Modeling the quarks' helicity flipping stimulated by their confinement

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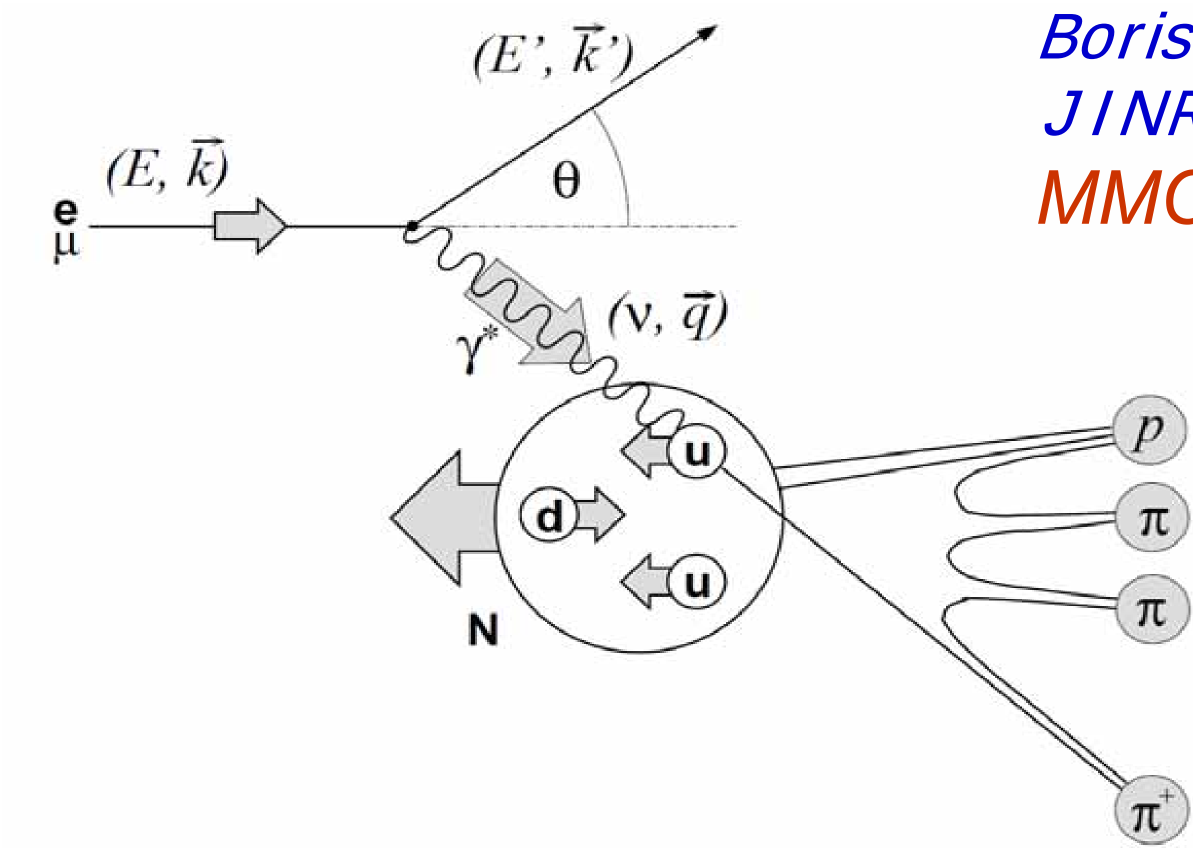


Experiments

HERMES, CLASS, HALL A

Proton spin puzzle

A general state of
$q-\bar{q}$ field:

$$\psi = \begin{pmatrix} q_+ \\ q_- \\ \bar{q}_+ \\ \bar{q}_- \end{pmatrix}$$

Conservation of charge and polarization
at Lorentz transform:

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \text{Lorentz transform} \rightarrow \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \dfrac{p}{E+m} \end{pmatrix}, \quad q_+{}^2 - \bar{q}_-{}^2 = 1$$

Experiment *EMC, Phys. Lett. B* ***206*** *(1988) 364*

Total contribution of quark helicities to the proton spin is consistent with ***zero*** *within the range of experimental errors*

Explanation - only total angular momentum survives

$$S_z + L_z = \text{const}$$

L_z should be a **conserved** quantum number

This force us to introduce **axial symmetry** into consideration. According to the parton model, quarks are **free particles**. All that results in Dirac equation for free quarks in the cylindrical coordinates

$$\left[i\left(\gamma^r \frac{\partial}{\partial r} + \gamma^\varphi \frac{\partial}{\partial \varphi} + \gamma^z \frac{\partial}{\partial z} \right) + \gamma^0 E - m \right] \psi(r, \varphi, z; E) = 0, \qquad \textbf{(1)}$$

Gamma matrixes in cylindrical coordinates

$$\gamma^r = \begin{pmatrix} 0 & \sigma^r \\ -\sigma^r & 0 \end{pmatrix}, \gamma^\varphi = \begin{pmatrix} 0 & \sigma^\varphi \\ -\sigma^\varphi & 0 \end{pmatrix}, \gamma^z = \begin{pmatrix} 0 & \sigma^z \\ -\sigma^z & 0 \end{pmatrix}, \gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\sigma^r = \begin{pmatrix} 0 & e^{-i\varphi} \\ e^{i\varphi} & 0 \end{pmatrix}, \sigma^\varphi = \begin{pmatrix} 0 & -ie^{-i\varphi} \\ ie^{i\varphi} & 0 \end{pmatrix}, \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The expressions for γ-matrices follows from their usual form *(Bjorken, Drell. Relativistic Quantum Mechanics)* and the vector nature of their change under coordinate transformations,

$$\gamma^s = \gamma^i \partial x^s / \partial x^i .$$

Substitution

$$\psi = \begin{pmatrix} q(r,\varphi) \\ \overline{q}(r,\varphi) \end{pmatrix} e^{ip_z z}$$

separates wave functions of quarks and antiquarks and reduces the Dirac equation to a system of two ordinary differential equations

$$\begin{cases} (E-m)q + \left[i\left(\sigma^r \dfrac{\partial}{\partial r} + \dfrac{\sigma^\varphi}{r}\dfrac{\partial}{\partial \varphi} - \sigma^z p_z \right)\right]\overline{q} = 0, \\ (E+m)\overline{q} + \left[i\left(\sigma^r \dfrac{\partial}{\partial r} + \dfrac{\sigma^\varphi}{r}\dfrac{\partial}{\partial \varphi} - \sigma^z p_z \right)\right] q = 0. \end{cases}$$

Transforms of Dirac equation (1)

1. Using the second equation of the system we express $\bar{q}$ in terms of q and substitute it in the first one. Then we obtain

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}\right)q+p_\perp^2 q=0.$$

2. Separation of variables by means of a substitution

$$q(r,\varphi)=u(r)e^{in\varphi}$$

where *n* is an integer due to the single-valuedness of $q(r,\varphi)$, leads to the **Bessel equation**

(2)

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{n^2}{r^2}u+p_\perp^2 u=0,$$

$$p_\perp^2=E^2-(p_z^2+m^2)$$

transverse momentum

Solution to Dirac equation (1)

Physically acceptable solutions of (2) describing quark distribution in the transverse plane is the Bessel functions of the first kind, $u(r) = Const \cdot J_n(p_\perp r).$

The corresponding wave function $q(r,\varphi) = u(r)e^{in\varphi}$ of quarks are eigenstates of *z*-component of orbital momentum operator

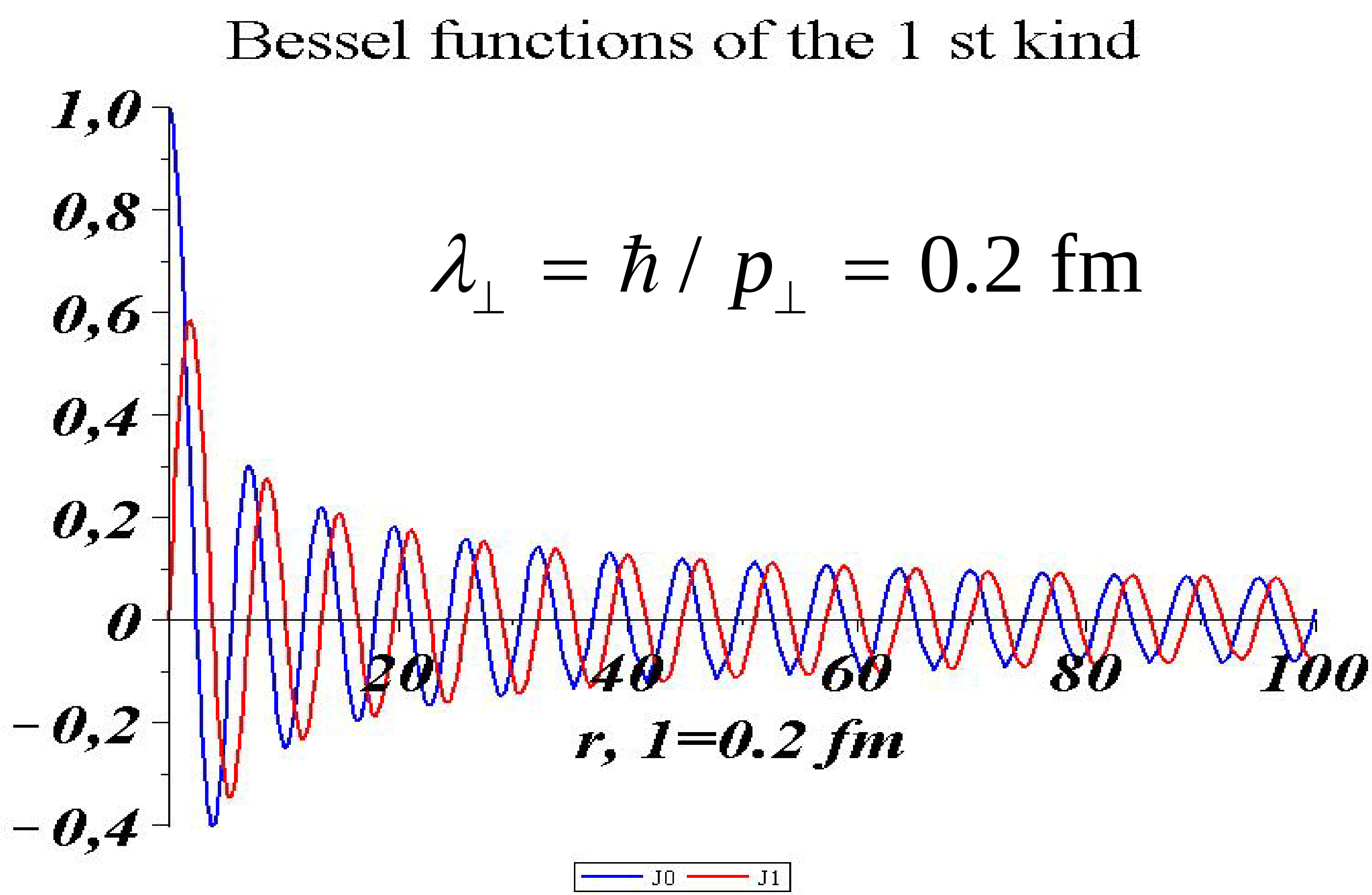
$$\hat{L}_z = -i\frac{\partial}{\partial\varphi}.$$

It gives a contribution to the total helicity of proton if

$$n \neq 0$$

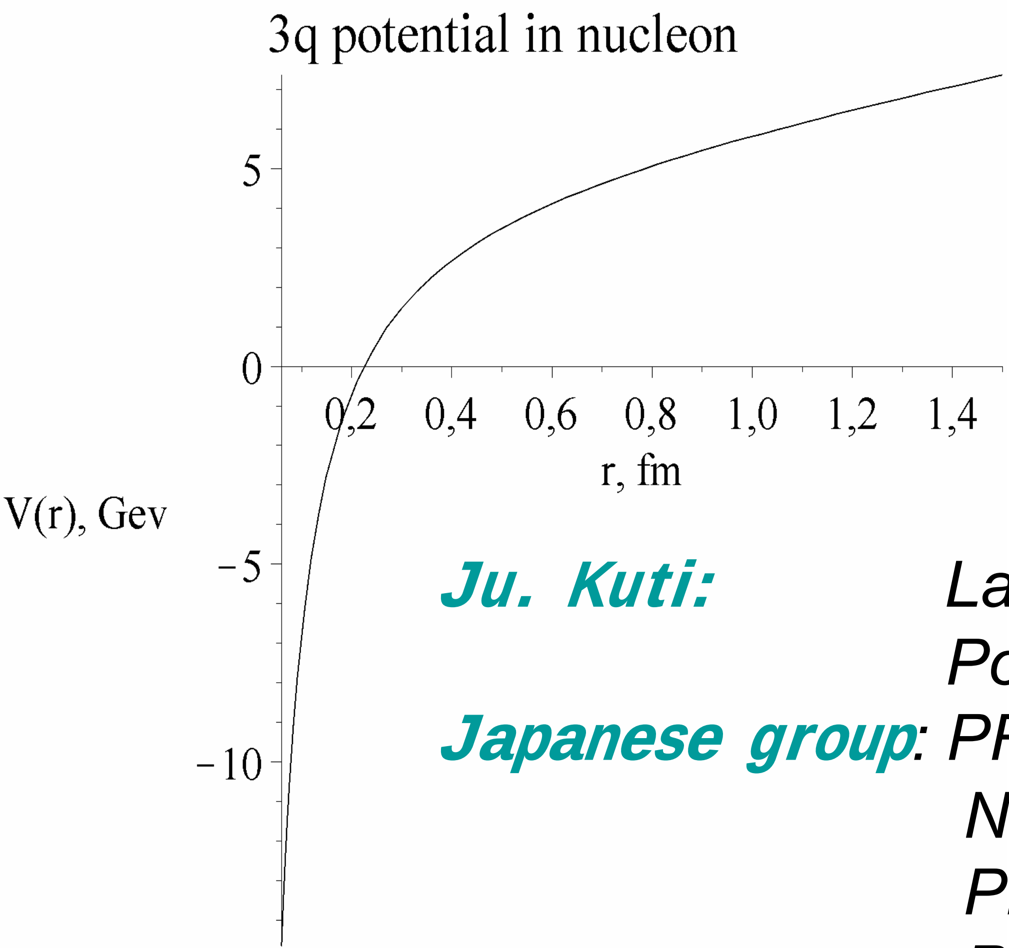
(and thus **can explain deficiency in proton spin** composed only of quark helicities).

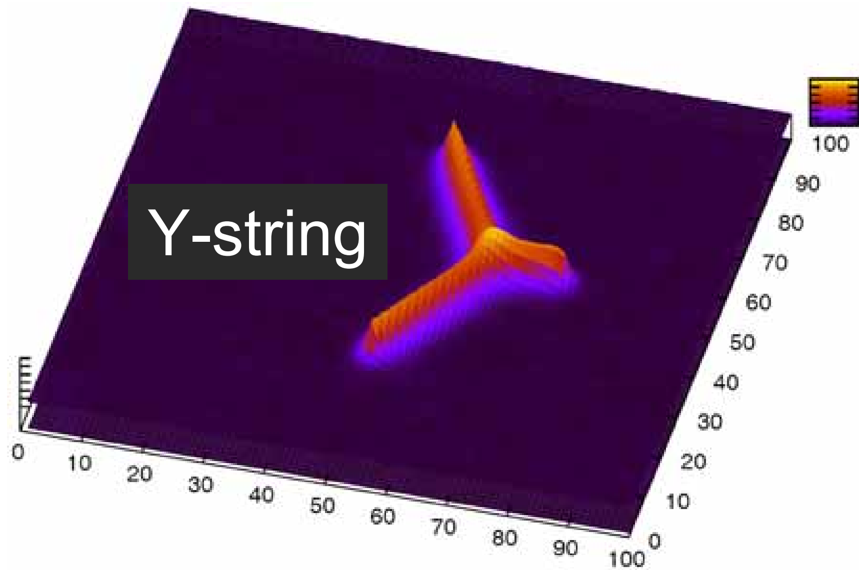
The problem



The solutions are badly localized in the vicinity of *r=0*.
Such a localization obviously **contradicts to the requirement of quark confinement** in the transverse plane.

Let us introduce the confinement potential





Ju. Kuti: *Lattice QCD and String Theory, PoS (LAT2005) 001*

Japanese group: *PRL 86 (2001) 18*

Nucl. Phys. A680 (2001) 159c

Phys. Rev. D65 (2002) 114509

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Dirac equation for *q* within V(r)

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}+p_\perp^2\right)q(r,\varphi)- \qquad \textbf{(3)}$$

$$\underbrace{i\sigma^r\frac{dV/dr}{E-V+m}\left[i\left(\sigma^r\frac{\partial}{\partial r}+\frac{\sigma^\varphi}{r}\frac{\partial}{\partial\varphi}\right)-\sigma^z p_z\right]q(r,\varphi)}_{\text{Additional term}}=0.$$

Compare with (2)

Key observation

Equation (3) contains a term proportional to

$$\sigma^r \sigma^z = -i\sigma^\varphi .$$

It destroys the initial polarizations of quarks,

$$-i\sigma^\varphi \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ e^{i\varphi} \end{pmatrix},$$

Helicity +1/2, orbital momentum 0

Helicity -1/2, orbital momentum +1

leaving the total angular momentum, ½, conserved:

$$(S_z + L_z)q = \frac{1}{2}q, (S_z + L_z)q' = \frac{1}{2}q', q' = -i\sigma^\varphi q.$$

Confinement potential may change the initial helicity of quarks with simultaneous change of their orbital angular momentum!

Instead of the Bessel equation, 1

Separation of variables may be performed now by a substitution

$$q(r,\varphi) = \begin{pmatrix} u_{+}(r)e^{in\varphi} \\ -iu_{-}(r)e^{i(n+1)\varphi} \end{pmatrix},$$

Restricting yourself with the case *n=0*, which indicates that **there are no the orbital excitations in the region of the asymptotic freedom and all of them appear only due to the confinement force**, we arrive to the following system of differential equations for the wave functions of quarks with positive and negative helicities:

Instead of the Bessel equation, 2

$$\left\{\begin{aligned}
&\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+p_\perp^2\right)u_+(r)-\\
&\frac{dV/dr}{E-V+m}\left[\frac{\partial u_+(r)}{\partial r}+p_z u_-(r)\right]=0,\\
&\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+p_\perp^2-\frac{1}{r^2}\right)u_-(r)-\\
&\frac{dV/dr}{E-V+m}\left[\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)u_-(r)+p_z u_+(r)\right]=0.
\end{aligned}\right. \tag{4}$$

Cauchy problem for eq.(4)

Initial conditions,

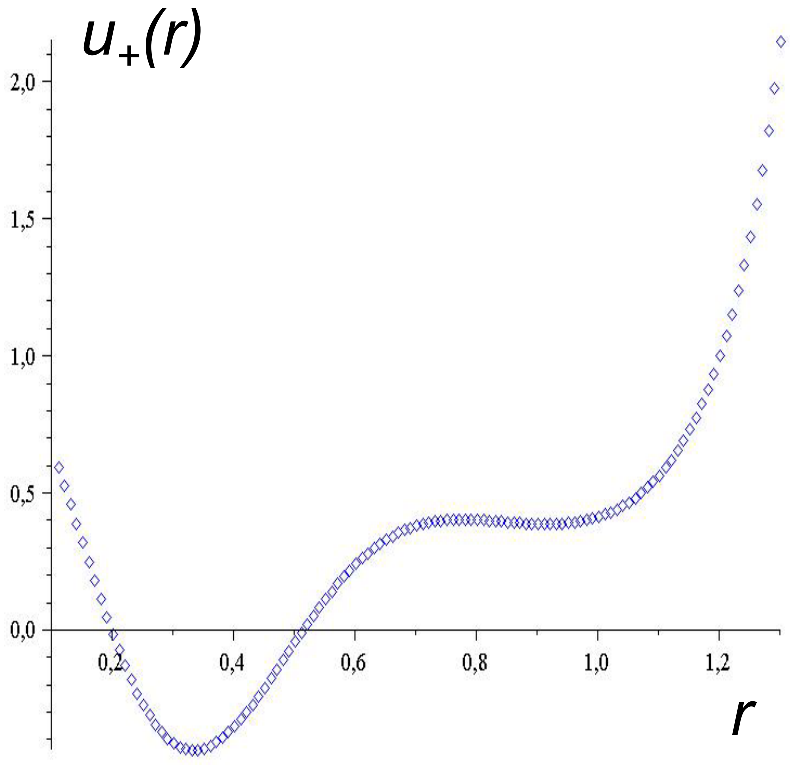
$$u_+(r) = J_0(p_\perp r)\Big|_{r=0.7}\,, u'_+(r) = J'_0(p_\perp r)\Big|_{r=0.7}$$

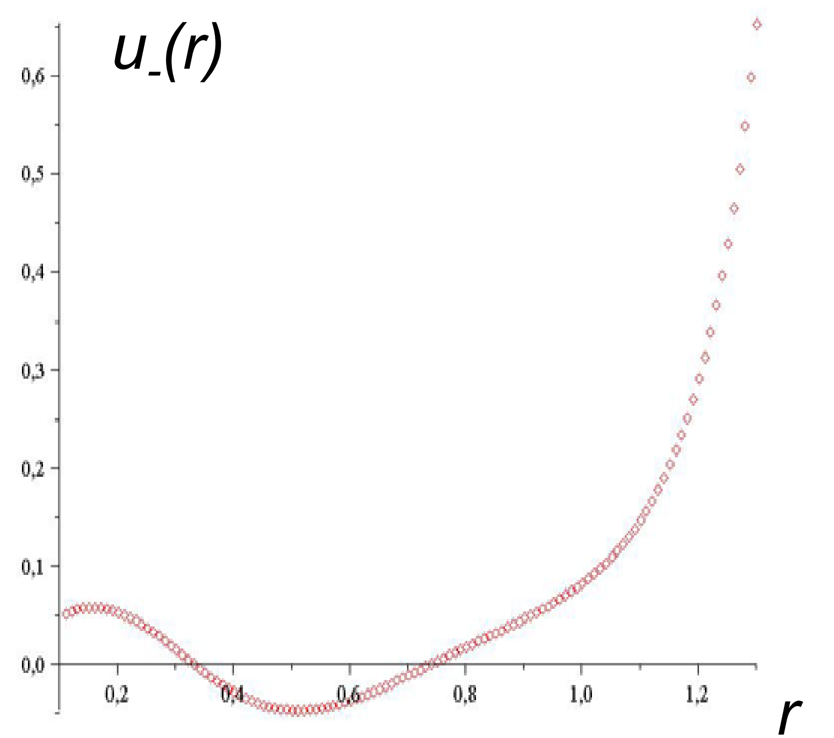
$$u_-(r) = 0\Big|_{r=0.7}\,, u'_-(r) = 0\Big|_{r=0.7},$$

suppose that quarks are free at $r < 0.7$ fm and therefore are described by the Bessel functions of the 1st kind (see solution of (1)) and that a contribution of negative helicity is negligible there. Now a solution of the Cauchy problem in the transverse plane, which corresponds to (4), may be found numerically.

Confusing surprise

MAPLE assisted **Runge-Kutta-Fehlberg** algorithm (rkf45 proc.) finds a solution inconsistent with the confinement condition

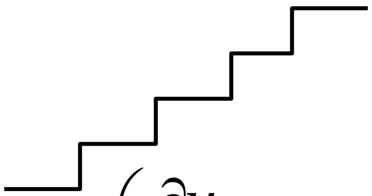




RKF-solution for simplified version of linear confinement potential with string tension 1GeV/fm defined for 0.1<r<1.4 fm.

Numerical solutions consistent with confinement

Ladder-shaped confinement potential:



$$\begin{cases} \dfrac{\partial^2 u_+}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u_+}{\partial r} + p_\perp^2 u_+ = \dfrac{dV/dr}{E - V(r) + m}\,\delta(r - r_i)\left(\dfrac{\partial u_+}{\partial r} + p_z u_-\right) \\ \dfrac{\partial^2 u_-}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u_-}{\partial r} - \dfrac{1}{r^2}u_- + p_\perp^2 u_- = \dfrac{dV/dr}{E - V(r) + m}\,\delta(r - r_i)\left(\dfrac{\partial u_-}{\partial r} + \dfrac{u_-}{r} + p_z u_+\right) \end{cases} \quad \textbf{(6)}$$

where $dV/dr = \Delta V_i \times \delta(r - r_i)$.

Everywhere, apart from points r_i of $V(r)$ breaking , system (6) has form of the

Bessel equations:

$$\begin{cases} \dfrac{\partial^2 u_+}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u_+}{\partial r} + p_\perp^2 u_+ = 0, \\ \dfrac{\partial^2 u_-}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u_-}{\partial r} - \dfrac{1}{r^2}u_- + p_\perp^2 u_- = 0 \end{cases} \quad \textbf{(7)}$$

Solutions to the Bessel equations (7)

$$\text{Solutions for } p_\perp^2 > 0: \quad \begin{aligned} u_+(r) &= A_1 J_0(p_\perp r) + A_2 Y_0(p_\perp r), \\ u_-(r) &= B_1 J_1(p_\perp r) + B_2 Y_1(p_\perp r), \end{aligned} \tag{8}$$

where J_i , Y_i are Bessel functions of the 1st and 2nd kind (Y_i ≡ Neumann functions).

$$\text{And for } p_\perp^2 < 0: \quad \begin{aligned} u_+(r) &= C_1 I_0(|p_\perp| r) + C_2 K_0(|p_\perp| r), \\ u_-(r) &= D_1 I_1(|p_\perp| r) + D_2 K_1(|p_\perp| r), \end{aligned} \tag{9}$$

where I_i,K_i are modified Bessel functions of the 1st and 2nd kind. The **idea** of our numerical solution: to **join the Bessel functions** imposing some conditions following from eqs. **(6)** at points r_i where $V(r)$ undergoes an abrupt change.

The “insuperable” difficulties

In the classically forbiden region, where $p_\perp^2 < 0$, functions $I_0(|p_\perp(r)|r)$ and $I_1(|p_\perp(r)|r)$ **rapidly increase** at moving off the boundary $r = r_{bound}$ of the classically permissible region. Therefore the **only** solutions consistent with confinement at far distances from the boundary are $K_0(|p_\perp(r)|r)$ and $K_1(|p_\perp(r)|r)$, where they rapidly vanish.

But these function possess the **infinite value** at the boundary, where $p_\perp(r) = 0$, because $K_0(p_\perp r), K_1(p_\perp r) \to \infty$ when $p_\perp r \to 0.$ Thus, **at first sight**, it is impossible to describe confinement of quarks using generally accepted potential for it. Analogous problem arise in the classically permissible region at $p_\perp^2 \to 0,$ because $Y_0(p_\perp r), Y_1(p_\perp r) \to \infty$ in this limit.

We shall see that the problem has a solution based on an unexpected property of system (6).

Matching conditions at points where *V(r)* undergoes an abrupt change are

$$\begin{cases} \dfrac{\partial^2 u_+}{\partial r^2} = \dfrac{\Delta V}{E - V(r) + m}\,\delta(r - r_i)\left(\dfrac{\partial u_+}{\partial r} + p_z u_-\right) \\[2ex] \dfrac{\partial^2 u_-}{\partial r^2} = \dfrac{\Delta V}{E - V(r) + m}\,\delta(r - r_i)\left(\dfrac{\partial u_-}{\partial r} + \dfrac{u_-}{r} + p_z u_+\right) \end{cases} \quad \textbf{(10)}$$

They follow from (6), as far as **contribution of all finite terms in (6) may be neglected at $r = r_i$.**

Matching conditions

Integration of equations **(10)** gives the matching conditions for solutions of Bessel equations at points r_i of the potential discontinuities:

$$\Delta u'_+(r_{i+\varepsilon}) = \frac{\Delta V}{E - V(r_{i-\varepsilon}) + m}\left(u'_+(r_{i-\varepsilon}) + p_z u_-(r_{i-\varepsilon})\right),$$

$$\Delta u'_-(r_{i+\varepsilon}) = \frac{\Delta V}{E - V(r_{i-\varepsilon}) + m}\left(u'_-(r_{i-\varepsilon}) + \frac{u_-(r_{i-\varepsilon})}{r_{i-\varepsilon}} + p_z u_+(r_{i-\varepsilon})\right), \qquad \textbf{(11)}$$

where $u' = du/dr$ and $\Delta u'_+, \Delta u'_-$ describe discontinuities of the first derivatives at $r = r_i$. We take $\varepsilon = 1$, that is matching conditions (11) are used as **recurrence relations** for calculations of jumps of u'_+ and u'_- at mesh points in our finite-difference scheme.

Matching conditions

See (8):

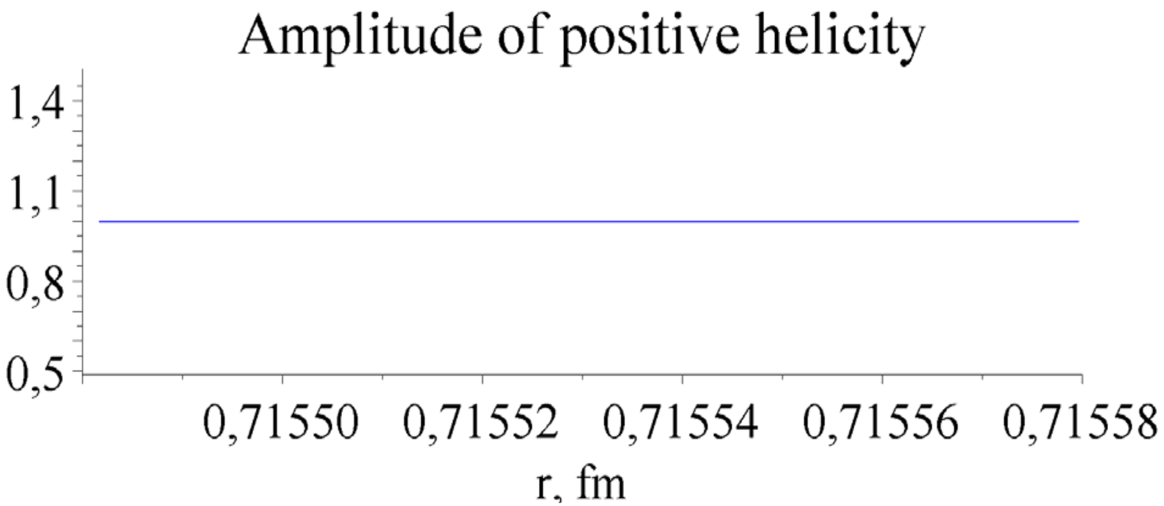
If all $A_i, B_i \neq 0,$ we may impose **additional conditions of continuity** of our solutions u_+ and u_- at $r = r_i.$ Then we have **4 equations**, (11) and the requirements of continuity, for determination of **4 coefficients** $A_i, B_i.$

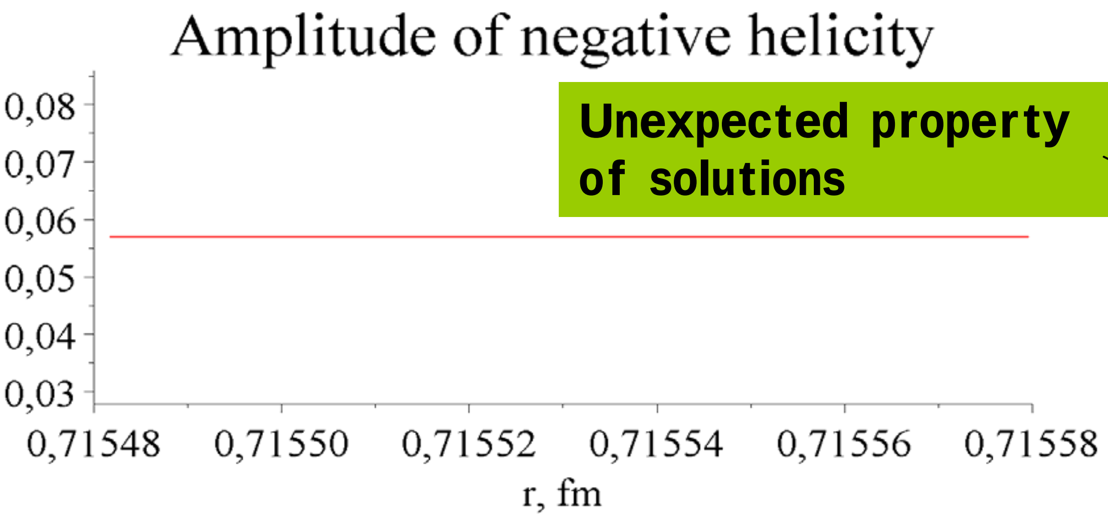
If one of the coefficients A_i **and** one of B_i are equal to zero the **functions** u_+ **and** u_- **itself should undergo breaks** at $r = r_i.$ In this case we have only two equations (11) for determinations of 2 nonzero coefficients $A_i, B_i.$

See (9):

The same prescription should be applied to the coefficients C_i and $D_i.$

Overcoming the stopping point of classical motion



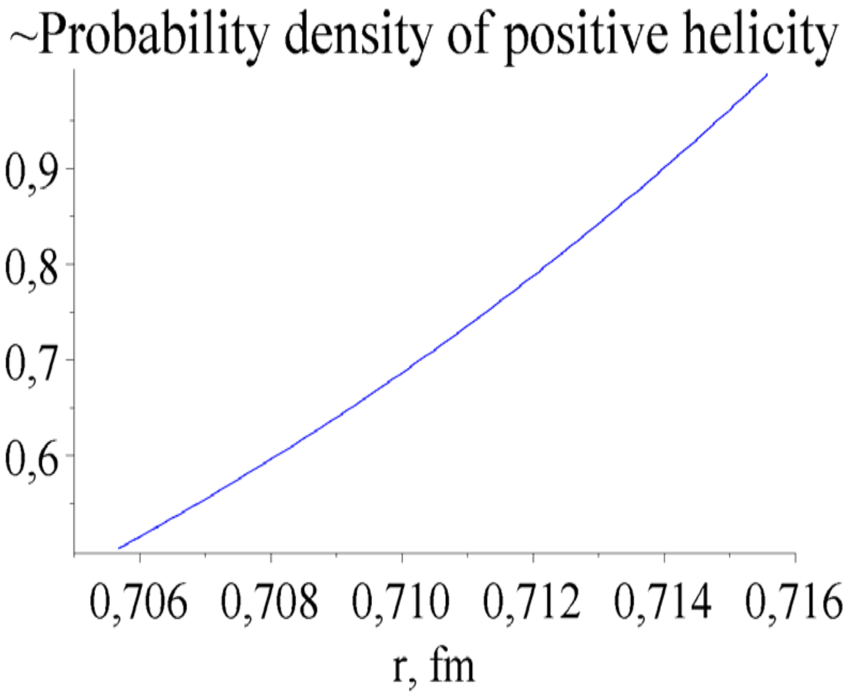


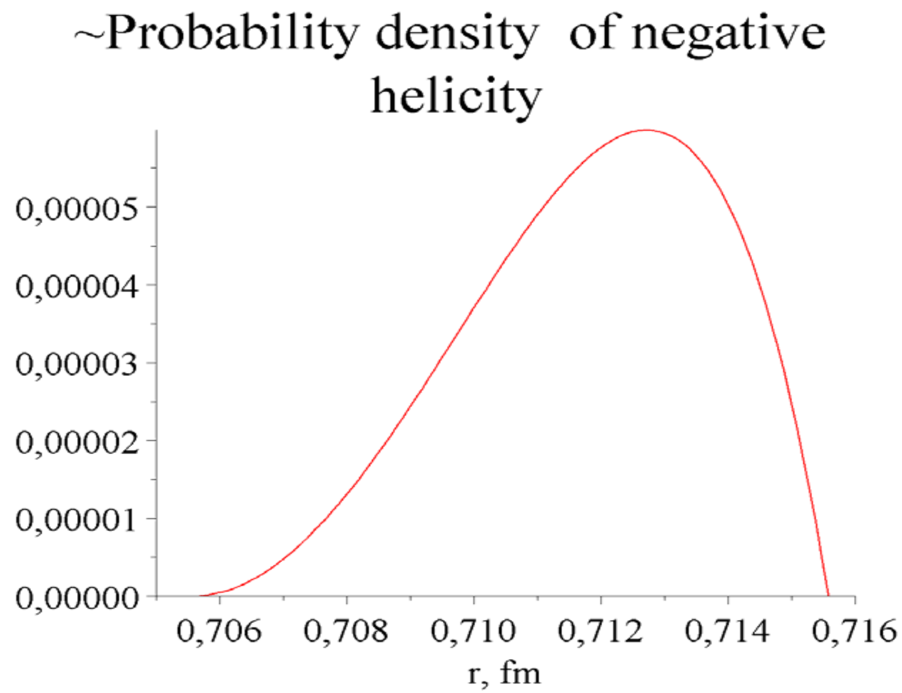
Unexpected property of solutions

Matching conditions use 4 equations for determinations of 4 nonzero coefficients. Similar solutions may be found for the Cauchy problem within the classically inaccessible region. A **negative feedback** between u_+ and u_- forces them to be constant! **The same is true for** $r > r_{stop}$.

Solutions for classically accessible region

Matching conditions use 2 equations (11) for determinations of 2 nonzero coefficients.

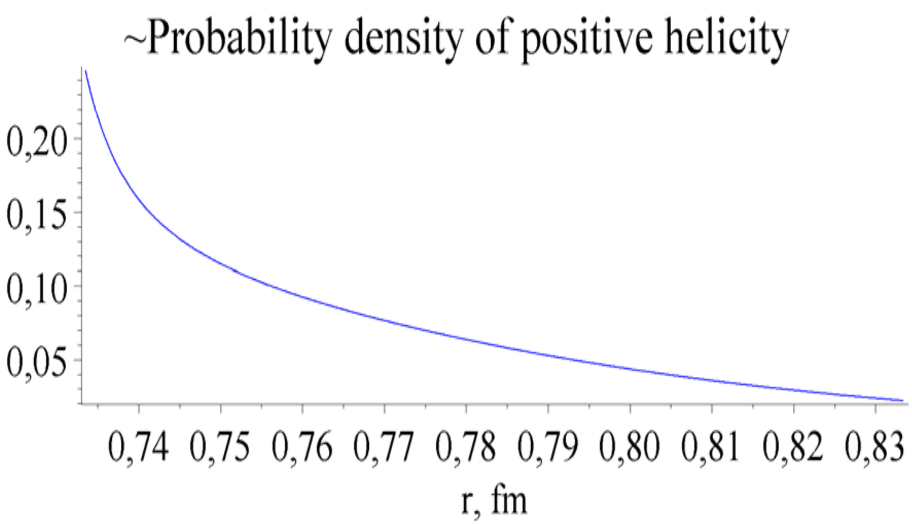


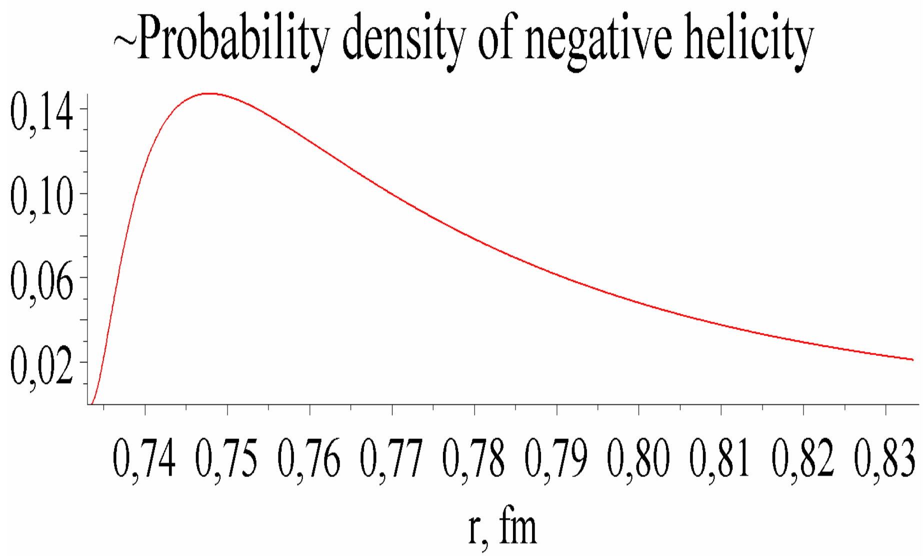


Confinement potential cannot stimulate significantly quark helicity flipping in the classically accessible region (CAR).

Solutions for classically inaccessible region – long jump

Matching conditions use 2 equations (11) for determinations of 2 nonzero coefficients. The initial data for the Cauchy problem are taken as continuation from of the solution in the classically accessible region according to the explanation in page 22. The distance of the continuation is Δr = 0.0178 fm.

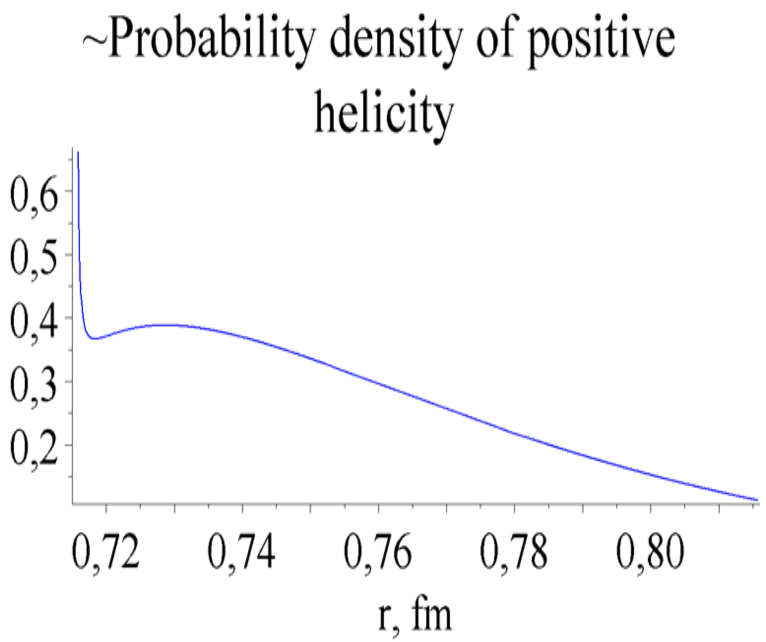


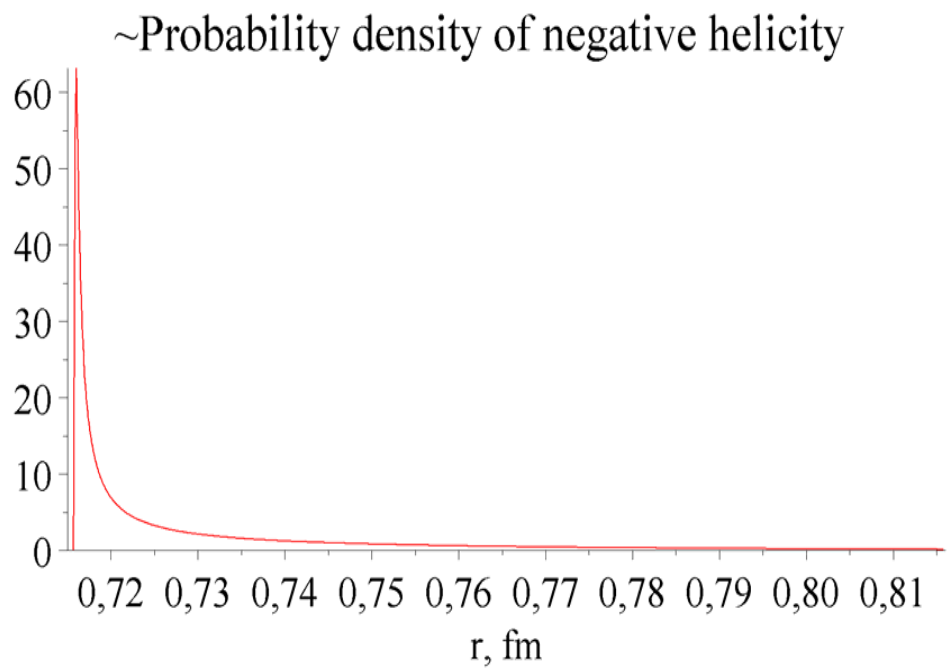


Confinement potential can stimulate *strongly* quark helicity flipping in the classically inaccessible region ($P_\perp^2$<0).

Solutions for classically inaccessible region – short jump

The distance of continuation is $\Delta r = 0.0002$ fm.





Confinement potential can stimulate *very strongly* quark helicity flipping in the classically inaccessible region for a short distance of continuation from CAR.

Estimation of depolarization values

Probabilities of different helicities in the classically inaccessible region:

$$P_{\pm} = 2\pi N \int_{r_0 > R_{stop}}^{\infty} u_{\pm}^2(r) r dr, \; N = \left(2\pi \int_{r_0 > R_{stop}}^{\infty} (u_{+}^2(r) + u_{-}^2(r)) r dr \right)^{-1}$$

Polarization= $\dfrac{P_+ - P_-}{P_+ + P_-} = \begin{cases} -0.0144 \leftarrow \text{long jump } \Delta r \\ -.71587 \leftarrow \text{short jump } \Delta r \end{cases}$

Conclusion: average polarisation of valence quark depends on unknown value of distance *Δr*, where regime $u_+(r), u_-(r) = const$ is realized. It may be near zero for the true choice of this value.

Thank you for your attention!