion:

Holographic models are some kind of phenomenological models with few number of parameters

We have considered the model that has:
anizotropic parameter, chemical potential, quark confinement.

To reproduce multiplicity we have to use the special anisotropic background

Anisotropy drastically change standard holographic calculations, in particular,

- **Wilson loops, and quark potential**
- **Jet quenching**
- **Drag forces**
- **sheet viscosity and therefore elliptic flows**
- **susceptibility**
- **thermalization time**

International on-line seminar
"Holography for NICA",

every 4-th/3-th Friday on even months, 2 pm (Mcs), www://mi.ras.ru

December 23, 2016,
February, 17 , 2017