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On Interpolational Approximation of Nonlinear Differential Operators of the Second Order in Partial Derivatives

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\def\title#1{\begin{center}#1\end{center}}
\def\author#1{\centerline{\small\textsf{#1}}}
\def\address#1#2{\begin{center}\small\emph{#1}\,\|\,E-mail: \texttt{#2}\end{center}}
\def\refname{\small References}
\begin{document}
\title{\textsf{On Interpolational Approximation of Nonlinear Differential Operators of the Second Order in Partial Derivatives}}\footnote{This work is supported by Belarusian Republican Foundation for Fundamental Research (project 16D-002).}
\author{L.~A.~Yanovich and M.~V.~Ignatenko}
\address{Institute of Mathematics, National Academy of Sciences of Belarus, Surganova Str. 11, 220072 Minsk, Belarus}{yanovich@im.bas-net.by, ignatenkomv@bsu.by}
\bigskip
\small
We consider the differential operators  $F : C^2(T \times S) \rightarrow Y$  of the second order in partial derivatives of the form  $F(x) = f(t, s, x(t, s), x'_t(t, s), x'_s(t, s), x''_{t2}(t, s), x''_{ts}(t, s), x''_{s2}(t, s))$ , where  $x(t, s)$  is defined on a rectangle  $\Omega = T \times S$ ,  $T$  is the space of two times continuously differentiable functions on  $S$ ,  $S \subset \mathbb{R}^2$ ,  $S$  is a set of the number line  $\{t_i\}_{i=0}^5$ ,  $S$  is a function space. The operator  $F$  is defined on  $\Omega$  and maps it into a function space  $Y$ .
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Here is the Lagrange interpolation formula for the operators (1): $L_n(F; x) = F(x_0) + \sum_{k=1}^n \int_0^1 \sum_{i,j=0; i+j \leq 2}^2 \frac{\partial}{\partial t^i \partial s^j} F(v_k) l_{n,k}(x) \tau^{i+j}$

eqno(2) where the functions $\{l_{n,k}\}$ are fundamental polynomials of the n -degree $\{k\}$
 $\{l_{n,k}\} = x_0 l(t,s) + \tau l(t,x) - l(t,s) + \tau l(s,x)$ for $k=1, 2, \dots, n$, and $\sigma_n(x) = \sum_{k=0}^n l_{n,k}(x)$.
 δ_{kj} is the Kronecker symbol ($k, j = 0, 1, \dots, n$), and $\sigma_n(x) = \sum_{k=0}^n l_{n,k}(x)$.
 σ_n is a constant or a variable value. The polynomial (2) satisfies the following interpolation conditions:
 $L_n(F; x_k) = F(x_k)$, ($k = 0, 1, \dots, n$).

For the interpolation error r_n

$|r_n| = |F(x) - L_n(x)|$, where $L_n(x)$ is the interpolation polynomial (2), the following representation holds:

$$r_n(x) = \sum_{k=1}^{n+1} \int_0^1 \sum_{i,j=0; i+j \leq 2}^2 \frac{\partial}{\partial t^i \partial s^j} F(v_k(t, s, \tau)) \times \\ \times \frac{\partial^{i+j}}{\partial t^i \partial s^j} \left\{ \left(\frac{l_{n+1,k}(x(t, s))}{\sigma_{n+1}(x(t, s))} - \frac{l_{n,k}(x(t, s))}{\sigma_n(x(t, s))} \right) (x_k(t, s) - x_0(t, s)) \right\} d\tau,$$

where $x_{n+1} = x$, $l_{n,n+1}(x) \equiv 0$.

Some other interpolation formulas for the operator (1) are also constructed.

\end{document}

Primary author: YANOVICH, Leonid (Institute of Mathematics National Academy of Sciences of Belarus)

Presenter: YANOVICH, Leonid (Institute of Mathematics National Academy of Sciences of Belarus)

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