Gaussian Quantum Steering of Two Bosonic Modes in a Squeezed Thermal Environment

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Entanglement

- **Definition (Separability).** The two-mode quantum system on $H = H_A \otimes H_B$ characterized by the state ρ is separable if and only if it can be written as:

$$\rho = \sum_k p_k \, \rho_k^{(A)} \otimes \rho_k^{(B)},$$

where $p_k \geq 0$ are the probabilities with $\sum_k p_k = 1$ and $\rho_k^{(A)}$, $\rho_k^{(B)}$ belong to H_A si H_B, respectively.

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- **Entanglement:** failure of quantum state separability.

ENTANGLEMENT

- **Operational definition:** Allow Alice and Bob the ability to measure a quorum of local observables, so that they can reconstruct the state ρ by tomography.

[1] R. F. Werner, Phys. Rev. A **40**, 4277 (1989).

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- **Operational definition:** Allow Alice and Bob the ability to measure a quorum of local observables, so that they can reconstruct the state ρ by tomography.
- Restrict to projective measurements: $\hat{A} \in \mathcal{D}_\alpha$, $\hat{B} \in \mathcal{D}_\beta$ - observables. $\lambda(\hat{A})$, $\lambda(\hat{B})$ set of eigenvalues a, b of $\hat{A}$ and $\hat{B}$ respectively.

$$P(a,b|\hat{A},\hat{B};\rho) = Tr[\rho(\hat{\Pi}_a^A \otimes \hat{\Pi}_b^B)],$$

where $\hat{\Pi}_a^A$ is the projector satisfying $\hat{A}\hat{\Pi}_a^A = a\hat{\Pi}_a^A$.

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where $\hat{\Pi}_a^A$ is the projector satisfying $\hat{A}\hat{\Pi}_a^A = a\hat{\Pi}_a^A$.
- Separability:

$$P(a, b|\hat{A}, \hat{B}; \rho) = \sum_k p_k P(a|\hat{A}; \rho_k^A) P(b|\hat{B}; \rho_k^B).$$

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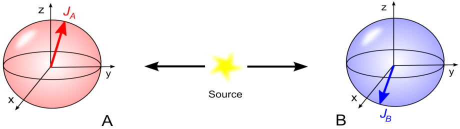
BELL NONLOCALITY

Consider experiment to measure spin correlation: spin ½ system



$$E(\theta,\phi) = \langle J^A_\theta J^B_\phi \rangle$$

IF we assign local hidden variables to each spin:

CHSH-Bell inequality

$$S = E(\theta,\phi) - E(\theta',\phi) + E(\theta,\phi') + E(\theta',\phi') \leq 2$$

*Quantum Mechanics predicts a **violation of Bell's inequality**!*

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \qquad \Rightarrow S = 2\sqrt{2}$$

(Tsirelson) maximum QM value

Bell Nonlocality

- **Local Hidden Variable model (LHV)**
  $\hat{A} \in \mathcal{M}_\alpha$, $\hat{B} \in \mathcal{M}_\beta$ - measurements $a \in \lambda(\hat{A})$, $b \in \lambda(\hat{B})$.

$$P(a, b|\hat{A}, \hat{B}; \rho) = \sum_k p_k \mathcal{P}(a|\hat{A}; k) \mathcal{P}(b|\hat{B}; k),$$

  where $\mathcal{P}(a|\hat{A}; k)$ denote some positive, normalized probability distributions, involving the LHV k.

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- **Bell Nonlocality**: Falure of LHV model, i.e., hidden variable separability.

THE CONCEPT OF STEERING

- Consider a general nonfactorizable pure state of two systems held by two parties (say Alice and Bob):

$$|\Psi\rangle = \sum_{n=1}^{\infty} c_n |\psi_n\rangle |u_n\rangle = \sum_{n=1}^{\infty} c_n |\phi_n\rangle |v_n\rangle,$$

where $\{|u_n\rangle\}$ and $\{|v_n\rangle\}$ two orthonormal bases for Alice's system.

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  where $\{|u_n\rangle\}$ and $\{|v_n\rangle\}$ two orthonormal bases for Alice's system.

- $\Rightarrow$ Choosing to measure in the $\{|u_n\rangle\}$ ($\{|v_n\rangle\}$) basis, Alice projects Bob's system into one of the states $\{|\psi_n\rangle\}$ ($\{|\phi_n\rangle\}$).

STEERABILITY?

- It is about whether Alice, by her choice of measurement $\hat{A}$, can collapse Bob's system into different types of states in the different ensembles $\{E^A : \hat{A} \in \mathcal{M}_\alpha\}$ into which she can steer Bob's state.

$$E^A \equiv \{\tilde{\rho}_a^A : a \in \lambda(\hat{A})\}, \quad \tilde{\rho}_a^A \equiv Tr_\alpha[\rho(\Pi_a^A \otimes I)],$$

where $\tilde{\rho}_a^A \in \mathcal{D}_\beta$ is Bob's state conditioned on Alice measuring $\hat{A}$ with result a.

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- **Operationaly**: Can Alice, with classical communication, convince Bob that they share an entangled state under the fact that Bob doesn't trust Alice?[1]

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STEERABILITY?

- If the correlations between Bob's measurement results and the results Alice reports *cannot* be explined by a *local hidden state* (LHS) ρ_k^B for Bob (if there is no such a prior ensemble $F = \{p_k \rho_k^B\}$ of LHS) , then Bob will be convinced that the state is entangled.

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- **Definition (Steering).**[2]: Alice's measurement strategy on ρ exhibits steering *iff* it is *not* the case that for all $a \in \lambda(A)$ and $b \in \lambda(B)$

$$P(a, b|A, B; \rho) = \sum_k p_k \mathcal{P}(a|\hat{A}; k) P(b|B; \rho_k^B),$$

where $F = \{p_k \rho_k^B\}$ some prior ensemble of LHS with $\rho = \sum_k p_k \rho_k^B$.

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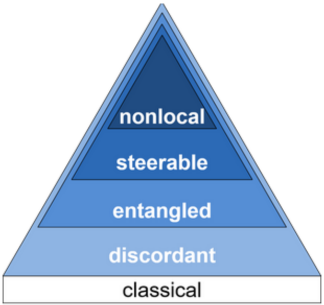
HIERARCHY OF NONLOCALITY

Generalised EPR paradox: for different measurements Alice appears to steer Bobs state from distant site.



Hierarchy of nonlocality:

- **Bell nonlocality:** Failure of Local Hidden Variable (LHV) model
- **EPR steering nonlocality:** Failure of Hybrid LHV-LQS model
- **Entanglement:** Failure of Local Quantum State (LQS) model

Gaussian states

- A general (multimode) bipartite Gaussian state W is defined by its covariance matrix (CM)[3]:

$$V_{\alpha\beta} = \begin{pmatrix} V_\alpha & C \\ C^T & V_\beta \end{pmatrix}, \quad \text{iff} \quad V_{\alpha\beta} + i\Sigma_{\alpha\beta} \geq 0$$

where $\Sigma_{\alpha\beta} = \Sigma_\alpha \oplus \Sigma_\beta$ is a smatrix.

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  - Consider that Alice can only make Gaussian measurements A described by a Gaussian positive operator with a CM satisfying $T^A + i\Sigma_\alpha \geq 0$
  - After such measurement Bob's conditioned state ρ_a^A is Gaussian with a CM

$$V_\beta^A = V_\beta - C(T^A + V_\alpha)^{-1}C^T$$

[4] G. Adesso and A. Datta, Phys. Rev. Lett. **105**, 030501 (2010).

Steerability Conditions and Quantification

- **Theorem:** The Gaussian state W is *not* steerable by a Gaussian measurement iff[4]

$$V_{\alpha\beta} + \mathbf{0}_{\alpha} \oplus i\Sigma_{\beta} \geq 0$$

or equivalently[5]

$$A > 0 \quad \text{and} \quad M^{\beta}_{\alpha\beta} + i\Sigma_{\beta} \geq 0,$$

where $M^{V_\beta}_{V_{\alpha\beta}} = V_\beta - C^T A^{-1} C$ is the Schur complement of V_α.

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- Quantification of steering

$$S^{A\to B}(V_{\alpha\beta}) = \max\{0, -\sum_{\nu_j^\beta 1} \ln\{\nu_j^\beta\}\}, \qquad (1)$$

where $\{\nu_j^\beta\}$ are the symplectic eigenvalues of $M^\beta_{\alpha\beta}$.

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THEORY OF OPEN QUANTUM SYSTEMS

The dynamics of the reduced density matrix ρ of the system interacting with an reservoir (bath) in *Markovian* approximation is described by the following master equation:

$$\dot{\rho} = \frac{\Gamma}{2}\left\{(N+1)\mathcal{L}[a] + N\mathcal{L}[a^\dagger] - M^*\mathcal{D}[a] - M\mathcal{D}[a^\dagger]\right\}\rho,$$

where Γ is overall damping rate, while $N \in R$ and $M \in C$ represent the effective photons number and the squeezing parameter of the bath respectively.
$\mathcal{L}[O]\rho = 2O\rho O^\dagger - O^\dagger O\rho - \rho O^\dagger O$ and $\mathcal{D}[O]\rho = 2O\rho O - OO\rho - \rho OO$ are the *Lindbladian superoperators*.

This is the axiomatic approach based on Completely Positive and Trace Preserving maps (that incorporates the dissipative and noisy effects due to the environment) $\rightarrow$ Noisy Channel (interaction with a squeezed thermal bath).

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[7]V. Gorini, A. Kossakowski, E. C. G. Sudarshan, J. Math. Phys. 17, 821 (1976);
G. Lindblad, Commun. Math. Phys. 48, 119 (1976)

Time evolution of the covariance matrix:

$$\gamma(t) = \exp(-\lambda t)\gamma(0) + (1 - \exp(-\lambda t))\gamma(\infty), \text{ and } \gamma(t) = \begin{pmatrix} A & C \\ C^{\mathrm{T}} & B \end{pmatrix},$$

where A, B, and C are 2×2 matrices, $\gamma(\infty)$ is the covariance matrix of the most general Gaussian reservoir:

$$\gamma(\infty) = \begin{pmatrix} \frac{1}{2}N + \Re[M] & \Im[M] & 0 & 0 \\ \Im[M] & \frac{1}{2} + N - \Re[M] & 0 & 0 \\ 0 & 0 & \frac{1}{2} + N + \Re[M] & \Im[M] \\ 0 & 0 & \Im[M] & \frac{1}{2} + N - \Re[M] \end{pmatrix}, \tag{2}$$

$$\begin{aligned} N &= n_{th}(\cosh[R]^2 + \sinh[R]^2) + \sinh[R]^2 \\ M &= -\cosh[R]\sinh[R]\exp i\varphi(2n_{th} + 1), \end{aligned} \tag{3}$$

where n_{th} is the thermal photon number of the bath, and φ is the squeezing phase. For $M = 0$ the bath is at thermal equilibrium, and N coincides with the average number of thermal photons in the bath. Otherwise, the bath is said to be âsqueezedâ, or phase sensitive.

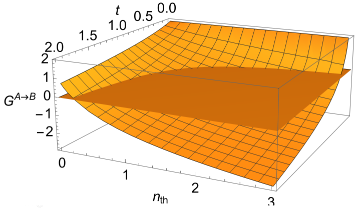

Figure 1. Evolution of quantum steering of a squeezed vacuum state with squeezing parameter $r = 1$ versus thermal photon number n_{th} of the squeezed thermal bath with squeezing parameter $R = 0.5$, dissipation coefficient $\lambda = 0.1$, and squeezing phase $\varphi = 0$.

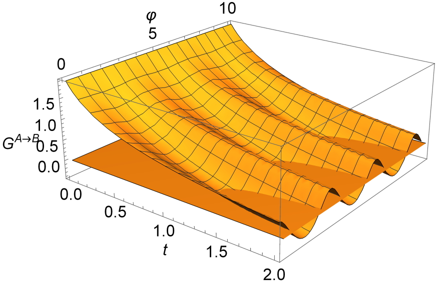

Figure 2. Evolution of quantum steering of a squeezed vacuum state $n_{th} = 0$, with squeezing parameter $r = 1$ versus phase φ of the squeezed thermal bath with squeezing parameter $R = 1$ and dissipation coefficient $\lambda = 0.1$.

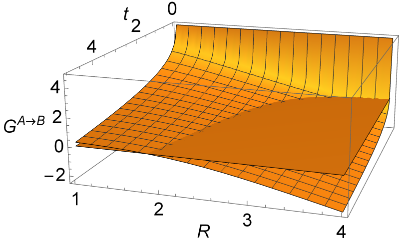

Figure 3. Evolution of quantum steering of a squeezed vacuum state with squeezing parameter $r = 2$ versus squeezing parameter R of the squeezed vacuum bath $n_{th} = 0$, dissipation coefficient $\lambda = 0.1$, and squeezing phase $\varphi = 0$.

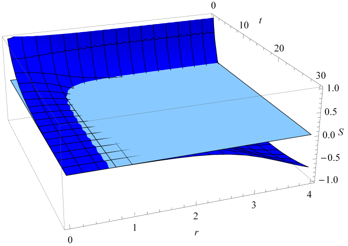

Figure 4. Gaussian quantum steering S versus time t and squeezing parameter r of a squeezed vacuum state, with temperature $\coth(\omega/2kT) = 2$, dissipation constant $\lambda = 0.1$ and $\omega = 1$.

Conclusion

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- Steerability is strictly stronger than nonseparability, but Bell nonlocality is strictly stronger than steerability.
- For a non-zero temperature of the thermal reservoir the initially steerable Gausian states become unsteerable in a finite time.