



Contribution ID: 75

Type: not specified

Diffraction of electromagnetic waves on a waveguide joint

Friday 7 July 2017 13:15 (15 minutes)

Consider a waveguide of a constant simply connected cross-section S with ideally conducting walls. The axis Oz is directed along the cylinder axis, the normal to ∂S will be denoted as \vec{n} , the tangent vector perpendicular to \vec{e}_z as $\vec{\tau}$. Let the filling ϵ, μ of this waveguide have the jump at $z = 0$ so this plane is the joint of two waveguide. For a while we will not make any assumptions about the dependence ϵ, μ on x, y .

For a basis we take the system of Maxwell's equations, from which we exclude E_z and H_z . A two-dimensional analogue of the Helmholtz decomposition makes it possible, without loss of generality, to seek a solution in the form

$$\begin{aligned} \vec{E}_{\perp} &= \nabla u_e + \nabla' v_e, \\ \vec{H}_{\perp} &= \nabla v_h + \nabla' u_h, \end{aligned}$$

where

$$\nabla = (\partial_x, \partial_y)^T, \quad \nabla' = (-\partial_y, \partial_x)^T.$$

The four scalar functions introduced here will be called potentials, and we'll always assume that they satisfy the boundary conditions

$$u_e = u_h = n \cdot \nabla v_e = n \cdot \nabla v_h = 0.$$

The boundary conditions for the field are automatically satisfied and Maxwell's equations give a system of four equations on the potentials. The solution of this system we consider as functions of variable z with values in appropriate Sobolev spaces. Thus Maxwell's equations give for vector $w = (u_e, u_h, v_e, v_h)$ the ordinary differential equation

$$B \frac{dw}{dz} + ikAw - \frac{1}{ik}Cw = 0,$$

where A, B, C are bounded operators. Any solution of this equation at $z < 0$ and at $z > 0$ can be represented as the superposition of normal modes with respect to the eigenvalue problem

$$\beta Bw = Aw + \frac{1}{k^2}Cw,$$

so the solving of diffraction problem is reduced to the matching of both solutions.

In our talk we want to show how to calculate the eigenmodes and to make the matching in Sage partially by the analytic way.

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Session Classification: Computer Algebra and Quantum Computing with Applications (II)