# Turbulent mixing of a critical fluid: the exact renormalization

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# Program

- *Model*: turbulently moving liquid close to its critical point
- *Aims*: study of large scale behaviour and calculation of the critical exponents
- *Method*: exact renormalization
- *Results*: phase diagram and critical exponents

# Equation of motion: the model $A$

The Langevin equation for the scalar field $\phi(x,t)$

$$\lambda\,\partial_t\phi(x,t) = -\frac{\delta H[\phi]}{\delta\phi(x,t)} + \eta(x,t) \tag{1}$$

$\lambda^{-1}$ is a kinetic coefficient.

The Gaussian white noise

$$\langle\eta(x,t)\rangle = 0 \tag{2}$$
$$\langle\eta(x,t)\eta(x',t')\rangle = 2\lambda\,\delta^d(x-x')\delta(t-t')$$

The Landau-Wilson Hamiltonian

$$H[\phi] = \int \mathrm{d}^d x \left\{ \frac{1}{2}(\nabla\phi)^2 + \frac{\tau_0}{2}\phi^2 + \frac{g_0}{4!}\phi^4 \right\} \tag{3}$$

$$\tau_0 \sim T - T_c, \quad g_0 > 0$$

- Ising model
- liquid-vapour critical point
- binary mixture

# Turbulent moving

$v_j(x,t)$ – chaotic velocity fluctuations
$\partial_j v_j(x,t) \neq 0$ – a compressible fluid

The velocity statistics

$$\langle v_j(x,t)\rangle = 0, \quad \langle v_j(x,t) v_i(x',t')\rangle = D_{ij},$$

$$D_{ij} = D_0\, \delta(t-t') \int \frac{\mathrm{d}^d p}{(2\pi)^d} \left[\delta_{ij} - (1-\alpha)\frac{p_j p_j}{p^2}\right] \frac{\exp[ip(x-x')]}{p^{d+\zeta}}$$

$$D_0 > 0, \alpha \geqslant 0.$$

The Kolmogorov spectrum
$E_p = \langle |v(p)|^2\rangle \sim p^{-5/3}$ for $\zeta = 4/3$

The inclusion of the velocity field

$$\partial_t \to \nabla_t = \partial_t + v_j \partial_j$$

# Effective field model

Stochastic equation

$$\lambda \nabla_t \phi(x,t) = -\frac{\delta H[\phi]}{\delta \phi(x,t)} + \eta(x,t) \tag{4}$$

The MSR action for $\Psi = \{\phi, \phi', v_j\}$

$$S[\Psi] = \int \mathrm{d}^d x \mathrm{d}t \left\{ \lambda \phi' \nabla_t \phi + \phi' \frac{\delta H[\phi]}{\delta \phi} - \lambda \phi' \phi' + \frac{1}{2} v_i D_{ij}^{-1} v_j \right\} \tag{5}$$

Statistical average = path integration

$$\langle \dots \rangle = \int \mathcal{D}\Psi \dots \exp\{-S[\Psi]\} \tag{6}$$

$$\mathcal{D}\Psi \equiv \mathcal{D}\phi \mathcal{D}\phi' \mathcal{D}v$$

$\Gamma_k[\Phi]$ – the scale dependent Effective Average Action
$\Phi = \langle\Psi\rangle$ – the order parameter

the ultra-violet limit $\Gamma_{k=\Lambda} = S$ – the microscopic action
the infra-red limit $\Gamma_{k=0} = \Gamma$ the macroscopic free energy

The exact Wetterich equation

$$\partial_s \Gamma_k = \frac{1}{2}\int \frac{\mathrm{d}^d p \mathrm{d}\omega}{(2\pi)^d}\left(\Gamma_k^{[2]}(p) + R_k(p)\right)^{-1}\partial_s R_k, \quad s = \ln(k/\Lambda)$$

$R_k(p) = (k^2 - p^2)\Theta(1 - k^2/p^2) -$ the infra-red regulator

$$(\Gamma_k^{[2]})_{ij} = \frac{\delta^2 \Gamma_k[\Phi]}{\delta\Phi_j \delta\Phi_i}$$

knowledge of $\Gamma_{k=0}$ = solution of the model

Field and derivative expansion

$$\Gamma_k = \int \mathrm{d}^d x \mathrm{d}t \left\{ X_k \, \varphi' \left\{ \nabla_t + A_k \left( \partial_i v_i \right) \right\} \varphi + \varphi' \frac{\delta H_k[\varphi]}{\delta \varphi} - Y_k \, \varphi' \varphi' + \frac{1}{2} v_i D_{ij}^{-1} v_j \right\}$$

$$H_k[\varphi] = \int \left\{ \frac{1}{2} Z_k \left( \nabla \varphi \right)^2 + \frac{\lambda_k}{2} \left( \frac{\varphi^2}{2} - \rho_k \right)^2 \right\} \mathrm{d}^d x$$

$A_k, X_k, Y_k, Z_k$ – renormalization functions
$\rho_k$ – a minimum of $H_k$

only the scale $k$ dependence – “Local Potential Approximation”

Close to criticality

$$X_k \sim k^{-\gamma_*^X}, \quad Y_k \sim k^{-\gamma_*^Y}, \quad Z_k \sim k^{-\eta_*}$$

Running anomalous dimensions

$$\gamma_k^X = -\partial_s \ln X_k, \quad \gamma_k^Y = -\partial_s \ln Y_k, \quad \eta_k = -\partial_s \ln Z_k \tag{7}$$

Dimensionless couplings

$$\begin{aligned} g_1 &= X_k^{-1} Y_k Z_k^{-2} k^{d-4} \lambda_k, \quad g_2 = X_k Z_k^{-1} k^{-\zeta} D_0, \\ g_3 &= X_k Y_k^{-1} Z_k k^{2-d} \rho_k, \quad g_4 = A_k, \end{aligned} \tag{8}$$

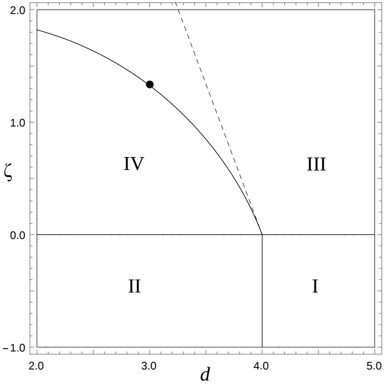
# Running couplings

The functional Wetterich equation $\Longrightarrow$ truncation $\Longrightarrow$ the system of ODE

$$\gamma_k^X(g_j, \alpha, d, \zeta), \quad \gamma_k^Y(g_j, \alpha, d, \zeta), \quad \eta_k(g_j, \alpha, d, \zeta)$$

$$\partial_s g_j = \beta_j(\{g_j\}, \alpha, d, \zeta)$$
$$g_j\big|_{s=0} = g_{j0}$$

We investigate the infra-red ($k/\Lambda \to 0$) stable fixed points of the equations.




Sets of IR stable fixed points in the model. The dashed line – one-loop approximation (by N. Antonov). The dot $(3, 4/3)$ is the physical point, $\alpha = 2.2$

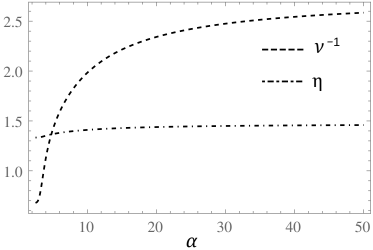
I. The Gaussian fixed point:
$g_{i*} = 0$.

II. The "pure" *A* model:
$g_{1*} \neq 0,\ g_{2*} = 0$.

III. Mixing of a passive scalar:
$g_{1*} = 0,\ g_{2*} \neq 0$.
$\nu^{-1} = 2 - \zeta,\ \eta = \zeta,\ z = 2 - \zeta$
$\alpha < 2.26$

IV. New regime:
$g_{1*} \neq 0,\ g_{2*} \neq 0$.
$z = 2 - \zeta$
$\alpha > 2.26$

# Results: critical exponents




The critical exponents for the scaling regime *IV*, $\alpha \gtrsim 2.26$.

The values of exponents can be extrapolated at $\alpha = \infty$:

$\eta \approx 1.47, \quad \nu^{-1} \approx 2.75,$
$z = 0.6(6)$

# Results

1. stochastic problem $\Longrightarrow$ field theory
2. the Wetterich equation $\Longrightarrow$ infra-red asymptotics
3. turbulence $+$ critical fluctuations $=$ New scaling regime

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