Directed Bond Percolation Process in the Presence of Velocity Fluctuations: Two-loop Approximation

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Percolation, turbulence and where to find them

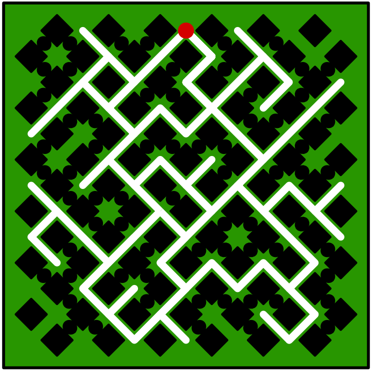
Percolation:

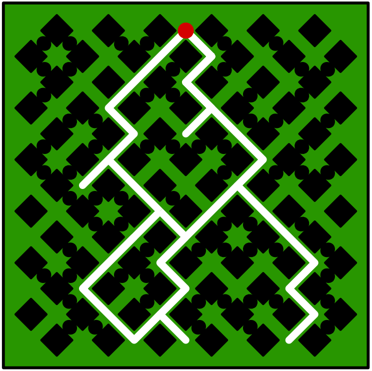
- infiltration of gas through gas masks
- flow of liquid in a porous medium
- conductor/insulator transition in composite materials
- polymer gelation, vulcanization
- from nonphysical applications: forest fires, biological evolution, epidemic process

Turbulence:

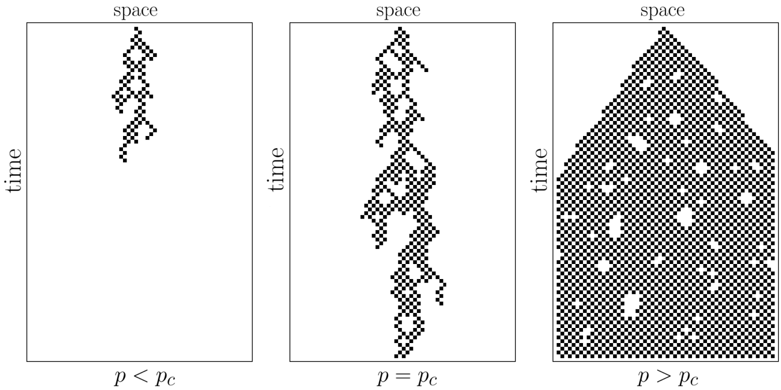
- heat fluctuation
- atmospheric turbulence

Isotropic percolation vs. directed percolation





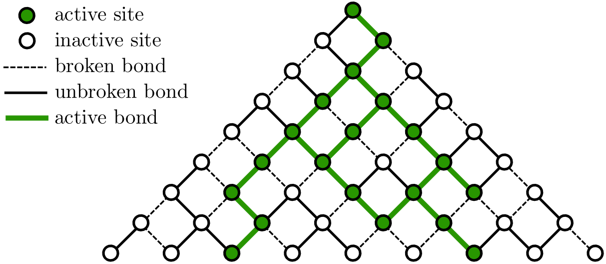
Directed bond percolation



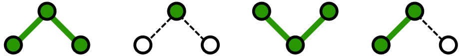
Phase transition between active and absorbing state for percolation probability $p = p_c$.

(Hinrichsen, 2000*)*

Reaction scheme for directed bond percolation



t	$N(t)$
1	1
2	1
3	1
4	2
5	3
6	3
7	4
8	3
9	2



From master equation to field-theoretical action

Master equation

$$\frac{\mathrm{d}P(\alpha,t)}{\mathrm{d}t} = \sum_{\beta} \Big[\underbrace{R_{\beta\to\alpha}P(\beta,t)}_{\text{flow into }\alpha} - \underbrace{R_{\alpha\to\beta}P(\alpha,t)}_{\text{flow out of }\alpha} \Big].$$

$P(\alpha,t)$ the probability of obtaining state α at time t, $R_{\alpha\to\beta}$ probability of transition between states

Integer changes in the values of the states $\to$ Doi approach (for example for single lattice site):

- introduce creation $a^\dagger$ and annihilation a operators with boson commutation relation $[a, a^\dagger] = 1$
- represent the state of zero particles $|0\rangle$, defined via $a|0\rangle = 0$.
- represent a state of n particles by $|n\rangle = a^{\dagger n}|0\rangle$.
- for this state

$$a^\dagger|n\rangle = |n+1\rangle, \quad a|n\rangle = n|n-1\rangle, \quad a^\dagger a|n\rangle = n|n\rangle,$$

- for multiple lattice sites: pair a_i and $a_i^\dagger$ for each lattice site i

(Doi, 1976*)*

From master equation to field-theoretical action

State vector

$$|\phi(t)\rangle = \sum_{\{n\}} P(\{n\}, t)|\{n\}\rangle$$

and rewrite the master equation in Schrödinger-like form

$$\frac{\mathrm{d}}{\mathrm{d}t}|\phi(t)\rangle = -H|\phi(t)\rangle$$

Each process contributes two terms to Hamiltonian of the form

$$(\text{rate})\Big[(\text{reactants}) - (\text{reaction})\Big]$$

(reactants) creation and annihilation operator for each reactant while normal ordered
(reaction) annihilation operator for each reactant, creation operator for each product, normal ordered

$$A \to A + A \qquad \rho[a^\dagger a - {a^\dagger}^2 a]$$

$$A + A \to A \qquad \kappa[{a^\dagger}^2 a^2 - a^\dagger a^2]$$

$$A \to \emptyset \qquad \sigma[a^\dagger a - a]$$

$$A_i + \emptyset_j \to \emptyset_i + A_j \qquad D_0[a_i^\dagger a_i - a_j^\dagger a_i]$$

Fluctuation of the velocity field

Real critical system $\to$ extremely sensitive to hardly avoidable external disturbances.
Effects of velocity field fluctuations $\to$ directed percolation process.

Kraichnan model: Galilean invariant model with velocity field $\boldsymbol{v}$ as a Gaussian variable with zero mean value and correlator in the form

$$\langle v_i(t,\boldsymbol{x})v_j(t',\boldsymbol{x}')\rangle = \delta(t-t')\int \frac{\mathrm{d}^d k}{(2\pi)^d}[P_{ij}(k)-\alpha Q_{ij}(k)]D_v(k)\mathrm{e}^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}$$

$$P_{ij}(k)=\delta_{ij}-\frac{k_ik_j}{k^2} \qquad Q_{ij}(k)=\frac{k_ik_j}{k^2} \qquad D_v(k)=g_0D_0k^{-d-\zeta}$$

$g_0>0$ is the positive parameter, the exponent $0<\zeta<2$ is a free parameter

Including the velocity field fluctuation to the model by replacing

$$\partial_t \to \partial_t + (\boldsymbol{v}\cdot\boldsymbol{\nabla}) + a_0(\boldsymbol{\nabla}\cdot\boldsymbol{v}) \quad \text{for compressible case } (\boldsymbol{\nabla}\cdot\boldsymbol{v}\neq\boldsymbol{0})$$

(N. V. Antonov, 2000)

Field-theoretical action

Action of the model: $\mathcal{S} = \mathcal{S}_{diff} + \mathcal{S}_{vel} + \mathcal{S}_{int}$

$$\mathcal{S}_{diff}(\psi,\psi^\dagger) = \int \mathrm{d}t \int \mathrm{d}^d\boldsymbol{x}\, \psi^\dagger(-\partial_t + D_0\boldsymbol{\partial}^2 - D_0\tau_0)\psi$$

$$\mathcal{S}_{int}(\psi,\psi^\dagger,v) = \int \mathrm{d}t \int \mathrm{d}^d\boldsymbol{x} \left\{ \frac{D_0\lambda_0}{2}[(\psi^\dagger)^2\psi - \psi^\dagger\psi^2] + \psi^\dagger\nabla\cdot(\boldsymbol{v}\psi) \right\}$$

$$\mathcal{S}_{vel}(v) = -\int \mathrm{d}t \int \mathrm{d}^d\boldsymbol{x} \int \mathrm{d}^d\boldsymbol{x}'\, \frac{1}{2}\boldsymbol{v}(t,\boldsymbol{x}) D_{\boldsymbol{v}}^{-1}(t-t',\boldsymbol{x}-\boldsymbol{x}')\boldsymbol{v}(t,\boldsymbol{x}')$$

Renormalization of the model

Expansion of correlation function $\rightarrow$ divergent contributions from the Feynman integrals

for $d < 4$: IR divergences ($x \rightarrow \infty$, $t \rightarrow \infty$)
for $d > 4$: UV divergences ($x \rightarrow 0$, $t \rightarrow 0$)

Elimination of the UV divergences, rescaling of fields and parameters of the model is needed
(dynamical model $\rightarrow$ time and spatial scales)

$$e_0 = e\mu^x Z_e, \quad \mathbf{\Phi} \rightarrow \mathbf{\Phi} Z_{\mathbf{\Phi}}, \; x \text{ canonical dimensions.}$$

$$e_0 = \{g_0, \lambda_0, D_0, \tau_0\}, \quad \mathbf{\Phi} = \{\psi, \psi^\dagger, \boldsymbol{v}\}$$

Replacing original parameters and field $\rightarrow$ renormalized action

$$\mathcal{S}_R(\mathbf{\Phi}_R, e).$$

Using MS scheme

Beta and gamma functions

1. Gamma functions: $\mu\partial_\mu \ln Z_e = \gamma_e$

2. Beta functions: $\mu\partial_\mu e = \beta_e$

3. Statistical physics $\to$ interested in behavior in $\boldsymbol{x} \to \infty$ and $t \to \infty$ and at the low dimensions ($d < d_c$) $\to$ associated with IR stable fixed points

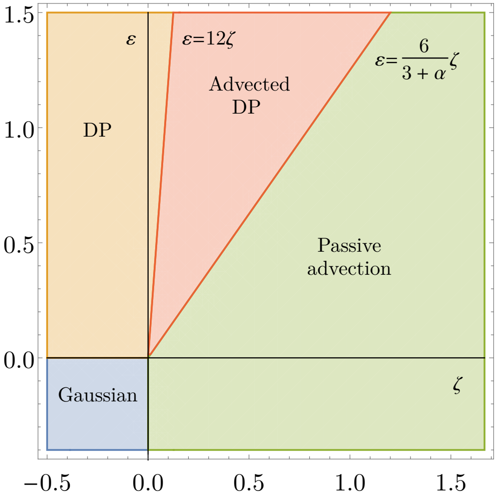
$$\beta(e^*) = 0$$

with positive eigenvalues of the matrix

$$\Omega_{ij} = \partial\beta_i/\partial e_j|_{e=e^*}$$

(Vasilev, 2004)

Phase diagrams up to 1-loop approximation



- in agreement with article *(Antonov, Kapustin, 2010)*

Conclusion and further goals

- Directed bond percolation process is studied in the presence of random velocity field generated by compressible Kraichnan model.
- Till now renormalizability of the model was shown and asymptotic behavior was studied up to 1-loop approximation within perturbation theory.
- Computing two loop diagrams is in process.
- Using velocity field modeled by Navier-Stokes equation

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Thank you for your attention!