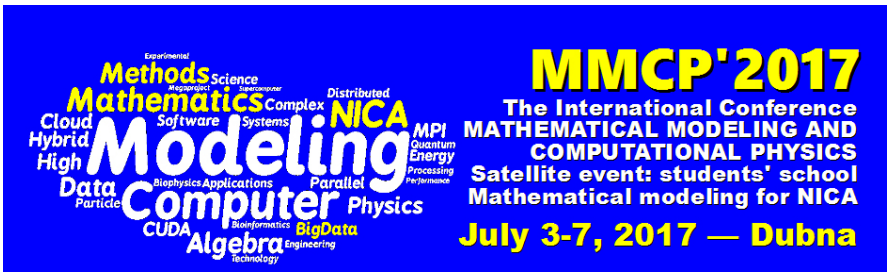

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Finite difference schemes as algebraic correspondences between layers

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Modern development of computer science revived the old investigations about the solvability of differential equations in the finite terms. There are remarkable differential equations which can be integrated in CAS. In our work we want to speak about remarkable differential equations in another sense: for these equations there are finite difference schemes which conserve algebraic properties of solutions exactly. It should be noted that these classes of differential equations are very similar.

As was shown by Painlev\'e, all ordinary differential equations which can be integrated in classical transcendental functions have a common algebraic property, namely, general solution of such differential equation depends on integration constant algebraically. So we can construct a version of Galois theory without fixation of allowed transcendental operations [1,2].

In term of Cauchy problem this property means that the differential equation defines an algebraic correspondence between initial and terminal values. For example Riccati equation $y' = p(x)y^2 + q(x)y + r(x)$ defines one-to-one correspondence between initial and terminal values of y on projective straight line. However, standard finite difference scheme don't conserve this algebraic property of exact solution. Further more, the scheme, which defines one-to-one correspondence between layers, truly describes solution not only before but also after mobile singularities and conserves algebraic properties of equations like the anharmonic ratio [3].

When the differential equation has a form $F(x, y, y') = 0$ we reduce it to equation of 2nd degree and then apply the finite difference method. In so doing, the expression $F(x, y, y')$ is the integral of the differential equation of 2nd degree and thus its quantity is conserved on approximate equation only with some small error. However if the equation $F(x, y, y') = 0$ defines one-to-one correspondence between initial and terminal values we can try to find the difference scheme which define one-to-one correspondence between layers, that is between algebraic curves. For curves with the genus $p > 0$ such correspondence can be constructed by pure algebraic way and at the same time periodical functions appear here naturally without any references on exact solution. In our talk such schemes will be presented.

Short biography note

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