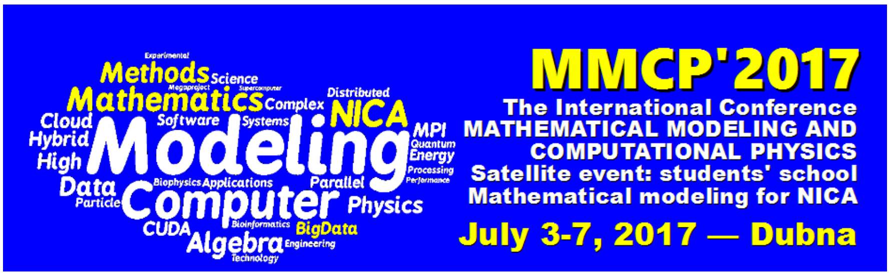

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Higher-order partial differential equations for description of the Fermi-Pasta-Ulam and the Kontorova-Frenkel models

Friday 7 July 2017 08:30 (30 minutes)

We consider the following dynamical system:
\begin{equation}\begin{gathered}
\label{I_1}
m\,\frac{d^2y_{i}}{d t}=F_{i+1,i}-F_{i,i-1}-f_0\,\sin{\left(\frac{2\,\pi\,y_i}{a}\right)}, \qquad
(i=1,\ldots,N),
\end{gathered}\end{equation}
where y_i measures the displacement of the i-th mass from equilibrium in time t, the force $F_{i+1,i}$ describes the nonlinear interaction between atpms dislocations in the crystal lattice in case of dislocations
\begin{equation}\begin{gathered}
\label{I_2}
F_{i+1,i}=\gamma\,(y_{i+1}-y_i)+\alpha\,(y_{i+1}-y_i)^2+\beta\,(y_{i+1}-y_i)^3,
\end{gathered}\end{equation}
and f_0, a, γ, α, β are constant parameters of system \eqref{I_1}.

The system of equations \eqref{I_1} is the generalization of some well-known dynamical systems. At $\alpha=0$ and $\beta=0$ the system of equations \eqref{I_1} is the mathematical model introduced by Frenkel and Kontorova for the description of dislocations in the rigid body \cite{Frenkel}. In this model it was suggested that the influence of atoms in the crystal is taken into account by term $f_0\,\sin\frac{2\,\pi\,y_i}{a}$ but the atoms in case of dislocations interact by means of linear low. Assuming that $N\rightarrow\infty$ and $h\rightarrow 0$ where h is the distance between atoms, we can get the Sine-Gordon equation.

In case of $f=0$ and $\beta=0$ system of equations \eqref{I_1} is the well-known Fermi-Pasta-Ulam model \cite{Fermi} which was studied many times. It is known that the Fermi-Pasta-Ulam model is transformed at $N\rightarrow\infty$ and $h\rightarrow 0$ to the Korteweg-de Vries equation \cite{Kruskal}.

The main result of work \cite{Kruskal} was the introduction of solitons as solutions of the Koryeweg-de Vries equation.
It was shown in 1967 that the Cauchy problem for this equation can be solved by the Inverse Scattering transform \cite{Gardner}.

Assuming $f_0=0$, $\alpha\neq 0$ and $\beta\neq 0$ at $N\rightarrow\infty$ and $h\rightarrow 0$ one can find the modified Korteweg-de Vries equation for the description of nonlinear waves.

In papers \cite{ Kudr15, Kudr17} the author took into account high order terms in the Taylor series for the description of nonlinear waves in the Fermi-Pasta-Ulam and the Kontorova-Frenkel models assuming that $\alpha\neq 0$ and $\beta\neq 0$ and did not obtain nonlinear integrable differential equations in mass chain. Here we assume that the interaction between dislocations in crystal is described by means of nonlinear low at $\alpha\neq 0$ and $\beta\neq 0$ and consider the other equations. The aim of this talk is to present the nonlinear partial differential equations corresponding to dynamical system \eqref{I_1} and to discuss the properties of these equations.

Short biography note

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