CHIRAL LIQUIDS. Part II

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Talk
at Dubna School of Physics, 30.01-04.02 (2017)

2 февраля 2017 г.

Derivation of Chiral Effects, (Son+Surowka,2009)

Hydrodynamics is a universal theoretical framework.

The only input is conservation laws and expansion in derivatives, i.e. long-wave approximatiion.

The reason: only perturbation of conserved quantities propagate far off. other perturbation die off fast (Feynman)

T.D. Son and P. Surowka derived ChME and ChVE using only hydrodynamics + anomaly

the best known derivation, although there were predecessors and many-many followers

Cnt'd

In presence of external electric and magnetic fields:

$$\partial_\mu T^{\mu\nu} = F^{\nu\rho} j^{el}_\rho$$

$$\partial_\mu j^\mu_{el} = 0, \quad \partial_\mu j^\mu_5 = \frac{\alpha_{el}}{4\pi} \epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta}$$

$$\partial^\mu s_\mu \geq 0$$

s_μ is the entropy current; $T_{\mu\nu}, j_\mu$ expanded in derivatives

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

$$j^\mu = n u^\mu + \nu^\mu$$

where $\tau^{\mu\nu}, \nu^\mu$ are of higher order in derivatives and incorporate dissipative effects

+

Effect of anomaly on hydrodynamcs

Standard expressions:

$$\tau_{\mu\nu} = -\eta(\partial^\alpha u^\beta + \partial^\beta u^\alpha)P_{\mu\alpha}P_{\nu\beta} - \ldots \qquad (1)$$

$$\nu^\mu = -\sigma P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu$$

$$s^\mu = su^\mu - \frac{\mu}{T}\nu^\mu$$

with $P^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ (other notations are rather obvious)
central point: anomaly invalidates $\partial_\mu s^\mu \geq 0$. Instead:

$$\nu^\mu = -\sigma P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu + +\xi\omega^\mu + \xi_B B^\mu$$

$$s^\mu = su^\mu - \frac{\mu}{T}\nu^\mu + D\omega^\mu + D_B B^\mu$$

Cnt'd

The central (and beautiful) point of Son+Surowka:

the extra terms are uniquely determined by the hydrodynamic equations in terms of the anomaly.

The energy-momentum tensor was chosen in its simplest form, zero-order in derivatives. No viscosity, in particular

Higher orders in hydrodynamic expansion are assumed to give small corrections.

Hydrodynamics as an effective theory

Hydrodynamics is an "effective theory" but not (only) in the sense of Wilson (integrating out short distances)
Rather, one changes Hamiltonian

$$H_0 \;\rightarrow\; H_0 - \mu Q$$

$Q = \int d^3x\, j_0$ is charge associated with chemical potential μ
Moreover,

$$\delta L \;=\; -\delta H$$

To observe relativistic covariance,

$$\begin{aligned} \mu j_0 \;&\rightarrow\; \mu u^\mu j_\mu \;,\;\; or \\ eA_\mu \;&\rightarrow\; eA_\mu + \mu u_\mu \end{aligned} \tag{2}$$

(once again: no actual local interaction behind μu_μ)

Chiral anomaly vs new conserved current

Standard chiral (U(1)) anomaly can be reformulated as an expression for a new conserved axial current:

$$Q^A_{conserved} = Q^A_{naive} + \frac{e^2}{4\pi^2}\mathcal{H}_{magn}, \qquad \frac{d}{dt}Q^A_{conserved} = 0$$

where Q^A_{naive} counts chiral constituents, $Q^A_{naive} = n_L - n_R$, and $\mathcal{H}$ is the so called magnetic helicity:

$$\mathcal{H}_{magn} = \int d^3x \vec{A}\cdot\vec{B}$$

Note that the magnetic helicity is gauge invariant although the new current density is not.
All this is known since Gell-Mann's times.

Reminder of basics

$$\mathcal{H}_{magn} \sim \int d^3x \epsilon_{0ijk} A^i \partial^j A^k$$

$$\mathcal{H}_{magn} \sim \int d^3x \vec{A} \cdot \vec{B}$$

is gauge invariant under $A^i \rightarrow A^i + \partial^i \Lambda(x)$ (easy to check)

$$\partial_0 \mathcal{H}_{magn} \sim \int d^3x \vec{E} \cdot \vec{B}$$

is a trivial algebraic statement

Total conserved axial current, hydrodynamics

Remember, there is a change brought in by hydrodynamics
As is discussed, $eA_\mu \;\rightarrow\; eA_\mu + \mu \cdot u_\mu$ Then:

$$Q^A_{hydro} \;=\; Q^A_{naive} + Q^A_{fluid\;\;helicity} + Q^A_{mixed} + Q^A_{magnetic\;\;helicity}$$

where $Q^A_{fluid\;\;helicity} \;=\; (1/4\pi^2) \int d^3x j^0_{fluid\;\;helicity}$

$$j^\mu_{fluid\;\;helicity} \;=\; (1/2)\epsilon^{\mu\nu\rho\sigma}\omega_{\nu\rho}(\mu u)_\sigma$$

$$\omega_{\nu\rho} \;= (\mu u_\nu)_{,\rho} - (\mu u_\rho)_{,\nu}$$

and Q^A_{mixed} involves interference of both eA_μ and μu_μ

The red-line eqn matches perfectly Son+Surowka

Non-renormaization theorems

In $Q^A_{fluid\ helicity}$ we recognize axial charge associated with the vortical effect, or helical macroscopic motion.

Thus, all non-renormalization theorems for the chiral effects become now a direct consequence of Adler-Bardeen theorem and hydro-extension $eA_\mu \ \rightarrow \ eA_\mu + \mu \cdot u_\mu$

Note:Only the last term, $Q^A_{magnetic\ helicity}$ is quadratic in electromagnetic coupling and related to the anomaly on fundamental level.

Other terms are anomalies induced by introduction of the effective theory (V.I. Shevchenko et al. (2011)).

Intermediate conclusions

- There are well defined derivations of chiral effects in lowest-order of the derivative expansion
- The chiral effects are protected against corrections by the extension of the Adler-Barden theorem
- Actually, there are many other derivations (like geometric formulation of thermodynamcs)
- Non-conservation of Q_{naive} implies transition of charge from microscopic to macroscopic d.o.f. and vice versa

All in all, a new interesting chapter on field theory

Scrutinizing the derivations

Let us pursue the questions touched above:

- How is it possibe to have anomalies in the effective theory which apparently do not match anomalies of the fundamental theory?
- If we look into the Son+Surowka result, ChVE survives even in the limit of external fields switched off.
  But then there is no anomaly, and how anomaly could fix the extra terms?

Classical physics and IR "divergences"

General remark:
Rather commonly, in classical physics there are hidden large parameters or infrared divergences (contrasted with UV problems of QFT)

(as is known since long: infinite amount of light in the sky:

$$\mathit{Intensity} \sim \int_0^{R_\infty} d^3 r \frac{1}{r^2} n(r) \sim R_\infty$$

In our case: large time needed to reach equilibrium might invalidate expansion in derivatives (?).

Conflict of symmetries in the ideal-liquid limit

The answers are actually not difficult to find:

For ideal liquid, there is an extra conserved current,
or charge which is the whole of extension of Q_{naive} that is

$$\frac{d}{dt}\left(Q_{fluid\ \ helicity} + Q_{mixed\ \ helicity} + Q_{magnetic\ \ helicity}\right) = 0$$

Or: ideal liquid is such an IR completion of field theory which screens the short-distance anomaly off

More on the screening

Nature of extra conservation law is readily recognizable:

For ideal conductor

$$\vec{j}_{el} = \sigma_E \vec{E}$$

in the limit $\sigma_E \to \infty$ reads as $\vec{E} = 0$, pushed to boundary

The conservation law above is a generalization
of this high-school statement

All currents considered might be pushed to the boundary,
see also the talk by O.V. Teryaev

Subtle point

Subtle point: Son+Surowka keep

$$\sigma_E \textit{ finite}, \;\; \textit{while } \eta \equiv 0$$

a kind of hybrid scheme (not a fully ideal liquid).
And reproduce the same current. That is, transfer from "micro" to "macro" is allowed.

if one keeps $\eta \neq 0$ the "classical" Son+Surowka result is strongly modified.

Thus, the predictions seem to be IR sensitive

Intermediate conclusion

- In the limit of ideal liquid transfer of charge from micro constituents to macro configurations is not allowed. This limit, however, is the only field theoretic starting point known.
- Probably, all currents are pushed to the boundary in this limit
- Next step: scrutinize the ideal-liquid field theory itself. Is there any sign of pushing to he boundary?

Field theoretic approach to hydrodynamics

Not to forget: the presentation was partly motivated by the fact that the effect of polarization of final particles (see the talk by O.V. Teryaev) has been just considered within this framework:

arXiv:1701.08263
The ideal relativistic fluid limit for a medium with polarization David Montenegro, Leonardo Tinti, Giorgio Torrieri

One of conclusions is similar to what we derived: no transfer from micro to macro in the ideal-liquid limit

Field theoretic approach to hydro, Cnt'd

Aimed at expressing all the quantities in field-theoretic variable, that is, fields

$$L(\phi^I, \partial_\mu \phi^I) \quad \to \quad T_{\mu\nu}, \quad J_\mu$$

Where ϕ^I are specific hydrodynamic d.o.f. in all generality.

Advantages: systematic way, quantization, (in)stability

However, dissipation is difficult for field theory.
Hence, mostly, perfect fluid.
Ways to introduce dissipation (not covered in this review):

- introduction of local operators, or interaction with heavy degrees of freedom
- introduction of a new dimension, moving away from the boundary corresponds to poor resolution

References

A few waves of literature ("Backward to theory of ...."):

A. Clebsch, "Über die Integration der Hydrodynamischen Gleichungen" J. Reine Angew. Math. 56 1 (1859)

Reproduced and renovated in:

B.F. Schutz, "Perfect fluids in general relativity: velocity potentials and a variational principle", Phys. Rev. D2, 2762 (1970); Hamiltonian theory of perfect fluid, Phys Rev D4, 3559, (1971)

G. Herglotz "Über die Mechanik des deformierbaren Körpers vom Standpunkte der Relativitätstheorie" Annalen Der Physik vol. 341, no. 13, pp. 493, (1911)

Reproduced (apparently) in

D. Soper, "Classical Field Theory", (2008) Dover.

References, New era

Beginning with $\sim$ year 2000 a "new era" of great popularity
D.T. Son, 23 PRL papers, 4700 citations
M. Stephanov, 12 PRL papers, 2700 citations
P. Kovtun, 5 PRL papers, 1800 citations
Ch. Herzog, 3 PRL papers, more than 1000 citations
A. Abanov, 4 PRL papers, very recent
Sean Hartnoll, "New Horizons" prize (2015)

- old results and extensions in modern language
- hydrodynamics and quantum anomaly
- geometry and thermodynamics
- geometry and dissipation
- New synthesis of condensed-matter and QFT, GR...

Number of hydrodynamic d.o.f.

perfect liquid, d (spatial) dimensions.

effective low-energy degrees of freedom can be chosen as

$$d \quad \text{scalar fields} \quad \phi^I, \quad (I = 1, ..., d).$$

scalars can be identified with co-moving coordinates of an element of the liquid. In equilibrium

$$\phi^I = x^I, \quad I = 1, ..., d$$

If there is a conserved current, need one more scalar.

Coordinates are components of a vector and do not look like scalars at all. Address this question first.

Effective action, reminder

Let us start with the action

$$S \;=\; \int d^4x \left(\partial_\mu\varphi^*\partial^\mu\varphi \;-\; (\varphi^*\varphi \;-v^2)^2 \right)$$

Substitute $\phi \;=\; v\exp(i\theta)$. For small momenta $p \ll v$

$$S \;\approx\; v^2(\partial_\mu\theta\partial^\mu\theta) \;+\; ..$$

Original invariance under

$$\varphi \;\to\; \varphi\exp(i\alpha)$$

becomes invariance under

$$\theta \;\to\; \theta \;+\; a$$

where α, a constants. Keeps θ massless.

Restoring symmetries of space

Postulate invariances under following transformations:

- $\phi^I \;\rightarrow\; \phi^I \;+\; a^I$, a^I *are constants*,
- $\phi^I \;\rightarrow\; R^I_J \phi^J$, $\quad R^I_J \;\in\; SO(d)$,
- $\phi^I \;\rightarrow\; \xi^I(\phi^I), \qquad det\left(\frac{\partial \xi^I}{\partial \phi^I}\right) \;=\; 1$.
- Poincare invariance in physical coodinates,
  $x_i, t \;\; (i = 1, ..., d)$,

the invariance under $\phi^I \;\rightarrow\; \xi(\phi^I)$ is most non-trivial and specific

Clebsch potentials

Dynamic 4 velocity

$$\xi_\lambda \;=\; \partial_\lambda \theta + \alpha \partial_\lambda \beta \quad , \tag{3}$$

where θ, α, β are Clebsch potential.

No action was considered, only kinematics, locally.
(notion of gauge invariance is not that simple since diffeomorphism is an infinite-dimensional group)

We will proceed to the effective action which incorporates symmetries of the problem.

Constructing action

Invariants are organized according to the number of derivatives. The lowest order invariant looks as

$$B \equiv det(B^{IJ}) \;, where \;\; B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J \;.$$

To this order in derivatives, in relativistic 4d case

$$S_{liquid} = \int d^4 x F(B) \;,$$

$$B = (const)\epsilon^{\mu\alpha\beta\gamma}\epsilon_\mu{}^{\rho\sigma\delta}\epsilon_{IJK}\epsilon_{LMN}\partial_\alpha\phi^I\partial_\beta\phi^J\partial_\gamma\phi^K\partial_\rho\phi^L\partial_\sigma\phi^M\partial_\delta\phi^N$$

$F(B)$ is an arbitrary function of the invariant B.
Can normalize $B = 1$ at the equilibrium.

Link to hydrodynamics

Knowing S_{liquid} allows determine energy-momentum tensor

$$T_{\mu\nu} \ = \ -2F^{'}(B)B(B^{-1})_{IJ}\partial_\mu\phi^I\partial_\nu\phi^J + \eta_{\mu\nu}F(B) \ (*).$$

The standard hydrodynamic expression is:

$$(T_{\mu\nu})_{hydro} \ = \ (\rho + p)u_\mu u_\nu + p\eta_{\mu\nu} \ , \qquad (**)$$

where $\eta_{\mu\nu} \ = \ (-1, 1, 1, 1)$ and u_μ is the 4-velocity of an element of the liquid, $u_\mu u^\mu = -1$.

To match (*) and (**) we need to identify field-theoretic expression for the velocity u_μ.

Matching hydrodynamics (cnt'd)

Since ϕ^I are comoving coordinates,

$$\frac{d}{d\tau}\phi^I(x) = 0 \,,$$

where τ parametrizes the streamline. In terms of the 4-velocity this derivative is given by:

$$\frac{d\phi^I}{d\tau} \equiv u^\mu \partial_\mu \phi^I(x) \,.$$

And we conclude:

$$u^\mu = -\frac{1}{\sqrt{B}}\epsilon^{\mu\alpha\beta\gamma}\partial_\alpha\phi^1\partial_\beta\phi^2\partial_\gamma\phi^3 \,, \tag{4}$$

where $\epsilon_{0123} = -\epsilon^{0123} = 1$.

Hydrodynamic excitations.

Deviations from the equilibrium parametrized as π^I:

$$\phi^I = x^I + \pi^I(x) .$$

To second order:

$$L^{(2)} = \frac{1}{2} w_0 \left(\dot{\pi}_L^2 - u_s^2 (\vec{\partial}\pi_L)^2 \right) + \frac{1}{2} w_0 \dot{\vec{\pi}}_T^2 ,$$

where π_L and π_T are longitudinal and transverse:
$\pi^I = \frac{\partial^I}{\sqrt{-\partial^2}} \pi_L + \pi_T^I$,
while w_0 is entalpy $w_0 = -2F'(1) = (\rho + p)_{B=1}$,
u_s^2 is speed of sound squared:

$$u_s^2 = \frac{dp}{d\rho}|_{B=1} = \frac{2F''(B)B + F'(B)}{F'(B)}|_{B=1} .$$

Problems in infrared?

For longitudinal excitations:

$$\omega_L = u_s p_L ,$$

For the transverse fields the dispersion relation is degenerate:

$$\omega_T = 0 .$$

Vortices do not propagate at large distances

Agrees with hydrodynamic theory,
with relation of propagation and conservation laws
(no symmetry of space behind a helical motion)

$\omega_T = 0$ is potential source of infrared problems

Problems in infrared

Solution for π_T:

$$\vec{\pi}_T = \vec{\nabla}\ x\ \left(\vec{a}(\vec{x}) + t\vec{b}(\vec{x})\right)$$

where $\vec{a}(\vec{x})$, $\vec{b}(\vec{x})$ are arbitrary.
Linearized version of a vortex in constant rotation.

QFT allows to calculate higher orders. For example,

$$\lim_{\omega\to\ 0}\langle\partial_i\pi^I,\ \partial_j\pi^J\rangle = \frac{P_T^{IJ}p^5}{w_0\omega} + \ldots\ ,$$

where P_T^{IJ} is the transverse projector.

Explicit demonstration of a region where the interaction is strong due to the pole at $\omega = 0$.

Problem with the problem in infrared

S. Endlich et al. 1011.6396 hep-th, argued that
IR problems of the S-matrix cannot be cured

This suggests that one should go to another vacuum

The problem is, however, that there is no reasonable "new vacuum" in sight. (see 1011.6396)

Two recent papers

There are two recent papers, both on the optimistic side

B. Gripaios, D, Sutherland,
"Quantum Field Theory of Fluids
Phys.Rev.Lett. 114 (2015) 7, 071601,
e-Print: arXiv:1406.4422 [hep-th].

T. Burch, G. Torrieri, " Indications of a non-trivial vacuum in the effective theory of perfect fluids"
Phys.Rev. D92 (2015) 1, 016009,
e-Print: arXiv:1502.05421 [hep-lat].

Consistent quantum theory of perfect liquid?

quote from the former paper:

"We assert that, in a general physical theory, only quantities that invariant under symmetries of the theory are observable. This is tautology.."

In reality, suggest to consider only
correlators of pressure p, energy density ρ, four velocity u_μ

In $(2+1)$ case have examples that this helps,
without damaging UV behaviour.

The $(3+1)$ case is not considered at all,
for "technical reasons".

Numericl study

Using lattice field theory techniques, we investigate the vacuum structure of the field theory corresponding to perfect fluid dynamics in the Lagrangian prescription. We find intriguing, but inconclusive evidence, that the vacuum of such a theory is non-trivial, casting doubts on whether the gradient expansion can provide a good effective field theory for this type of system. The non-trivialvacuum looks like a "turbulent" state where some of the entropy is carried by macroscopic degrees of freedom. We describe further steps to strengthen or falsify this evidence.

$F(B) \sim B^{2/3}$
as for ultrarelativistic gas

Intermediate conclusions III

- On classical level, remarkable simplicity in identifying universal scalar degrees of freedom, symmetries of the action
- On quantum level, difficult to summarize
- Possible reasons: complicated structure of gauge transformations (diffeomorphism); topological nature of vortices on large distances....

Next time consider superfluids, where vortices are presumably gapped. Promise of easier life.