Numerical methods for the prediction and optimization of the cryosurgery operations

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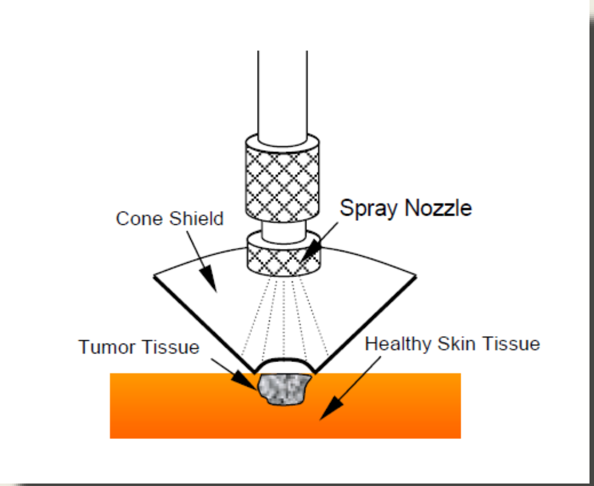
Introduction



1. Wide range of applicability of the cryosurgery
2. Difficulties of cryosurgery planning on the real tumors of an arbitrary shape
3. Finding of the ways to improve the effectiveness and safety of carrying out the cryosurgery

Aims and problems solved

Aim:

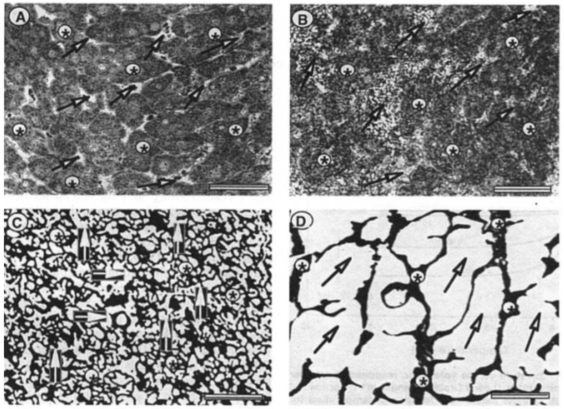
The development of effective numerical algorithms and program complexes for the planning and optimization of cutaneous cryosurgery operations

Problems solved:

1. Macroscopically averaged model for bioheat transfer processes taking into account the peculiarities of cell-scale cryogenic processes
2. Numerical algorithms and computerized tools for three dimensional macroscopic modeling of cutaneous cryosurgery combined with cryoprobe correction

Introduction. Cell scale.
Cryogenic processes in biotissues

1. Intracellular crystallization
2. Cell dehydratation
3. Extracellular crystallization



Influence of cooling rate on the cell processes:

a) Low cooling rate: extracellular crystallization is dominant (cryoconservation)

b) High cooling rate: intracellular crystallization is dominant (cryusurgery)

►►Dependence of freezing interval on the cooling rate

Phase-averaged model for cryogenic bioheat transfer

$$C\frac{\partial T}{\partial t} = -div\mathbf{Q} + \rho_b\omega_b C_b(T_b - T) + q_{met}, \mathbf{Q} = -\kappa\nabla T,$$

$$0 < t < T_{\mathrm{f}}, \quad \mathbf{r} \in H \subset R^3.$$

\+ initial and boundary conditions

Averaging parameters:

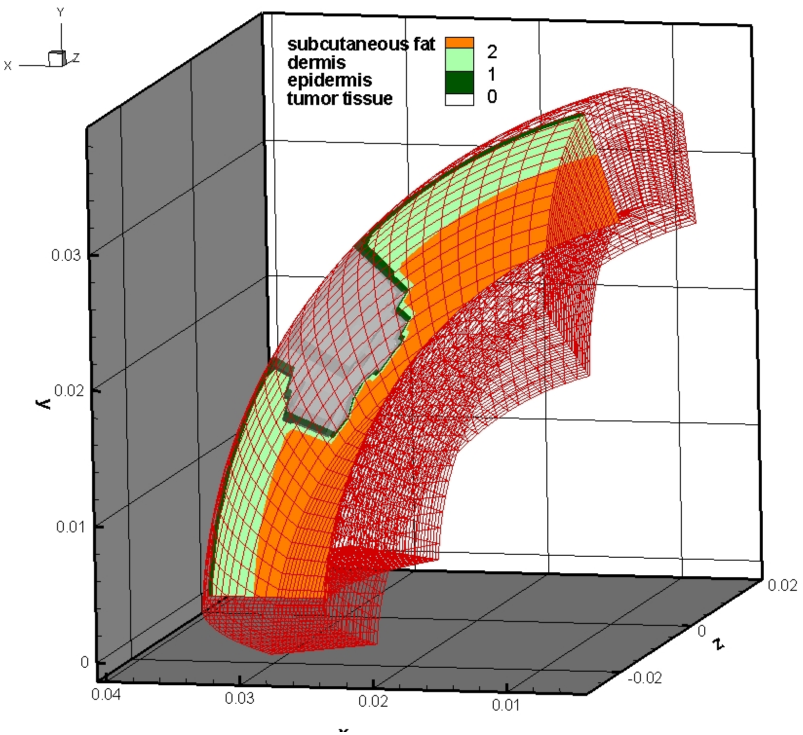
1. T_α - characteristic temperature of full freezing

2. $\delta = (T^* - T_\alpha)/(T_c - T_\alpha)$, $T^* \in (T_\alpha, T_c)$

$$\overline{C}(T; T_\alpha, T^*) = \begin{cases} C_1(T), & T \le T_\alpha \\ \frac{\rho L}{T_c - T_\alpha} + \delta C_1(T_\alpha) + (1-\delta)C_2 & T_\alpha < T < T_c \\ C_2, & T \ge T_c \end{cases}$$

Three dimensional numerical modeling of the cryosurgery
Main assumptions

1. Skin tissue consists of 3 layers: epidermis (0.5 mm), dermis (3.5 mm), subcutaneous fat (5 mm)
2. Tumor tissue bio-physical parameters are the same with the dermis tissue
3. Computation area allows the construction of indexed mesh
4. Correcting cryotip's working surface is assumed to have a sphere shape (radius is 2mm)
5. Phase averaging parameters are piecewise constant



Three dimensional numerical modeling of the cryosurgery
Numerical algorithm

$$\rho_{ijk}^{n} C_{ijk}^{n} \frac{T_{ijk}^{n+1} - T_{ijk}^{n}}{\tau} V_{ijk} = - \oint\limits_{\partial v_{ijk}} (\mathbf{Q}, \mathbf{n}) dS + \rho_b \omega_b C_b (T_b - T_{ijk}^{n}) V_{ijk}$$

$$\oint\limits_{\partial v_{ijk}} (\mathbf{Q}, \mathbf{n}) dS = \sum_{l=1}^{6} \oint\limits_{\Gamma_l} (\mathbf{Q}, \mathbf{n}) dS \equiv \sum_{l=1}^{6} Q_l$$

Flux relaxation method for the improving the scheme stability conditions:

$$\rho C \frac{\partial T}{\partial t} = -div\mathbf{Q} + \rho_b \omega_b C_b (T_b - T),$$

$$\mathbf{Q} + \tau_r \frac{\partial}{\partial t} \mathbf{Q} = \mathbf{Q}_F.$$

$$\mathbf{Q}_F = -\kappa \nabla T$$

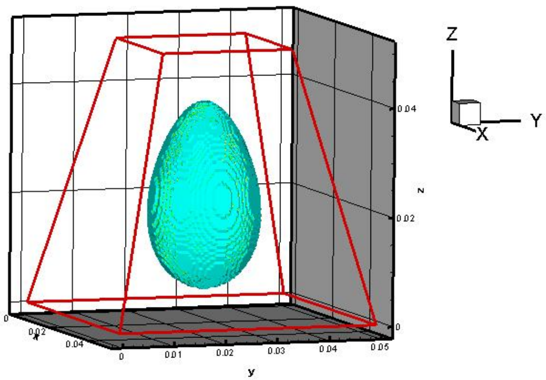
$$\mathbf{Q}(\mathbf{r}, t = 0) = \mathbf{0}$$

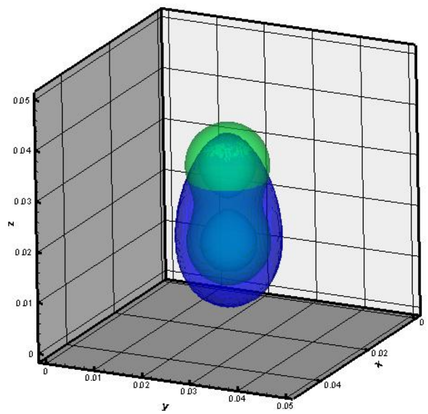
$$\mathbf{Q}^n = D\mathbf{Q}^{n-1} + (1 - D)\mathbf{Q}_F^n, \quad D = e^{-\frac{\tau}{\tau_r}}$$

Chetverushkin B.N., Shilnikov E.V. (2010)

Three dimensional numerical modeling of the cryosurgery

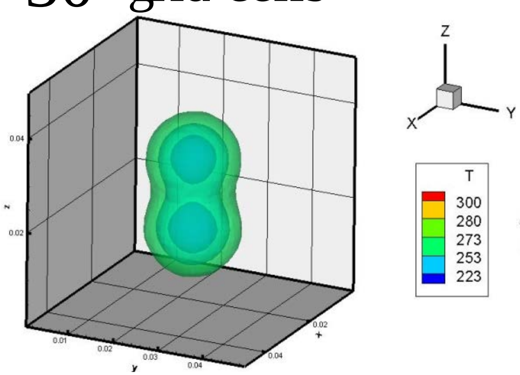
Grid convergence



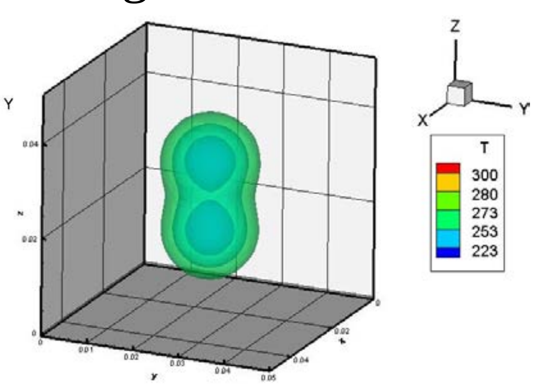


Step, м	Healthy tissue destructed, м^3	Tumor tissue undestructed, м^3
0.00167	5.44e-7	3.41e-6
0.00125	5.27e-7	3.28e-6
0.001	5.17e-7	3.35e-6
0.00083	5.21e-7	3.25e-6
0.00071	5.24e-7	3.3e-6
0.00063	5.15e-7	3.31e-6
0.0005	5.2e-7	3.29e-6

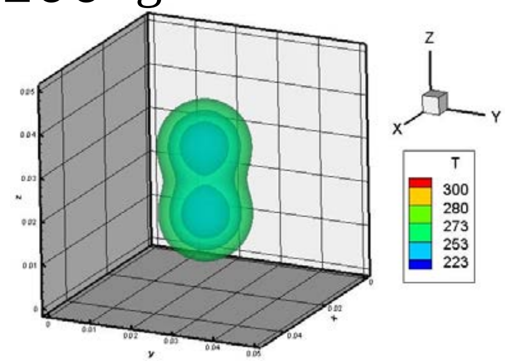
30^3 grid cells



40^3 grid cells



100^3 grid cells



Pareto optimization of the cryosurgery operations

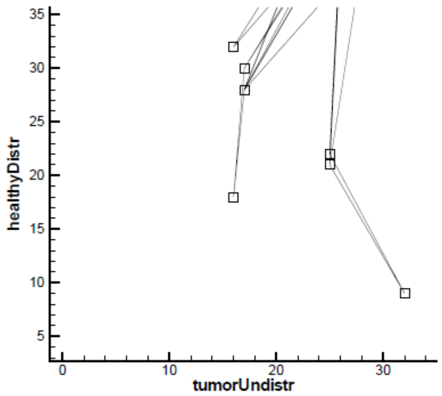
Objective variables:

$$\mathfrak{R} = (\mathbf{R}_1, \mathbf{R}_2, \ldots \mathbf{R}_N)^T$$

Problem statemanent:

$$\mathfrak{R}^* = \arg\min_{\mathfrak{R}} (V_H, V_T)^T$$

Objective functions: volume of injured healthy tissue (V_H) and volume of undestructed tumor tissue(V_T)

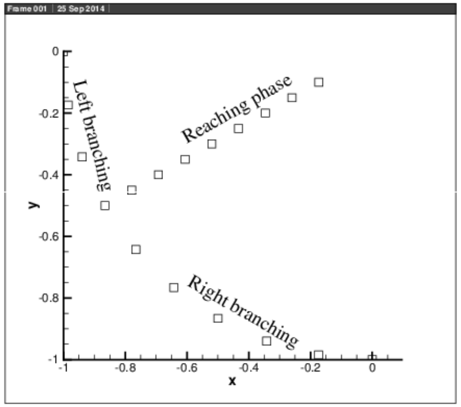


Quasi-gradient scheme:

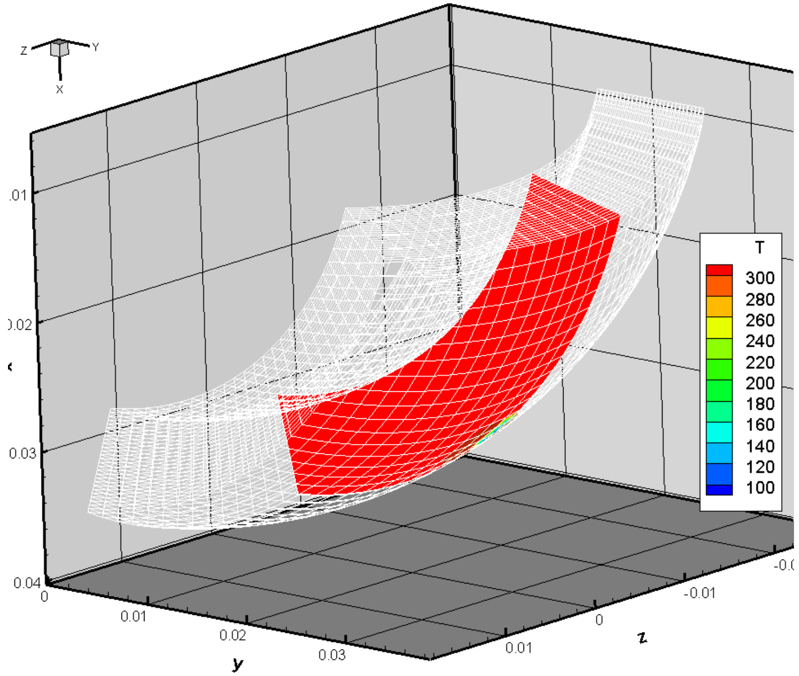
$$\mathfrak{R}_s^{jk\pm} = \left(\mathbf{R}_1, \mathbf{R}_2, \ldots, \mathbf{R}_j^{k\pm}, \ldots, \mathbf{R}_N\right)$$

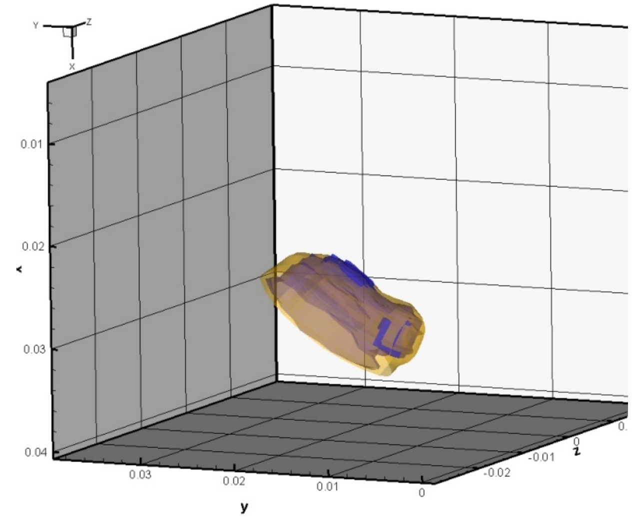
$$\mathbf{R}_j^{k\pm} = \left(x^1, x^2, \ldots, x^k \pm \delta, \ldots, x^M\right)$$

$$\mathfrak{R}_{s+1} = \mathbf{R}_s^{j^* k^* \{+|-\}}$$



Three dimensional numerical modeling of the cryosurgery
Numerical results

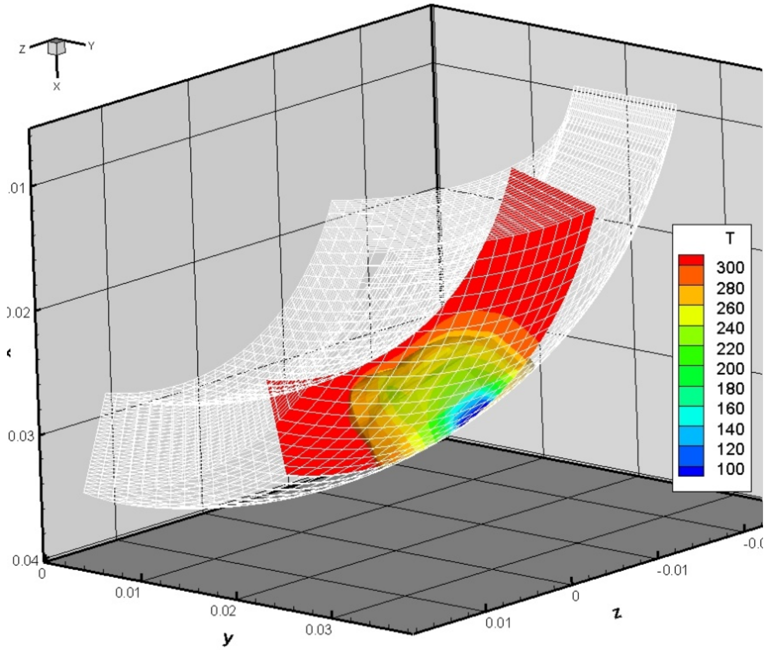




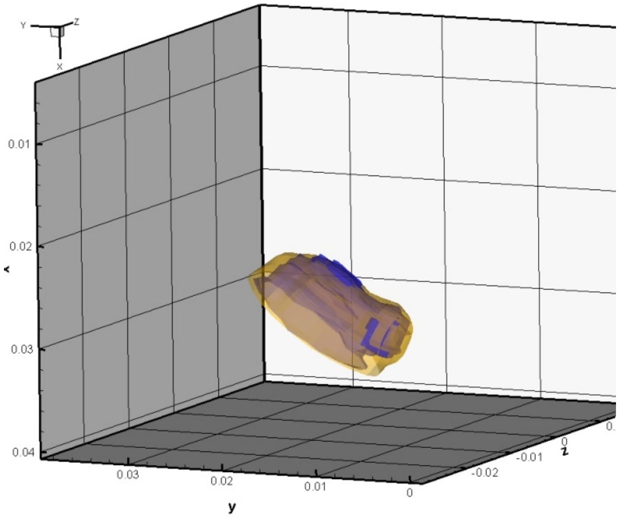
Resulting tissue necrosis region (yellow) and tumor tissue (blue)

Three dimensional numerical modeling of the cryosurgery

Numerical results



$$\tau \sim h^{\frac{3}{2}}$$

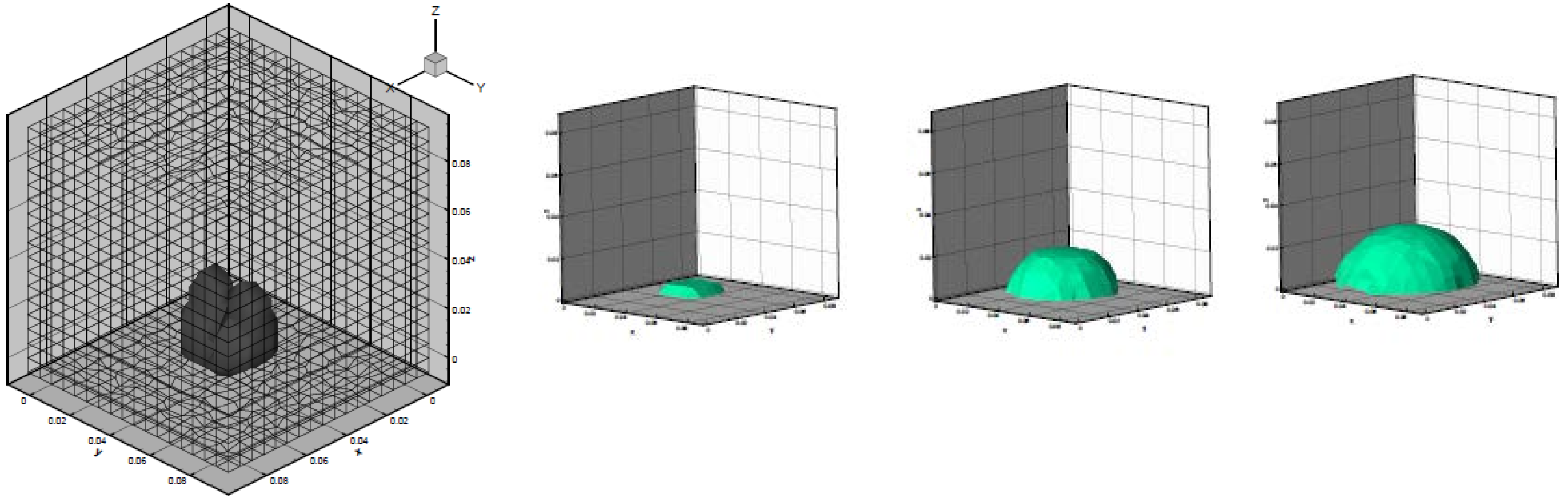


Resulting tissue necrosis region (yellow) and tumor tissue (blue)

Optimization of the cutaneous cryosurgery
Suppressing heaters method(1)

Boundary conditions: $(\mathbf{Q}, \mathbf{n})|_{\Gamma \backslash \Omega} = 0 \quad T(t, \mathbf{r} \in \Omega) = T_{LN_2}$

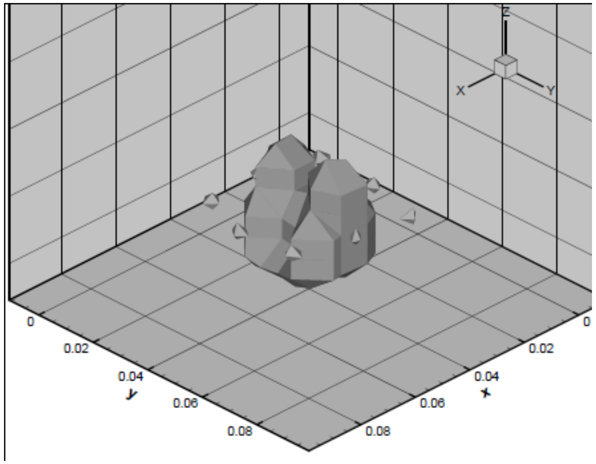
Free propagation of the tissue necrosis front

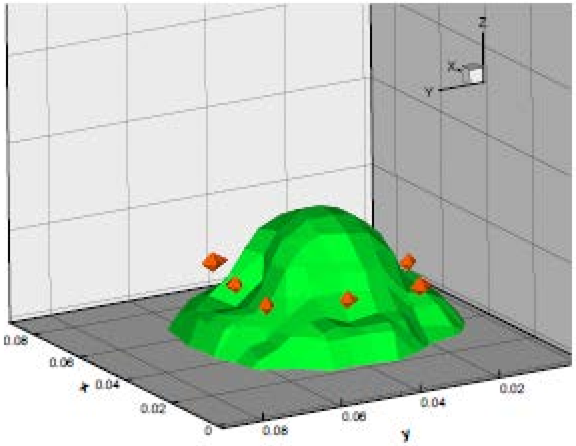


Optimization of the cutaneous cryosurgery
Suppressing heaters method(2)

$$V_H,\ V_T - \min_{\mathfrak{R}},\ \mathfrak{R} = \left(\widehat{\mathbf{R}}_1, \widehat{\mathbf{R}}_2, \ldots, \widehat{\mathbf{R}}_P\right) \qquad \mathfrak{R}^+ = (\mathfrak{R}, T) \quad V_H - \min_{\mathfrak{R}^+},\ V_T = 0.$$

Resulting tissue necrosis front localization





Healthy tissue injury decreased by 30% against the increasing of duration of the operation by 25%

Conclusion

1. The phase averaged macroscopic model with scalar averaging fields for the cryogenic bioheat transfer is proposed as the tools for predicting the results of cryosurgery with respect to peculiarities of phase change processes in biological solutes
2. The quasi-gradient method is adopted for the optimization of the cryosurgery operations
3. Introducing of the additional cryotip into the skin tissue during the cutationeous cryosurgery is considered
4. Numerical experiments on the model problems showed that the developed planning tools can cause a valuable increasing of effectiveness and safety of cryosurgery operations performing

Thank you for your attention