Phase Space of Instantaneous Cardiac Rhythm is a Fractal

A.N. Kudinov, S.A. Mikheyev, V.P. Tsvetkov, I.V. Tsvetkov

Tver State University

E-mail: mancu@mail.ru

We present the instantaneous cardiac rhythm (ICR) function *y(t)* and its difference derivative *v(t)* constructed based on the data of day-long Holter monitoring. These functions contain complete patient cardiovascular system state information. In order to analyze the behavior of these functions, there was introduced a concept of phase space (PS) of ICR the points of which had coordinates *y(t)* and *v(t)* in space R^2 and moved along the phase trajectories. In order to visualize the ICR dynamics, there was introduced a concept of extended phase space (EPS) of ICR. The examples of PS and EFS of ICR for specific patients of the Tver Cardiology Health Center are shown visually.

Within 24-48 hours we calculated the volumes of PS and EFS filled with the phase trajectories of ICR with different sizes of cells covering the phase trajectories. The calculation results showed that a phase volume of ICR had a power law dependence on cell size within the accuracy of 1-2%. That established the fractality of PS of ICR.

Taking into account the fractal properties of PS of ICR, volume Γ of PS of ICR is introduced. Γ is a volume filled with the phase trajectories of ICR in the D-dimensional space. As in statistical physic, the log Γ can be referred to as fractal entropy S of ICR.

We undertook a study of $\Gamma(t)$ and $S(t)$ behavior dynamics for several patients the results of which are given in the tables.

We argued in favor of use of parameters Γ and S as cardiovascular system state markers.

We cite an example of construction of a PS of ICR for one of the patients of Tver Regional Cardiology Health Center for a HM time interval of 1.5 min. It is demonstrated in Figure .1

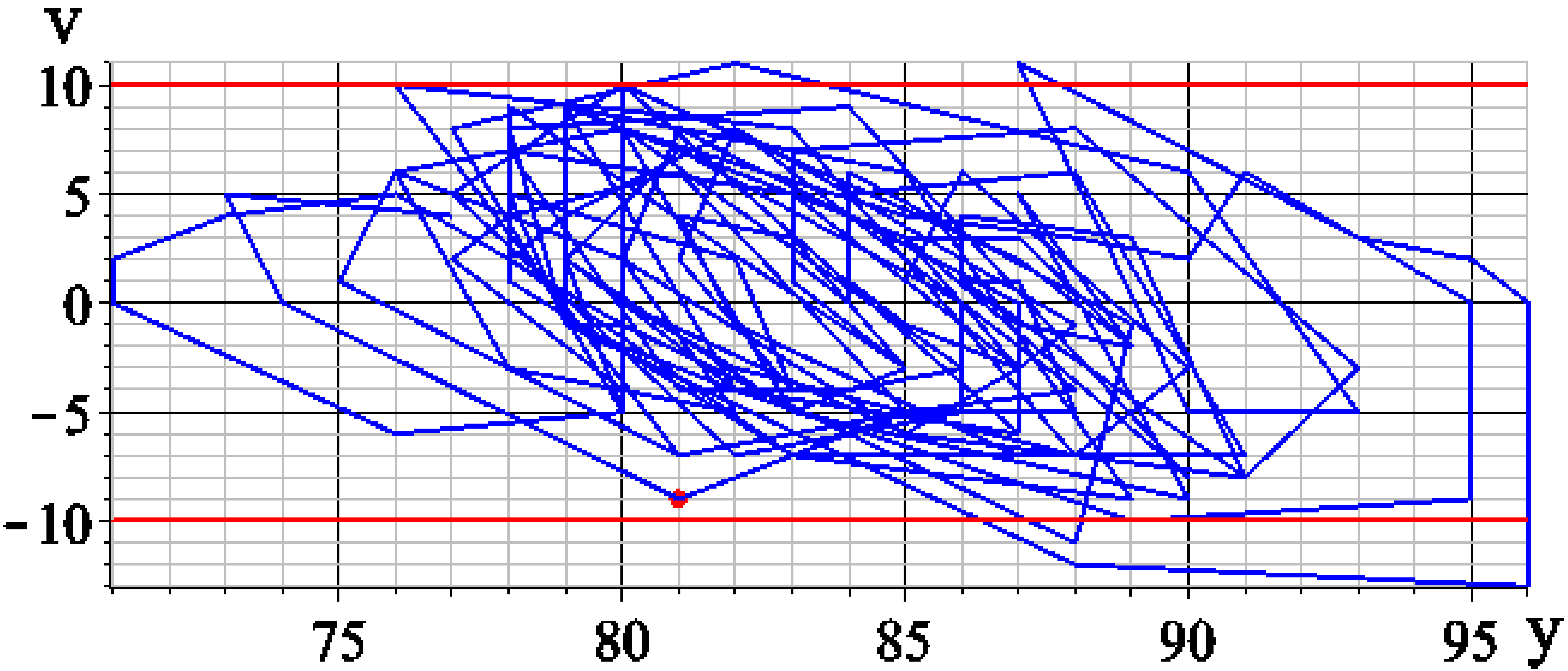

Fig. 1

In this figure, the heavy point reflects the phase point position at an arbitrary point of observation time. Directly in Figure 1, it is difficult to observe some phase point dynamics. As a consequence we wrote down the real-time ICR phase point motion animation application. Visual observation of phase point motion pattern allows performing detailed analysis of ICR dynamics and, consequently, the patient cardiovascular system state. As is seen from Fig. 1, the phase point trajectory has a Brownian pattern in a quality manner and shall be a fractal curve. Hereafter we will confirm this property by character-numeric calculations.

As mentioned above, the qualitative assessments are indicative of the fractal structure of PS of ICR. We will give the strong evidence of this conclusion by the example of the above PSes of ICR of the patients under examination. If the phase trajectories of ICR represent fractal curves, that the power law of *N* meshes with dimensions of $\delta = 1/B$ [5] that cover them takes place:

$$N(\delta) = N_0 \delta^{-D} = N_0 B^D = N(B), \tag{1}$$

where *D* – the value of fractal dimension of the PS of ICR.

In order to calculate *N(B)*, we wrote down the computer program enabling to find the *lnN*-*lnB* relationship which is approximated by the following equation of line:

$$lnN(B) = DlnB + \ln(N_0)\,. \tag{2}$$

Then the fractal dimension *D* of the PS of ICR shall be defined as a slope ratio in (2).

For the first patient, we calculate *N(B)* during the HM period of 1.21 hr in case where *B*=5, 10, 15, 20, 25. The results are set out in Figure 2 as a log-log chart.

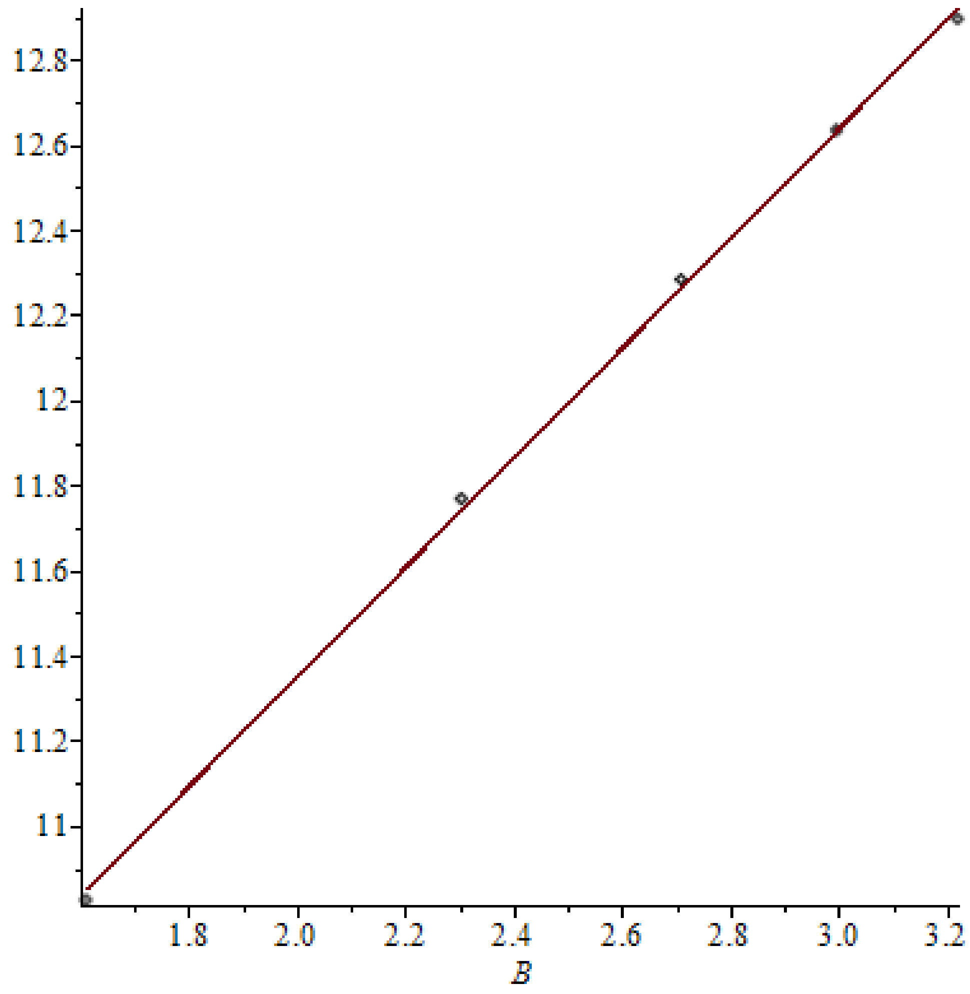

Fig. 2

At that, the approximation accuracy was $2.73 \cdot 10^{-2}$ in the *C*-metric. It follows therefrom that the PS of ICR generated by the phase trajectory in the space of 1.21 hr is a fractal set within the same accuracy and with the same fractal dimension *D*=1.289.

In statistical physics, the critical characteristic of PS is the phase volume Γ. As far as the PS of ICR is a fractal phase trajectory, it is impossible to define Γ as an area of covering squares of side *B* in the general case.

Therefore in the general case, as is the practice in theory of fractals, we will define the fractal phase-space volume of ICR as a volume to be filled by the phase trajectory of ICR in *D*-dimensional space.

$$\Gamma(t) = \lim_{B\to\infty} \frac{N(B,t)}{B^D} = N_0(t). \qquad (3)$$

Only in this instance, $\Gamma(t)$ will not depend on the size of mesh of 1/*B* and will have a physical significance.

As in statistical physics, let us than define the fractal entropy of ICR *S(t)* as the Napierian logarithm of fractal phase-space volume $\Gamma(t)$:

$$S(t) = ln\Gamma(t) = lnN_0(t). \qquad (4)$$

From the definition of $\Gamma(t)$ and *S(t)* it follows that these values can be accepted as a true measure of ICR variability.

We will illustrate this conclusion by calculations of *D*, Γ, and *S* of the patients under examination for different time intervals of HM. The calculation results for the first and the second patients are given in Tables 1.

Table 1

t (hr) XM	0.292	0.553	1.216	1.947	2.578	3.877	12.282	24.865
D	1.125	1.151	1.288	1.362	1.393	1.447	1.533	1.581
Γ	3514.6	5947.7	6459.3	7174.5	8101.9	8268.0	10872.9	12005.0
S	8.164	8.690	8.773	8.878	8.999	9.020	9.294	9.393

thank for your attention