Введение в непертурбативную кинетическую теорию кварк-глюонной плазмы - II

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1. Почему необходима КХД кинетика.
2. Специфика КХД кинетики в условиях соударения ультрарелятивистских тяжелых ядер
3. Абелева проекция в кинетике КГП
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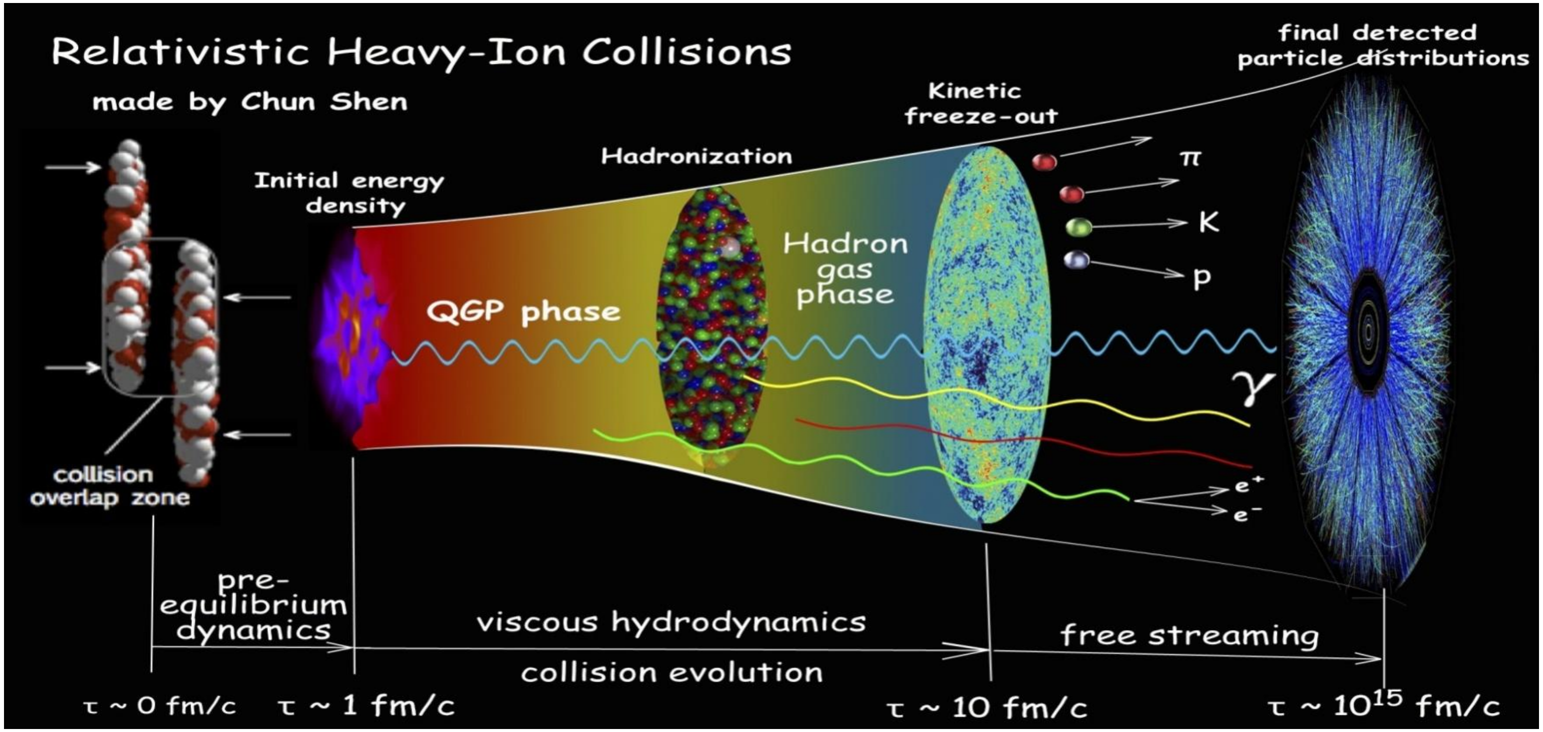
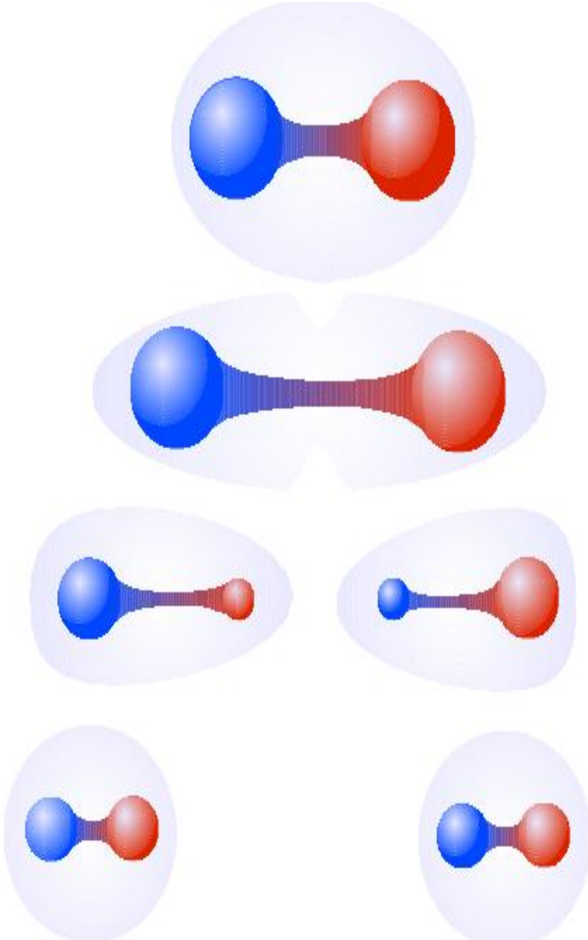
6. Проблема начальных условий



Вакуумное рождение $q\tilde{q}$ плазмы в сильных глюонных полях



1. **Почему необходима КХД кинетика ?**

Боголюбовская иерархия времен релаксации:

$$\tau_1 < \tau_2 < \tau_3 < \cdots .$$

2. **Специфика КХД кинетики в условиях соударения ультрарелятивистских тяжелых ядер**

2.1. **Различия в постановке задач и начальных условиях:**

- **в КЭД электромагнитное поле задаётся как внешнее,** $A^{\mu}(t) = A^{\mu}_{ext}(t)$ **В начальном состоянии электрон-позитронная плазма отсутствует** $f(\bar{p}, t = t_0) = 0$.

- В рассматриваемой ситуации столкновения ультрарелятивистских ядер сильные токи индуцируют внутренние электрическое $A^{\mu}(t)$ и цветовое $B_a^{\mu}(t)$ поля.

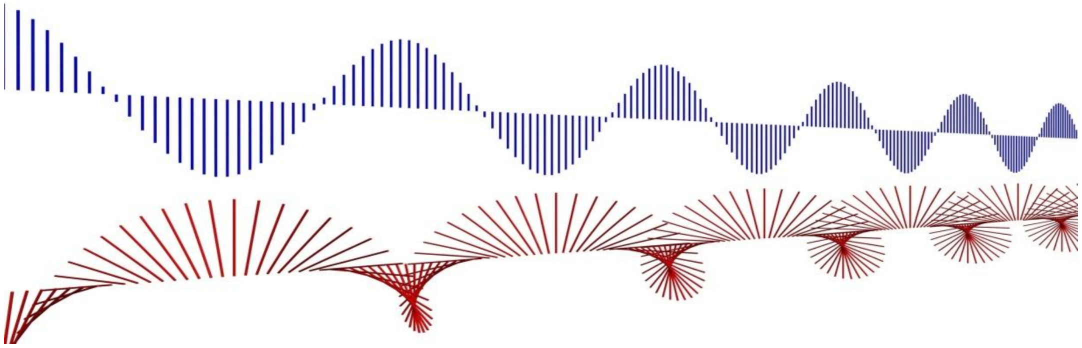
  В начальном состоянии необходимо определить:

  а) электромагнитное поле $A^{\mu}(t = t_0) =?$

  и цветовые поля $B_a^{\mu}(t = t_0) =?$

  б) функции распределения кварков различных ароматов и цветов $f_{fj}(\bar{p}, t = t_0)$.

Вывод: поляризации полей $A^{\mu}(t)$ и $B_a^{\mu}(t)$ являются динамическими. Всякое ограничение поляризации этих полей приводит к модельному ограничению.



3. Абелева проекция в кинетике КГП

Простейшей является моель линейно поляризованных полей:

$$A^{\mu}(t) = (0,0,0, A^{3}(t) = A(t))$$

$$B_a^{\mu}(t) = (0,0,0, B_a^{3}(t) = B_a(t))$$

При таком определении цветовое поле становится Абелевым. Рассмотрим не абелево и абелево поля в сравнении

$$\mathcal{L}_{JM} = -\frac{1}{4}\sum G_a^{\mu\nu} G_{\mu\nu}^{a}$$

$$G_a^{\mu\nu} = B_a^{\mu,\nu} - B_a^{\nu,\mu} + g\sum f_{abc} B_b^{\mu} B_c^{\nu}$$

f_{abc} **- структурные константы SU(3) группы. Они полностью антисимметричны.**

В случае линейной поляризации:

$$G_a^{30} = -G_a^{03} = \dot{B}_a \,,$$

$$\mathcal{L}_M = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \,,$$

$$F^{\mu\nu} = A^{\nu,\mu} - A^{\mu,\nu}$$

$$F^{30} = -F^{03} = \dot{A}$$

1-е условие Абелевости: модельное ограничение линейной поляризацией.

4. Кинетическая теория в квазичастичном представлении

Let us write the initial definitions: the Lagrangian function of the quark sector of QCD in an external classical gluon (B) and electromagnetic (A) fields ($\hat{m}$ is the mass matrix)

$$\mathcal{L} = i\bar{q}\gamma^{\mu} D_{\mu} q - \bar{q}\hat{m}q, \quad (1)$$

equation of motion

$$\left(iD_{\mu}\gamma^{\mu} - \hat{m}\right) q = 0, \quad (2)$$

energy-momentum tensor

$$T_{\mu\nu} = \frac{i}{2}\left[\bar{q}\gamma_{\mu} D_{\mu} q - (D_{\nu}^{+}\bar{q})\gamma_{\mu} q\right] \quad (3)$$

and densities of electromagnetic

$$j_{\mu} = \bar{q}\gamma_{\mu} q \quad (4)$$

and color

$$J_{\mu}^{a} = \bar{q}\lambda^{a}\gamma_{\mu} q \quad (5)$$

currents. The electromagnetic A^{μ} and gluon fields B_a^{μ} bring in theory by standard way $(e > 0, g > 0)$

$$D^{\mu} q^{j} = \sum_{j'}\left[\left(\partial^{\mu} + ie_{f}A^{\mu}\right)\delta_{jj'} - \frac{i}{2} g \sum_{a} B_{a}^{\mu}\lambda_{jj'}^{a}\right] q^{j'}. \quad (6)$$

Here $f, f', \ldots$ are flavour and $j, k, \ldots$ are color index; e_f is the electric charge of the quark with flavour f, g is the color charge. The corresponding Hamiltonian function is equal

after exception of the time derivative of the quark field with help of equation of motion

$$\mathcal{H} = T_{00} = \frac{i}{2}\left[\bar{q}\vec{\gamma}\vec{D}q - \left(\vec{D}^{+}\bar{q}\right)\vec{\gamma}q\right]. \qquad (7)$$

In general case transition in the quasiparticle representation (QPR) implies diaganolization of the Hamiltonian in the Fock representation relative to the quark and antiquark creation and annihilation. Bogolubov's canonical transformation method is used for this aim. In QED it is possible for the category of space homogeneous time dependent classical electromagnetic fields, which provides locality in the momentum space of the Hamiltonian and does not violate of homogeneity of the quadratic form of the creation and annihilation operators. However the last feature of the QED interaction is violated in QCD, where the nondiagonal color terms in the quark quadratic form in the Hamiltonian which do not change color of quarks in act interaction with the gluon field to the diagonal elements of the Gell-Mann matrix λ. Thus, the Hamiltonian function can be represented in the form:

$$\mathcal{H} = \mathcal{H}_Q + \mathcal{H}_{int}, \qquad (8)$$

where $\mathcal{H}_Q$ is the homogeneous flavour and color quadratic form.

$$\mathcal{H}_Q = \frac{i}{2}[\bar{q}\vec{\gamma}\vec{\mathfrak{D}}q - (\vec{\mathfrak{D}}\bar{q})\vec{\gamma}q] \qquad (9)$$

where $\mathfrak{D}^{\mu}_{fj}$ contains interaction of the electromagnetic and gluon fields with the quark of a flavour f and a color j only,

$$\mathfrak{D}^{\mu}_{fj} = \partial^{\mu} + ie_f A^{\mu} - \frac{i}{2} g \sum B^{\mu}_a \lambda^a_{jj} = \partial^{\mu} + ie_f A^{\mu} - \frac{i}{2} g \mathcal{A}^{\mu}_j \qquad (10)$$

where the gluon fields $\mathcal{A}_j^\mu$ do not change the quark color,

$$\mathcal{A}_j^\mu = \begin{cases} B_3^\mu + \frac{1}{\sqrt{3}} B_8^\mu, & j = 1; \\ -B_3^\mu + \frac{1}{\sqrt{3}} B_8^\mu, & j = 2; \\ -\frac{2}{\sqrt{3}} B_8^\mu, & j = 3. \end{cases} \qquad (11)$$

Just the Hamiltonian function (9) is the basis for transition in QPR in the quark Fock space.

2-е условие Абелевости:

выделение диагональнй части матрицы Гелл-Манна

$$\lambda^a_{jj'} \to \lambda^a_{jj}$$

The Hamiltonian function $\mathcal{H}_{nt}$ in (8) describes the residual part of the Hamiltonian function (7) which connect with the color changing interaction of a quark with the gluon field.

In order to the Hamiltonian operator would be local in the second quantization representation, it is necessary to introduce a model simplification about space homogeneous of the system, i.e. the gluon and electromagnetic fields will depend on the time only. It is the most essential limitation of the method. In this case, the every Hamiltonian corresponding to functions (9) allows transition in QPR and, consequently, diaganolization with help of the Bogoliubov canonical transformation. The similar problem was solved primarily in QED for the linear polarized external electric field and after that in general case. Bellow we will consider also the case of the linear polarized electric and gluon fields. We select the Hamiltonian gauge of the electromagnetic field $(A^0 = 0)$ and, correspondingly, $B^0 = 0$ for the gluon field. Thus,

$$A^\mu(t) = \big(0, 0, 0, A^3(t) = A(t)\big), \qquad B^\mu(t) = \big(0, 0, 0, B^3(t) = B(t)\big). \qquad (12)$$

Transition to QPR relates with transition from stationary vacuum $|in>$ to non-stationary unstable vacuum $|t>$. Operators of creation and annihilation of quarks $q^{\pm}_{fj}(\vec{p},t)$ and antiquark $\tilde{q}^{\pm}_{fj}(\vec{p},t)$ are defined on this quasiparticle vacuum and describe the quasiparticle excitations with the quasienergy $\varepsilon_{fj}(\vec{p},t) = \sqrt{E_f^2 + P_{fj}^2}$ with the quasimomentum

$$\vec{P}_{fj} = \left(p^1, p^2, P^3 = p^3 - e_f A(t) + \frac{1}{2} g \mathcal{A}_j(t)\right) \quad (13)$$

and the transversal quark energy $\varepsilon_f^2 = m_f^2 + p_1^2 + p_2^2.$

Thus, the quasiparticle excitations in $q - \tilde{q}$ plasma are result of joint action of the electric and chromo-electric fields.

The further procedure of derivation of the KE's in the quark sector QCD does not differ from the well known one in QED. The resulting system of KE's relative to the distribution functions of quarks

$$f_{fj}(\vec{p},t) =< t|q^{+}_{fj}(\vec{p},t) q^{-}_{fj}(\vec{p},t)|t> \quad (14)$$

and antiquarks

$$\tilde{f}_{fj}(\vec{p},t) =< t|\tilde{q}^{+}_{fj}(\vec{p},t) \tilde{q}^{-}_{fj}(\vec{p},t)|t> \quad (15)$$

has the following form

$$\begin{aligned} \dot{f}_{fj}(\vec{p},t) &= \dot{\tilde{f}}_{fj}(\vec{p},t) = \frac{1}{2} \lambda_{fj}(\vec{p},t) \int_{t_0}^{t} dt' \lambda_{fj}(\vec{p},t') \times \\ &\times \left[1 - f_{fj}(\vec{p},t) - \tilde{f}_{fj}(-\vec{p},t)\right] \cos\theta_{fj}(\vec{p};t,t'), \end{aligned} \quad (16)$$

where

$$\lambda_{fj}(\vec{p},t) = \frac{F_{fj}(\vec{p},t)}{\varepsilon^2_{fj}(\vec{p},t)} \quad (17)$$

is the amplitude of vacuum transitions,

$$F_{fj}(\vec{p},t) = -e_f \dot{A}(t) + \frac{1}{2} g \dot{A}_j(t) \quad (18)$$

is the generalized force acting on the quark with the quantum numbers $f,\ j,\ \vec{p};$ finally,

$$\theta_{fj}(\vec{p};t,t') = 2\int_{t'}^{t} d\tau \varepsilon_{fj}(\vec{p},\tau) \quad (19)$$

is the phase of the high frequency oscillations with the gap energy $2\varepsilon_{fj}(\vec{p},t)$ in the presence of the classical electric and chromo-electric fields.

KE's (16) for quarks and antiquarks are the same, but the functions $f_{fj}(\vec{p},t)$ and $\tilde{f}_{fj}(-\vec{p},t)$ can differ because of difference of the initial conditions, $f_{fj}(\vec{p},t_{in}) \neq \tilde{f}_{fj}(-\vec{p},t_{in}),$ that corresponds to rather real conditions in the moment of merging of the nuclear matter by collision two ultrarelativistic heavy ions. In QED the the equality $f = \tilde{f}$ is using, as a rule, that corresponds to the electroneutrality condition.

The system (16) contains 2 x 3N integro-differential KE's of the non-Markovian type for the three combinations of the color fields (11). For numerical investigations it is convenient to rewrite each KE (16) in the form of system of the three ordinary differential equations

$$\begin{aligned} \dot{f} &= \dot{\tilde{f}} = \frac{1}{2}\lambda u, \\ \dot{u} &= \lambda(1 - f - \tilde{f}) - 2\varepsilon v, \\ \dot{v} &= 2\varepsilon u. \end{aligned} \quad (20)$$

Adequate dynamical description of QCP is impossible without electromagnetic constituent of dynamics. It can wait strong electromagnetic fields are generated at the same time with strong gluon fields. It leads to vacuum EPP creation and strong electromagnetic currents in EPP Thus, the quark KE system (16) or (20) should be added by the corresponding KE for description of vacuum EPP production. Thus, the KE has the form analogical to KE (16)

$$\dot{f}(\vec{p},t) = \dot{\tilde{f}}(\vec{p},t) = \frac{1}{2}\lambda(\vec{p},t)\int_{t_0}^{t} dt'\lambda(\vec{p},t')\left[1 - f(\vec{p},t) - \tilde{f}(-\vec{p},t)\right]\cos\theta(\vec{p};t,t') \quad \textbf{(21)}$$

or system (20) with

$$\lambda(\vec{p},t) = \frac{E(t)}{\varepsilon^2(\vec{p},t)}, \quad \textbf{(22)}$$

where $\varepsilon(\vec{p},t) = \sqrt{\varepsilon_\perp^2 + P^2}$ **is the electron quasienergy,** $\varepsilon_\perp = \sqrt{m^2 + p_\perp^2}$ **is the transversal electron energy. The phase** $\theta(\vec{p};t,t')$ **in (21) is defined by analogy with (19).**

5. Динамика калибровочных полей

Самосогласованный характер эволюции КГП.

Поля $A(t)$ **и** $B_3(t), B_8(t)$ **- внутренние, их динамика самосогласована с кинетикой вакуумного рождения кварков.**

Уравнения типа Максвелла для электрических и хромоэлектрических полей:

$$\dot{E}(t) = -\ddot{A}(t) = -4\pi j\,,$$

$$\ddot{\mathcal{A}}_j(t) = 4\pi j_j(t)\,,$$

$j(t) = j_{cond}(t) + j_{pol}(t)$ **- плотность электрического тока,**

$j_j(t) = {j^j}_{cond}(t) + {j^j}_{pol}(t)$ **- плотность хромоэлектрического тока.**

Здесь $j_{cond}(t) = j_{cond}(f)$, $j_{pol}(t) = j_{pol}(u)$.

6. Проблема начальных условий

ИТОГИ: получена замкнутая самосогласованная система интегро-дифференциальных уравнений, описывающих динамику КГП в Абелевой модели КХД.

Основные ограничения модели:

1. Пространственная однородность $\varphi(\bar{x},t) \to \varphi(t)$.
2. Абелевость, что подразумевает
   2.1. Линейную поляризацию классических бозонных полей
   2.2. Выделение в Гамильтониане кваркового сектора диагональной по цвету части, описывающей взаимодействие кварков с YM – полем
3. Дополнительное предположение: симметрия относительно P^3 - инверсии. Например,

$$P^3 f(p_\perp, p^3 = p_\parallel) = f(p_\perp, -p^3) = f(p_\perp, p^3)$$

иначе, $f(\bar{p}) = f(p_\perp, (p^3)^2)$ - в любой момент времени.

Следствие: любые векторные величины должны обращаться в нуль.

Пример: Back reaction in QED

Strong currents generated by string-like superstrong "chromo-electric" field Es(t) can excite the internal field Ein(t) which can be comparable with Es(t). It defined by the Maxwell equation with the regularized polarization current,

$$\dot{E}_{in} = -2g \int \frac{d^3p}{(2\pi)^3} \frac{p_\parallel}{\varepsilon} \left[f + \frac{u}{2} \frac{\varepsilon_\perp}{p_\parallel} - g\dot{E} \frac{\varepsilon_\perp^2}{8\varepsilon^4 p_\parallel} \right] .$$

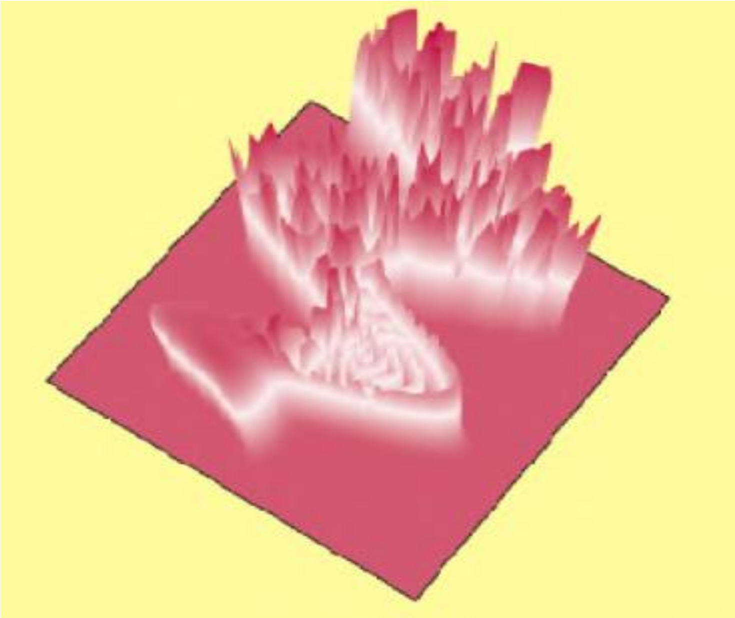
The total field here is equal

$$E(t) = E_s(t) + E_{in}(t).$$

The total equation system of the back reaction problem will be

KE + Maxwell Eq.

Back reaction mechanism leads to exhaustion of the color field and chaotization of the quark component of QGP (D.V. Vinnik, V.A. Miserny, A.V. Prozorkevich, S.A. Smolyansky, V.D. Toneev, Phys. of Atom. Nucl., **64**, 775 (2001)).



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