



**JINR, BLTP**



# **Modelling of the High Temperature Superconductors at Nonequilibrium Conditions**

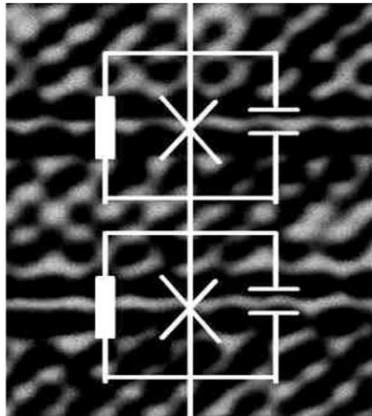
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# Coupled System of Josephson Junctions

## RCSJ - model



Current flowing through the system of Josephson junctions

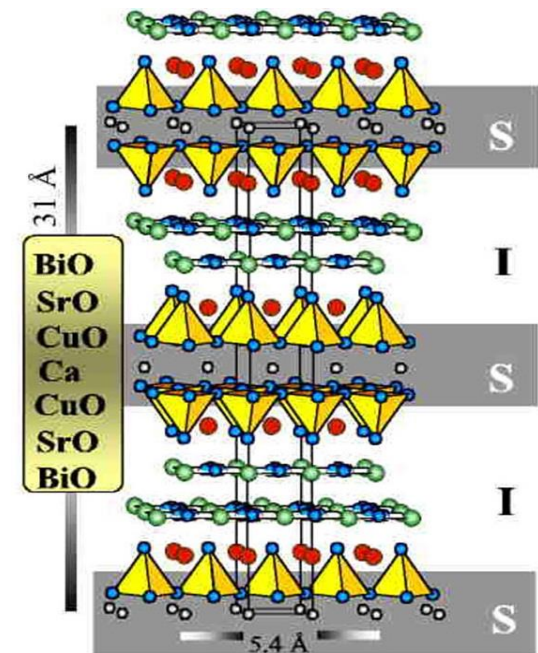
$$I_j = C_j \frac{dV_l}{dt} + I_c \sin \varphi_l + \frac{\hbar}{2eR_j} \frac{\partial \varphi_l}{\partial t}$$

Layered system, superconductor-insulator-superconductor (SIS).

One SIS layer form a Josephson junction.

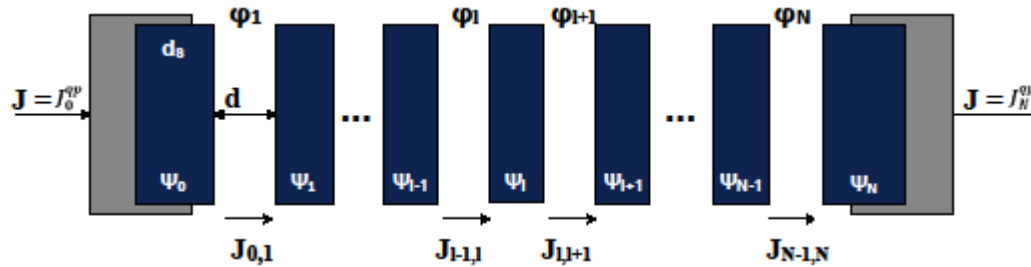
The capacitive coupling between adjacent junctions leads to generalised Josephson relation.

$$\frac{\hbar}{2e} \frac{d\varphi_l}{dt} = V_l - \alpha(V_{l+1} + V_{l-1} - 2V_l)$$



**Bi<sub>2</sub>Sr<sub>2</sub>CaCu<sub>2</sub>O<sub>8+x</sub> (BSCCO)**

# Charge imbalance in the System of Junctions



Nonperiodic boundary conditions

$$\gamma = \frac{d_s}{d_s^0} = \frac{d_s}{d_s^n}$$

The charge imbalance potentials  $\Psi_l$  are defined by the kinetic equations

$$\begin{aligned}\dot{\Psi}_0 &= \frac{4\pi r_D^2}{d_s^0} \left( J - \frac{\hbar}{2eR} \dot{\varphi}_1 + \frac{\Psi_1 - \Psi_0}{R} \right) - \frac{\Psi_0}{\tau_{qp}^0} \\ \dot{\Psi}_l &= \frac{4\pi r_D^2}{d_s} \left( \frac{\hbar}{2eR} \dot{\varphi}_{l-1} - \frac{\hbar}{2eR} \dot{\varphi}_l + \frac{\Psi_{l-1} + \Psi_{l+1} - 2\Psi_l}{R} \right) - \frac{\Psi_l}{\tau_{qp}} \\ \dot{\Psi}_N &= \frac{4\pi r_D^2}{d_s^N} \left( \frac{\hbar}{2eR} \dot{\varphi}_N - J + \frac{\Psi_{N-1} - \Psi_N}{R} \right) - \frac{\Psi_N}{\tau_{qp}^N}\end{aligned}$$

The total current density  $J_{l-1,l} \equiv J_l$  through each S-layer is given as a sum of displacement, superconducting, quasiparticle and diffusion terms

$$J_l = C \frac{dV_l}{dt} + J_c \sin \varphi_l + \frac{\hbar}{2eR} \dot{\varphi}_l + \frac{\Psi_{l-1} - \Psi_l}{R}$$

# System of equations

## Normalized system of equations

$$\dot{v}_l = \left[ I - \sin \varphi_l - \beta \dot{\varphi}_l + A \sin \omega \tau + I_{noise} + \psi_l - \psi_{l-1} \right]$$

$$\dot{\varphi}_1 = v_1 - \alpha(v_2 - (1 + \gamma)v_1) + \frac{\psi_1 - \psi_0}{\beta}$$

$$\dot{\varphi}_l = (1 + 2\alpha)v_l - \alpha(v_{l-1} + v_{l+1}) + \frac{\psi_l - \psi_{l-1}}{\beta}$$

$$\dot{\varphi}_N = v_N - \alpha(v_{N-1} - (1 + \gamma)v_N) + \frac{\psi_N - \psi_{N-1}}{\beta}$$

$$\zeta_0 \dot{\psi}_0 = \eta_0 (I - \beta \dot{\varphi}_{0,1} + \psi_1 - \psi_0) - \psi_0$$

$$\zeta_l \dot{\psi}_l = \eta_l (\beta [\dot{\varphi}_{l-1,l} - \dot{\varphi}_{l,l+1}] + \psi_{l-1} + \psi_{l+1} - 2\psi_l) - \psi_l$$

$$\zeta_N \dot{\psi}_N = \eta_N (-I + \beta \dot{\varphi}_{N-1,N} + \psi_{N-1} - \psi_N) - \psi_N$$

## Normalization parameters

$$V/v = \hbar \omega_p / 2e,$$

$$\psi_l = \Psi_l / J_c R,$$

$$\omega_p = \sqrt{2eJ_c / \hbar C},$$

$$\tau = \omega_p t,$$

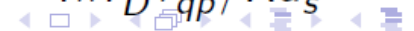
$$\zeta_l = \omega_p \tau_{qp}^l,$$

$$\alpha = \epsilon \epsilon_o / 2e^2 N(0) d,$$

$$\beta = \hbar \omega_p / 2e R J_c,$$

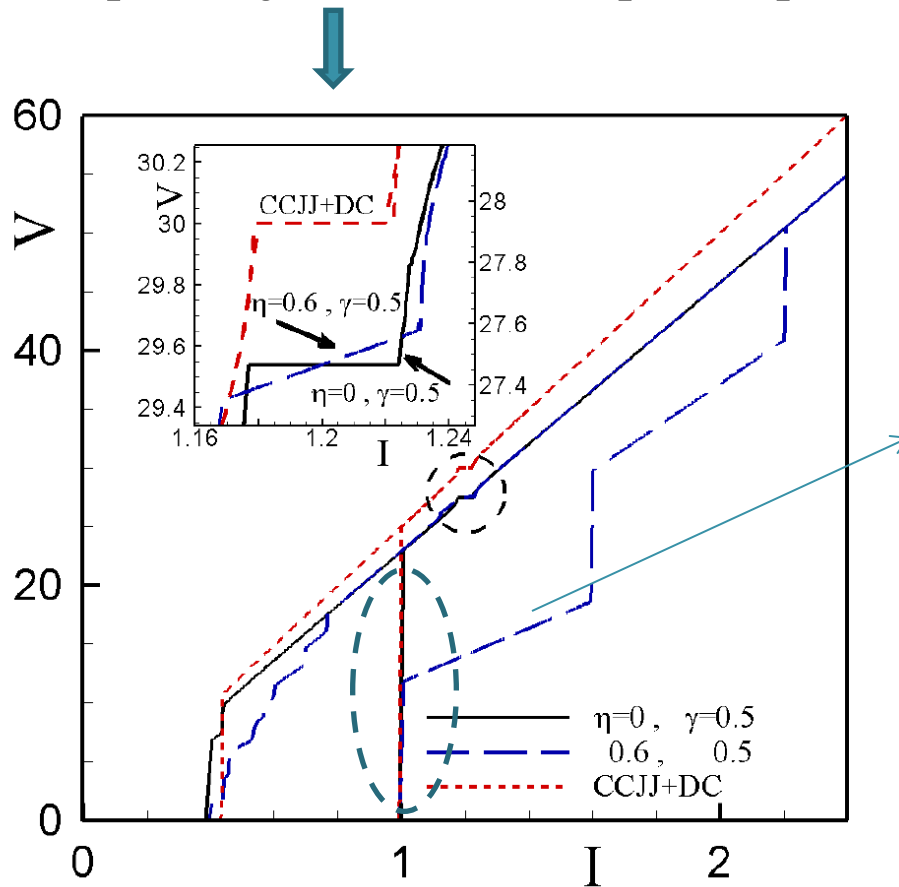
$$\eta_l = \nu^l \tau_{qp}^l =$$

$$4\pi r_D^2 \tau_{qp}^l / R d_s^l$$



# Current-voltage characteristics

The nonperiodic boundary conditions shift the outermost branch relatively to the curve of CCJJ+DC model leading to the corresponding shift of the Shapiro steps.

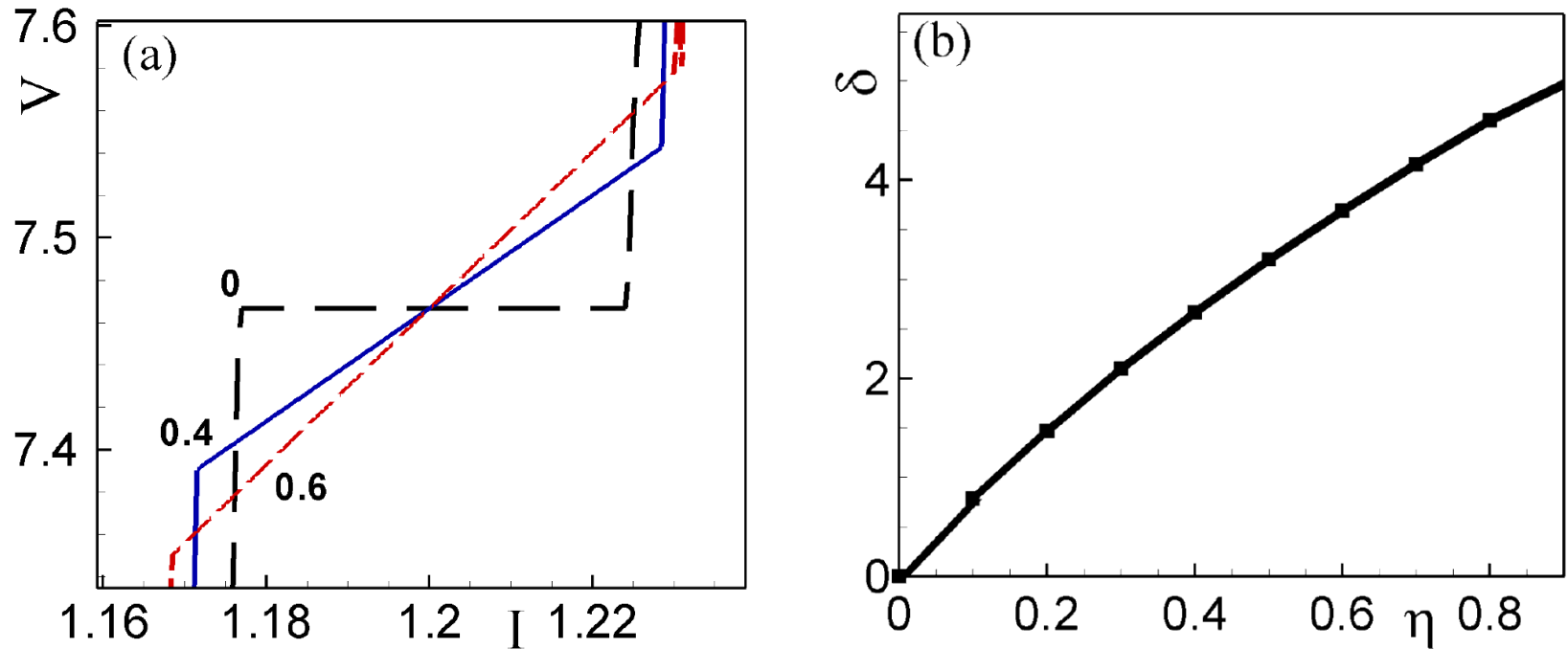


The coupled Josephson junctions at the nonequilibrium conditions are described by IV-characteristic with intensive branching near the critical current.

The charge imbalance manifests itself as appearance of the slope of the Shapiro step, which is clearly demonstrated in the inset.



# Slope value

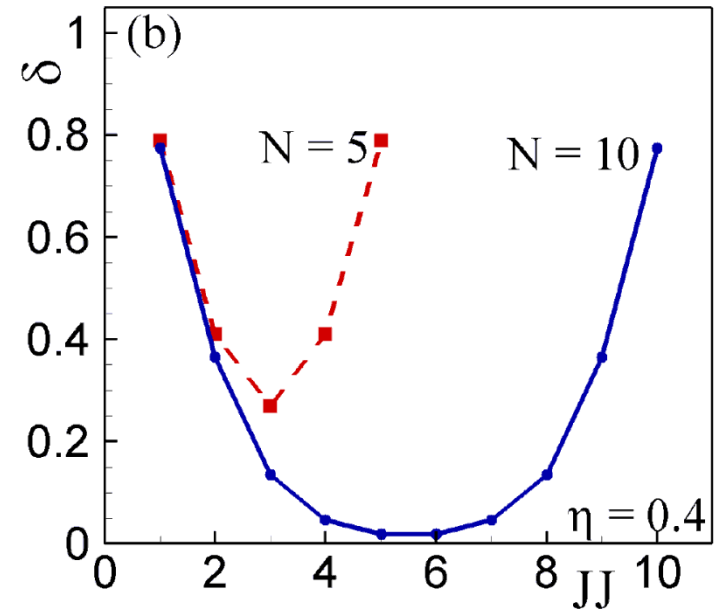
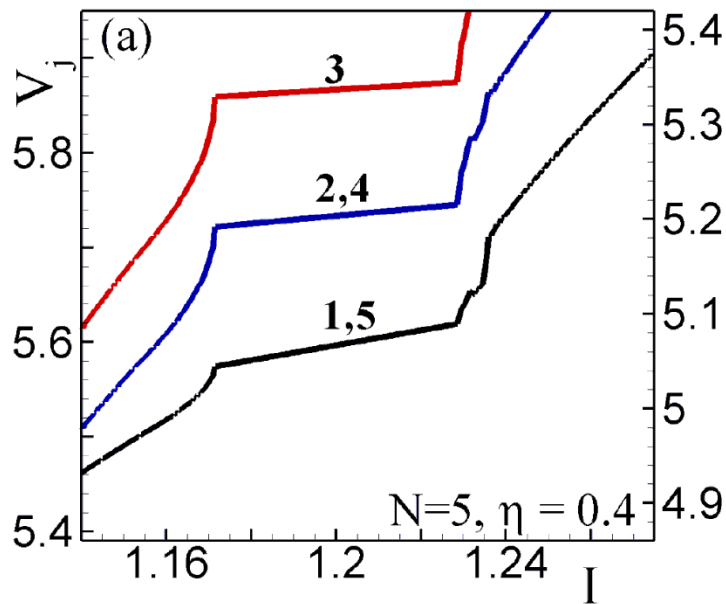


Increasing in nonequilibrium parameter  $\eta$  leads to enhance in slope value  $\delta$ .

Fitting for the chosen parameters of the junction give us

$$\delta = 8.18664\eta - 1.93047\eta^2$$

# Distribution of the slope value along the stack



Symmetric distribution

The largest slope value can be observed in the first and the last junction

# Summery

As summary, we have investigated the effect of the charge imbalance on the Shapiro step in the outermost branch at the nonequilibrium conditions. Two important features for the Shapiro step are predicted.

First, the Shapiro step demonstrates a shift of its position from the canonical value  $N\omega$ , where  $N$  is the number of junctions in the stack and  $\omega$  is the frequency of the external radiation. The value of this shift depends on the boundary conditions and coupling between Josephson junctions.

Second, the Shapiro step demonstrates a finite slope in the IV-characteristics of a stack of coupled junctions. The value of the slope depends on the value of the nonequilibrium parameter.

Thank you for attention