QUASI-VECTOR MODEL OF PROPAGATION OF POLARIZED LIGHT IN A THIN-FILM WAVEGUIDE LENS

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Session: Mathematical methods and application software for modeling complex systems and engineering

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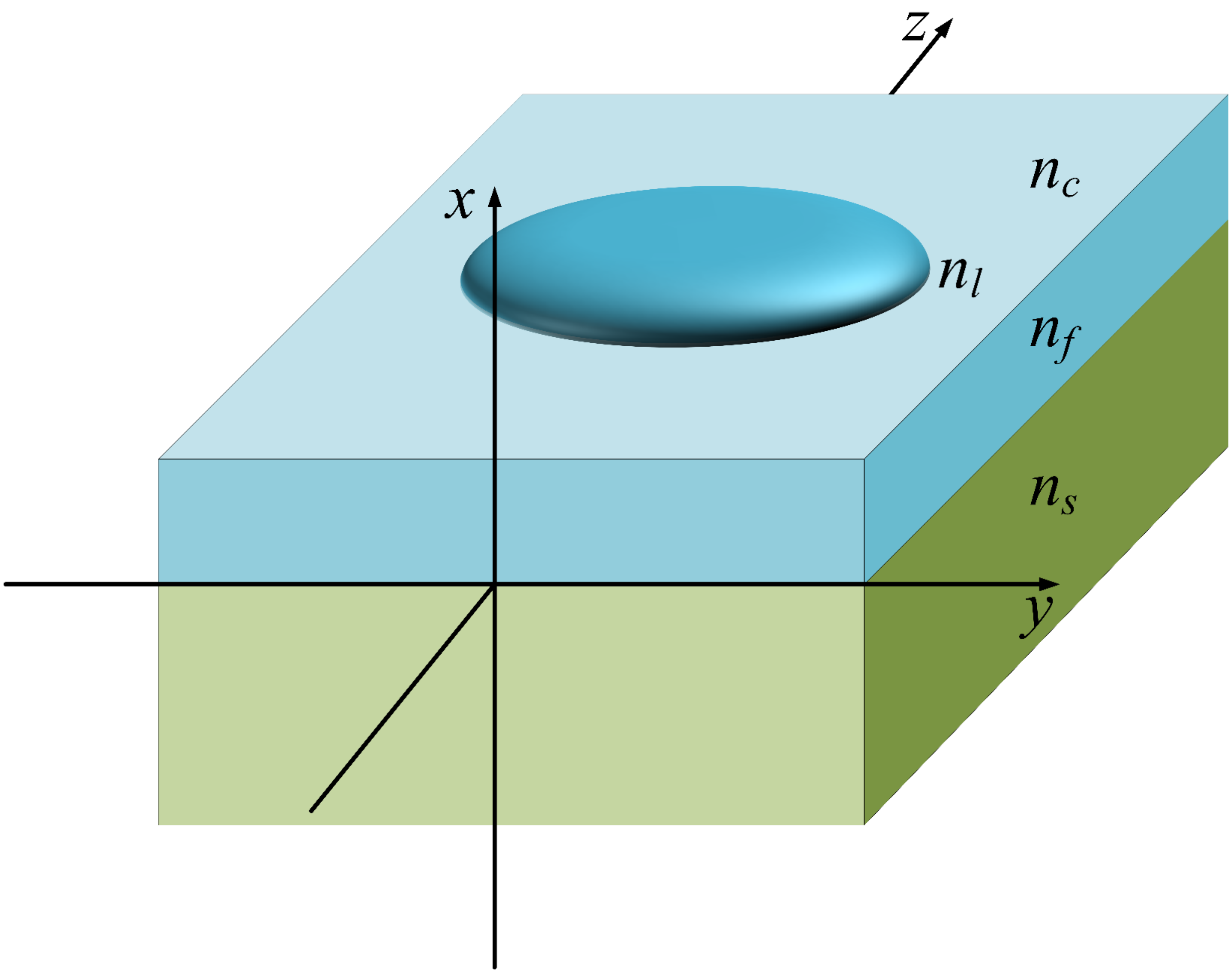
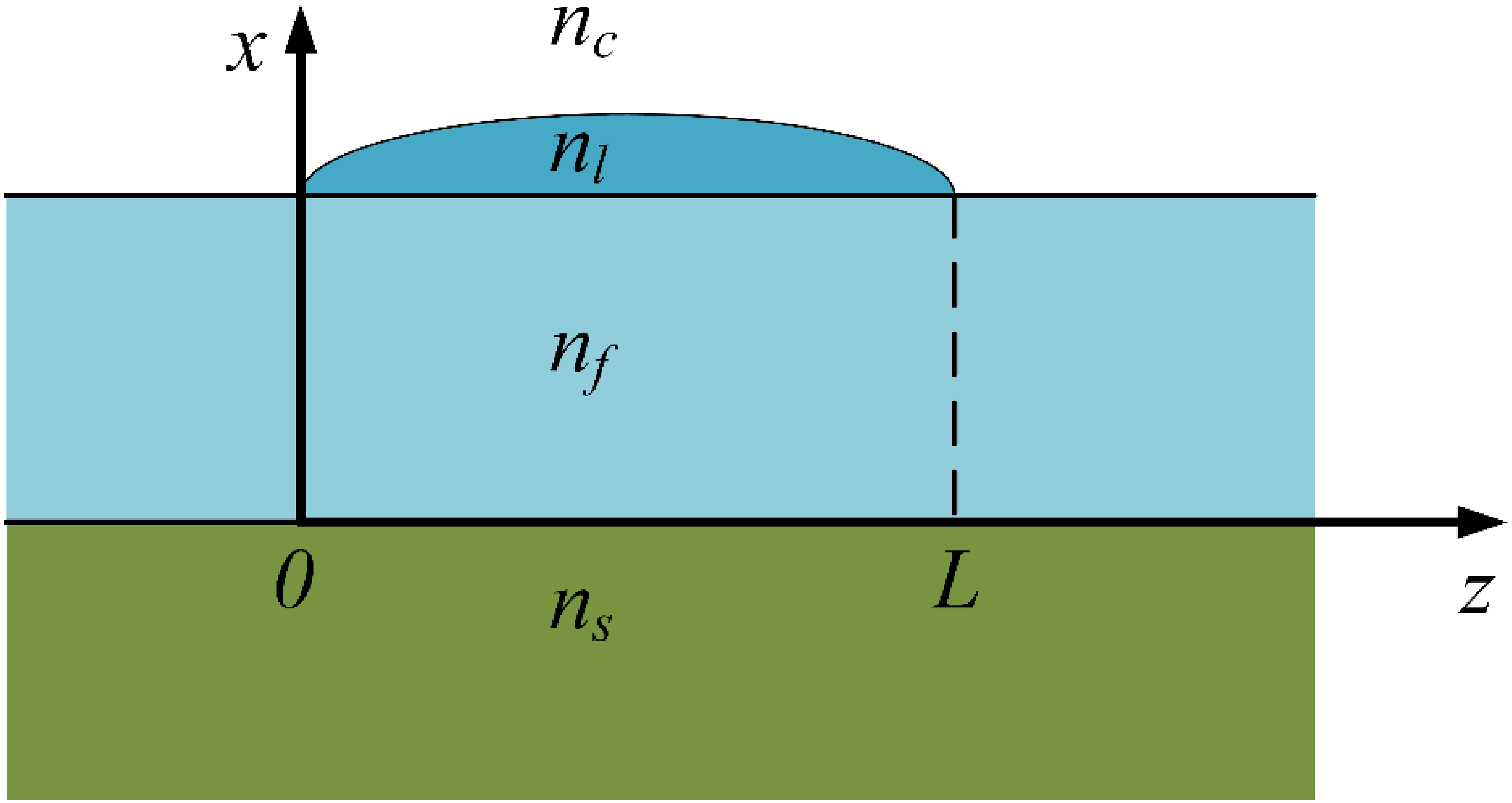
Simulation object



Simulation object



Maxwell's equations

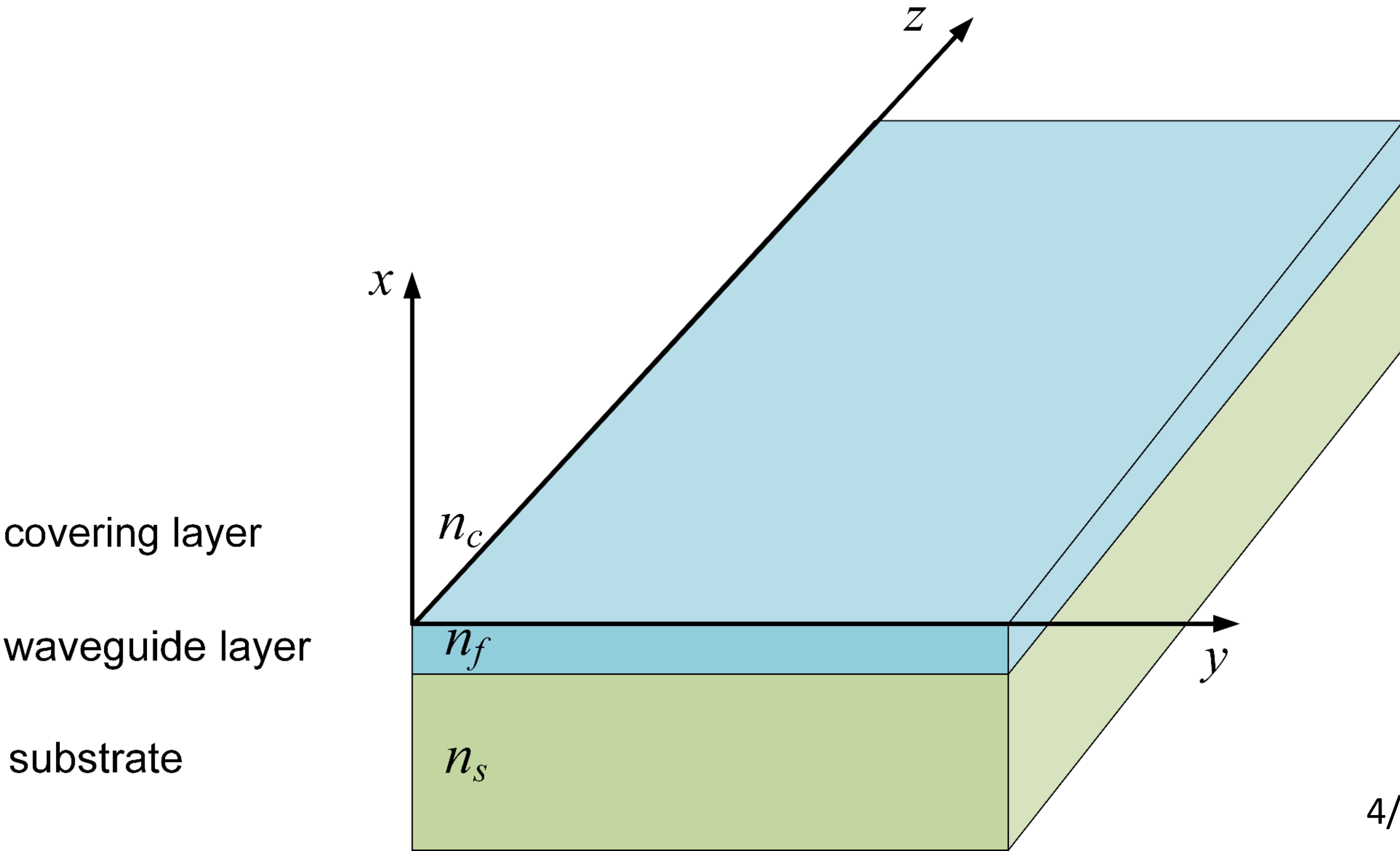
Maxwell's equations in the case of scalar waveguide diffraction problems are reduced to the Helmholtz equations:

$$\left(\Delta + k_0^2 n^2\right) E_y = 0, \ \left(\Delta + k_0^2 n^2\right) H_y = 0,$$

$$H_x = \frac{-1}{ik_0 \mu} \frac{\partial E_y}{\partial z}, \qquad E_x = \frac{1}{ik_0 \varepsilon} \frac{\partial H_y}{\partial z},$$

$$H_z = \frac{1}{ik_0 \mu} \frac{\partial E_y}{\partial x}, \qquad E_z = \frac{-1}{ik_0 \varepsilon} \frac{\partial H_y}{\partial x}.$$

Three-layer waveguide

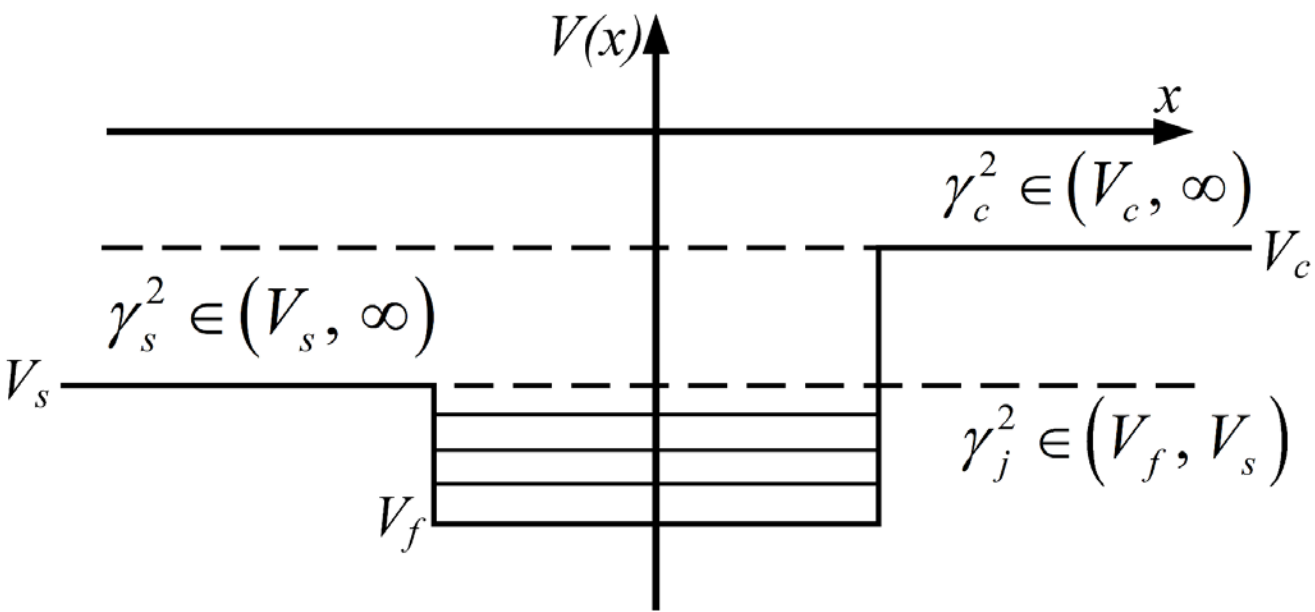


The Sturm-Liouville problem

The Sturm-Liouville problem for operator

$$-\frac{d^2\psi}{dx^2} + V(x)\psi = \gamma^2\psi$$

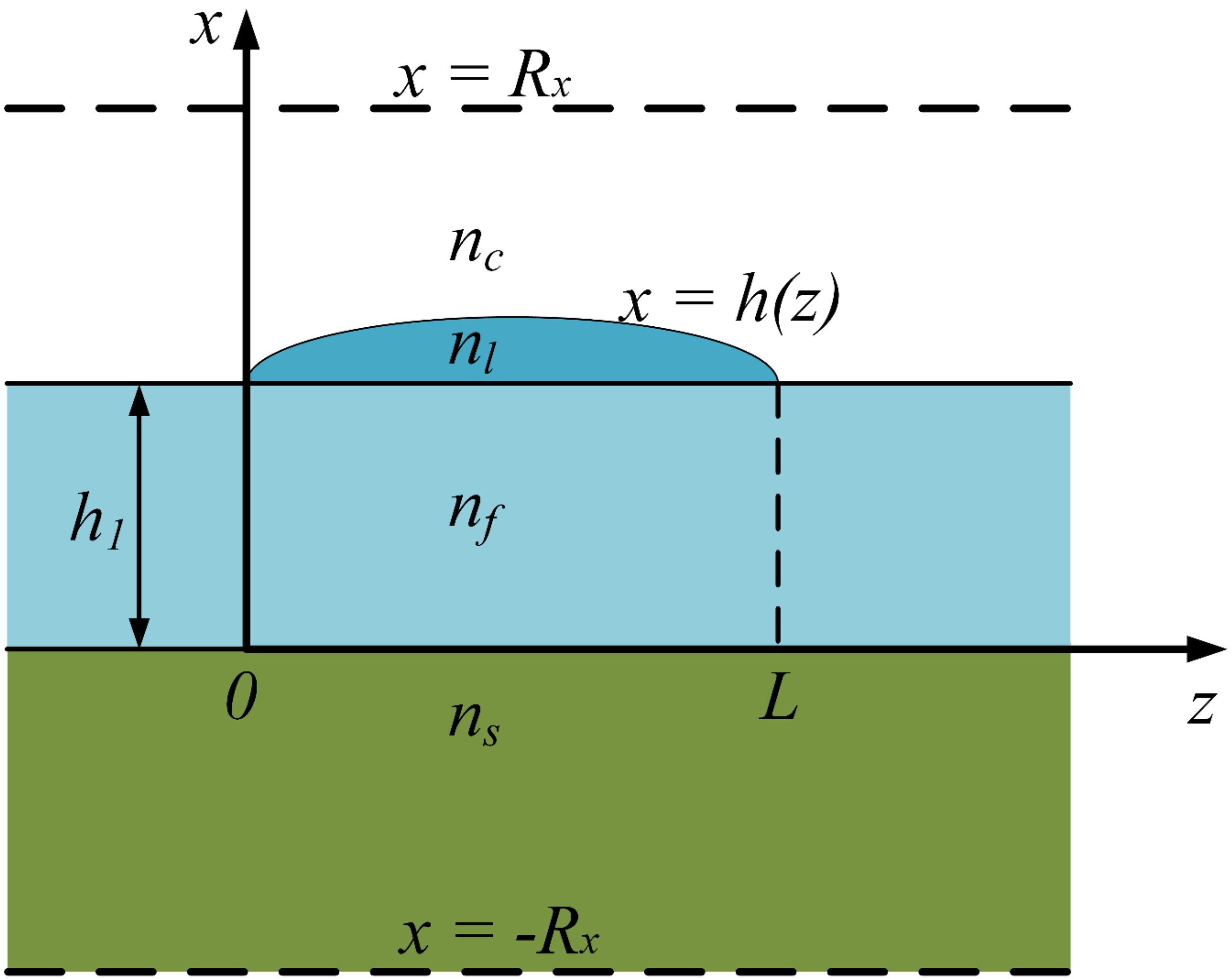
has a mixed spectrum



$$V(x) = -k_0^2 n^2(x)$$

$$\gamma^2 = -k_0^2 \beta^2$$

Diffraction problem in a "Dirichlet's box"



Diffraction problem in a "Dirichlet's box"

$$\begin{cases} \Delta u + k_0^2 n^2(x,z) u = 0, \\ u\big|_{x=-R_x} = u\big|_{x=R_x} = 0, \\ u\big|_{z\le 0} = A_{n_0}\varphi_{n_0}(x) e^{ik_0\beta_{n_0} z} + \sum_{n=1}^{\infty} R_n \varphi_n(x) e^{-ik_0\beta_n z}, \\ u\big|_{z\ge L} = \sum_{n=1}^{\infty} T_n \varphi_n(x) e^{ik_0\beta_n z}, \end{cases}$$

where φ_n are normalized eigenfunctions of the problem for a regular waveguide:

$$\begin{cases} \varphi'' + k_0^2 n_0^2(x)\varphi = k_0^2 \beta^2 \varphi, \\ \varphi\big|_{-R_x} = \varphi\big|_{R_x} = 0. \end{cases}$$

Analogue of incomplete Galerkin's method

We seek an approximate solution of the problem in the form of an expansion:

$$u^N(x,z) = \sum_{n=1}^{N} V_n(z)\varphi_n(x),$$

Applying the projection scheme of the Galerkin method, we obtain:

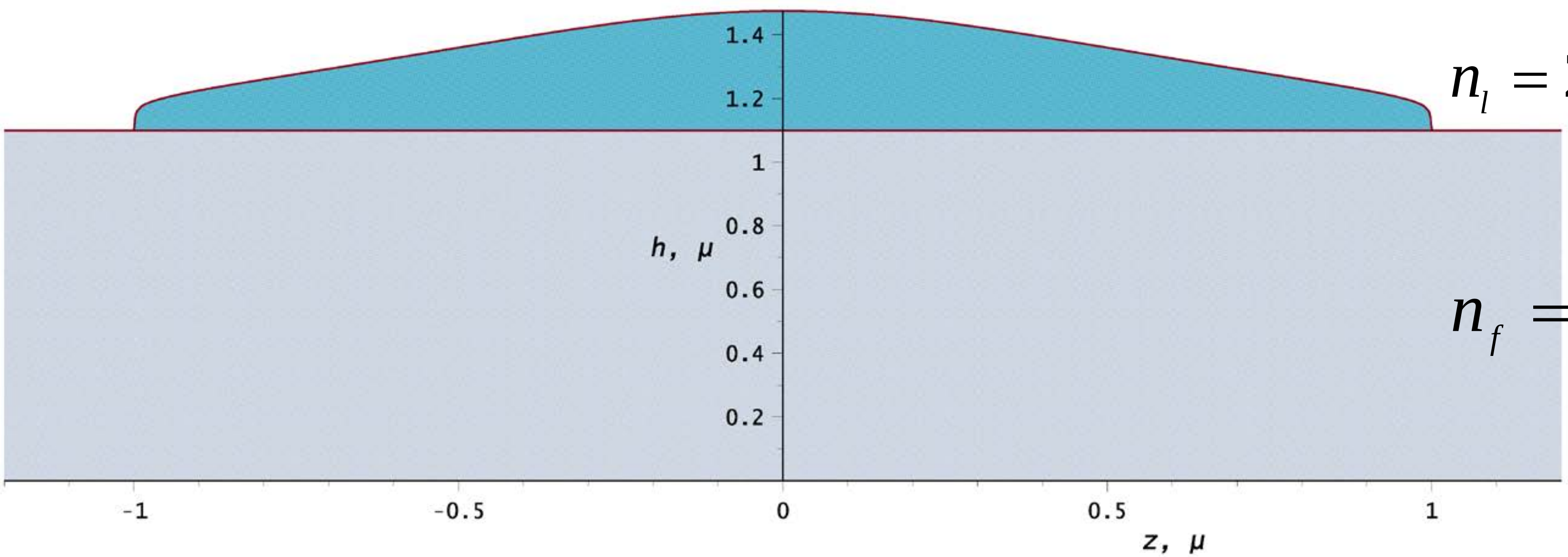
$$\begin{cases} \vec{v}'' + \mathbf{Q}(z)\vec{v} = \vec{0} \\ \vec{v}'(0) + ik_0\mathbf{D}\vec{v}(0) = 2ik_0\mathbf{D}\vec{a}_{n_0} \\ \vec{v}'(L) - ik_0\mathbf{D}\vec{v}(L) = \vec{0} \end{cases},$$

$$\vec{v} = \left(V_1(z), V_2(z), \ \ldots \ V_N(z)\right)^T.$$

Numerical experiment

$n_c = 1.000$

$n_l = 2.100$

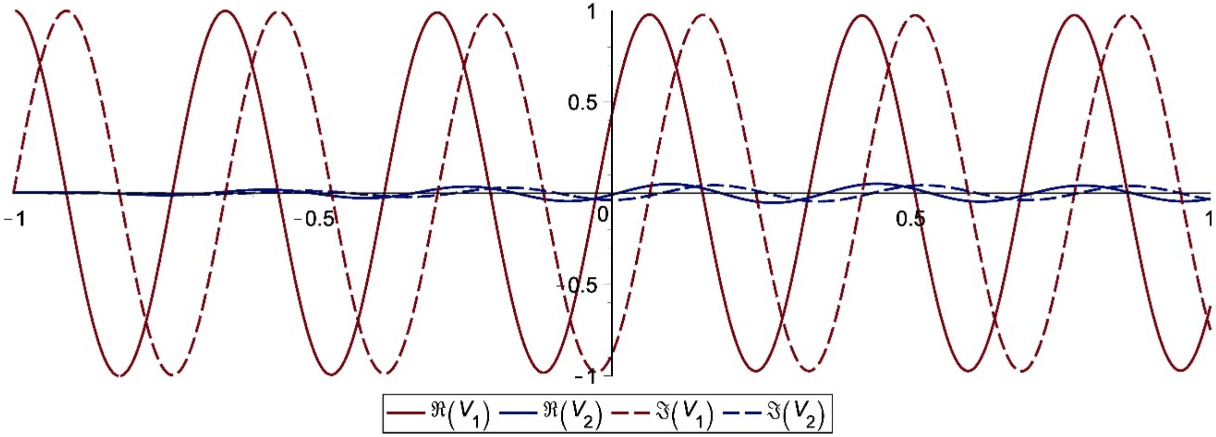


$n_f = 1.565$

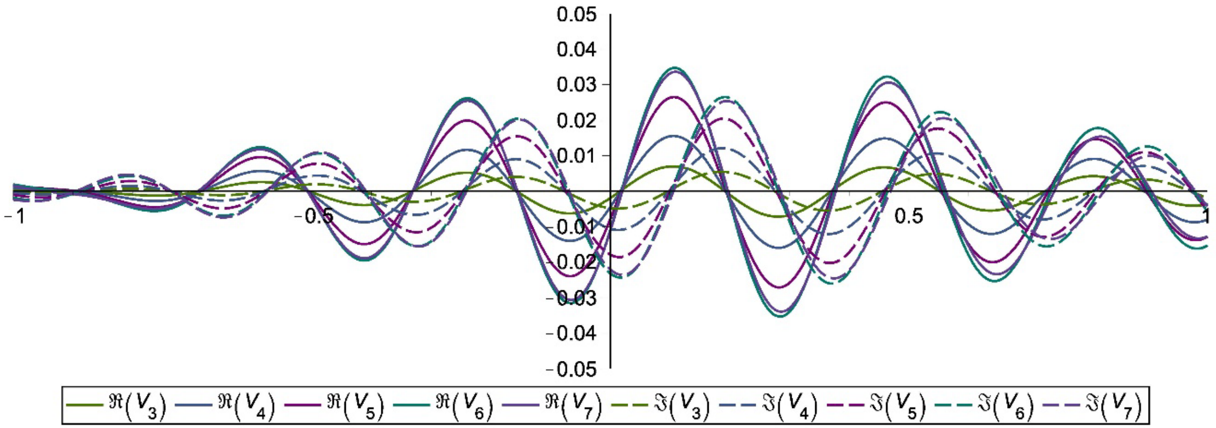
$n_s = 1.470$

Longitudinal section of a waveguide lens placed on a waveguide layer. The lens is designed to focus mode $TE_0(A_0 = 1)$, $\lambda = 0.633\mu$.

Numerical experiment

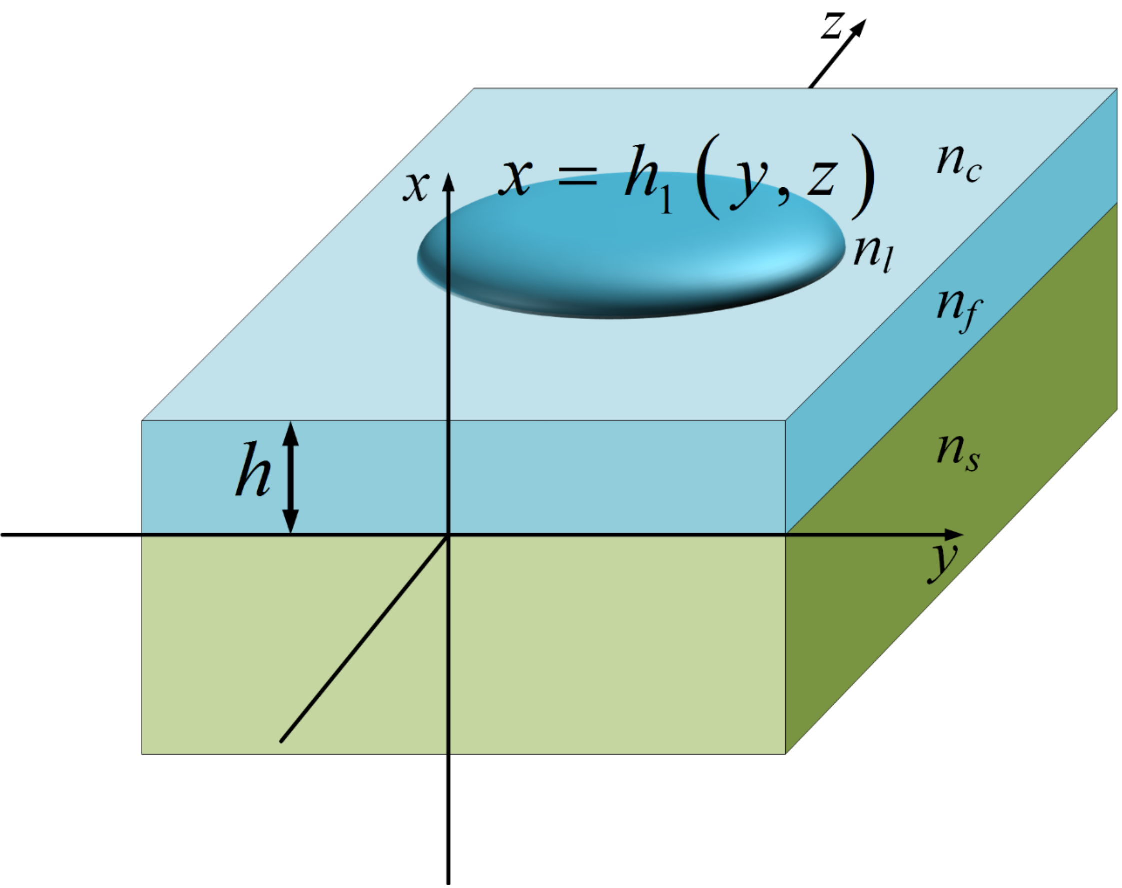


Coefficient functions that determine guided modes of an open waveguide



Coefficient functions that determine substrate radiation modes of an open waveguide

Luneburg lens



$$n(x,y,z) = \begin{cases} n_c, & x > h_1(y,z) \\ n_l, & h < x < h_1(y,z) \\ n_f, & 0 < x < h \\ n_s, & x < 0 \end{cases}$$

Small parameter

We assume a small change of the electromagnetic field along y-axis, which is characterized by small parameter δ:

$$\delta = \max\left\{\frac{\partial E_x}{\partial y}, \frac{\partial E_y}{\partial y}, \dots\ , \frac{\partial H_z}{\partial y}\right\}$$

In the zeroth approximation with respect to the small parameter δ, we obtain the Helmholtz equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k_0^2 n^2\left(x, y, z\right)\right) E_y^{(0)} = 0$$

Diffraction problem in the zeroth approximation

$$\begin{cases} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k_0^2 n^2(x,y,z) \right) E_y^{(0)} = 0, \\ E_y^{(0)}\Big|_{x=\pm R_x} = 0, \\ E_y^{(0)}\Big|_{z\le -R} = A_{n_0}(y)\varphi_{n_0}(x)e^{ik_0\beta_{n_0}z} + \sum_{n=1}^{\infty} R_n(y)\varphi_n(x)e^{-ik_0\beta_n z}, \\ E_y^{(0)}\Big|_{z\ge R} = \sum_{n=1}^{\infty} T_n(y)\varphi_n(x)e^{ik_0\beta_n z}, \end{cases}$$

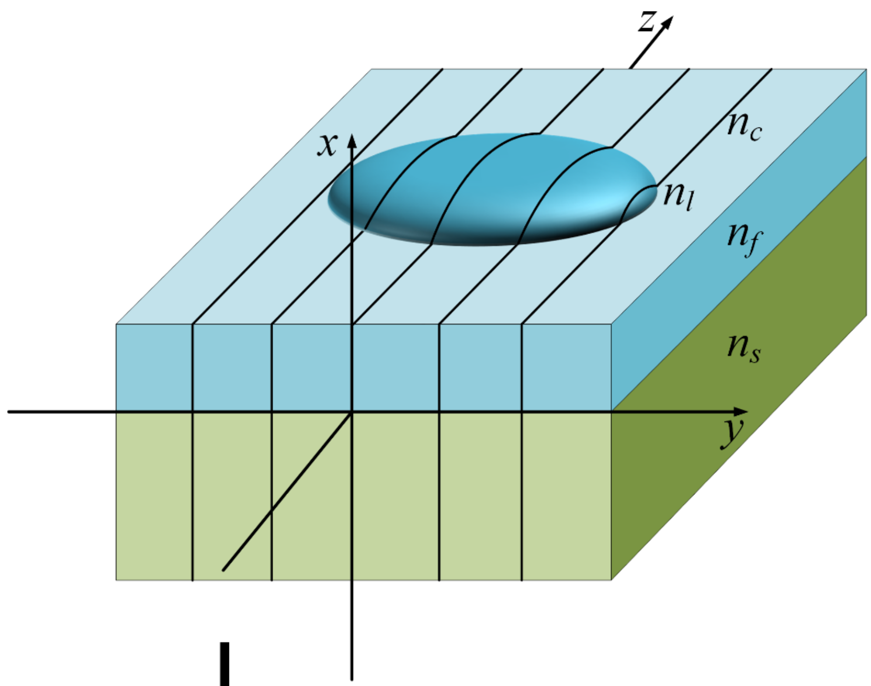
where R is the radius of lens, $E_y^{(0)} = E_y^{(0)}(x, z; y)$ depends on y parametrically.

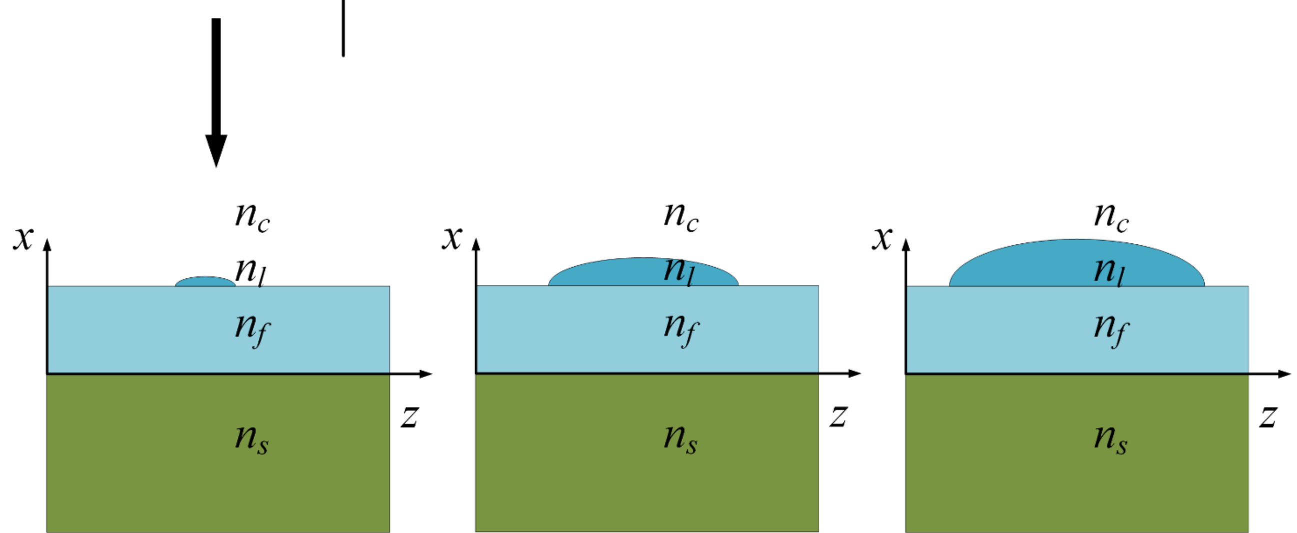
Solution of the diffraction problem

For each fixed value of y_j, we obtain a two-dimensional diffraction problem

$$
\begin{cases}
\left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial z^2}+k_0^2 n^2\left(x,y_j,z\right)\right)u_j^{(0)}=0,\\
u_j^{(0)}\Big|_{x=\pm R_x}=0,\\
u_j^{(0)}\Big|_{z\le -R}=A_{n_0}\left(y_j\right)\varphi_{n_0}\left(x\right)e^{ik_0\beta_{n_0}z}+\sum\limits_{n=1}^{\infty}R_n\left(y_j\right)\varphi_n\left(x\right)e^{-ik_0\beta_n z},\\
u_j^{(0)}\Big|_{z\ge R}=\sum\limits_{n=1}^{\infty}T_n\left(y_j\right)\varphi_n\left(x\right)e^{ik_0\beta_n z},
\end{cases}
$$

Solution of the diffraction problem





Solution of the diffraction problem