Nonlinear spinor field in non-diagonal Bianchi type space-time

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Corresponding literature

Bijan Saha *Some remarks on Bianchi type-II, VIII and IX models* **Grav. & Cosmology 19(1), 65 - 69 (2013)**

Bijan Saha *Nonlinear Spinor Fields in Bianchi type-I space-time reexamined* **Int. J. Theor. Phys. 53, 1109 (2014)**

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Bijan Saha *Nonlinear Spinor Fields in Bianchi type-III space-time* **Int. J. Theor. Phys. 55, 2259 (2016)**

Corresponding literature

Bijan Saha *Spinor Field with Polynomial Nonlinearity in LRS Bianchi type-I space-time* **Canadian J. Phys. 93, 1 (2015)**

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Bijan Saha *Spinor field in Bianchi type-IX space-time* **ArXiv: 1705.07773 [gr-qc] (2017)**

Introduction

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(iii) Light solitons in waveguide arrays and experimental realization of an optical analog for relativistic quantum mechanics;
(iv) Bose-Einstein condensate in honeycomb optical lattices;
(v) Phenomenological models in quantum chromodynamics;
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(vii) Chiral models of Skyrme and Faddeev;
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(i) eliminate the problem of initial singularity giving rise to a regular solution;(ii) accelerate the isotropization process of an initially anisotropic space-time;(iii) generate late time acceleration of the expansion of the Universe.

Moreover, thanks to its flexibility nonlinear spinor field can simulate the different characteristics of matter from perfect fluid to dark energy and describe the different stages of the evolution of the Universe.

Some recent study suggests that flexible though it is, the presence of non-diagonal components of the energy-momentum tensor of the spinor field together with Fierz identity impose very severe restrictions on the geometry of the Universe as well as on the spinor field, thus justifying our previous claim that spinor field is very sensitive to the gravitational one

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But those studied were performed within the scope of diagonal metrics. The purpose of this report is to extend that study to non-diagonal cases and clarify the role of spinor field in the evolution of the Universe.

Basic Equations

The action we choose in the form

$$\mathcal{S}(g;\psi,\bar\psi) = \int \left(L_{\rm g} + L_{\rm sp}\right)\sqrt{-g}d\Omega \tag{1}$$

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$$\mathcal{S}(g; \psi, \bar{\psi}) = \int (L_{\rm g} + L_{\rm sp}) \sqrt{-g} d\Omega \qquad (1)$$

Here $L_{\rm g}$ corresponds to the gravitational field

$$L_{\rm g} = \frac{R}{2\kappa}, \qquad (2)$$

where R is the scalar curvature, $\kappa = 8\pi G$ with G being Einstein's gravitational constant and $L_{\rm sp}$ is the spinor field Lagrangian.

Basic Equations

We consider the spinor field Lagrangian given by

$$L_{\rm sp} = \frac{\imath}{2}\left[\bar\psi\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma^\mu\psi\right] - m_{\rm sp}\bar\psi\psi - F. \qquad (3)$$

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To maintain the Lorentz invariance of the spinor field equations the self-interaction (nonlinear term) F is constructed as some arbitrary functions of invariants generated from the real bilinear forms.

Thanks to Fierz identity, without losing generality we can choose $F = F(K)$, with K taking one of the following expressions $\{I,\ J,\ I+J,\ I-J\}$. It describes most general form of spinor field nonlinearity.
$I = S^2 = (\bar\psi\psi)^2, \quad J = P^2 = (\imath\bar\psi\gamma^5\psi)^2$

Basic equations

The energy momentum tensor (EMT) of the spinor field is given by

$$
\begin{aligned}
T_{\mu}{}^{\rho} &= \frac{\imath g^{\rho\nu}}{4}\left(\bar\psi\gamma_\mu\nabla_\nu\psi + \bar\psi\gamma_\nu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma_\nu\psi - \nabla_\nu\bar\psi\gamma_\mu\psi\right) - \delta_\mu^\rho L_{\rm sp}\\
&= \frac{\imath}{4}g^{\rho\nu}\left(\bar\psi\gamma_\mu\partial_\nu\psi + \bar\psi\gamma_\nu\partial_\mu\psi - \partial_\mu\bar\psi\gamma_\nu\psi - \partial_\nu\bar\psi\gamma_\mu\psi\right)\\
&- \frac{\imath}{4}g^{\rho\nu}\bar\psi\left(\gamma_\mu\Gamma_\nu + \Gamma_\nu\gamma_\mu + \gamma_\nu\Gamma_\mu + \Gamma_\mu\gamma_\nu\right)\psi\\
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&- \frac{\imath}{4}g^{\rho\nu}\bar\psi\left(\gamma_\mu\Gamma_\nu + \Gamma_\nu\gamma_\mu + \gamma_\nu\Gamma_\mu + \Gamma_\mu\gamma_\nu\right)\psi\\
&- \delta_\mu^\rho\left(2KF' - F\right).
\end{aligned} \tag{4}
$$

where $F' = F_K, \quad \nabla_\nu\psi = \partial_\nu\psi - \Gamma_\nu\psi, \quad \nabla_\nu\bar\psi = \partial_\nu\bar\psi + \bar\psi\Gamma_\nu$. The spinor affine connection is defined as

$$
\Gamma_\mu = \frac{1}{4}\bar\gamma_a\gamma^\nu\partial_\mu e_\nu^{(a)} - \frac{1}{4}\gamma_\rho\gamma^\nu\Gamma_{\mu\nu}^{\rho}. \tag{5}
$$

Basic equations

As one sees, the energy momentum tensor of spinor field depends on the spinor-affine connection, which is closely related to the metric itself. As a result, depending on the type and form of the metric we have different set of components.

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It was found that in case of a FRW metric EMT of spinor field is diagonal, while in other cases there occurs not-trivial non-diagonal elements.

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It was found that in case of a FRW metric EMT of spinor field is diagonal, while in other cases there occurs not-trivial non-diagonal elements.

It should be noted that while $T_{\mu\nu} = T_{\nu\mu}$ and $T^{\mu\nu} = T^{\nu\mu}$, for the mixed tensor generally $T_\nu^\mu \neq T_\mu^\nu$.

The non-diagonal Bianchi spacetime is given by

$$ds^2 = dt^2 - a_1^2(t)dx_1^2 - [h^2(x_3)a_1^2(t) + f^2(x_3)a_2^2(t)]dx_2^2 - a_3^2 dx_3^2 + 2a_1^2(t)h(x_3)dx_1 dx_2, \quad (6)$$

$a_1(t)$, $a_2(t)$, $a_3(t)$ - functions of time and $f(x_3)$ and $h(x_3)$ - some some functions of x_3. Depending on the value of δ

$$\delta = -\frac{f''}{f}, \quad (7)$$

we have different cosmological models. Namely, $\delta = 0$ corresponds to Bianchi type-II model; $\delta = -1$ describes Bianchi type-$VIII$ model and finally $\delta = 1$ gives rise to Bianchi type-IX model.

Basic Equations

Spinor affine connections corresponding to metric (6) are

$$\Gamma_1 = \frac{1}{2}\dot{a}_1 \bar{\gamma}^1 \bar{\gamma}^0 - \frac{1}{4}\frac{a_1^2 h'}{a_2 a_3 f}\bar{\gamma}^2 \bar{\gamma}^3, \tag{8a}$$

$$\Gamma_2 = \frac{1}{2} f \dot{a}_2 \bar{\gamma}^2 \bar{\gamma}^0 - \frac{1}{2} h \dot{a}_1 \bar{\gamma}^1 \bar{\gamma}^0 - \frac{1}{4}\frac{a_1 h'}{a_3}\bar{\gamma}^1 \bar{\gamma}^3 + \frac{1}{2}\frac{a_2 f'}{a_3}\bar{\gamma}^2 \bar{\gamma}^3 + \frac{1}{4}\frac{a_1^2 h h'}{a_2 a_3 f}\bar{\gamma}^2 \bar{\gamma}^3, \tag{8b}$$

$$\Gamma_3 = \frac{1}{2}\dot{a}_3 \bar{\gamma}^3 \bar{\gamma}^0 + \frac{1}{4}\frac{a_1 h'}{a_2 f}\bar{\gamma}^1 \bar{\gamma}^2, \tag{8c}$$

$$\Gamma_0 = 0. \tag{8d}$$

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$$\Gamma_0 = 0. \tag{8d}$$

We consider $\psi = \psi(t)$. On account of (8) one finds the nontrivial components of the EMT.

It can be shown that Einstein tensor and energy-momentum tensor of spinor field in in case of the metric given by (6) beside nontrivial diagonal elements contains nonzero non-diagonal components as well. Moreover, unlike the diagonal Bianchi metrics in this case the components of energy-momentum tensor along the principal diagonal are not equal, i.e.

$$T_1^1 \neq T_2^2 \neq T_3^3, \tag{9}$$

even in absence of spinor field nonlinearity. This very fact plays crucial role in the evolution of non-diagonal Bianchi universes.

From the Einstein and energy-momentum tensors one finds that

$$G_2^1 = h\left(G_2^2 - G_1^1\right) + \left(h^2 + \frac{a_2^2 f^2}{a_1^2}\right) G_1^2. \qquad (10)$$

$$T_2^1 = h\left(T_2^2 - T_1^1\right) + \left(h^2 + \frac{a_2^2 f^2}{a_1^2}\right) T_1^2. \qquad (11)$$

$$T_2^3 = \frac{a_2^2 f^2}{a_3^2} T_3^2 - h T_1^3, \qquad (12)$$

$$T_3^1 = h T_3^2 + \frac{a_3^2}{a_1^2} T_1^3, \qquad (13)$$

$$T_0^1 = h T_0^2 - \frac{1}{a_1^2} T_1^0, \qquad (14)$$

$$T_2^0 = -a_2^2 f^2 T_0^2 - h T_1^0. \qquad (15)$$

On account of these relations we write the system of Einstein equations

$$G_\mu^\nu = -\kappa T_\mu^\nu, \qquad (16)$$

having the following linearly independent components

$$\left(\frac{\ddot a_2}{a_2} + \frac{\ddot a_3}{a_3} + \frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3}\right) - \frac{1}{2}\frac{a_1^2 h^2}{a_2^2 a_3^2 f^2}\left(\frac{h''}{h} - \frac{h'}{h}\frac{f'}{f} + \frac{3}{2}\frac{h'^2}{h^2}\right)$$
$$-\frac{1}{a_3^2}\frac{f''}{f} = \kappa\left[(F(K) - 2KF_K) + \frac{1}{4}\frac{a_1 h}{a_2 f}\left(\frac{\dot a_2}{a_2} - \frac{\dot a_1}{a_1}\right)A^3\right.$$
$$\left. + \frac{1}{4}\frac{a_1 h}{a_2 a_3 f}\left(\frac{f'}{f} - \frac{h'}{h}\right)A^0\right], \quad (1,1) \qquad (17)$$

Basic Equations

$$\left(\frac{\ddot{a}_3}{a_3}+\frac{\ddot{a}_1}{a_1}+\frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1}\right)+\frac{1}{2}\frac{a_1^2h^2}{a_2^2a_3^2f^2}\left(\frac{h''}{h}-\frac{h'}{h}\frac{f'}{f}+\frac{1}{2}\frac{h'^2}{h^2}\right)$$
$$=\kappa\left[(F(K)-2KF_K)-\frac{1}{4}\frac{a_1h}{a_2f}\left(\frac{\dot{a}_2}{a_2}-\frac{\dot{a}_1}{a_1}\right)A^3\right.$$
$$\left.-\frac{1}{4}\frac{a_1h}{a_2a_3f}\left(\frac{f'}{f}-\frac{h'}{h}\right)A^0\right],\quad (2,2) \tag{18}$$

$$\left(\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2}\right)+\frac{1}{4}\frac{a_1^2h'^2}{a_2^2a_3^2f^2}$$
$$=\kappa\left[(F(K)-2KF_K)+\frac{1}{4}\frac{a_1h'}{a_2a_3f}A^0\right],\quad (3,3) \tag{19}$$

Basic Equations

$$\left(\frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2} + \frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3} + \frac{\dot a_3}{a_3}\frac{\dot a_1}{a_1}\right) - \frac{1}{4}\frac{a_1^2 h'^2}{a_2^2 a_3^2 f^2} - \frac{1}{a_3^2}\frac{f''}{f}$$

$$= \kappa\left[m_{\rm sp} S + F(K) - \frac{1}{4}\frac{a_1 h'}{a_2 a_3 f} A^0\right], \quad (0,0) \qquad (20)$$

$$\frac{1}{2}\frac{a_1^2 h}{a_2^2 a_3^2 f^2}\left(\frac{h''}{h} - \frac{h'}{h}\frac{f'}{f}\right)$$

$$= -\kappa\frac{1}{4}\frac{1}{a_2 f}\left[\left(\frac{\dot a_2}{a_2} - \frac{\dot a_1}{a_1}\right) A^3 + \frac{a_1}{a_3}\frac{f'}{f} A^0\right], \quad (2,1) \qquad (21)$$

Basic Equations

$$\left(\frac{\dot a_2}{a_2} - \frac{\dot a_3}{a_3}\right)\frac{f'}{f} = 0, \quad (0,3) \tag{22}$$

$$0 = \left(\frac{\dot a_2}{a_2} - \frac{\dot a_3}{a_3}\right)A^1, \quad (2,3) \tag{23}$$

$$0 = \left(\frac{\dot a_1}{a_1} - \frac{\dot a_3}{a_3}\right)A^2, \quad (3,1) \tag{24}$$

$$0 = \left[\frac{a_1 h'}{a_2 f}A^1 + f'A^2\right], \quad (0,1) \tag{25}$$

$$0 = \left(1 + \frac{a_1}{a_2 f}\right)\frac{f'}{f}A^1. \quad (2,0) \tag{26}$$

Basic Equations

In view of $\frac{f'}{f} \neq 0$ from (22) we find $\left(\frac{\dot a_2}{a_2} - \frac{\dot a_3}{a_3}\right) = 0$, which means the space rotationally symmetric.

On the other hand for same reason (26) yields $A^1 = 0$, on account of which from (25) we obtain $A^2 = 0$. Thus in this case from (22) - (26) we have

$$A^1 = 0, \quad A^2 = 0, \quad \left(\frac{\dot a_2}{a_2} - \frac{\dot a_3}{a_3}\right) = 0. \qquad (27)$$

Basic Equations

In view of $A^2 = 0$ the equation (24) yields two possibilities:

$$\left(\frac{\dot a_1}{a_1} - \frac{\dot a_3}{a_3}\right) \neq 0, \tag{28}$$

which means the model is rotationally symmetric, or

$$\left(\frac{\dot a_1}{a_1} - \frac{\dot a_3}{a_3}\right) = 0, \tag{29}$$

which means the model is isotropic.

Basic Equations

To solve the Einstein equation we still need some additional conditions. One of the conditions extensively used in literature is proportionality conditions which connects the share with expansion. Assuming the proportionality condition $\sigma_1^1 = q_1 \vartheta, \quad q_1 = \text{const.}$ Then for the metric functions we find

$$a_i = X_i V^{Y_i}, \quad \prod_{i=1}^{3} X_i = 1, \quad \sum_{i=1}^{3} Y_i = 0. \tag{30}$$

In this concrete case we have $X_1 = q_2, \ X_2 = \sqrt{q_3/q_2}$, $X_3 = 1/\sqrt{q_2 q_3}$, $Y_1 = q_1 + 1/3, \ Y_2 = Y_3 = 1/3 - q_1/2, \quad q_2, \ q_3 - \text{const.}$ Here we define volume scale

$$V = a_1 a_2 a_3. \tag{31}$$

Basic Equations

Exploiting the diagonal Einstein Eqs. let us now find the Eq. for V.

$$\frac{\ddot V}{V} - \frac{1}{2}\frac{a_1^2}{a_2^2 a_3^2}\left(\frac{h'}{f}\right)^2 - \frac{2}{a_3^2}\frac{f''}{f} = \frac{3\kappa}{2}\left[m_{\rm sp} S + 2\left(F - K F_K\right) - \frac{1}{2}\frac{a_1 h'}{a_2 a_3 f} A^0\right], \qquad (32)$$

which on account of (30) can be rewritten as

$$\frac{\ddot V}{V} - \frac{1}{2} q_2^4 V^{4q_1 - 2/3}\left(\frac{h'}{f}\right)^2 - 2 q_2 q_3 V^{q_1 - 2/3}\frac{f''}{f} = \frac{3\kappa}{2}\left[m_{\rm sp} S + 2\left(F - K F_K\right) - \frac{1}{2} q_2^2 V^{2q_1 - 1/3} A^0\right], \qquad (33)$$

Basic Equations

The spinor field equations in this case take the form

$$
\begin{aligned}
\imath\bar{\gamma}^0\dot{\psi} &+ \frac{\imath}{2}\frac{\dot{V}}{V}\bar{\gamma}^0\psi + \frac{1}{4}\frac{a_1 h'}{a_2 a_3 f}\bar{\gamma}^5\bar{\gamma}^0\psi + \frac{\imath}{2}\frac{f'}{a_3 f}\bar{\gamma}^3\psi \\
&- \left[m_{\rm sp} + \mathcal{D}\right]\psi - \imath\mathcal{G}\bar{\gamma}^5\psi = 0, \qquad (34a)\\
\imath\dot{\bar{\psi}}\bar{\gamma}^0 &+ \frac{\imath}{2}\frac{\dot{V}}{V}\bar{\psi}\bar{\gamma}^0 - \frac{1}{4}\frac{a_1 h'}{a_2 a_3 f}\bar{\psi}\bar{\gamma}^5\bar{\gamma}^0 + \frac{\imath}{2}\frac{f'}{a_3 f}\bar{\psi}\bar{\gamma}^3 \\
&+ \left[m_{\rm sp} + \mathcal{D}\right]\bar{\psi} + \imath\mathcal{G}\bar{\psi}\bar{\gamma}^5 = 0. \qquad (34b)
\end{aligned}
$$

From (34) it can be shown that the spinor field invariants in this case obey the following equations:

$$\dot{S}_0 + \frac{1}{2}\frac{a_1 h'}{a_2 a_3 f} P_0 + 2\mathcal{G} A_0^0 = 0, \quad (35a)$$

$$\dot{P}_0 + \frac{1}{2}\frac{a_1 h'}{a_2 a_3 f} S_0 - 2\left[m_{\rm sp} + \mathcal{D}\right] A_0^0 = 0, \quad (35b)$$

$$\dot{A}_0^0 + \frac{1}{2}\frac{f'}{a_3 f} A_0^3 + 2\left[m_{\rm sp} + \mathcal{D}\right] P_0 + 2\mathcal{G} S_0 = 0, \quad (35c)$$

$$\dot{A}_0^3 + \frac{1}{2}\frac{f'}{a_3 f} A_0^0 = 0, \quad (35d)$$

where $\mathcal{D} = 2SF_K K_I$, $\mathcal{G} = 2PF_K K_J$. From (35) it can be easily shown that

Basic Equations

$$P_0^2 - S_0^2 + \left(A_0^0\right)^2 - \left(A_0^3\right)^2 = \text{const.}, \qquad (36)$$

On the other hand from Fierz theorem we have

$$I_A = \left(A^0\right)^2 - \left(A^1\right)^2 - \left(A^2\right)^2 - \left(A^3\right)^2 = -\left(S^2 + P^2\right), \qquad (37)$$

On account of $A^1 = 0$ and $A^2 = 0$ from (37) and (36) yields

$$S = \frac{V_0}{V}, \quad V_0 = \text{const.} \qquad (38)$$

For diagonal Bianchi metrics (38) fulfills only if $K = I$

Let us rewrite (21) on account of (35d)

$$\frac{1}{f}\left(h'' - \frac{f'}{f}h'\right) = -\frac{\kappa}{2}\frac{a_3}{a_1^2}\left[\left(\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_1}{a_1}\right)A_0^3 - 2a_1\dot{A}_0^3\right]. \qquad (39)$$

Now the left hand side of (39) depends of x_3 only, while the right hand side depends on only t, hence can be written as

$$\frac{1}{f}\left(h'' - \frac{f'}{f}h'\right) = b, \qquad (40a)$$

$$\frac{\kappa}{2}\frac{a_3}{a_1^2}\left[\left(\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_1}{a_1}\right)A_0^3 - 2a_1\dot{A}_0^3\right] = -b, \qquad (40b)$$

Basic Equations

We will consider (40a) **and** (40b) **for both** $b = 0$ **and** $b \neq 0$.
In case of $b = 0$ **one dully finds that**

$$h' = c_1 f, \quad c_1 = \mathrm{const.} \tag{41}$$

$$A_0^3 = C_1 \exp\left[\frac{9q_1}{4q_2(3q_1+1)} V^{-(q_1+1/3)}\right]. \tag{42}$$

In view of (35d) **equation for** V (33) **can be rewritten as**

$$\ddot{V} - \frac{3\kappa}{2}\sqrt{\frac{q_2^3}{q_3}}\frac{h'}{f'}\dot{A}_0^3 V^{3q_1/2} - \frac{1}{4}q_2^4 V^{4q_1+1/3}\left(\frac{h'}{f}\right)^2 - \frac{1}{2}q_2 q_3 V^{q_1+1/3}\frac{f''}{f} = \frac{3\kappa}{2}\left[m_{\mathrm{sp}} S + 2(F - KF_K)\right] V. \tag{43}$$

In order to solve the equation (43) **we have to define** $f(x_3)$ **from** (7), $h(x_3)$ **from** (40a) **and** A_0^3 **from** (40b). **Beside that we have to give the concrete form of spinor field nonlinearity as well. Earlier we have considered the spinor field nonlinearity as a power law of** K. **In this report following some recent paper we choose the nonlinearity to be the polynomial of** S **only, having the form**

$$F = \sum_k \lambda_k I^{n_k} = \sum_k \lambda_k S^{2n_k}. \tag{44}$$

Basic Equations

We can finally write (43) and (40b) as follows

$$\dot{V} = Y, \qquad (45a)$$

$$\dot{Y} = \frac{3\kappa}{2}\sqrt{\frac{q_2^3}{q_3}}\frac{h'}{f'}V^{3q_1/2}\Phi_1(V, A_0^3, Y) + \Phi_2(V, A_0^3, Y), \qquad (45b)$$

$$\dot{A}_0^3 = \Phi_1(V, A_0^3, Y), \qquad (45c)$$

$$\Phi_1 \quad (V, A_0^3, Y) = -\frac{3q_1}{4q_2}\, A_0^3\, V^{-(q_1+4/3)}Y + \frac{b}{\kappa}\sqrt{q_2^5 q_3}\,V^{5q_1/2-2/3},$$

$$\Phi_2 \quad (V, A_0^3, Y) = \frac{1}{4}q_2^4 V^{4q_1+1/3}\left(\frac{h'}{f}\right)^2 + \frac{1}{2}q_2 q_3 V^{q_1+1/3}\frac{f''}{f}$$

$$+ \quad \frac{3\kappa}{2}\left[(m_{\rm sp} + \lambda_0) + 2\lambda_1(1-n_1)V^{1-2n_1} + 2\lambda_2(1-n_2)V^{1-2n_2}\right].$$

In the foregoing system for simplicity we consider only three terms of the sum. We set $n_k = n_0 : 1 - 2n_0 = 0$ which gives $n_0 = 1/2$. In this case the corresponding term can be added with the mass term. We assume that q_1 is a positive quantity, so that $4q_1 + 1/3$ is positive too. For the nonlinear term to be dominant at large time, we set $n_k = n_1 : 1 - 2n_1 > 4q_1 + 1/3$, i.e.,$n_1 < 1/3 - 2q_1$. And finally, for the nonlinear term to be dominant at the early stage we set $n_k = n_2 : 1 - 2n_2 < 0$, i.e., $n_2 > 1/2$. Since we are interested in qualitative picture of evolution, let us set $q_2 = 1,\ q_3 = 1$ and $\kappa = 1$. We also assume $V_0 = 1$. the initial values are taken as $V(0) = 0.01, \quad \dot{V}(0) = 0.1$. We have also set $t \in [0,\ 2]$ with step size 0.001. We set $x_3 = [0,\ 1] = 0.2k$ with step $k = 0..5$.

Numerical solution to the field equations

We consider three different models. In case of *BII*, *BVIII* and *BIX* we have following expressions for $f(x_3)$ and $h(x_3)$, respectively

$$\begin{aligned} f(x_3) &= px_3 + q, \quad BII \\ h(x_3) &= (1/3)bpx_3^3 + \frac{1}{2(c_1 p + bq)} x_3^2 + c_1 q x_3 + c_2, \end{aligned} \tag{46}$$

$$\begin{aligned} f(x_3) &= \sinh(x_3), \quad h(x_3) = \cosh(x_3), \quad b = 0, \quad BVIII \\ h(x_3) &= b\,(x_3 \cosh(x_3) - \sinh(x_3)) + c_1 \cosh(x_3) + c_2, \quad b \neq 0, \end{aligned} \tag{47}$$

$$\begin{aligned} f(x_3) &= \sin(x_3), \quad h(x_3) = \cos(x_3), \quad b = 0, \quad BIX \\ h(x_3) &= b\,(\sin(x_3) - x_3 \cos(x_3)) - c_1 \cos(x_3) + c_2, \quad b \neq 0. \end{aligned} \tag{48}$$

$c_1 = \text{const.}, \quad c_2 = \text{const.}$

Numerical solution to the field equations

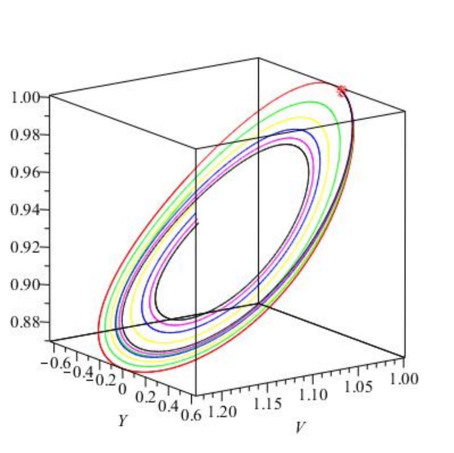

Figure: Phase diagram of $[V,\ \dot{V},\ A_0^3]$ in case of $b = 0$, $\lambda_1 = 1$ and $\lambda_2 = 1$ for *BVIII* model.

Numerical solution to the field equations

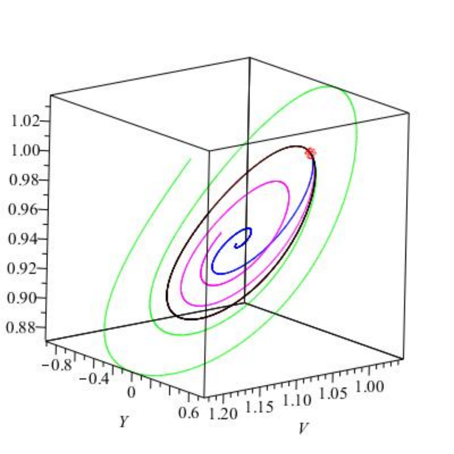

Figure: Phase diagram of $[V,\ \dot{V},\ A_0^3]$ in case of $b = 0$, $\lambda_1 = 1$ and $\lambda_2 = 1$ for *BIX* model.

Numerical solution to the field equations

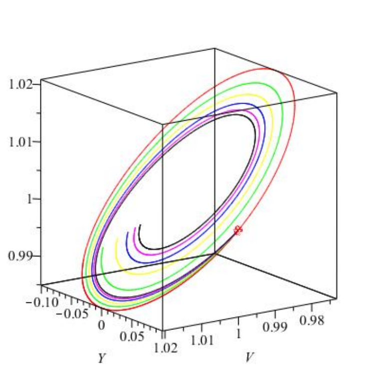

Figure: Phase diagram of $[V,\ \dot{V},\ A_0^3]$ in case of $b = 0$, $\lambda_1 = 1$ and $\lambda_2 = -0.1$ for *BVIII* model.

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Numerical solution to the field equations

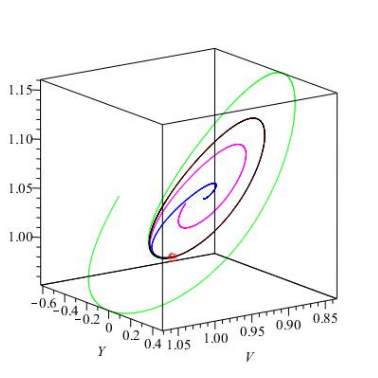

Figure: Phase diagram of $[V,\ \dot{V},\ A_0^3]$ in case of $b = 0$, $\lambda_1 = 1$ and $\lambda_2 = -0.1$ for *BIX* model.

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Numerical solution to the field equations

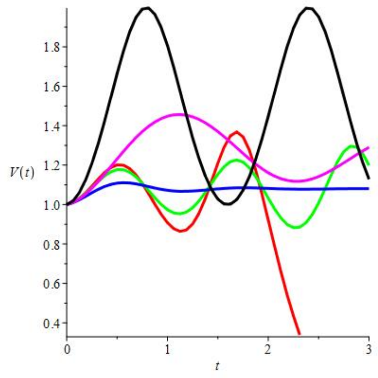

Figure: Evolution of V in case of $b = 0$, $\lambda_1 = 1$ and $\lambda_2 = 1$ for *BIX* model.

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Numerical solution to the field equations

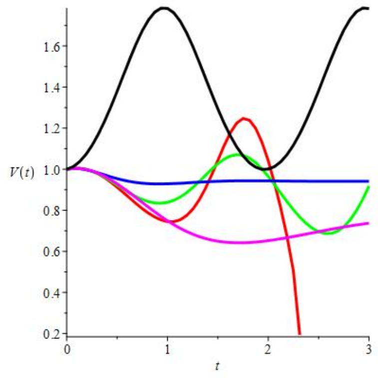

Figure: Evolution of V in case of $b = 0$, $\lambda_1 = 1$ and $\lambda_2 = -0.1$ for *BIX* model.

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Numerical solution to the field equations

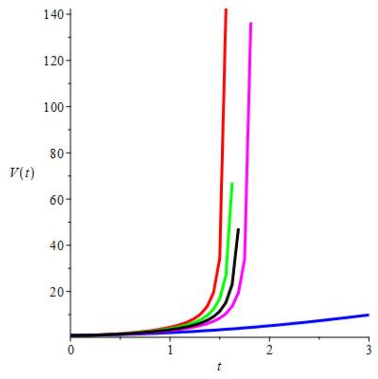

Figure: Evolution of V in case of $b = 0$, $\lambda_1 = 0$ and $\lambda_2 = 0$ for *BIX* model.

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Numerical solution to the field equations

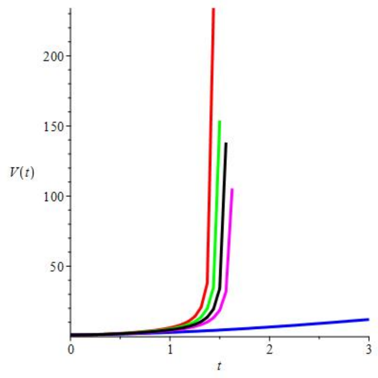

Figure: Evolution of V in case of $b = 0$, $\lambda_1 = 0$ and $\lambda_2 = 1$ for *BIX* model.

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Numerical solution to the field equations

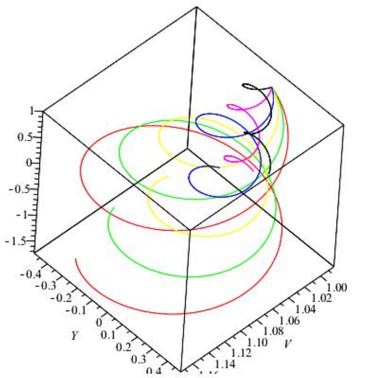

Figure: Phase diagram of $[V, \dot{V}, A_0^3]$ in case of $b = 1$, $\lambda_1 = 1$ and $\lambda_2 = 1$ for *BII* model.

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Numerical solution to the field equations

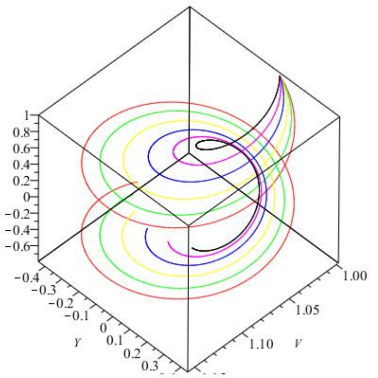

Figure: Phase diagram of $[V,\ \dot{V},\ A_0^3]$ in case of $b = 1$, $\lambda_1 = 1$ and $\lambda_2 = 1$ for *BVIII* model.

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Numerical solution to the field equations

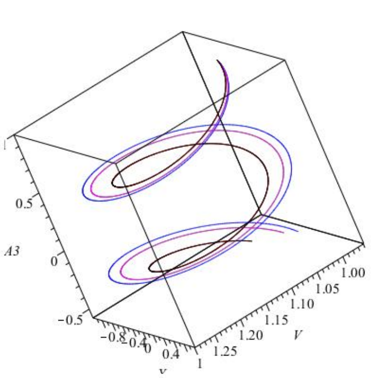

Figure: Phase diagram of $[V,\ \dot{V},\ A_0^3]$ in case of $b = 1$, $\lambda_1 = 1$ and $\lambda_2 = 1$ for *BIX* model.

Numerical solution to the field equations

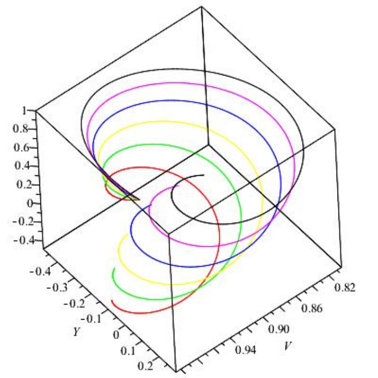

Figure: Phase diagram of $[V, \dot{V}, A_0^3]$ in case of $b = 1$, $\lambda_1 = 1$ and $\lambda_2 = -0.1$ for *BII* model.

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Numerical solution to the field equations

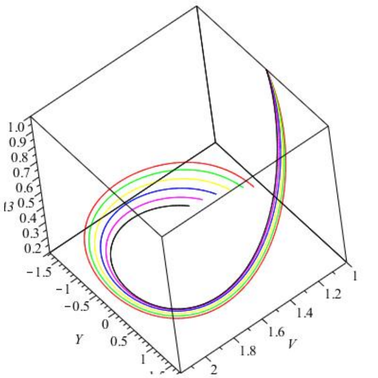

Figure: Phase diagram of $[V,\ \dot{V},\ A_0^3]$ in case of $b = 1$, $\lambda_1 = 1$ and $\lambda_2 = -0.1$ for *BVIII* model.

Numerical solution to the field equations

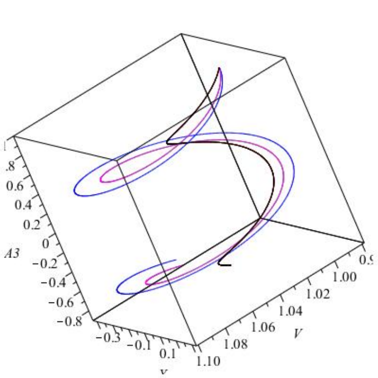

Figure: Phase diagram of $[V,\ \dot{V},\ A_0^3]$ in case of $b = 1$, $\lambda_1 = 1$ and $\lambda_2 = -0.1$ for *BIX* model.

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Numerical solution to the field equations

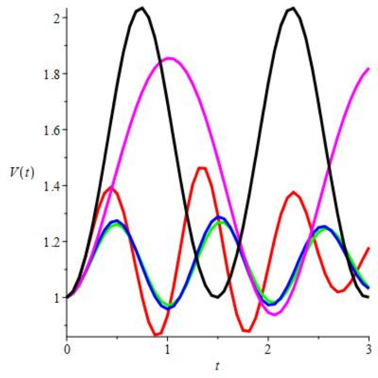

Figure: Evolution of V in case of $b = 1$, $\lambda_1 = 1$ and $\lambda_2 = 1$ for *BIX* model.

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Numerical solution to the field equations

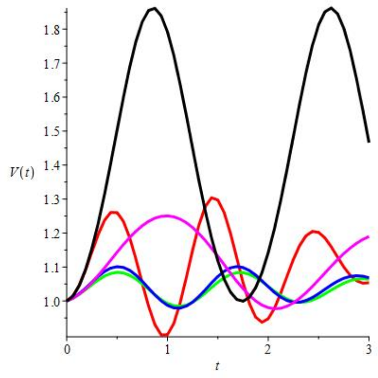

Figure: Evolution of V in case of $b = 1$, $\lambda_1 = 1$ and $\lambda_2 = -0.1$ for *BIX* model.

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Numerical solution to the field equations

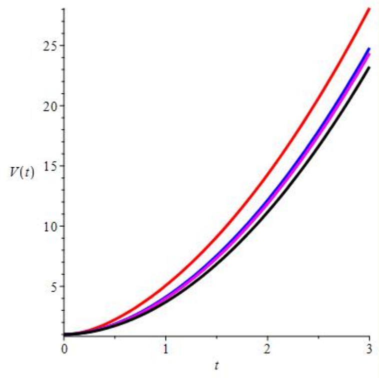

Figure: Evolution of V in case of $b = 1$, $\lambda_1 = 0$ and $\lambda_2 = 0$ for *BIX* model.

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Numerical solution to the field equations

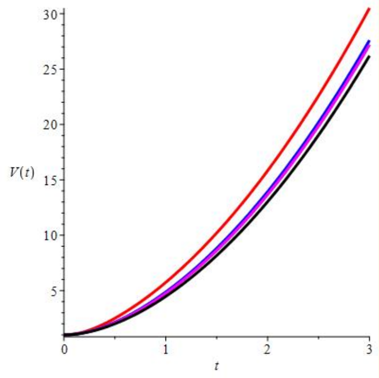

Figure: Evolution of V in case of $b = 1$, $\lambda_1 = 0$ and $\lambda_2 = 1$ for BIX model.

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Concluding remarks

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Unlike the previously mentioned models in the present model the equation for volume scale contains $\dot{V}$ term explicitly and does not allow exact solution in quadrature.

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Basic Equations

In absence of non-diagonal components of energy-momentum tensor the non-diagonal Bianchi space-time becomes locally rotationally symmetric which is true for spinor field.

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