

QGP, holography and all that

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based on works with

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Heavy Ion Physics: from LHC to NICA,
Dubna 2017

The quark-gluon plasma

Experiments on Heavy Ion Collisions at **RHIC** and **LHC**:

- QGP – quarks, antiquarks, gluons in deconfinement

The quark-gluon plasma

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- Spatially anisotropic at early stage.

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Experiments on Heavy Ion Collisions at **RHIC** and **LHC**:

- QGP – quarks, antiquarks, gluons in deconfinement
- QGP does not behave like a weakly coupled gas of quarks and gluons, but a strongly coupled fluid.
- Spatially anisotropic at early stage.
- Hadronization, freezing out: QGP is a particle factory
- Early anisotropic stage: responsible for multiplicity of produced particles

The quark-gluon plasma (2005)

$$\tau_{\text{therm}}(0.1 \text{ fm}/c) < \tau_{\text{hydro}} < \tau_{\text{hard}}(10 \text{ fm}/c) < \tau_f(20 \text{ fm}/c)$$

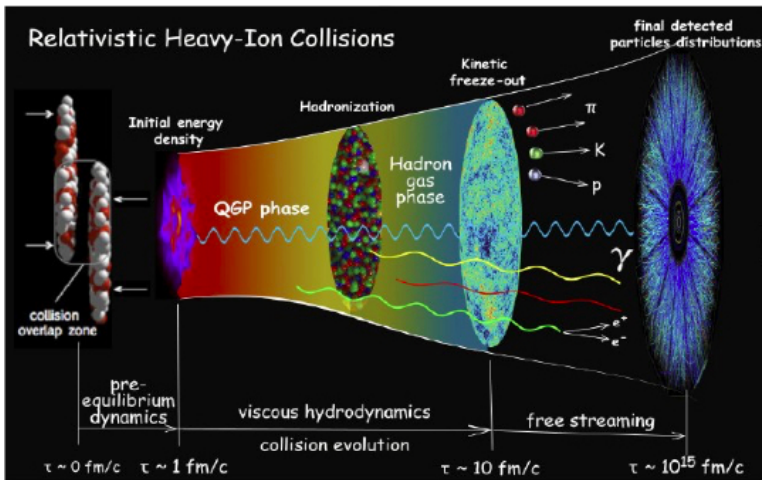


Figure: Picture from: P.Sorensen, C.Shen

- $D = 4$ Multiplicity is proportional to entropy of $D = 5$ BH Gubser'08

Experiment:

$$S_{data} = s_{NN}^{0.155}$$

ALICE collaboration'15

Modified AdS:

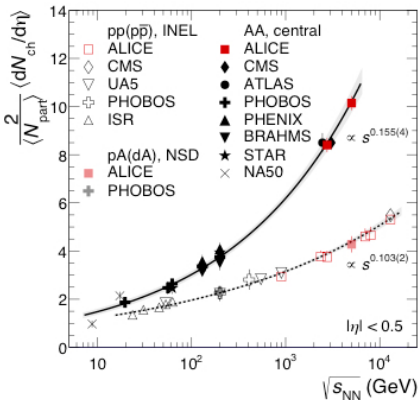
$$S_{data} = s_{NN}^{0.12}$$

Kiritis & Taliotis'11

Modified AdS+ ghosts:

$$S_{data} = s_{NN}^{0.16}$$

Aref'eva et al.'14



ALICE collaboration'15

Broken scaling

$$S_{data} = s_{NN}^{0.16}$$

Aref'eva & A.G.'14

Outline

- ① Part I: QGP
- ② Part II: Holography
 - The AdS/CFT conjecture
 - Holographic dictionary
 - Holographic models
- ③ Part III: All that
 - Colliding shocks
 - Holographic Wilson loops
 - WL and thermalization
- ④ Aferword



The AdS/CFT conjecture

The AdS/CFT correspondence

Maldacena'98

The AdS/CFT conjecture claims exact equivalence between the theory in the bulk, which is a low energy approximation to $D = 10$ IIB string theory on $AdS_5 \times S^5$ the theory defined on the boundary, which is $\mathcal{N} = 4$ supersymmetric Yang-Mills with gauge group $SU(N_C)$ at large N_C .

- The strong coupling regime of one theory reflects the weak coupling regime of the other one

Gubser, Klebanov, Polyakov, Witten'98

$$Z_4[\phi_0(x)] = \int \mathcal{D} \exp\{iS_4 + \int_{x^4} \phi_0 \mathcal{O}\}, \quad S_4 = \int d^4x \mathcal{L}$$



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$$Z_5[\phi_0(x)] = \int_{\phi(x,\varepsilon)=\phi_0(x)} \mathcal{D}[\phi] e^{iS_5[\phi]}$$

$$Z_4 = Z_5$$

Generating functional[4d sources $\phi_0(x)] =$ Effective action[fields $\phi_0(x)]$

Finite temperature

$T \neq 0,$

Witten'98

Finite temperature Yang-Mills theory in 4d dual to a $AdS_5 \times S^5$ set up with a black hole (Schwarzschild- $AdS_5 \times S^5$)

$$ds^2 = \frac{1}{z^2} \left[-f(z)dt^2 + (dx^i)^2 + \frac{dz^2}{f(z)} \right], \quad f(z) = 1 - \left(\frac{z}{z_h} \right)^4.$$

- The temperature of the the Yang-Mills theory is identified with the Hawking temperature of the black hole.

Holographic dictionary for QGP

We have to "mimic" the heavy ions collision

Models:

- shock waves collision in AdS
- infalling shell (Vaidya solutions)

Holographic dictionary

- $4d$ Multiplicity in HIC = BH entropy in AdS_5 Gubster et al.'08

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- two point correlators = geodesics
- Wilson loops = 2-dimensional minimal surfaces
- Entanglement entropy = 3-dimensional minimal surfaces

Holographic models

Holographic models

- Top-down approach: low-energy approximation of string theory (supergravity model) in asymptotically AdS backgrounds trying to reproduce features similar to QCD

Examples: Sakai-Sugimoto model ($D4 - D8 - \bar{D}8$ -branes),
Mateos-Trancanelli model ($D3 - D7$ -branes).

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- **Bottom-up approach:** effective $5D$ gravitational theory with matter in
 - asymptotically AdS spacetimes
 - non-conformal backgrounds

Examples: wall models (Karch et al., Erlich et al.), improved holographic QCD model (Kiritsis et al.)

Bottom-up: Breaking scale invariance

The AdS/CFT correspondence:

The Field Theory

- the conformal group $SO(D, 2)$

of a D-dimensional CFT

$$(t, x_i) \rightarrow (\lambda t, \lambda x_i), \quad i = 1, \dots, d - 1$$

The Gravitational Background

- the group of isometries

of AdS_{D+1}

$$ds^2 = r^2 (-dt^2 + d\vec{x}_{d-1}^2) + \frac{dr^2}{r^2}$$

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Generalizations?



Lifshitz scaling: $t \rightarrow \lambda^\nu t, \quad \vec{x} \rightarrow \lambda \vec{x}, \quad r \rightarrow \frac{1}{\lambda} r,$

where ν is the Lifshitz dynamical exponent

Lifshitz metric: $ds^2 = -r^{2\nu} dt^2 + \frac{dr^2}{r^2} + r^2 d\vec{x}_{d-1}^2$

Kachru, Liu, Millgan '08

Lifshitz-like spacetimes for holography

- A spatial extension of the Lifshitz scaling

$$(t, x, y, r) \rightarrow (\lambda^\nu t, \lambda^\nu x, \lambda y_1, \lambda y_2, \frac{r}{\lambda})$$

Lifshitz-like spacetimes for holography

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- Lifshitz-like backgrounds

$$ds^2 = r^{2\nu} (-dt^2 + dx^2) + r^2 dy_1^2 + r^2 dy_2^2 + \frac{dr^2}{r^2},$$

M. Taylor'08, Pal'09.

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The 5d Lifshitz-like metrics

Type – **(1, 2)** $ds^2 = r^{2\nu} (-dt^2 + dx^2) + r^2 (dy_1^2 + dy_2^2) + \frac{dr^2}{r^2}.$

Type – **(2, 1)** $ds^2 = r^{2\nu} (-dt^2 + dx_1^2 + dx_2^2) + r^2 dy^2 + \frac{dr^2}{r^2}.$

Type IIB SUGRA, $D3 - D7$ -branes

$\mathcal{M} = M_5 \times X_5$: M_5 is a $5d$ Lifshitz-like metric, X_5 is an Einstein manifold.

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D3-D7 system, Azeanagi, Lin, Takayanagi'09

$$ds^2 = \tilde{R}^2 \left[\rho^2 (-dt^2 + dx^2 + dy^2) + \rho^{4/3} dw^2 + \frac{d\rho^2}{\rho^2} \right] + R^2 ds_{X_5}^2.$$

$$\nu = 3/2, \quad r \equiv \rho^{2/3}, \quad (t, x, y, w, \rho) \rightarrow \left(\lambda t, \lambda x, \lambda y, \lambda^{2/3} w, \frac{\rho}{\lambda} \right)$$

$\mathcal{M}_5 \times X_5$	t	x	y	r	w	s_1	s_2	s_3	s_4	s_5
D3	×	×	×		×					
D7	×	×	×			×	×	×	×	×

Type IIB SUGRA, $D3 - D7$ -branes: Anisotropic QGP

Mateos&Trancanelli'11

$$ds^2 = \frac{1}{u^2} \left(-\mathcal{F}\mathcal{B}dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{H}dx_3^2 + \frac{du^2}{\mathcal{F}} \right) + \mathcal{Z}d\Omega_{S^5}^2.$$

The functions $\mathcal{F}, \mathcal{B}, \mathcal{H}$ depend on the radial direction u and the anisotropy α . At high temperatures $\alpha \ll T$:

$$\mathcal{F}(u) =$$

$$1 - \frac{u^4}{u_h^4} + \frac{\alpha^2}{24u_h^2} \left[8u^2(u_h^2 - u^2) - 10u^4 \log 2 + (3u_h^4 + 7u^4) \log\left(1 + \frac{u^2}{u_h^2}\right) \right]$$

$$\mathcal{B}(u) = 1 - \frac{\alpha^2}{24u_h^2} \left[\frac{10u^2}{u_h^2 + u^2} + \log\left(1 + \frac{u^2}{u_h^2}\right) \right], \quad \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}}.$$

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$D7$ -probes in $D3$ -background $Lif_{IR}/AdS_{5,UV} \times X_5 \Rightarrow$ deformed SYM.

$\alpha = 0 \Rightarrow$ isotropic $D3$ -brane AdS/CFT : $AdS_5 \times S^5 \Rightarrow \mathcal{N} = 4$ SYM.

Jet quenching, drag force, potentials... see [Giataganas et al.'12](#)

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Jet quenching, drag force, potentials... see **Giataanas et al.'12**

- difficult to study
- not cover experimental data

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 - Holographic models
- ③ Part III: All that
 - Colliding shocks
 - Holographic Wilson loops
 - WL and thermalization
- ④ Afterword



Construction of a shock wave in Lifshitz-like spacetimes

The 5d Lifshitz-like metrics, $z = \frac{1}{r^\nu}$

$$\text{Type - (1, 2)} \quad ds^2 = L^2 \left[\frac{(-dt^2 + dx^2)}{z^2} + \frac{(dy_1^2 + dy_2^2)}{z^{2/\nu}} + \frac{dz^2}{z^2} \right].$$

$$\text{Type - (2, 1)} \quad ds^2 = L^2 \left(\frac{(-dt^2 + dx_1^2 + dx_2^2)}{z^2} + \frac{dy^2}{z^{2/\nu}} + \frac{dz^2}{z^2} \right).$$

A massless particle located at $u = 0$ and moving with the speed of light in the v -direction:

$$ds^2 = 2A(u, v)dudv + g(u, v)h_{ij}(x)dx^i dx^j$$

$$ds^2 = 2A(u, v)du(dv - f(x^i)\delta(u)du) + g(u, v)h_{ij}(x)dx^i dx^j,$$

$$v \rightarrow v + f(x).$$



T. Dray & G. 't Hooft, '85; Hotta & Tanaka'93; Sfetsos'95.

The shock wave metric

$$ds^2 = \frac{\phi(y_1, y_2, z)\delta(u)}{z^2} du^2 - \frac{1}{z^2} dudv + \frac{1}{z^{2/\nu}} (dy_1^2 + dy_2^2) + \frac{dz^2}{z^2}, \quad (1)$$

$u = t - x$ and $v = t + x$ – light cone coordinates.

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$u = t - x$ and $v = t + x$ – light cone coordinates.

The equation for the profile

$$\left[\square_{Lif_3} - \left(1 + \frac{2}{\nu} \right) \right] \frac{\phi(y_1, y_2, z)}{z} = -2zt_{uu}, \quad T_{uu} = t_{uu}\delta(u), \quad (2)$$

$$\square_{Lif_3} = \frac{1}{\nu} \left(z^2 \nu \frac{\partial^2}{\partial z^2} + \nu z \frac{\partial}{\partial z} - 2z \frac{\partial}{\partial z} + z^{2/\nu} \nu \frac{\partial^2}{\partial y_1^2} + \nu z^{2/\nu} \frac{\partial^2}{\partial y_2^2} \right).$$

$$ds_{Lif_3}^2 = \frac{dy_1^2 + dy_2^2}{z^{2/\nu}} + \frac{dz^2}{z^2}.$$

Domain-walls in Lifshitz-like spacetimes

$$\phi(y_1, y_2, z) = \phi(z), \quad \text{Lin \& Shuryak'09}$$

The equation for the profile

$$\frac{\partial^2 \phi(z)}{\partial z^2} - \left(1 + \frac{2}{\nu}\right) \frac{1}{z} \frac{\partial \phi(z)}{\partial z} = -16\pi G_5 E \left(\frac{z}{L}\right)^{1+2/\nu} \delta(z - z_*).$$

$$\phi = \phi_a \Theta(z_* - z) + \phi_b \Theta(z - z_*),$$

$$\phi_a(z) = C_0 z_a z_b \left(\frac{z_*^{2(\nu+1)/\nu}}{z_b^{2(\nu+1)/\nu}} - 1 \right) \left(\frac{z^{2(\nu+1)/\nu}}{z_a^{2(\nu+1)/\nu}} - 1 \right),$$

$$\phi_b(z) = C_0 z_a z_b \left(\frac{z_*^{2(\nu+1)/\nu}}{z_a^{2(\nu+1)/\nu}} - 1 \right) \left(\frac{z^{2(\nu+1)/\nu}}{z_b^{2(\nu+1)/\nu}} - 1 \right),$$

$$C_0 = -\frac{8\nu\pi G_5 E z_a^{1+2/\nu} z_b^{1+2/\nu}}{(\nu+1)L^{3+2/\nu} (z_b^{2(\nu+1)/\nu} - z_a^{2(\nu+1)/\nu})}.$$

Wall-on-wall collisions

$$ds^2 = -\frac{1}{z^2}dudv + \frac{\phi_1(z)}{z^2}\delta(u)du^2 + \frac{\phi_2(z)}{z^2}\delta(v)dv^2 + \frac{dy^2}{z^2} + \frac{dw^2}{z^{4/3}} + \frac{dz^2}{z^2}.$$

The trapped surface for wall-on-wall shock wave collision is $z_a < z_* < z_b$

$$(\partial_z \phi)|_{z=z_a} = 2, \quad (\partial_z \phi)|_{z=z_b} = -2$$



$$\frac{8\pi G_5 E z_a^{8/3} \left(1 - \frac{z_b^{11/3}}{z_*^{11/3}}\right)}{\tilde{R}^{14/3} \left(\frac{z_b^{11/3}}{z_*^{11/3}} - \frac{z_a^{11/3}}{z_*^{11/3}}\right)} = -1, \quad \frac{8\pi G_5 E z_b^{8/3} \left(1 - \frac{z_a^{11/3}}{z_*^{11/3}}\right)}{\tilde{R}^{14/3} \left(\frac{z_b^{11/3}}{z_*^{11/3}} - \frac{z_a^{11/3}}{z_*^{11/3}}\right)} = 1,$$

$$z_a = \left(\frac{z_b^{8/3}}{-1 + z_b^{8/3} C}\right)^{3/8}, \quad z_* = \left(\frac{z_a^{8/3} z_b^{8/3} (z_a - z_b)}{z_a^{8/3} - z_b^{8/3}}\right)^{3/8}.$$

The area of the trapped surface

$$S = \frac{1}{2G_5} \int_C \sqrt{\det|g_{Lif_3}|} dz dy_1 dy_2,$$

where $\det g_{Lif_3}$ is the metric determinant for

$$ds_{Lif_3}^2 = z^{-2/\nu} (dy_1^2 + dy_2^2) + \frac{dz^2}{z^2}.$$

The relative area s of the trapped surface defined by

$$s = \frac{S_{\text{trap}}}{\int dy_1 dy_2} = \frac{\nu}{4G_5} \left(\frac{1}{(z_a)^{2/\nu}} - \frac{1}{(z_b)^{2/\nu}} \right).$$

$$s(C, z_b) = \left(\frac{C}{2}\right)^{\frac{2}{2+\nu}} - \left(\frac{1}{z_b}\right)^{\frac{2}{\nu}} - \frac{2}{(\nu+2)} \left(\frac{C}{2}\right)^{\frac{2}{2+\nu}} \left(\frac{1}{z_b}\right)^{\frac{2+\nu}{\nu}} + \dots$$

The maximum value at infinite z_b

$$s|_{z_b \rightarrow \infty} = \frac{\nu}{4G_5} (8\pi G_5)^{2/(\nu+2)} E^{2/(\nu+2)}$$

$D = 5$ $Lif_{(1,2)}$ with $\nu = 4$

The trapped surface area

$$S = \left(\frac{C}{2}\right)^{\frac{1}{3}} - \left(\frac{1}{z_b}\right)^{\frac{1}{2}} - \frac{1}{3} \left(\frac{2}{C}\right)^{\frac{1}{3}} \left(\frac{1}{z_b}\right)^{\frac{3}{2}} + \dots$$

$$s|_{z_b \rightarrow \infty} = \frac{(8\pi)^{1/3}}{G_5^{2/3}} E^{1/3}.$$

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AdS-background deformed by $b = \frac{L}{z^a}$ with $a = 1/2$

The area of trapped surface

Aref'eva et al.'14

$$s|_{z_b \rightarrow \infty} = \frac{(8\pi E)^{1/3}}{G_5^{2/3}}$$

Black branes in Lifshiz-like spacetimes

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{|g|} \left(R[g] + \Lambda - \frac{1}{2} \partial_M \phi \partial^M \phi - \frac{1}{4} e^{\lambda\phi} F_{(2)}^2 \right),$$

Λ is negative cosmological constant.

The Einstein equations

$$R_{mn} = -\frac{\Lambda}{3} g_{mn} + \frac{1}{2} (\partial_m \phi) (\partial_n \phi) + \frac{1}{4} e^{\lambda\phi} (2F_{mp} F_n^p) - \frac{1}{12} e^{\lambda\phi} F^2 g_{mn}.$$

The scalar field equation

$$\square\phi = \frac{1}{4} \lambda e^{\lambda\phi} F^2, \quad \text{with} \quad \square\phi = \frac{1}{\sqrt{|g|}} \partial_m (g^{mn} \sqrt{|g|} \partial_n \phi).$$

The gauge field

$$D_m (e^{\lambda\phi} F^{mn}) = 0.$$

Gravity duals: black brane

The Lifshitz-like black brane

$$ds^2 = e^{2\nu r} \left(-f(r) dt^2 + dx^2 \right) + e^{2r} \left(dy_1^2 + dy_2^2 \right) + \frac{dr^2}{f(r)},$$

where $f(r) = 1 - m e^{-(2\nu+2)r}$. **Aref'eva, AG, Gourgoulhon'16**

$$F_{(2)} = \frac{1}{2} q dy_1 \wedge dy_2, \quad \phi = \phi(r), \quad e^{\lambda\phi} = \mu e^{4r}.$$

The Hawking temperature of the black brane:

$$T = \frac{1}{\pi} \frac{(\nu + 1)}{2\nu} m^{\frac{\nu}{2\nu+2}}.$$

Gravity duals: black brane

The Lifshitz-like black brane

$$ds^2 = e^{2\nu r} (-f(r)dt^2 + dx^2) + e^{2r} (dy_1^2 + dy_2^2) + \frac{dr^2}{f(r)},$$

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$$ds^2 = \frac{(-f(z)dt^2 + dx^2)}{z^2} + \frac{(dy_1^2 + dy_2^2)}{z^{2/\nu}} + \frac{dz^2}{z^2 f(z)},$$

$$f(z) = 1 - mz^{2+2/\nu}, \quad z = \frac{1}{r^\nu}.$$

Gravity duals: black brane

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5d GAUGED $U(1)^3$ SUGRA

Donos, Gauntlett'12, Hoker, Kraus'12

Holographic Wilson Loops

- The expectation value of WL in the fundamental representation calculated on the gravity side:

$$W[C] = \langle \text{Tr}_F e^{i \oint_C dx_\mu A_\mu} \rangle = e^{-S_{string}[C]}.$$

The Nambu-Goto action is

$$S_{string} = \frac{1}{2\pi\alpha'} \int d\sigma^1 d\sigma^2 \sqrt{-\det(h_{\alpha\beta})}, \quad (3)$$

with the induced metric of the world-sheet $h_{\alpha\beta}$ given by

$$h_{\alpha\beta} = g_{MN} \partial_\alpha X^M \partial_\beta X^N, \quad \alpha, \beta = 1, 2, \quad (4)$$

g_{MN} is the background metric, $X^M = X^M(\sigma^1, \sigma^2)$ specify the string, σ^1, σ^2 parametrize the worldsheet.

- The potential of the interquark interaction

$$W(T, X) = \langle \text{Tr} e^{i \oint_{T \times X} dx_\mu A_\mu} \rangle \sim e^{-V(X)T}.$$

Holographic Wilson Loops

A similar operator to probe QCD is **the spatial rectangular Wilson loop** of size $X \times Y$ (for large Y)

$$W(X, Y) = \langle \text{Tr} e^{i \oint_{X \times Y} dx_\mu A_\mu} \rangle = e^{-\mathcal{V}(X)Y}$$

defines the so called pseudopotential \mathcal{V} :

$$\mathcal{V}(X) = \frac{S_{string}}{Y}.$$

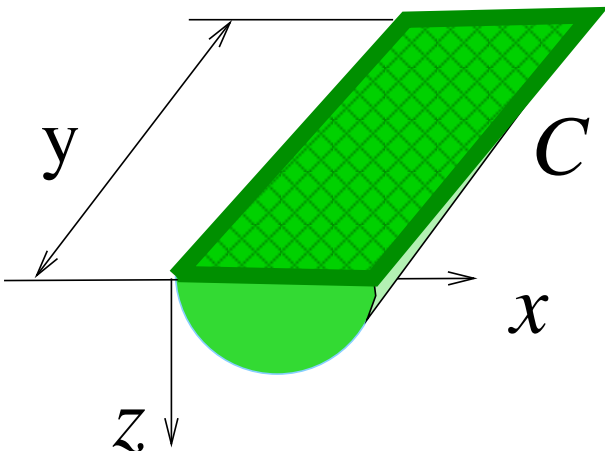
The spatial Wilson loops obey **the area law at all temperature**, i.e.

$$\mathcal{V}(X) \sim \sigma_s X,$$

where σ_s defines the spatial string tension

$$\sigma_s = \lim_{X \rightarrow \infty} \frac{\mathcal{V}(X)}{X}.$$

Holographic spatial Wilson loops



Spatial WL in Lifshitz-like backgrounds

Rectangular WL in the spatial planes xy_1 (or xy_2) and y_1y_2 .

Possible configurations: [D.Ageev, I.Ya.Aref'eva, AG, E.Gourgoulhon'16](#)

- a rectangular loop in the xy_1 (or xy_2) plane with a short side of the length ℓ in the longitudinal x direction and a long side of the length L_{y_1} along the transversal y_1 direction

$$x \in [0, \ell < L_x], \quad y_1 \in [0, L_{y_1}];$$

- a rectangular loop in the xy_1 plane with a short side of the length ℓ in the transversal y_1 direction and a long side of the length L_x along the longitudinal x direction:

$$x \in [0, L_x], \quad y_1 \in [0, \ell < L_{y_1}];$$

- a rectangular loop in the transversal y_1y_2 plane with a short side of the length ℓ in one of transversal directions (say y_1) and a long side of the length L_{y_2} along the other transversal direction y_2

$$y_1 \in [0, \ell < L_{y_1}], \quad y_2 \in [0, L_{y_2}].$$

Static WL. Case 1: $\sigma^1 = x$, $\sigma^2 = y_1$, $z = z(x)$, $v = v(x)$.

The renormalized Nambu-Goto action

$$S_{x,y_1(\infty),ren} = \frac{L_{y_1}}{2\pi\alpha'} \frac{1}{z_*^{1/\nu}} \int_0^1 \frac{dw}{w^{1+1/\nu}} \left[\frac{1}{\sqrt{f(z_*w)(1-w^{2+2/\nu})}} - 1 \right] - \frac{\nu}{z_*^{1/\nu}},$$

where $w = z/z_*$. The length scale is

$$\frac{\ell}{2} = 2z_* \int_{z_0/z_*}^1 \frac{w^{1+1/\nu} dw}{f(z_*w)(1-w^{2+2/\nu})}.$$

Then pseudopotential $\mathcal{V}_{x,y_1(\infty)} = \frac{S_{x,y_1(\infty),ren}}{L_{y_1}}$.

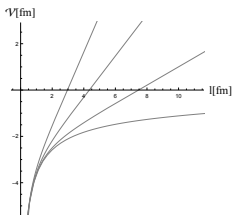
For small ℓ – the deformed Coulomb part

$$\mathcal{V}_{x,y_1(\infty)}(\ell, \nu) \underset{\ell \sim 0}{\sim} -\frac{\mathcal{C}_1(\nu)}{\ell^{1/\nu}}.$$

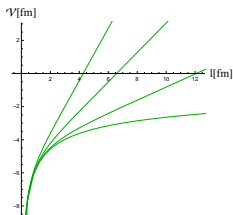
For large ℓ

$$\mathcal{V}_{x,y_1(\infty)}(\ell, \nu) \underset{\ell \rightarrow \infty}{\sim} \sigma_{s,1}(\nu) \ell.$$

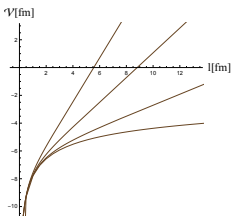
Static WL. Case 1: $\sigma^1 = x$, $\sigma^2 = y_1$, $z = z(x)$, $v = v(x)$.



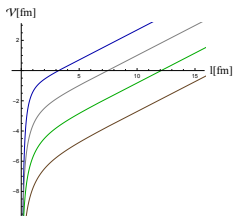
(a)



(b)



(c)



(d)

Figure: $\mathcal{V}_{x,y_1(\infty)}$ as a function of ℓ , $\nu = 2, 3, 4$ ((a),(b),(c)). The temperature $T = 30, 100, 150, 200$ MeV (from down to top) for all. In (d): $\mathcal{V}_{x,y_1(\infty)}$ for $\nu = 1, 2, 3, 4$ (from top to down) at $T = 100$ MeV.

Static WL. Case 2: $\sigma^1 = x$, $\sigma^2 = y_1$, $z = z(y_1)$, $v = v(y_1)$

The renormalized Nambu-Goto action

$$S_{y_1, x(\infty), ren} = \frac{L_x}{2\pi\alpha'} \frac{1}{z_*} \int_{z_0/z_*}^1 \frac{dw}{w^2} \left[\frac{1}{\sqrt{f(z_* w) (1 - w^{2+2/\nu})}} - 1 \right] - \frac{1}{z_*}.$$

The length scale is

$$\ell = 2z_*^{1/\nu} \int_0^1 \frac{w^{2/\nu} dw}{f(z_* w) (1 - w^{2+2/\nu})}$$

The pseudopotential $\mathcal{V}_{y_1, x(\infty)} = \frac{S_{y_1, x(\infty), ren}}{L_x}$.

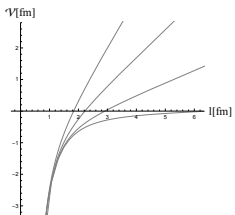
For small ℓ – the deformed Coulomb part

$$\mathcal{V}_{y_1, x(\infty)} \underset{\ell \sim 0}{\sim} -\frac{\mathcal{C}_2(\nu)}{\ell^\nu}.$$

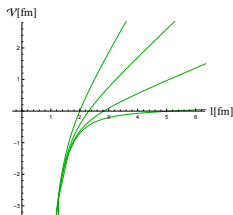
For large ℓ

$$\mathcal{V}_{y_1, x(\infty)}(\ell, \nu) \underset{\ell \rightarrow \infty}{\sim} \sigma_{s,2}(\nu) \ell.$$

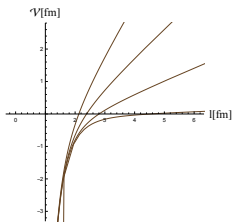
Static WL. Case 2: $\sigma^1 = x$, $\sigma^2 = y_1$, $z = z(y_1)$, $v = v(y_1)$



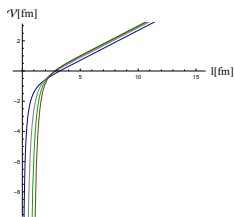
(a)



(b)



(c)



(d)

Figure: $\mathcal{V}_{y_1, x(\infty)}$ as a function of ℓ for $\nu = 2, 3, 4$ ((a),(b),(c)).

$T = 30, 100, 150, 200$ MeV from down to top, respectively, for all. In (d) \mathcal{V} for $\nu = 1, 2, 3, 4$ (from left to right, respectively) at $T = 100$ MeV.

Static WL. Case 3: $\sigma^1 = y_1$, $\sigma^2 = y_2$, $z = z(y_1)$, $v = v(y_1)$

The renormalized Nambu-Goto action

$$S_{y_1 y_2(\infty), ren} = \frac{L_{y_2}}{2\pi\alpha'} \frac{1}{z_*^{1/\nu}} \int_{\frac{z_0}{z_*}}^1 \frac{dw}{w^{1+1/\nu}} \left[\frac{1}{\sqrt{f(z_* w) (1 - w^{4/\nu})}} - 1 \right] - \frac{\nu}{z_*^{1/\nu}}.$$

The length scale is

$$\ell = z_*^{1/\nu} \int \frac{dw}{w^{1-3/\nu} \sqrt{f(z_* w) (1 - w^{4/\nu})}}.$$

The pseudopotential $\mathcal{V}_{y_1, y_2(\infty)} = \frac{S_{y_1, y_2(\infty)}}{L_{y_2}}$.

For small ℓ –

$$\mathcal{V}_{y_1, y_2(\infty)} \underset{\ell \rightarrow 0}{\sim} -\frac{\mathcal{C}_3(\nu)}{\ell}.$$

For large ℓ

$$\mathcal{V}_{y_1, y_2(\infty)}(\ell, \nu) \underset{\ell \rightarrow \infty}{\sim} \sigma_{s,3}(\nu) \ell.$$

Static WL. Case 3: $\sigma^1 = y_1$, $\sigma^2 = y_2$, $z = z(y_1)$, $v = v(y_1)$

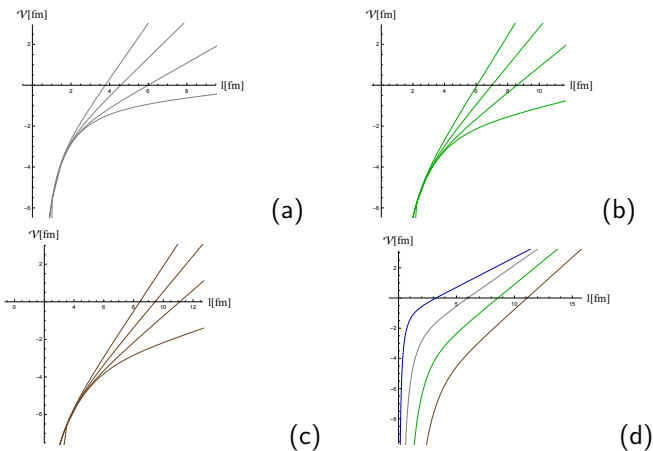


Figure: $\mathcal{V}_{y_1, y_2(\infty)}$ as a function of ℓ for $\nu = 2, 3, 4$ ((a),(b),(c), respectively). We take $T = 30, 100, 150, 200$ MeV from down to top, for (a),(b) and (c). In (d) $\mathcal{V}_{y_1, y_2(\infty)}$ for $\nu = 1, 2, 3, 4$ (from left to right) at $T = 100$ MeV.

Spatial string tension

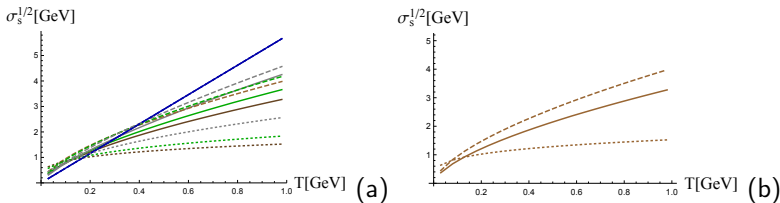


Figure: The dependence of the spatial string tension $\sqrt{\sigma_s}$ on orientation and temperature. The solid lines corresponds to the rectangular Wilson loop with a short extent in the x -direction, while the dashed lines correspond to a short extent in the y -direction. The dotted lines correspond to the rectangular Wilson loop in the transversal $y_1 y_2$ plane. (a) Blue line corresponds to $\nu = 1$, gray lines correspond to $\nu = 2$, green lines correspond to $\nu = 3$ and the brown ones correspond to $\nu = 4$. (b) The spatial string tension $\sqrt{\sigma_s}$ for different orientations for $\nu = 4$.

Alanen et al.'09, A. Dumitru et al.'13-14

The infalling shell background

The ingoing Eddington-Finkelstein coordinates

$$dv = dt + \frac{dz}{f(z)}.$$

The Vaidya solution in Lifshitz background

$$ds^2 = -z^{-2} f(z) dv^2 - 2z^{-2} dv dz + z^{-2} dx^2 + z^{-2/\nu} (dy_1^2 + dy_2^2),$$

$$f = 1 - m(v) z^{2+2/\nu}, v < 0 - \text{inside the shell}, v > 0 - \text{outside},$$

$$f(v, z) = 1 - \frac{M}{2} \left(1 + \tanh \frac{v}{\alpha} \right) z^{2+\frac{2}{\nu}}$$

The infalling shell background

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$$f(v, z) = 1 - \frac{M}{2} \left(1 + \tanh \frac{v}{\alpha} \right) z^{2+\frac{2}{\nu}}$$

- The Vaidya solution interpolates between the black hole (outside the shell) and the Lifshitz-like vacuum (inside the shell).

WL in time-dependent backgrounds. Case 1

$$v = v(x), \quad z = z(x), \quad f = f(v, z).$$

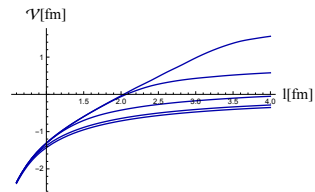
$$S_{x, y_1(\infty)} = \frac{L_y}{2\pi\alpha'} \int \frac{dx}{z^{1+1/\nu}} \sqrt{1 - f(z, v)v'^2 - v'z'}, \quad ' \equiv \frac{d}{dx}.$$

The corresponding equations of motion are

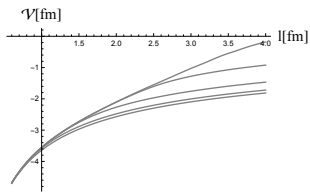
$$\begin{aligned} v'' &= \frac{1}{2} \frac{\partial f}{\partial z} v'^2 + \frac{(\nu+1)}{\nu z} (1 - f v'^2 - 2v'z'), \\ z'' &= -\frac{\nu+1}{\nu} \frac{f}{z} + \frac{\nu+1}{\nu} \frac{f^2 v'^2}{z} - \frac{1}{2} \frac{\partial f}{\partial v} v'^2 - \frac{1}{2} f v'^2 \frac{\partial f}{\partial z} - v'z' \frac{\partial f}{\partial z}, \\ &+ 2 \frac{(\nu+1)}{\nu z} f v'z'. \end{aligned}$$

The boundary conditions $z(\pm\ell) = 0$, $v(\pm\ell) = t$. The initial conditions $z(0) = z_*$, $v(0) = v_*$, $z'(0) = 0$, $v'(0) = 0$. The pseudopotential is

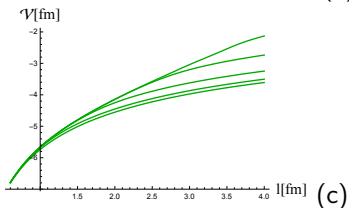
$$\mathcal{V}_{x, y_1(\infty)} = \frac{S_{x, y_1(\infty), ren}}{L_{y_1}}$$



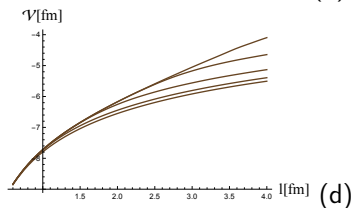
(a)



(b)



(c)



(d)

Figure: $\mathcal{V}_{x,y_1(\infty)}$ as a function of ℓ at fixed values of t for $\nu = 1, 2, 3, 4$ ((a),(b),(c),(d), respectively). Different curves correspond to time $t = 0.1, 0.5, 0.9, 1.4, 2$ (from down to top, respectively).

$$\delta\mathcal{V}_1(x, t) = \mathcal{V}_{x, y_1(\infty)}(x, t) - \mathcal{V}_{x, y_1(\infty)}(x, t_f).$$

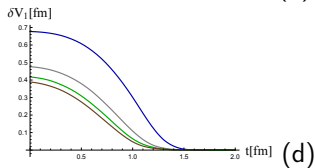
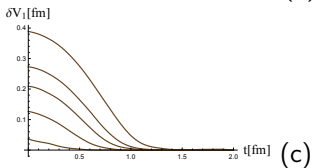
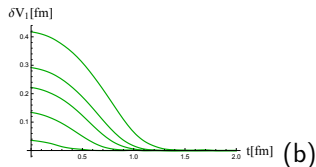
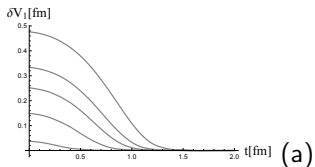


Figure: The time dependence of $-\delta\mathcal{V}_1(x, t)$, for different values of the length ℓ , $\nu = 2, 3, 4$ ((a),(b),(c), respectively). Different curves correspond to $\ell = 0.7, 1.2, 1.5, 1.7, 2$ (from down to top, respectively). In (d) we have shown $-\delta\mathcal{V}_1(x, t)$ as a function of t at $\ell = 2$ for $\nu = 1, 2, 3, 4$ (from top to down).

WL in time-dependent backgrounds. Case 2

$$v = v(y_1), \quad z = z(y_1), \quad f = f(v, z)$$

$$S_{y_1, x(\infty)} = \frac{L_x}{2\pi\alpha'} \int dy_1 \frac{1}{z^2} \sqrt{\left(\frac{1}{z^{2/\nu-2}} - f(z, v)(v')^2 - 2v'z' \right)}, \quad ' \equiv \frac{d}{dy_1}.$$

The corresponding equations of motion are

$$v'' = \frac{1}{2} \frac{\partial f}{\partial z} v'^2 + \frac{\nu+1}{\nu z} \left(z^{2-2/\nu} - \frac{2\nu}{(1+\nu)} f v'^2 - 2v'z' \right),$$

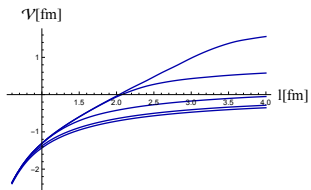
$$z'' = -\frac{\nu+1}{\nu} f z^{1-2/\nu} + \frac{2(\nu-1)z'^2}{\nu} + \frac{2}{\nu} \frac{f^2 v'^2}{z} - \frac{1}{2\nu} \frac{\partial f}{\partial v} v'^2 - \frac{1}{2\nu} f \frac{\partial f}{\partial z} v'^2$$

$$- z'v' \frac{\partial f}{\partial z} + \frac{4}{z} f z'v'.$$

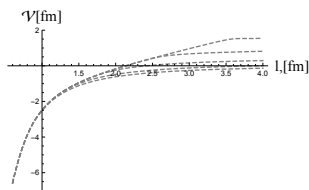
The boundary conditions $z(\pm\ell) = 0$, $v(\pm\ell) = t$. The initial conditions $z(0) = z_*$, $v(0) = v_*$, $z'(0) = 0$, $v'(0) = 0$. The pseudopotential is

$$\mathcal{V}_{y_1, x(\infty)} = \frac{S_{y_1, x(\infty), ren}}{L_{y_1}}.$$

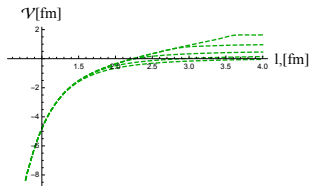
WL in time-dependent backgrounds. Case 2



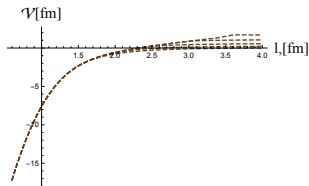
(a)



(b)



(c)



(d)

Figure: $\mathcal{V}_{y_1 x_\infty}$ as a function of ℓ at fixed values of t for $\nu = 1, 2, 3, 4$ ((a),(b),(c),(d), respectively). Different curves correspond to $t = 0.1, 0.5, 0.9, 1.4, 2$ from down to top.

WL in time-dependent backgrounds. Case 2

$$\delta\mathcal{V}_{y_1, x(\infty)}(x, t) = \mathcal{V}_{y_1, x(\infty)}(x, t) - \mathcal{V}_{y_1, x(\infty)}(x, t_f).$$

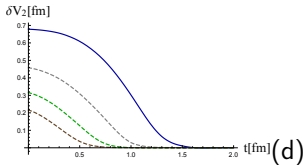
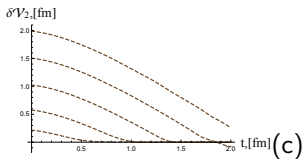
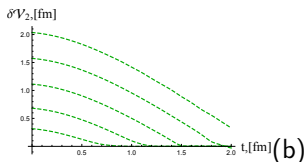
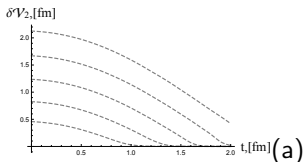


Figure: The time dependence of $-\delta\mathcal{V}_{y_1, x(\infty)}(x, t)$ for different values of the length ℓ , $\nu = 2, 3, 4$ ((a),(b),(c), respectively). Different curves correspond to $\ell = 2, 2.5, 3, 3.5, 4$ (from down to top, respectively). In (d) $-\delta\mathcal{V}_2(x, t)$ as a function of t at $\ell = 2$ for $\nu = 1, 2, 3, 4$ (from top to down, respectively).

WL in time-dependent backgrounds. Case 3

$$v = v(y_1), \quad z = z(y_1), \quad f = f(v, z).$$

$$S_{y_1, y_2, (\infty)} = \frac{L_{y_2}}{2\pi\alpha'} \int dy_1 \frac{1}{z^{1+1/\nu}} \sqrt{\left(\frac{1}{z^{2/\nu-2}} - f(v')^2 - 2v'z' \right)}.$$

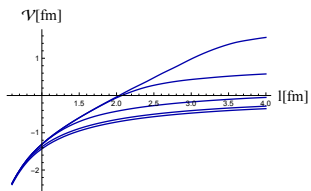
The corresponding equations of motion are

$$\begin{aligned} v'' &= \frac{1}{2} \frac{\partial f}{\partial z} v'^2 + \frac{2}{z\nu} \left(z^{2-2/\nu} - \frac{\nu+1}{2} f v'^2 - 2v'z' \right), \\ z'' &= -\frac{2}{\nu} f z^{1-2/\nu} + 2 \frac{\nu-1}{\nu} \frac{z'^2}{z} + \frac{\nu+1}{\nu z} f^2 v'^2 - \frac{1}{2} \frac{\partial f}{\partial v} v'^2 - \frac{1}{2} f \frac{\partial f}{\partial z} v'^2 \\ &\quad - z' v' \frac{\partial f}{\partial z} + \frac{2(\nu+1)}{\nu z} f v' z'. \end{aligned} \quad (5)$$

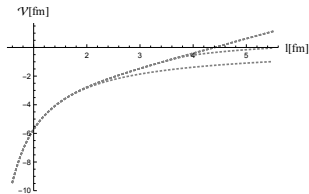
The boundary conditions $z(\pm\ell) = 0$, $v(\pm\ell) = t$. The initial conditions $z(0) = z_*$, $v(0) = v_*$, $z'(0) = 0$, $v'(0) = 0$. The pseudopotential is

$$\mathcal{V}_{y_1, y_2, (\infty)}(t, \ell) = \frac{S_{y_1, y_2, (\infty), ren}}{L_{y_2}}.$$

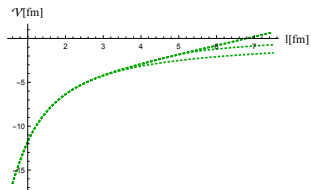
WL in time-dependent backgrounds. Case 3



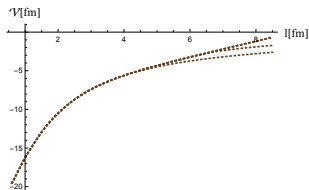
(a)



(b)



(c)



(d)

Figure: $\mathcal{V}_{y_1, y_2, (\infty)}(l, t)$ as a function of the length l at fixed values of t , $\nu = 1, 2, 3, 4$ ((a),(b),(c),(d)). (a): we take $t = 0.1, 0.5, 0.9, 1.4, 2$ from down to top, respectively; for plots (b),(c),(d): $t = 0.4, 1.5, 2.5, 3.34, 4$ from down to top, respectively.

WL in time-dependent backgrounds. Case 3

$$\delta\mathcal{V}_{y_1, y_2, (\infty)}(x, t) = \mathcal{V}_{y_1, y_2, (\infty)}(x, t) - \mathcal{V}_{y_1, y_2, (\infty)}(x, t_f).$$

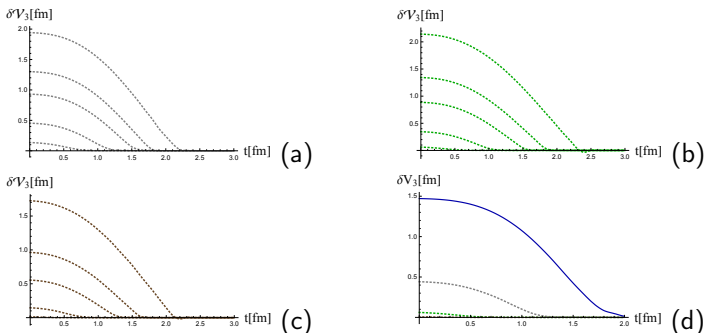


Figure: $-\delta\mathcal{V}_{y_1, y_2, (\infty)}(x, t)$ on t for different l , $\nu = 2, 3, 4$ ((a),(b),(c)). (a): $l = 2.2, 3, 3.85, 4.4, 5.2$ from top to down; (b): $l = 3, 4.1, 5.2, 6, 7.1$ from top to down; (c): $l = 3.4, 4.6, 5.9, 6.8, 8$ from top to down. In (d): $-\delta\mathcal{V}_3(x, t)$ as a function of t at $l = 3$ for $\nu = 1, 2, 3, 4$ (from top to down, respectively).

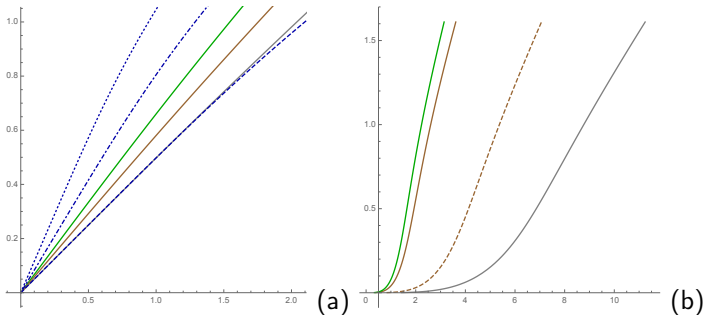


Figure: The thermalization times of the two-point correlators, holographic entanglement entropy and WL. (a) The solid lines (from left to right) correspond to the entropy (green), the Wilson loop (brown) and the two-point correlator (gray) with the dependences on the longitudinal direction x in the anisotropic background with $\nu = 4$, while the dashed, dash-dotted and dotted lines represent the behaviour of the two-point correlator, Wilson loop and entropy in the isotropic spacetime, respectively. (b) The solid curves correspond to the entropy (green), the Wilson loop (brown) on the xy_1 -plane and the two-point correlator (gray) with the dependences on the transversal direction y_1 . All plots in (b) are for the anisotropic background with $\nu = 4$.

Summary and Outlook

Done

- 1 Black brane and shell solutions with Lifshitz-like asymptotics
- 2 Wilson loops in the Lifshitz-like backgrounds
- 3 Pseudopotentials and spatial string tensions

Summary and Outlook

Done

- 1 Black brane and shell solutions with Lifshitz-like asymptotics
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Open questions

- 1 Time-like Willson loops, potentials, quarkonium spectrum that CAN FIT experemental data for multiplicity
- 2 Jet quenching parameter for anizotropic background that CAN FIT experemental data for multiplicity
- 3 Generalization for non-zero chemical potential (non-zero barionic density)
- 4 Isotropization, holographic RG-flow from $AdS_3 \times R^2$ to Lif_5 ?
- 5 Any supergravity embeddings?

Outline

- 1 Part I: QGP
- 2 Part II: Holography
 - The AdS/CFT conjecture
 - Holographic dictionary
 - Holographic models
- 3 Part III: All that
 - Colliding shocks
 - Holographic Wilson loops
 - WL and thermalization
- 4 Aferword

Afterword

ROTATION

- Top-down models: solutions of 10d supergravity – spinning D-branes (see works by **Tseytlin et.'98**, **Russo'99**)
- Bottom-up approach: 5d Kerr solution – (**Hawking et al.'99**)



- T-D approach: supersymmetry \rightarrow *R-symmetry* \rightarrow 10d- $>$ 5d **chemical potential** !
- B-U model: *J* – for non-central collisions, but **WHICH** gauge theory?

