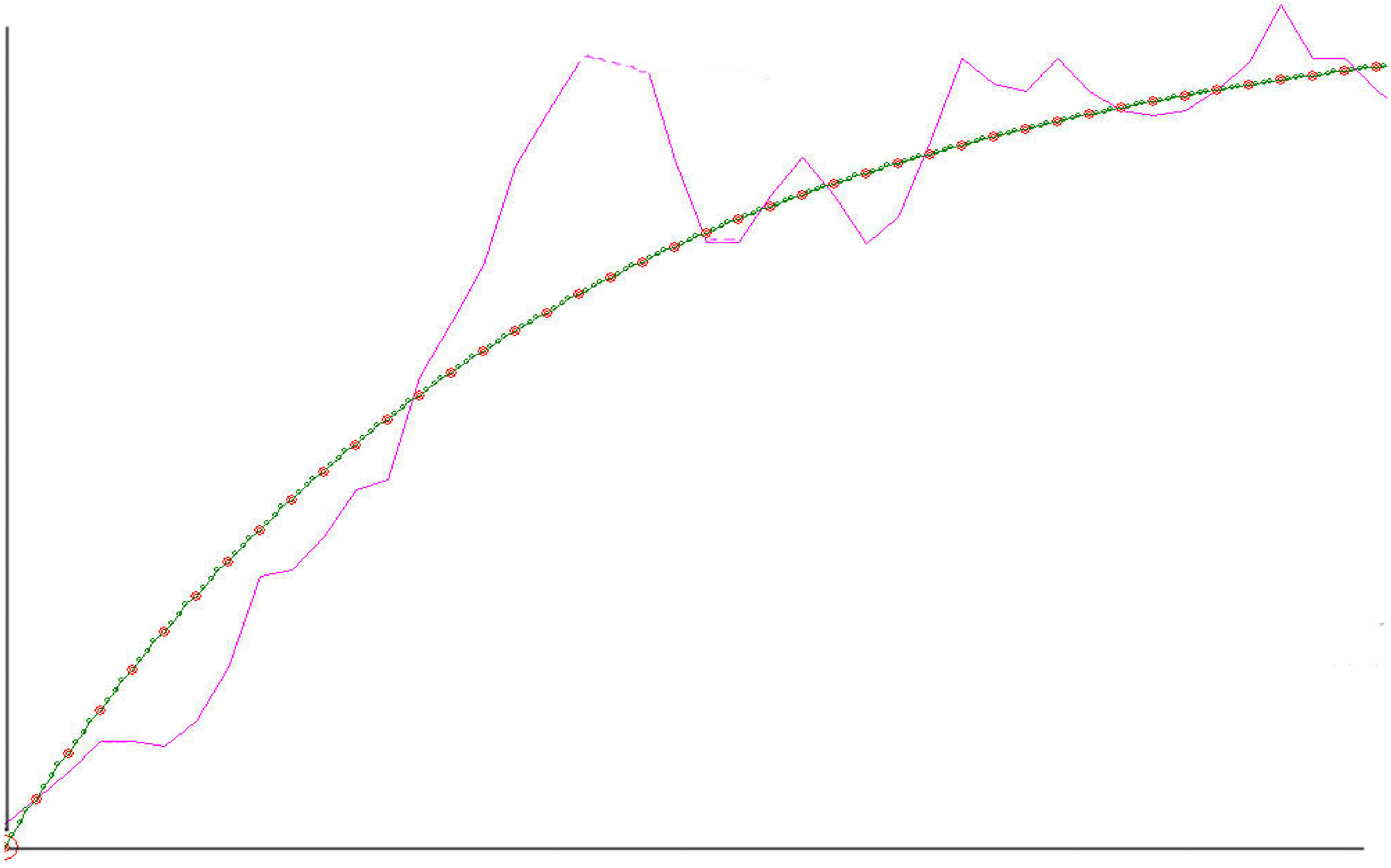
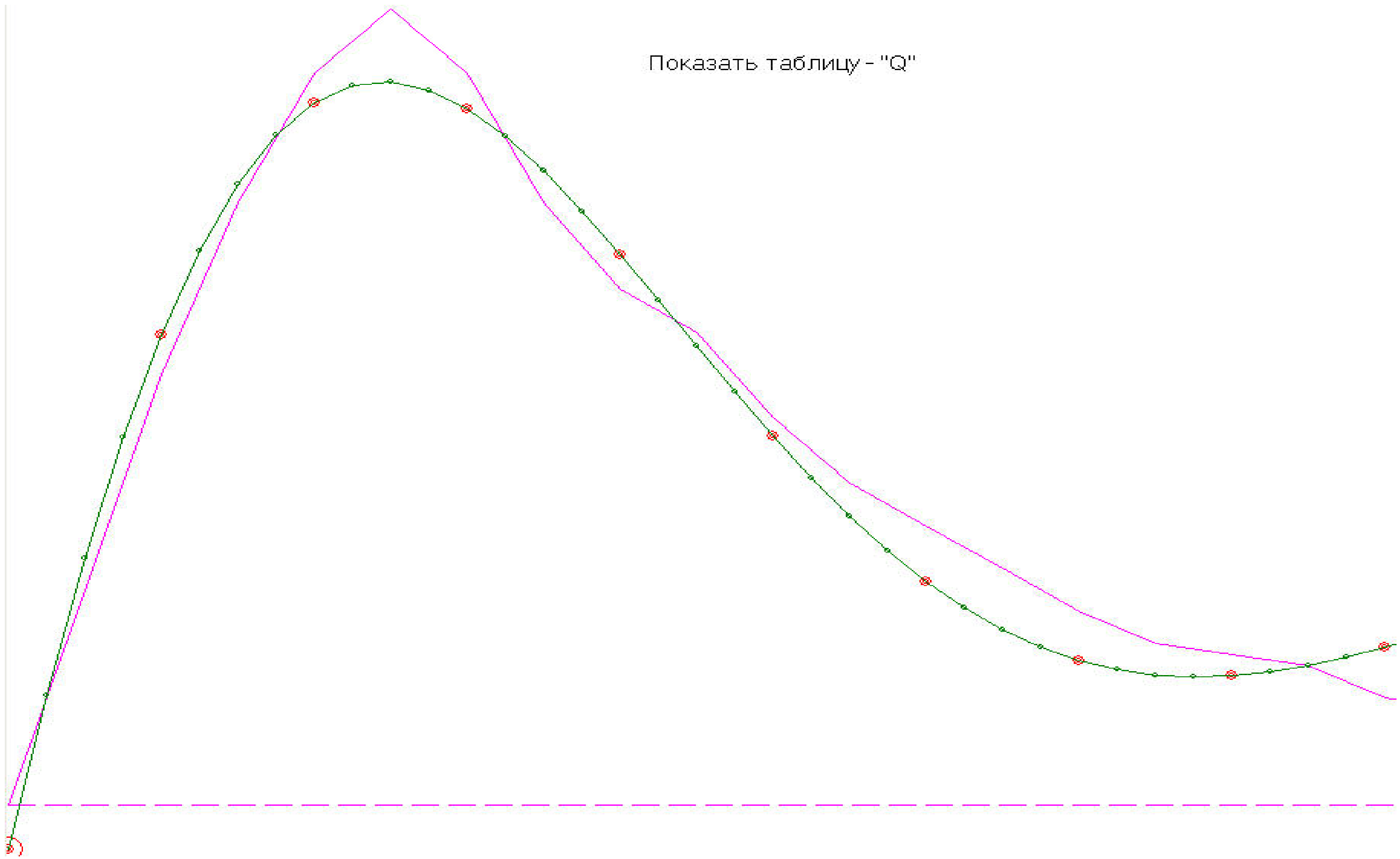
Approximation of Experimental Data by Solving Linear Difference Equations with Constant Coefficients

Experimental date – population of Amur tiger by years:



Dynamics of concentration of substance in human blood



Desirable types of Approximation:

1-st order:

$$X(j) = A \cdot Exp(\alpha \cdot j \cdot \Delta t) + B$$

2-nd order:

$$X(j) = A_1 \cdot Exp(\alpha_1 \cdot j \cdot \Delta t) + A_2 \cdot Exp(\alpha_2 \cdot j \cdot \Delta t) + B$$

$$X(j) = A_1 \cdot Exp(\alpha_1 \cdot j \cdot \Delta t) + A_2 \cdot j \cdot \Delta t \cdot Exp(\alpha_1 \cdot j \cdot \Delta t) + B$$

$$X(j) = A_1 \cdot Exp(\alpha_1 \cdot j \cdot \Delta t) \cdot Cos(\omega \cdot j \cdot \Delta t + \varphi) + B$$

Approximation of Data by Linear Difference Equations with Constant Coefficients:

1-st order:

$$X(j) = c_1 \cdot X(j\text{-}1) + b$$

2-nd order:

$$X(j) = c_1 \cdot X(j\text{-}1) + c_2 \cdot X(j\text{-}2) + b$$

Coefficients of Approximation :

1-st order:

$$c_1 \cdot \sum_{j=n-m}^{n} X^*(j-1)^2 + b \cdot \sum_{j=n-m}^{n} X^*(j-1) = \sum_{j=n-m}^{n} X^*(j) \cdot X^*(j-1)$$

$$c_1 \cdot \sum_{j=n-m}^{n} X^*(j-1) + b \cdot \sum_{j=n-m}^{n} 1 = \sum_{j=n-m}^{n} X^*(j)$$

2-nd order:

$$c_1 \cdot \sum_{j=n-m}^{n} X^*(j-1)^2 + c_2 \cdot \sum_{j=n-m}^{n} X^*(j-1) \cdot X^*(j-2) + b \cdot \sum_{j=n-m}^{n} X^*(j-1) = \sum_{j=n-m}^{n} X^*(j) \cdot X^*(j-1)$$

$$c_1 \cdot \sum_{j=n-m}^{n} X^*(j-1) \cdot X^*(j-2) + c_2 \cdot \sum_{j=n-m}^{n} X^*(j-2)^2 + b \cdot \sum_{j=n-m}^{n} X^*(j-2) = \sum_{j=n-m}^{n} X^*(j) \cdot X^*(j-2)$$

$$c_1 \cdot \sum_{j=n-m}^{n} X^*(j-1) + c_2 \cdot \sum_{j=n-m}^{n} X^*(j-2) + b \cdot \sum_{j=n-m}^{n} 1 = \sum_{j=n-m}^{n} X^*(j)$$

Initial conditions (for 2-nd order):

$$X(j) = c_1 \cdot X(j-1) + c_2 \cdot X(j-2) + b$$

$$X(j) = A(j) \cdot X_1 + B(j) \cdot X_0 + C(j)$$

For $j > 1$:

$$A(j+1) = c_1 \cdot A(j) + c_2 \cdot A(j-1)$$

$$B(j+1) = c_1 \cdot B(j) + c_2 \cdot B(j-1)$$

$$C(j+1) = c_1 \cdot C(j) + c_2 \cdot C(j-1) + b$$

$$j = 0 \quad A(0)=0; \; B(0)=1; \; C(0)=0$$

$$j = 1 \quad A(1)=1; \; B(1)=0; \; C(1)=0$$

Calculation of initial conditions

X_0 and X_1 (for 2-nd order):

$$X_1 \cdot \sum_{j=n-m}^{n} A(j) \cdot A(j) + X_0 \cdot \sum_{j=n-m}^{n} A(j) \cdot B(j) = \sum_{j=n-m}^{n} X^*(j) \cdot A(j) - \sum_{j=n-m}^{n} A(j) \cdot C(j)$$

$$X_1 \cdot \sum_{j=n-m}^{n} A(j) \cdot B(j) + X_0 \cdot \sum_{j=n-m}^{n} B(j) \cdot B(j) = \sum_{j=n-m}^{n} X^*(j) \cdot B(j) - \sum_{j=n-m}^{n} B(j) \cdot C(j)$$

The approximating curve with *initial conditions X0, X1 and* coefficients *c1, c2, and b,* will pass through the experimental points *X*(j) while minimizing the* RMS deviation with respect to both the coefficients *and the initial conditions.*

Initial conditions (for 1-st order):

For j > 0 :

$$X(j) = c_1 \cdot X(j\text{-}1) + b$$

$$X(j) = B(j) \cdot X_0 + C(j)$$

$$B(j) = c \cdot B(j\text{-}1) \qquad B(0)=1$$

$$C(j) = c \cdot C(j\text{-}1) + b \qquad C(0)=0$$

$$X_0 \cdot \sum_{j=n-m}^{n} B(j) \cdot B(j) = \sum_{j=n-m}^{n} X^*(j) \cdot B(j) - \sum_{j=n-m}^{n} B(j) \cdot C(j)$$

Initial conditions (for 3-d order):

$$X(j) = c_1 \cdot X(j\text{-}1) + c_2 \cdot X(j\text{-}2) + c_3 \cdot X(j\text{-}3) + b$$

$$X(j) = A(j) \cdot X_2 + B(j) \cdot X_1 + C(j) \cdot X_0 + D(j)$$

For j > 2 :

$$A(j+1) = c_1 \cdot A(j) + c_2 \cdot A(j\text{-}1) + c_3 \cdot A(j\text{-}2)$$

$$B(j+1) = c_1 \cdot B(j) + c_2 \cdot B(j\text{-}1) + c_3 \cdot B(j\text{-}2)$$

$$C(j+1) = c_1 \cdot C(j) + c_2 \cdot C(j\text{-}1) + c_3 \cdot C(j\text{-}2)$$

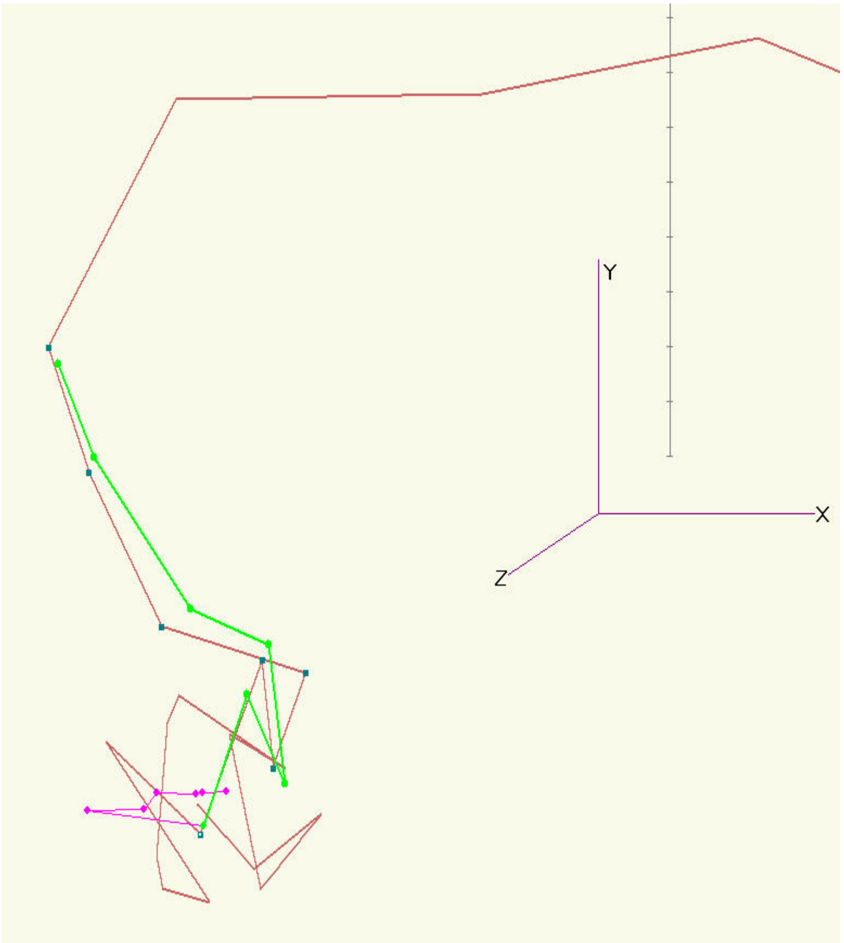
$$D(j+1) = c_1 \cdot D(j) + c_2 \cdot D(j\text{-}1) + c_3 \cdot D(j\text{-}2) + b$$

For $j = 0$ $A(0)=0;\ B(0)=0;\ C(0)=1;\ D(0)=0$

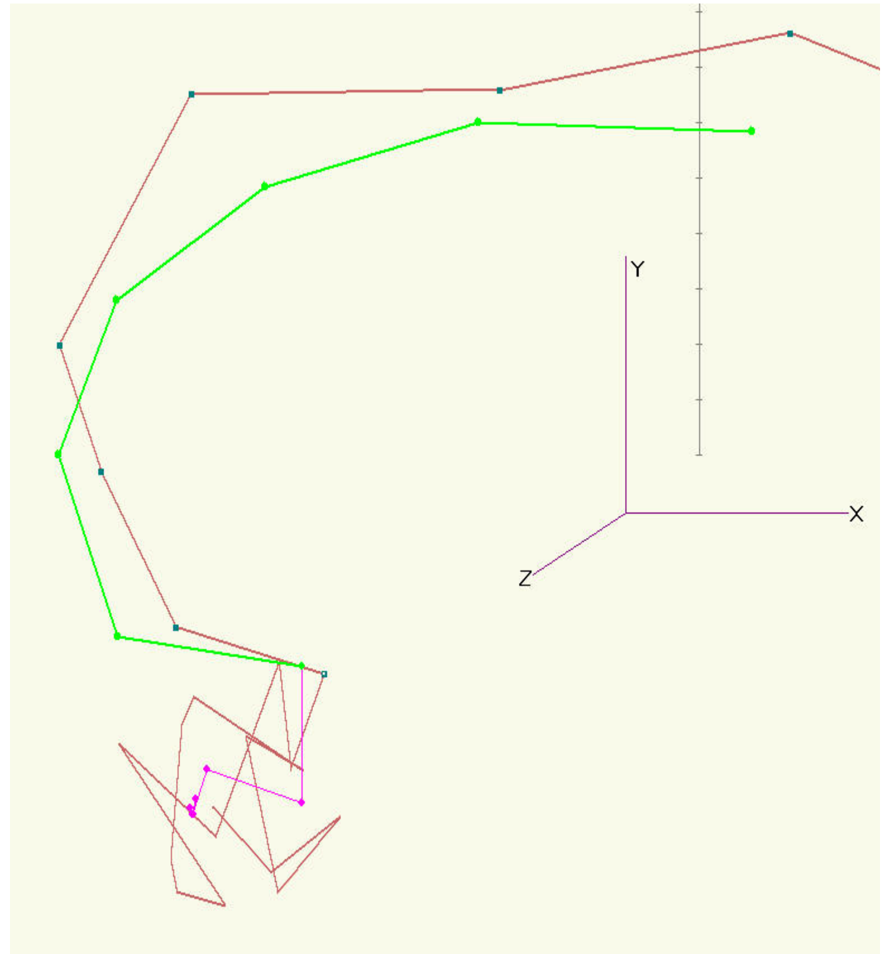
For $j = 1$ $A(1)=0;\ B(1)=1;\ C(1)=0;\ D(1)=0$

For $j = 2$ $A(2)=1;\ B(2)=0;\ C(2)=0;\ D(2)=0$

Approximation of 3-dimential ECG



3 points before beginning of QRS



At the beginning of QRS

The questions of:

- Getting the intermediate points
- The steadiness of approximation in form of solutions of Linear Difference Equations with Constant Coefficients

are in:

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