Quantum correlations in remote state creation. Information exchange with vanishing entanglement.

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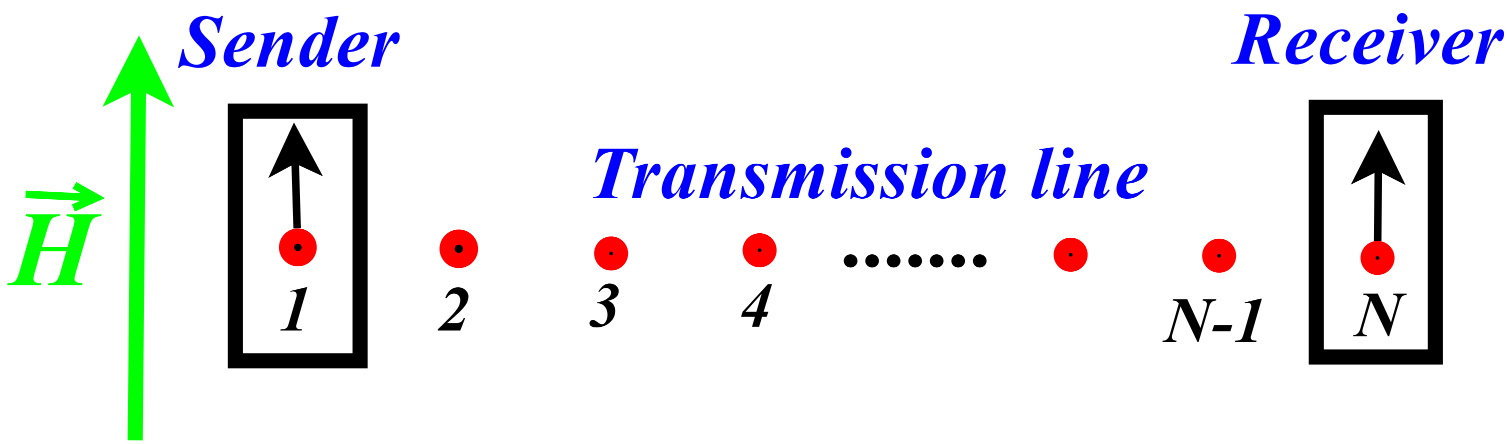
I. Communication line: control of entanglement



The Hamiltonian H conserves the z-projection of the total spin momentum: $[H, I_z] = 0$

Tensor-product initial state

$$\rho_0 = \rho_0^S \otimes \rho_0^{TL} \otimes \rho_0^R,$$

$$\rho^{TL} = \mathrm{diag}(1, 0, 0, \ldots) - \text{ground state},$$

$$\rho_0^S = U^S \Lambda^S (U^S)^+, \quad \rho_0^R = U^R \Lambda^R (U^R)^+.$$

$$\Lambda^S = \mathrm{diag}(\lambda^S, 1 - \lambda^S), \quad \Lambda^R = \mathrm{diag}(\lambda^R, 1 - \lambda^R),$$

$$U^S = \begin{pmatrix} \cos\frac{\pi\alpha_1}{2} & -e^{-2i\pi\alpha_2} \sin\frac{\pi\alpha_1}{2} \\ e^{2i\pi\alpha_2} \sin\frac{\pi\alpha_1}{2} & \cos\frac{\pi\alpha_1}{2} \end{pmatrix},$$

$$U^R = \begin{pmatrix} \cos\frac{\pi\beta_1}{2} & -e^{-2i\pi\beta_2} \sin\frac{\pi\beta_1}{2} \\ e^{2i\pi\beta_2} \sin\frac{\pi\beta_1}{2} & \cos\frac{\pi\beta_1}{2} \end{pmatrix}.$$

The density matrix of the subsystem SR

$$\rho^{SR}(t) = \mathrm{Tr}_{TL}\Big(V(t)\rho_0 V^+(t)\Big), \quad V(t) = e^{-iHt},$$

Three types of control parameters

1. The two eigenvalue parameters λ^S and λ^R.

2. Strong parameters α_1 and β_1.

3. Weak parameters α_2 and β_2.

The mean value of a E with respect to some parameter γ:

$$\langle E \rangle_\gamma = \int_0^1 d\gamma E(\gamma),$$

The mean value $\bar{E}$ with respect to all eigenvector parameters $\tilde{\Gamma} = (\alpha_1, \alpha_2, \beta_1, \beta_2)$:

$$\bar{E}(\lambda^S, \lambda^R) = \left\langle \left\langle \left\langle \langle E \rangle_{\alpha_1} \right\rangle_{\beta_1} \right\rangle_{\alpha_2} \right\rangle_{\beta_2}.$$

Standard deviation with respect to the particular parameter γ:

$$\delta_\gamma^{(E)}(\lambda^S, \lambda^R) = \sqrt{\left\langle \left(\bar{E}(\lambda^S, \lambda^R) - \langle E \rangle_{\tilde{\Gamma}_\gamma} \right)^2 \right\rangle_\gamma}.$$

Quantum entanglement E and concurrence C

$$E = -\frac{1+\sqrt{1-C^2}}{2}\log_2\frac{1+\sqrt{1-C^2}}{2} - \frac{1-\sqrt{1-C^2}}{2}\log_2\frac{1-\sqrt{1-C^2}}{2}$$

$$C = \max(0, 2\lambda_{max} - \sum_{i=1}^{4}\lambda_i),$$
$$\lambda_{max} = \max(\lambda_1, \lambda_2, \lambda_3, \lambda_4),$$

where λ_i are the eigenvalues of the following matrix

$$\tilde{\rho}^{(SR)} = \sqrt{\rho^{SR}(\sigma_y \otimes \sigma_y)(\rho^{SR})^*(\sigma_y \otimes \sigma_y)}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

II. A particular model of communication line: spin-1/2 chain of $N = 40$ nodes

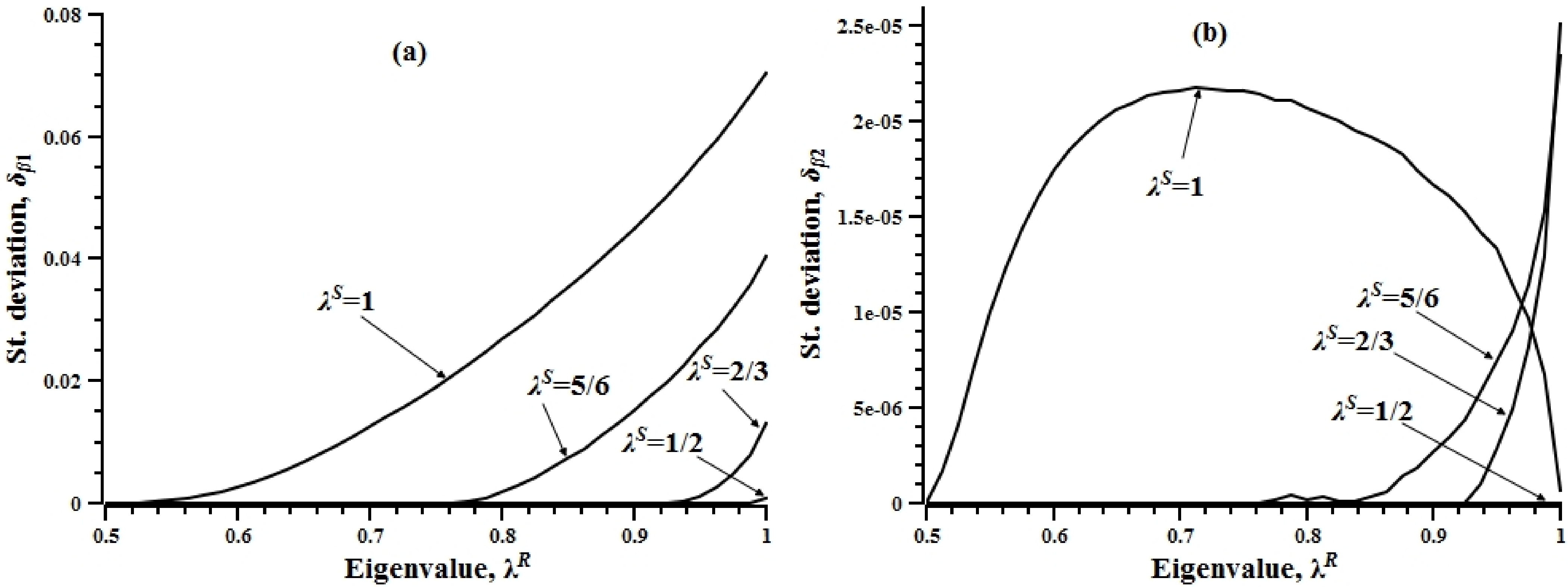
Hamiltonian

$$H = \sum_{i=1}^{39} D(I_{ix}I_{(i+1)x} + I_{iy}I_{(i+1)y}),$$

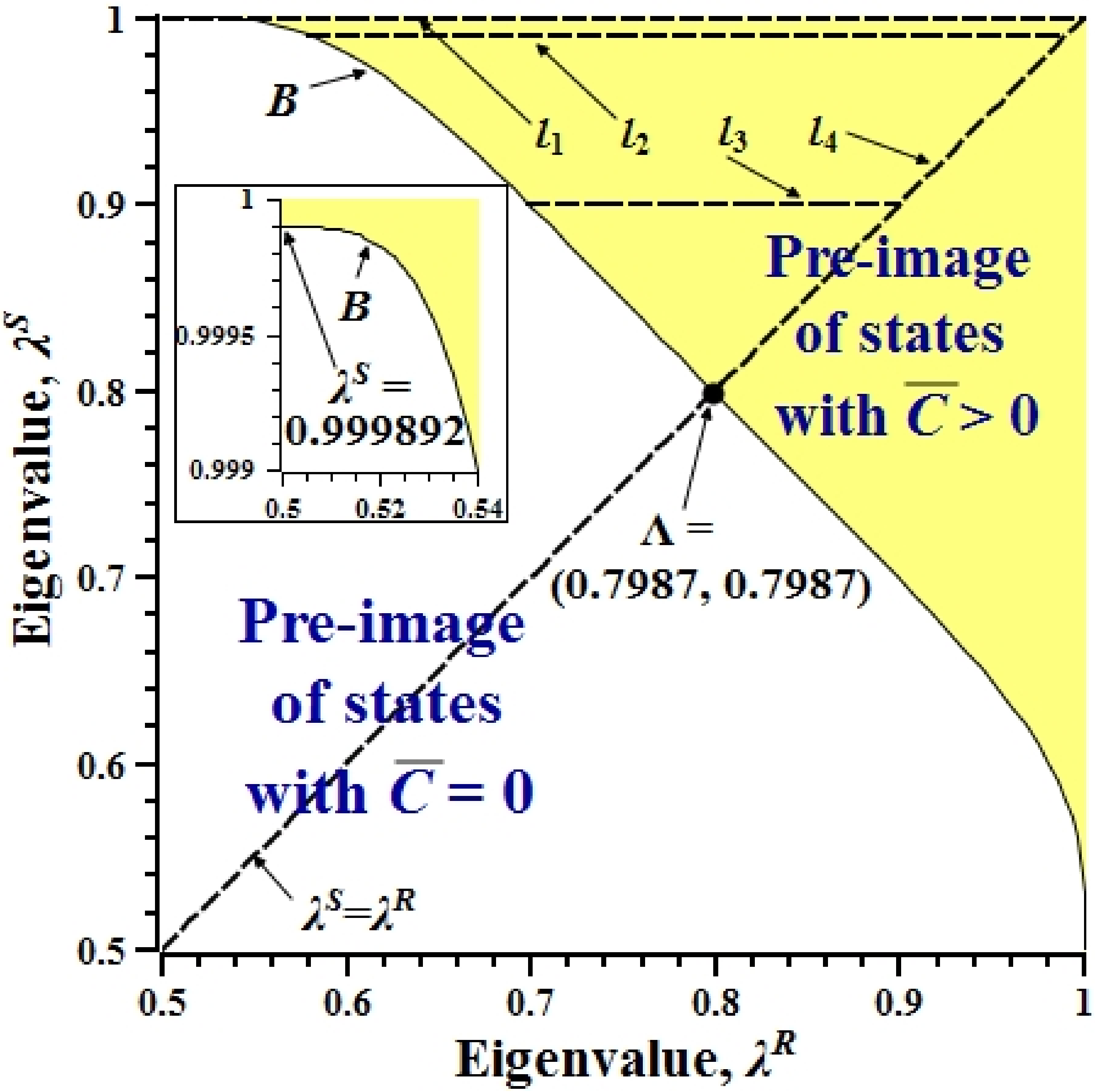
D − coupling constant, $I_{j\alpha}$ ($\alpha = x, y, z$) − jth spin projection on the α-axis.

Choice of the time instant for state registration: C averaged over the control parameters is maximal: $t = 43.442$.

Effect of eigenvector initial parameters

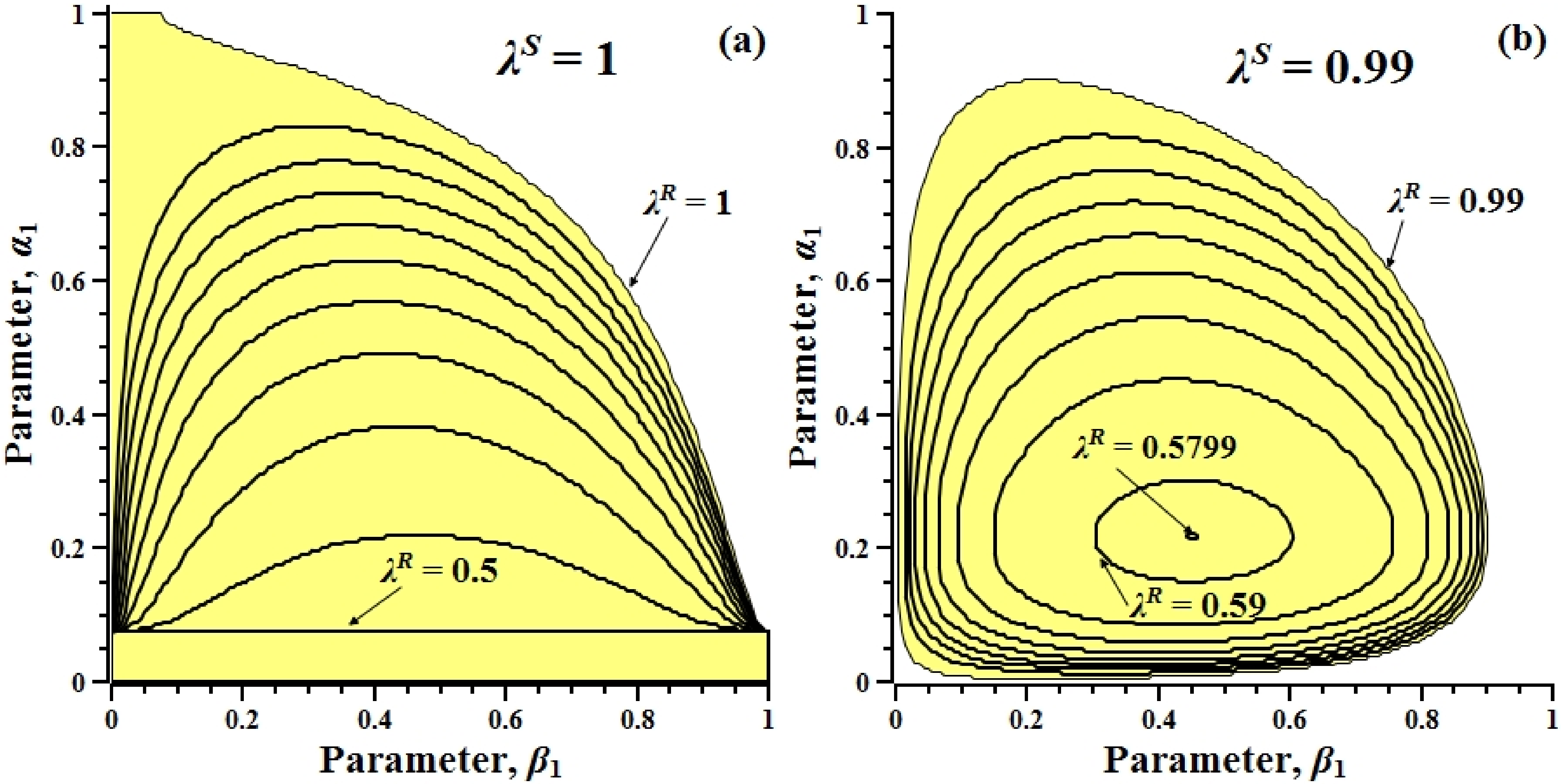


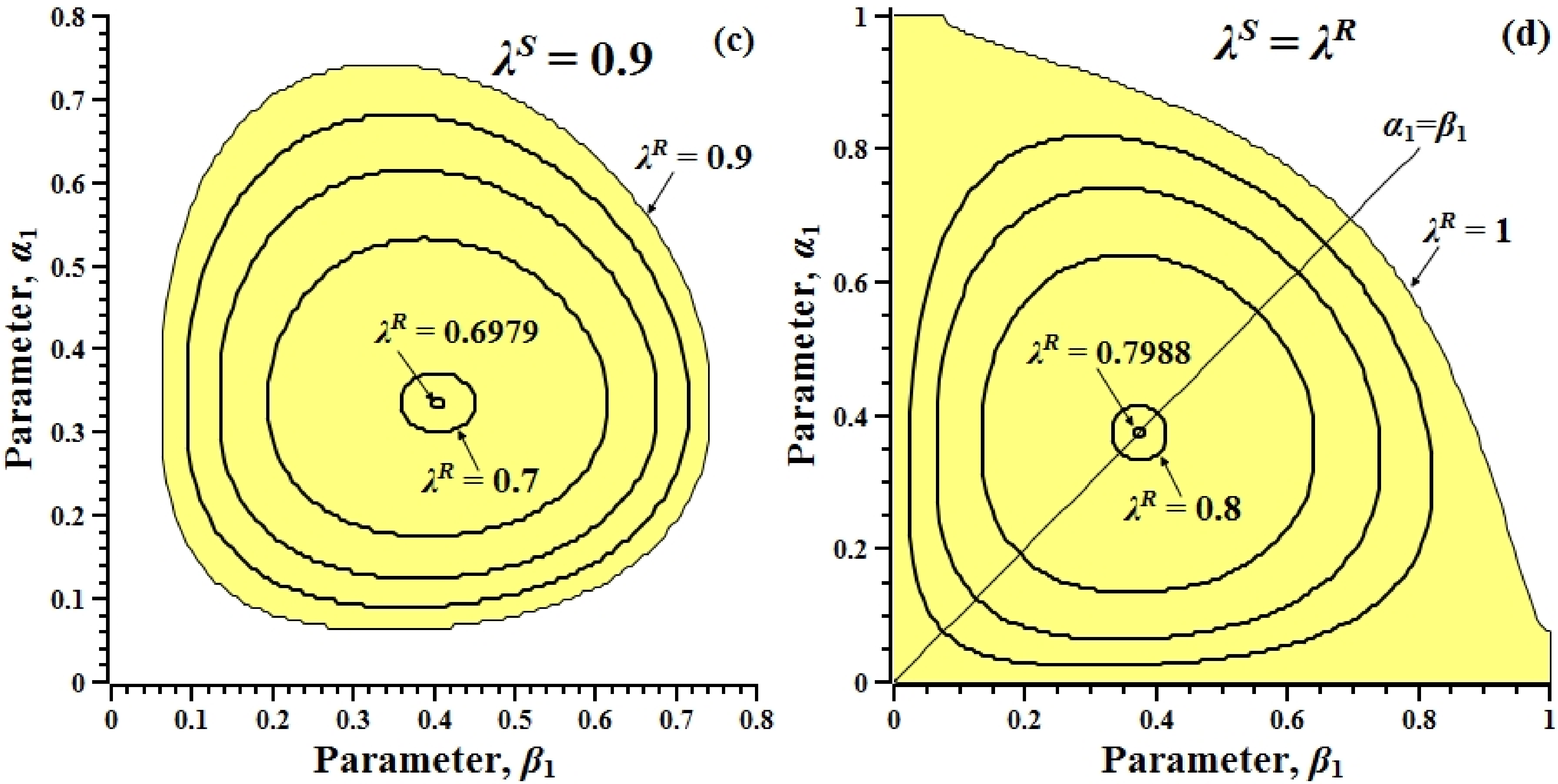
Concurrence of ρ^{SR} on (λ^R, λ^S)-plane



Concurrence of ρ^{SR} on (β_1, α_1)-plane.

$\Delta\lambda^R = 0.05$.





$\lambda^S = 0.9$
(c)
$\lambda^R = 0.9$
$\lambda^R = 0.6979$
$\lambda^R = 0.7$
Parameter, α_1
Parameter, β_1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$\lambda^S = \lambda^R$
(d)
$\alpha_1=\beta_1$
$\lambda^R = 1$
$\lambda^R = 0.7988$
$\lambda^R = 0.8$
Parameter, α_1
Parameter, β_1
0
0.2
0.4
0.6
0.8
1

III. Informational correlation

$$\rho_0^S = \begin{pmatrix} 1 - x_1 & x_2 + ix_3 \\ x_2 - ix_3 & x_1 \end{pmatrix},$$

$$\rho^R(t,x) = \begin{pmatrix} 1 - y_1(t,x) & y_2(t,x) + iy_3(t,x) \\ y_2(t,x) - iy_3(t,x) & y_1(t,x) \end{pmatrix},$$

$$x_1 = \frac{1}{2}\Big(1 + (1 - 2\lambda^S)\cos(\alpha_1\pi)\Big),$$

$$x_2 = -\frac{1}{2}(1 - 2\lambda^S)\sin(\alpha_1\pi)\cos(2\alpha_2\pi),$$

$$x_3 = \frac{1}{2}(1 - 2\lambda^S)\sin(\alpha_1\pi)\sin(2\alpha_2\pi),$$

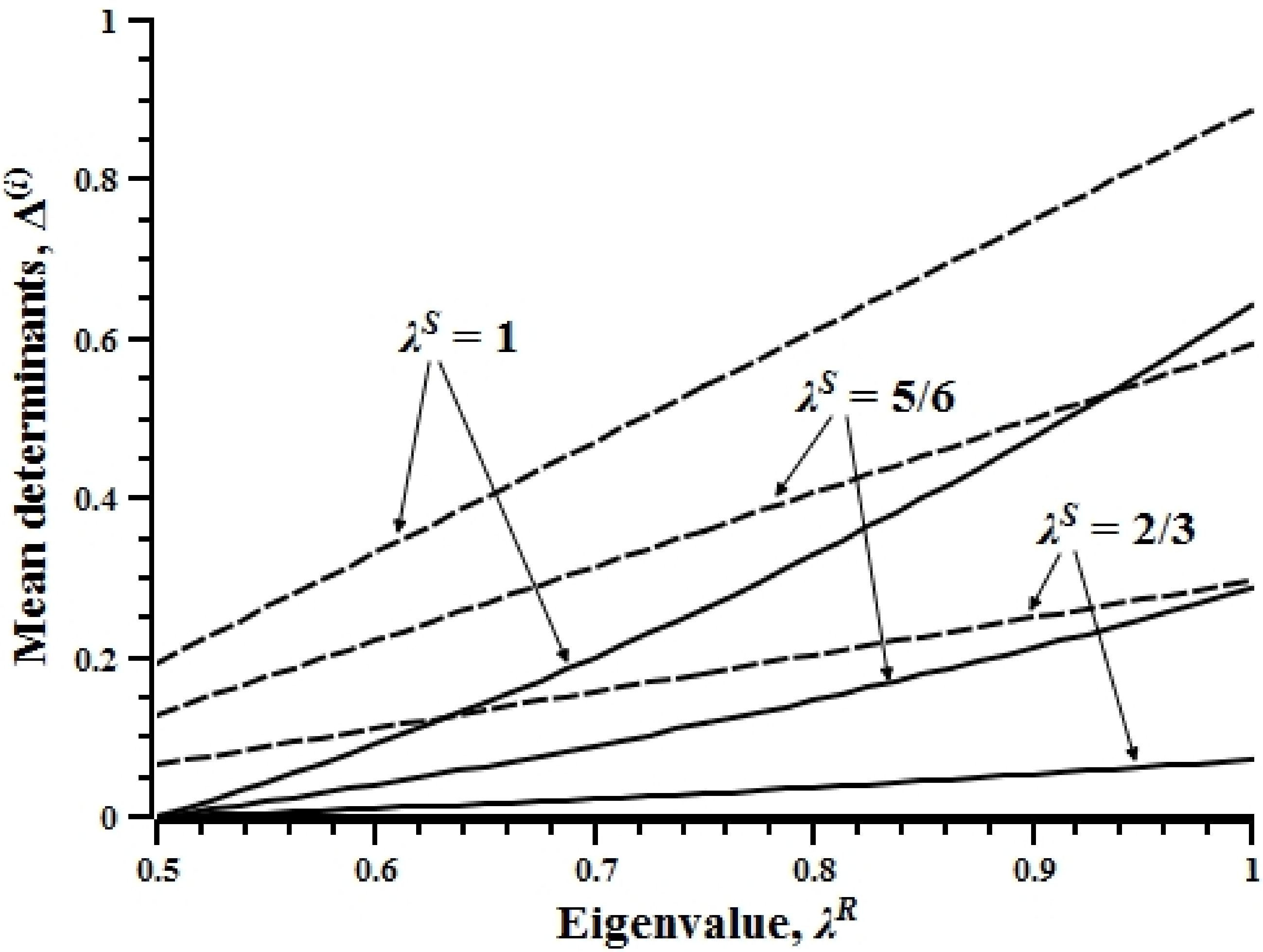
$\mathbf{E^{SR} = 2}$: Both parameters α_1 and α_2 can be found from y_i:

$$\Delta^{(2)} = \frac{1}{\Delta_0^{(2)}} \sum_{\substack{n,m=1 \\ m>n}}^{3} \sum_{\substack{i,j=1 \\ j>i}}^{3} \left| \frac{\partial(y_i, y_j)}{\partial(x_n, x_m)} \right| \left| \frac{\partial(x_n, x_m)}{\partial(\alpha_1, \alpha_2)} \right| \neq 0.$$

$\mathbf{E^{SR} = 1}$: Either α_1 or α_2 can be found from y_i:

$$\Delta^{(1)} = \frac{1}{\Delta_0^{(1)}} \sum_{i,n=1}^{3} \left| \frac{\partial y_i}{\partial x_n} \right| \left(\left| \frac{\partial x_n}{\partial \alpha_1} \right| + \left| \frac{\partial x_n}{\partial \alpha_2} \right| \right) \neq 0.$$

Mean determinants on (λ^R, λ^S) plane: $\bar{\Delta}^{(2)}$ (solid line) and $\bar{\Delta}^{(1)}$ (dashed line)



Unlike the concurrence, there is no domain on the plane (λ^R, λ^S) resulting in the zero mean determinants.

IV. Conclusions

1. There are strong eigenvector control-parameters which can significantly change quantum correlations.

2. The eigenvalues are most important parameters which strongly effect the quantum correlations and, in principle, they might be joined to the above strong control parameters.

3. There is a large domain in the control parameter space mapped into the non-entangled states of the subsystem SR, but nevertheless these parameters can be exchanged between sender and receiver.