High precision computer simulation of cyclotrons

KARAMYSHEVA T., AMIRKHANOV I.
MALININ V., POPOV D.

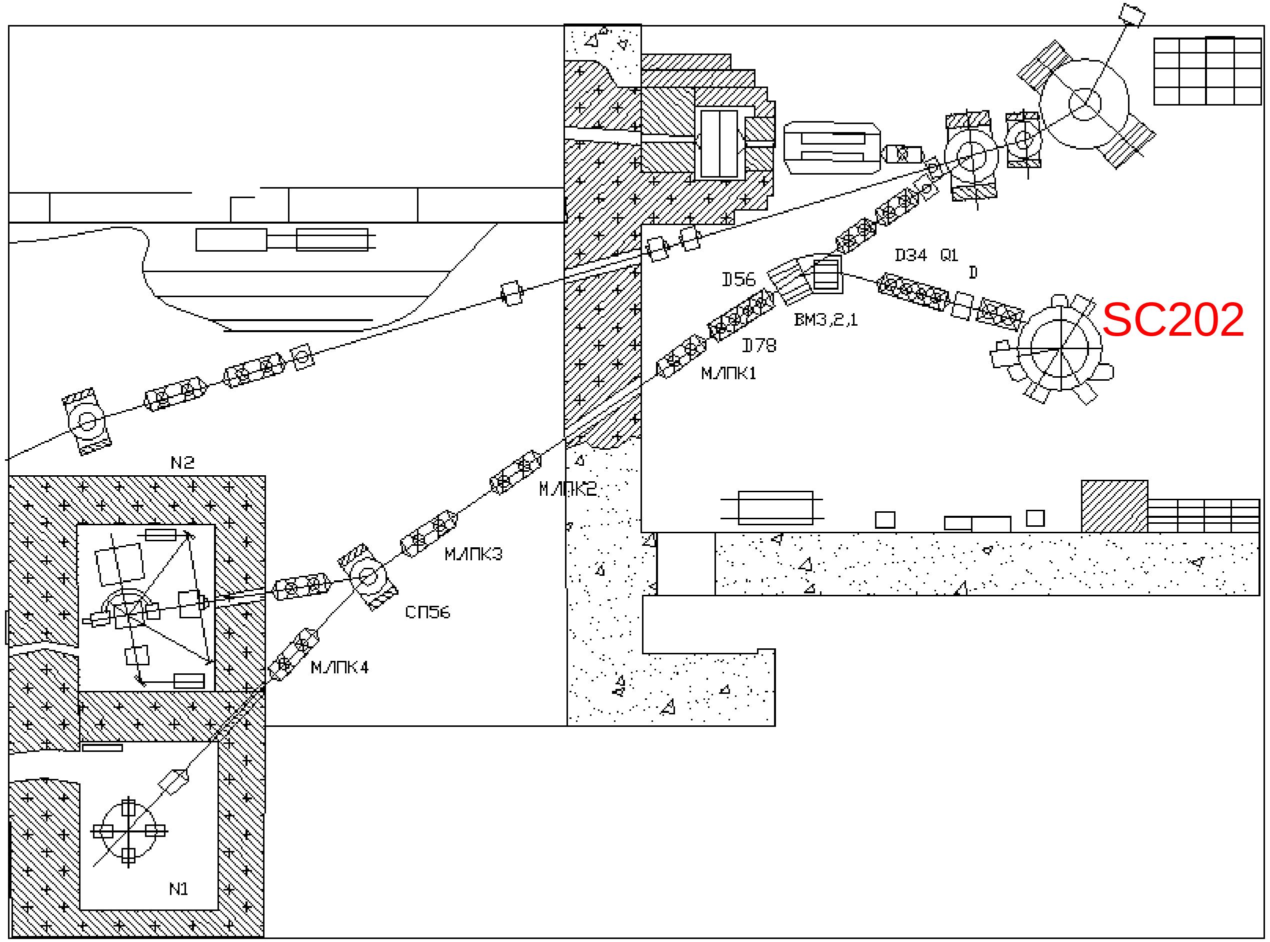
Abstract

Effective and accurate computer simulations are highly important in accelerators design and production. The most difficult and important task in cyclotron development is the magnetic field simulations. It is necessary to achieve the accuracy of the model that is higher than the tolerance for the magnetic field in the real magnet.

An accurate model of the magnet and other systems of the cyclotron allows us to perform beam tracking through the whole accelerator from the ion source to the extraction. While high accuracy is necessary in the late stages of research and development works, high performance of the simulations and ability to swiftly analyze and apply changes to the project play a key role in the early stages of the project. Techniques and algorithms for high accuracy and performance of the magnet simulations have been created and used for development of the SC202 cyclotron for proton therapy, which is under production by collaboration between JINR (Dubna, Russia) and ASIPP (Hefei, China).

JINR MEDICAL TECHNICAL COMPLEX with the future SC202 cyclotron

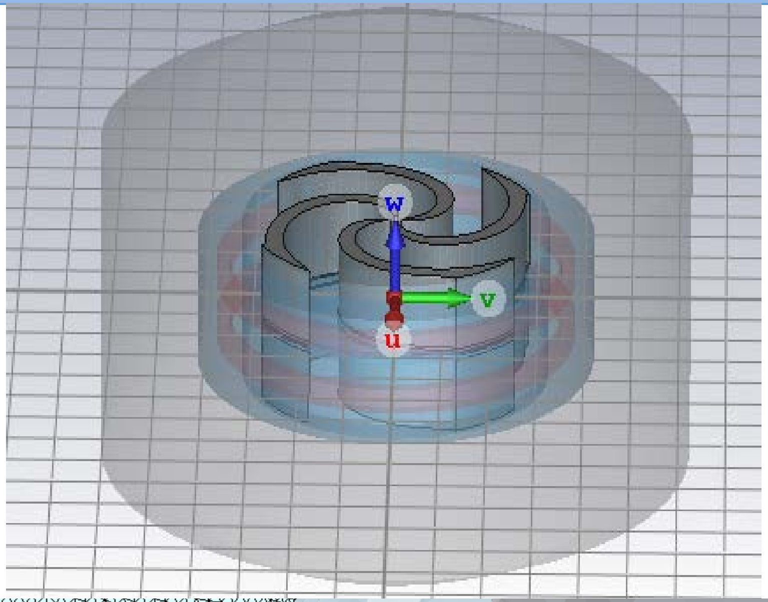
It is planned to manufacture two cyclotrons in China : one will operate in Hefei cyclotron medical center, the other will replace Phasotron in Medico-technical complex JINR Dubna and will be used to treat cancer with protons.



Dubna, “Phasotron”.
Originally built in 1949,
modernized in 1984

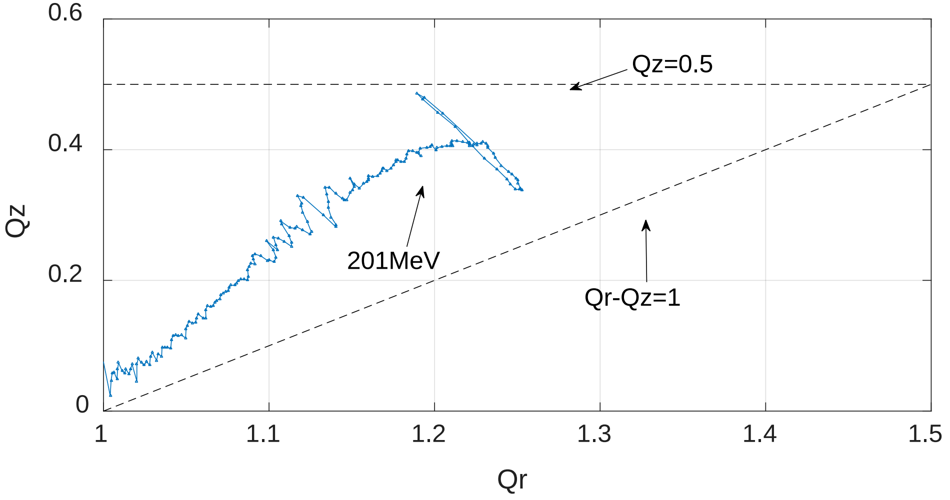
SC202 magnet design

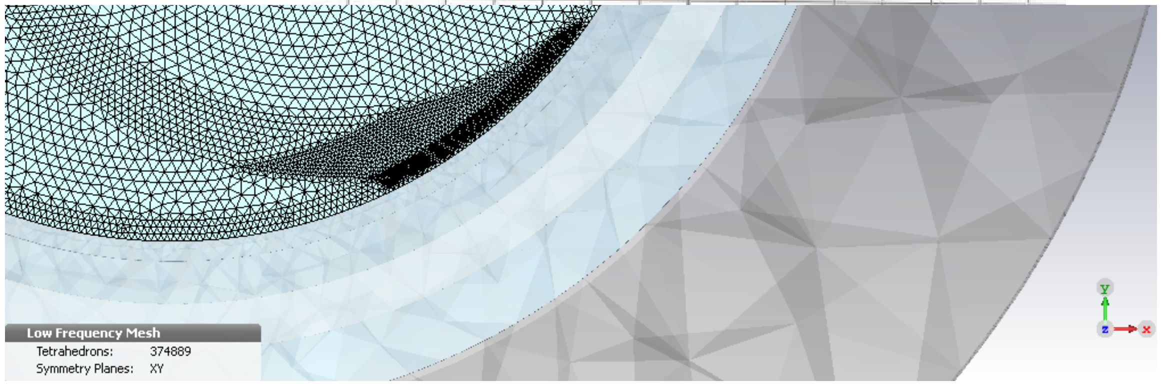
Computer model of the magnet



3D magnet simulation is the most important part when it comes to the development of an isochronous superconducting cyclotron as it defines the particle motion in the accelerator.

Working diagram



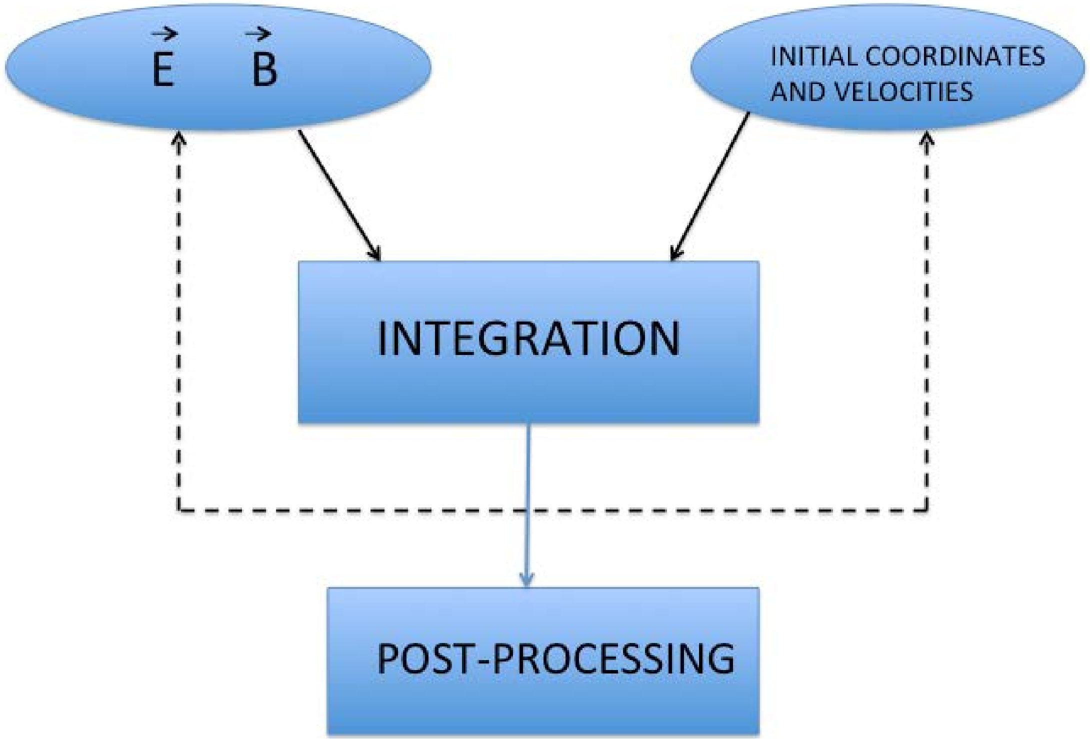


Meshed model

Magnet modeling goes hand in hand with beam dynamics modeling.

Beam dynamics simulations

Beam dynamics simulation solves the system of differential equations of particle motion with field maps from FEM (finite element method) codes or field measurements with different initial coordinates and velocities.



Runge–Kutta fourth-order method

Motion of charged particles in electromagnetic fields

Systems of differential equations of motion of charged particles in an electromagnetic field are derived from the Newton-Lorentz equations:

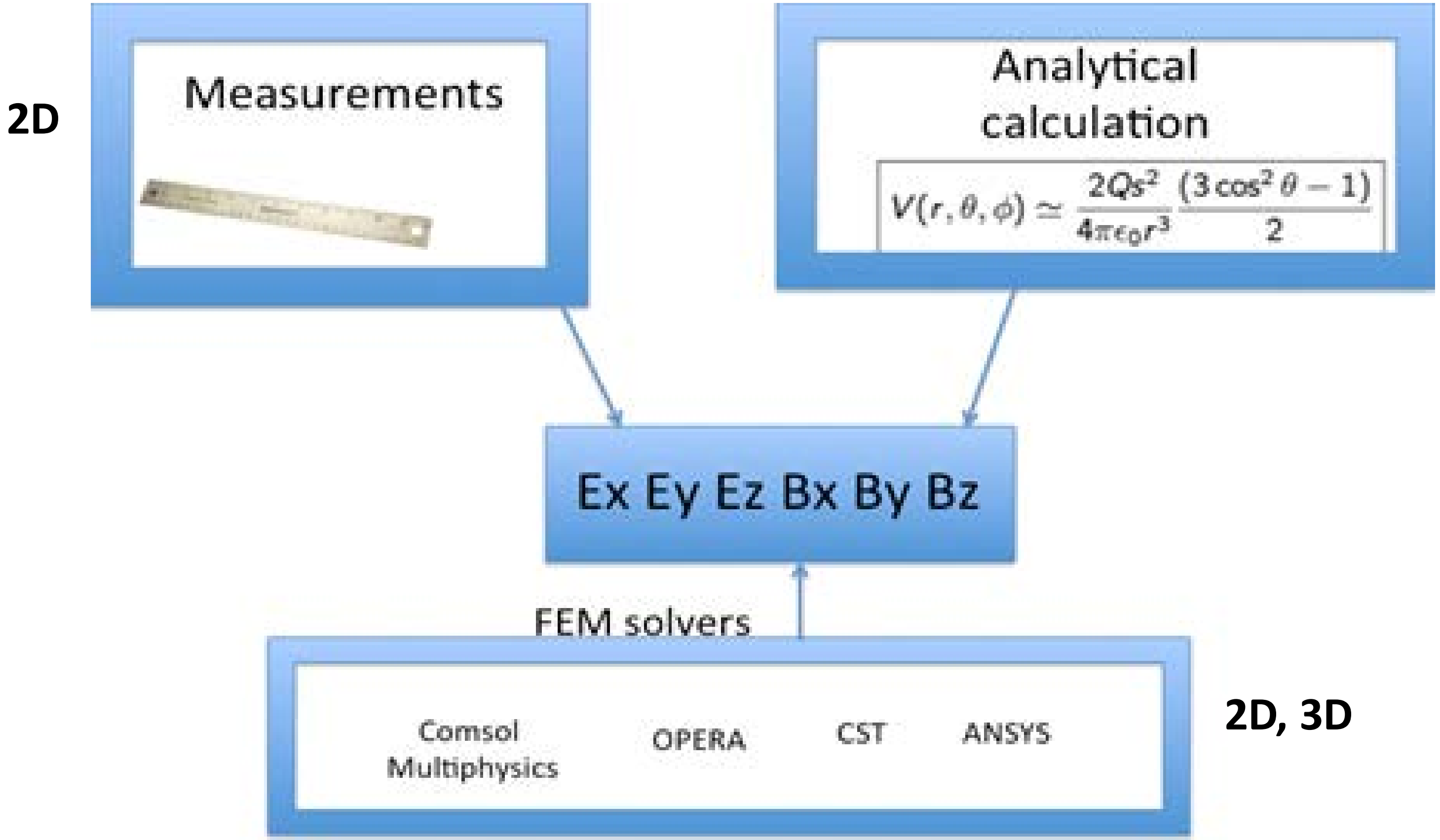
$$\frac{d\vec{V}}{dt}=\frac{q}{m}\sqrt{1-\frac{V^2}{c^2}}\left\{\vec{E}+\left[\vec{V}\vec{B}\right]-\frac{1}{c^2}\vec{V}(\vec{V}\vec{E})\right\}$$

$$\begin{cases} r''-\dfrac{2r'^2}{r}-r=\dfrac{q\sqrt{r^2+r'^2+z'^2}}{mcV}(rB_z-z'B_\varphi-\dfrac{r'z'}{r}B_r+\dfrac{r'^2}{r}B_z)+\dfrac{q}{m}\dfrac{r^2+r'^2+z'^2}{V^2}(\varepsilon_r-\dfrac{r'}{r}\varepsilon_\varphi) \\ z''-\dfrac{2r'z'}{r}=\dfrac{q}{mc}\dfrac{\sqrt{r^2+r'^2+z'^2}}{V}(r'B_\varphi-rB_r-\dfrac{z'^2}{r}B_r+\dfrac{r'z'}{r}B_z)+\dfrac{q}{m}\dfrac{r^2+r'^2+z'^2}{V^2}(\varepsilon_z-\dfrac{z'}{r}\varepsilon_\varphi) \\ t'=\dfrac{1}{c\beta}(r^2+r'^2+z'^2)^{\frac{1}{2}}=\dfrac{1}{V}(r^2+r'^2+z'^2)^{\frac{1}{2}} \end{cases}$$

In cyclotrons, it is most convenient to use the azimuth angle as a variable.

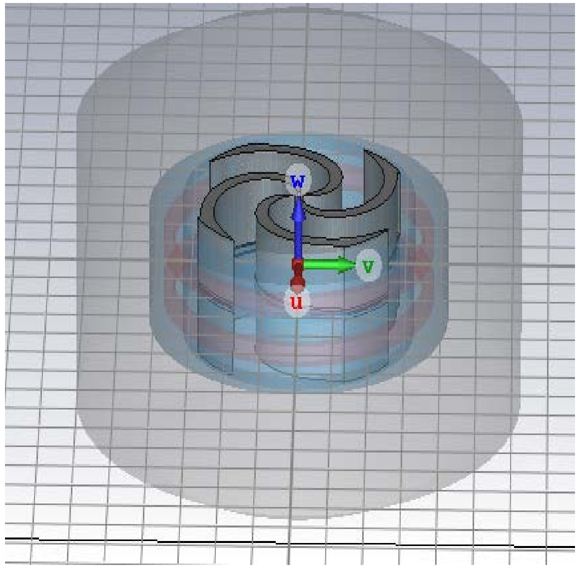
So the representation of the equations of motion with an independent variable azimuth angle is widely used in the design of cyclotrons.

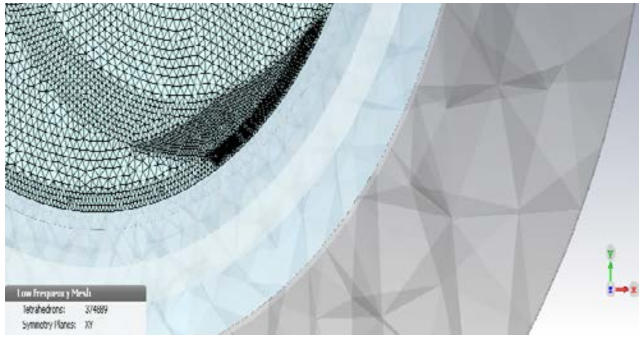
Representation of the electromagnetic field

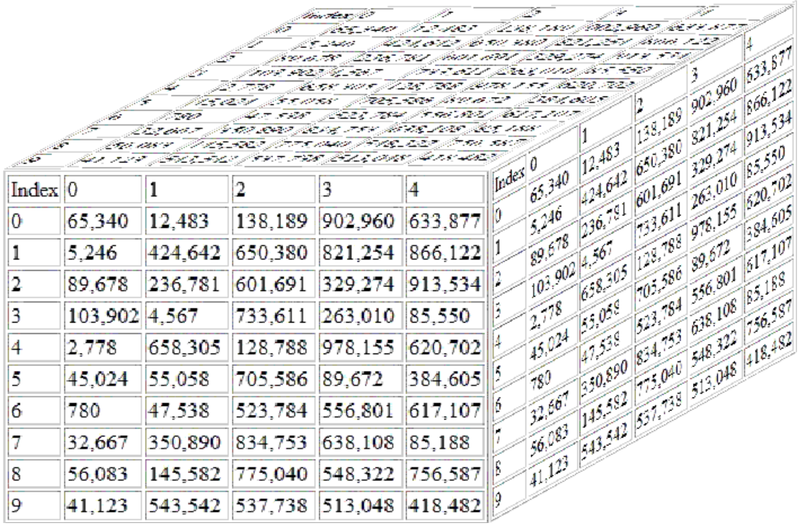


When calculating particle trajectories, an electromagnetic field can be specified analytically, in the form of a field map obtained as a result of measurements (2D maps) or as a result of model calculations (3D field maps).

Motion of charged particles in an electromagnetic field.







Index	0	1	2	3	4
0	65,340	12,483	138,189	902,960	633,877
1	5,246	424,642	650,380	821,254	866,122
2	89,678	236,781	601,691	329,274	913,534
3	103,902	4,567	733,611	263,010	85,550
4	2,778	658,305	128,788	978,155	620,702
5	45,024	55,058	705,586	89,672	384,605
6	780	47,538	523,784	556,801	617,107
7	32,667	350,890	834,753	638,108	85,188
8	56,083	145,582	775,040	548,322	756,587
9	41,123	543,542	537,738	513,048	418,482

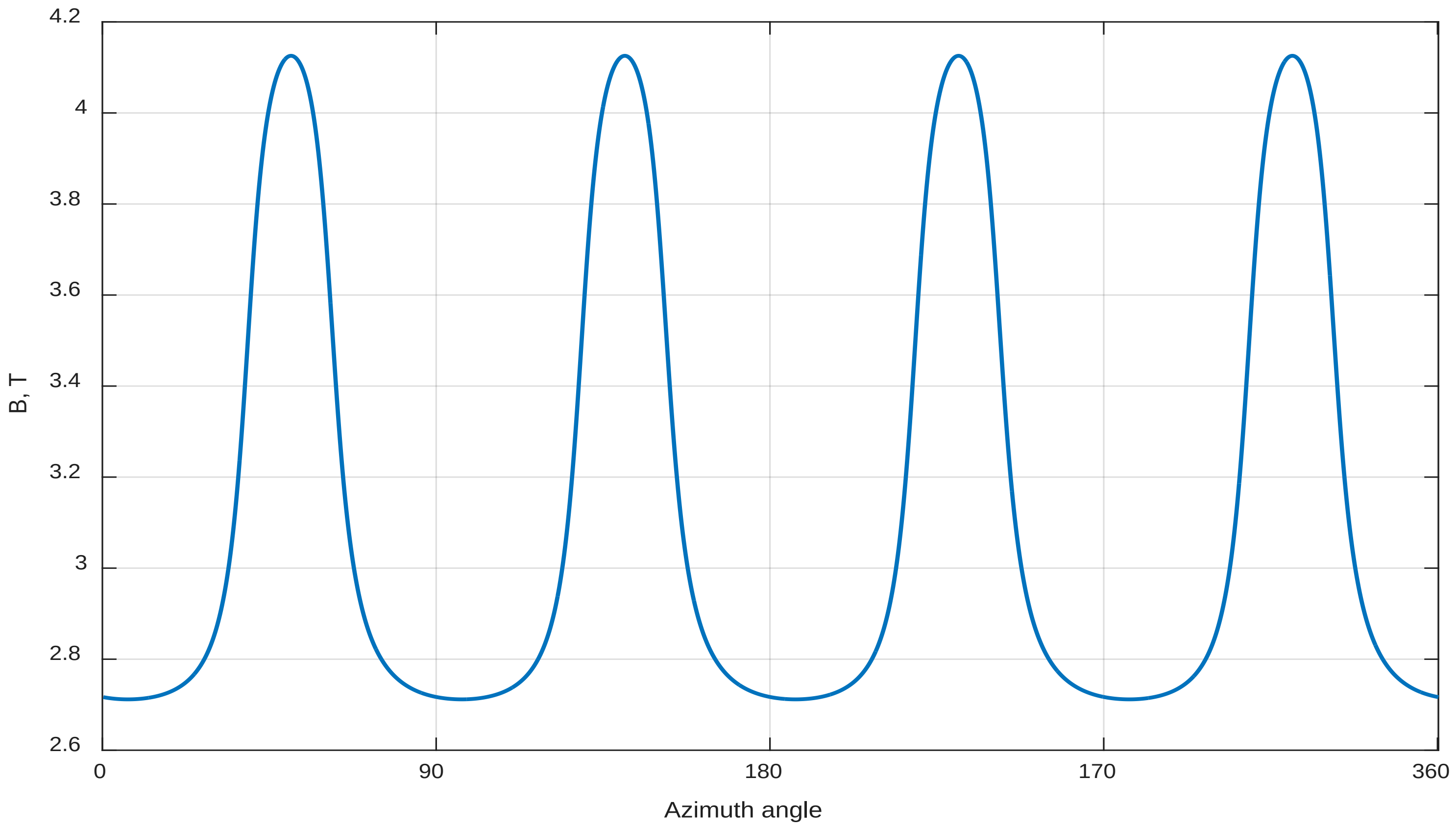
E-Field
B-Field

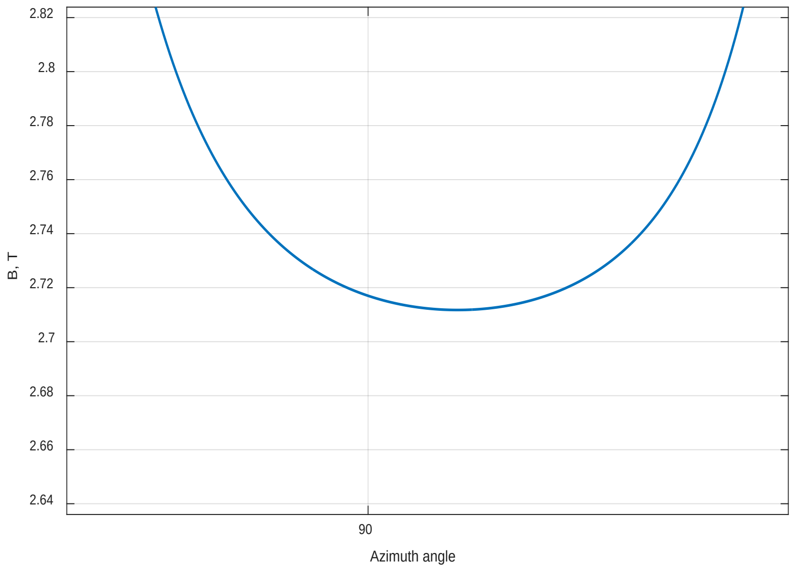
Up to 30 GB

The representation in the form of a 3D field map is obviously the most acceptable for calculations of beam dynamics, but maps can be huge when high accuracy is needed.

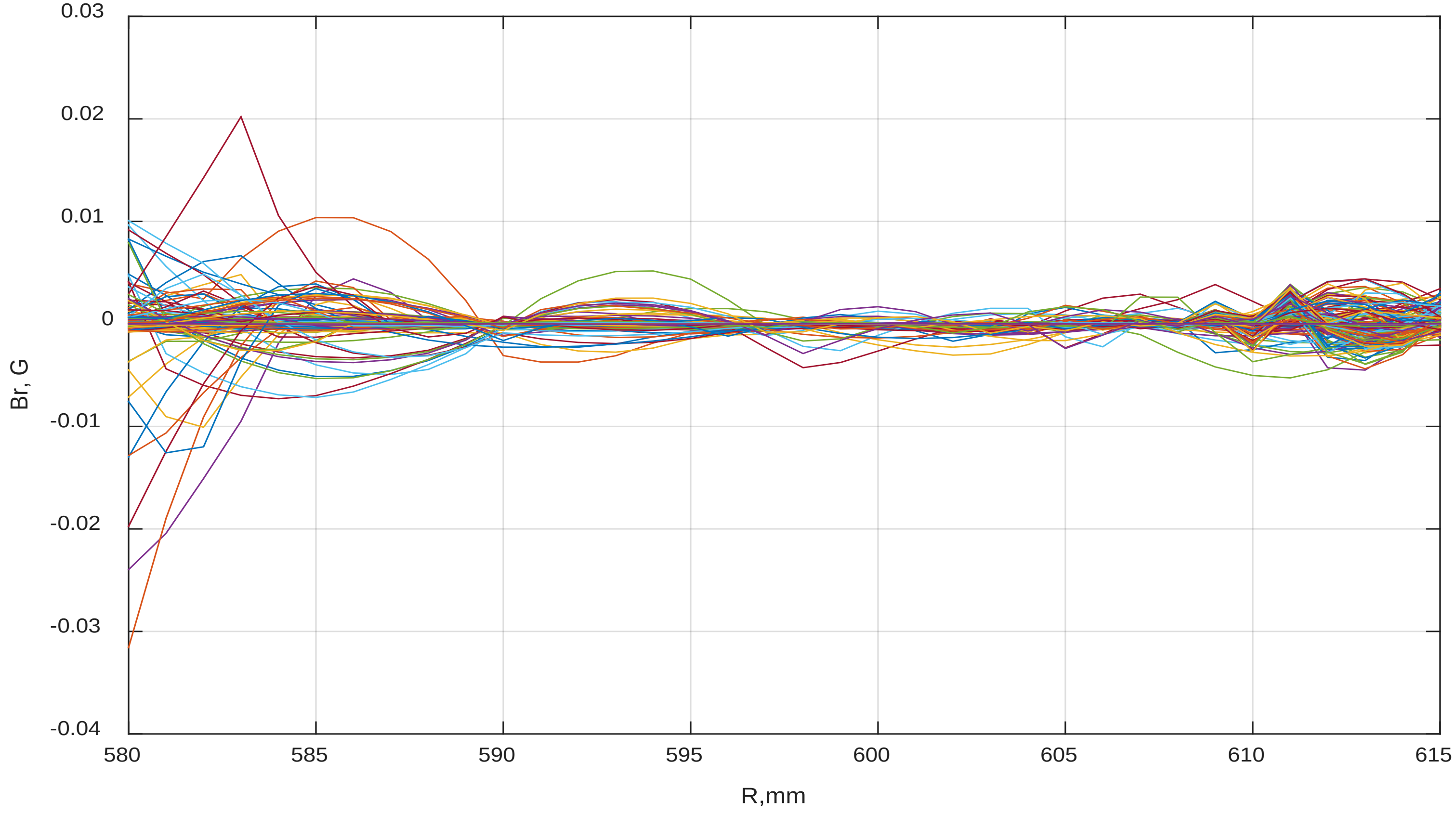
About the accuracy of the simulations

Also the accuracy of the simulations has been increased and the numerical error is lower than 0.1 Gs. This accuracy, of course, cannot be achieved in production, however when the accuracy of the model is much higher than the accuracy of manufacturing, it becomes possible to simulate tolerances and errors and the major weak spots that would help on the production stage.



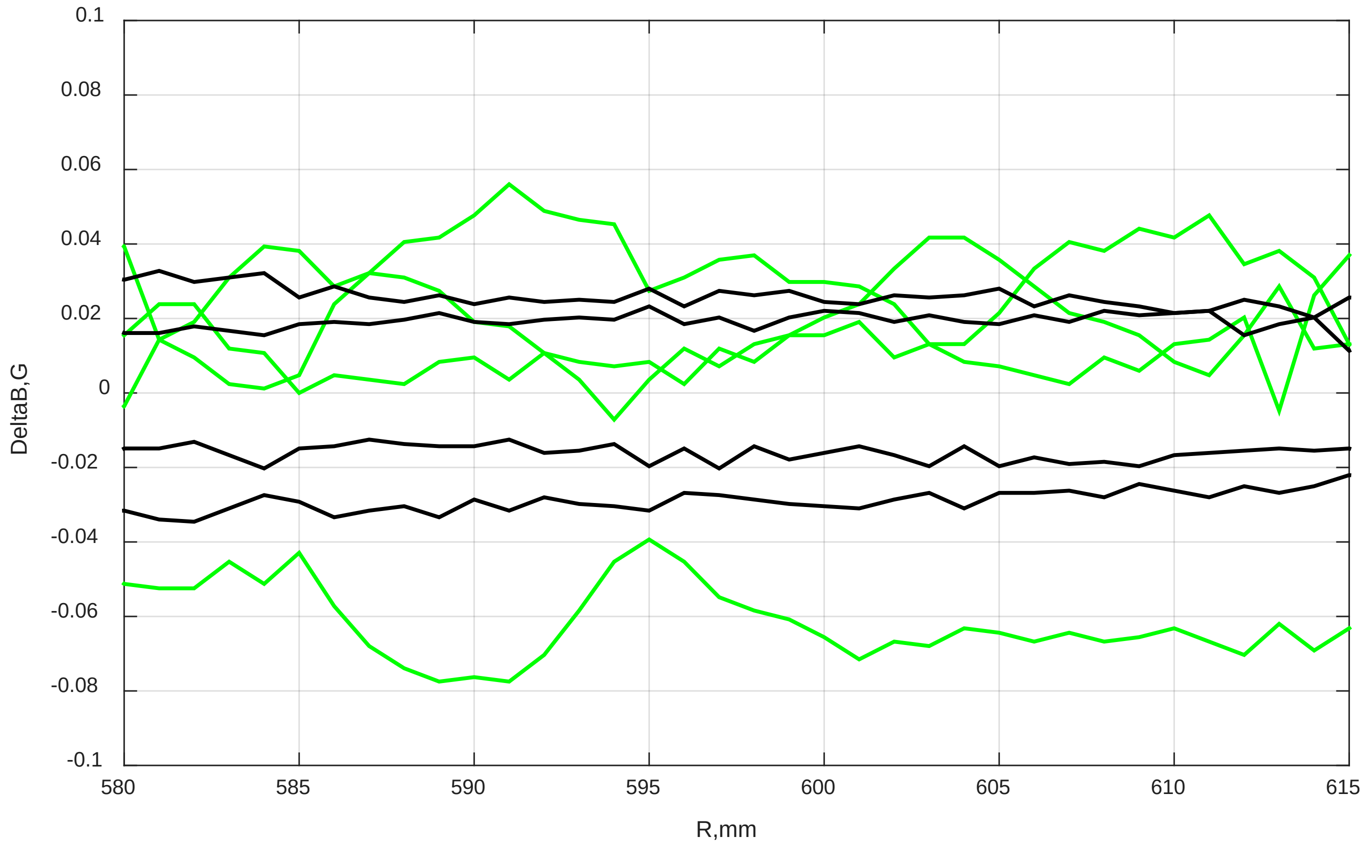


Radial component of the magnetic field in the median plane as a function of the radius (different curves correspond to different values of the azimuth angle).



Theoretically, the components Bφ and Br are equal to zero in the median plane, the presence of components indicates an error value that is ±0.03 Gs in the extraction zone.

Black curves - the deviation of the magnetic field from the average value in valleys (minimum points), green lines - deviation from the average value in sectors (maximum points).



The four-sector structure of the magnet makes it possible to estimate the accuracy of the model calculation by comparing the magnetic field for identical parts of the magnet sectors (calculation in the 360-degree model).

Magnet system design

- The major problem of R&D phase of the SC200 project was that 3D simulations of the magnet were very time-consuming and it was difficult to achieve an acceptable accuracy of the simulated field. In order to fix this issue CST studio macros and quick analysis of the magnet field map in MATLAB have been developed.
- Scripts for producing 3D magnetic field maps in CST studio and for reading them into the Matlab workspace were written.
- This technique allowed us to increase the number of simulated magnet geometries to over 5000 in half a year. Such fine tuning allowed us to optimize the magnet in very short time.

Using a 2D field map in the median plane for beam dynamics simulations.

In order to study non-linear resonances and define the conditions of their appearance we have performed simulations with a 2D field map in the median plane used to perform particle tracking. This is a traditional method and it is widely used in cyclotron physics. Only a vertical component exists in the median plane of the cyclotron (due to the symmetry) and the other components of the magnetic field can be calculated by:

$$B_r = z\frac{\partial B}{\partial r} - z^3 f_3$$

$$rB_\varphi = z\frac{\partial B}{\partial \varphi} - z^3 g_3$$

$$B_z = B - z^2 f_2 + z^4 f_4$$

$$f_2(r,\varphi) = \frac{1}{2}\left(\frac{\partial^2 B}{\partial r^2} + \frac{1}{r}\frac{\partial B}{\partial r} + \frac{1}{r^2}\frac{\partial^2 B}{\partial \varphi^2}\right)$$

$$f_3(r,\varphi) = \frac{1}{6}\left(\frac{\partial^3 B}{\partial r^3} + \frac{1}{r}\frac{\partial^2 B}{\partial r^2} - \frac{1}{r^2}\frac{\partial B}{\partial r} + \frac{1}{r^2}\frac{\partial^3 B}{\partial r \partial \varphi^2} - \frac{2}{r^3}\frac{\partial^2 B}{\partial \varphi^2}\right)$$

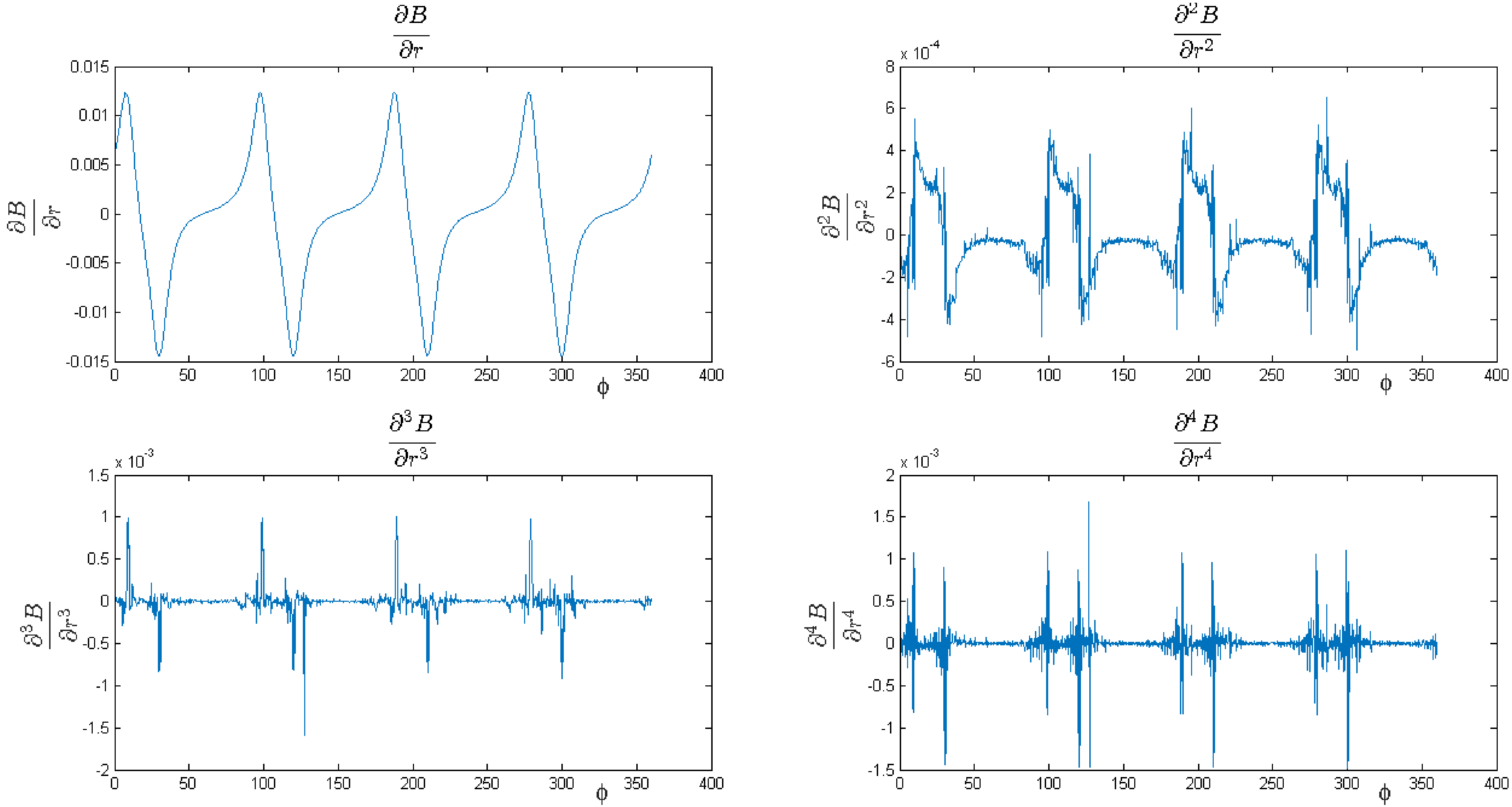
$$g_3(r,\varphi) = \frac{1}{6}\left(\frac{\partial^3 B}{\partial r^2 \partial \varphi} + \frac{1}{r}\frac{\partial^2 B}{\partial r \partial \varphi} + \frac{1}{r^2}\frac{\partial^3 B}{\partial \varphi^3}\right)$$

$$f_4(r,\varphi) = \frac{1}{24}\left(\frac{\partial^4 B}{\partial r^4} + \frac{1}{r^4}\frac{\partial^4 B}{\partial \varphi^4} + \frac{2}{r^2}\frac{\partial^4 B}{\partial r^2 \partial \varphi^2} + \frac{2}{r}\frac{\partial^3 B}{\partial r^3} - \frac{2}{r^3}\frac{\partial^3 B}{\partial r \partial \varphi^2} - \frac{1}{r^2}\frac{\partial^2 B}{\partial r^2} + \frac{4}{r^4}\frac{\partial^2 B}{\partial \varphi^2} + \frac{1}{r^3}\frac{\partial B}{\partial r}\right)$$

The formulas are presented for the cylindrical coordinate system, which is most suitable for cyclotrons. It is important to be able to calculate all derivatives for correct beam dynamics simulations during research and development and, especially on commissioning stage, when real magnetic field will be measured only on the median plane.

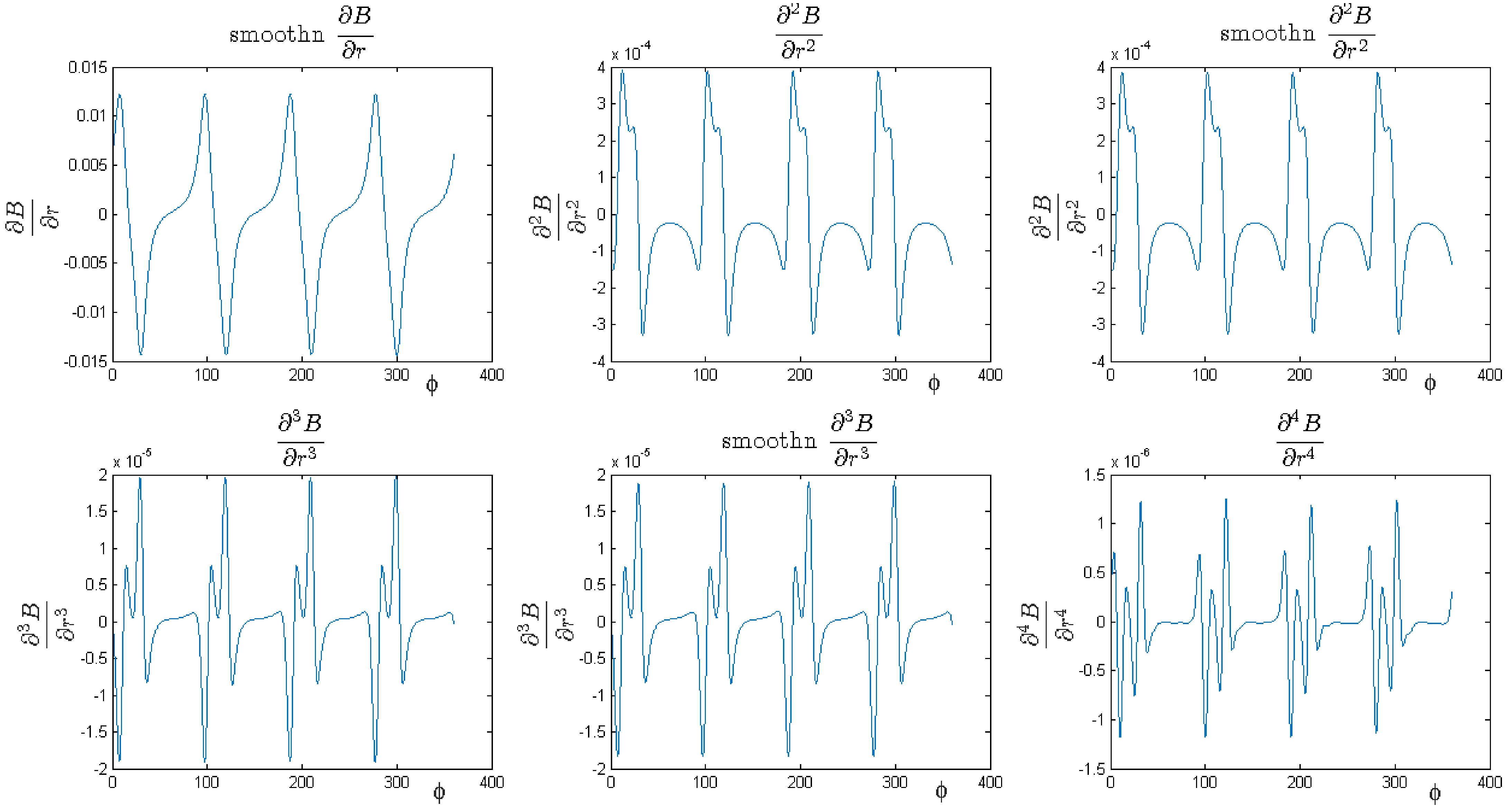
Derivatives of the magnetic field in the median plane taken without smoothing algorithms.

The biggest problem of these calculation is that the derivatives must be taken using the measured or calculated field map, which already contains error. Special mathematical algorithms were developed in order to obtain smooth and realistic derivatives of the magnetic field on the median plane.



Derivatives of the magnetic field in the median plane taken with smoothing algorithms.

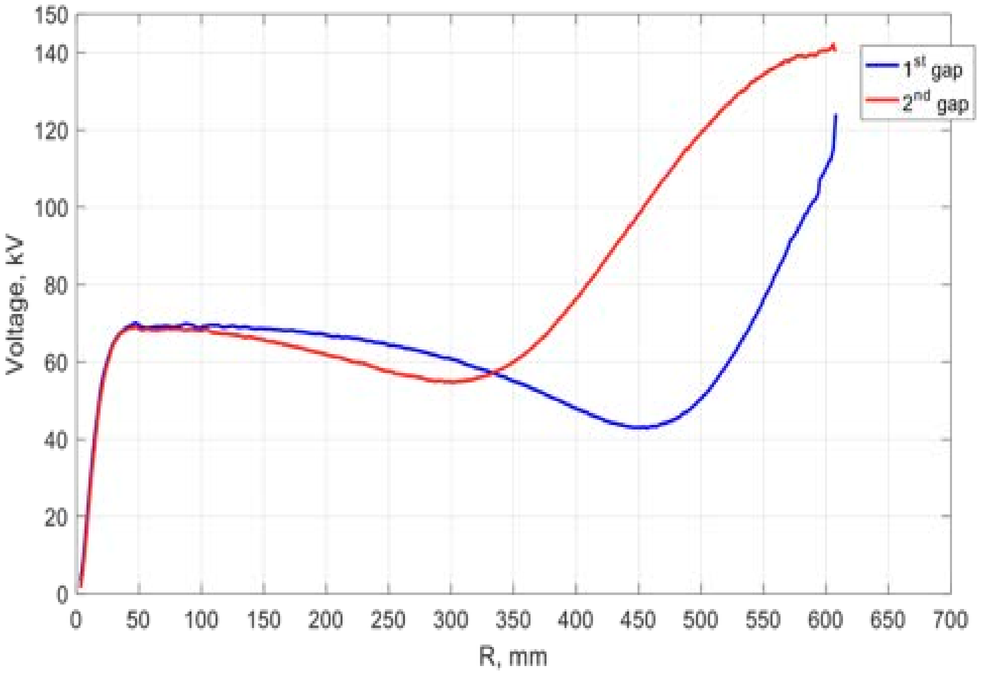
Such results were obtained by combining fitting of the field map by spline surface together with smoothing algorithms.



The developed software creates 3D field maps which can be used for beam dynamic simulations.

Accelerating RF system design

- It is also important for beam dynamics simulations to use an adequate representation of the accelerating field. We simulate the accelerating RF cavity in CST Studio.
- **Both electric and magnetic fields of RF cavity** from 3D simulations were used in particle tracking from the ion source to the extraction point of the SC202. High accuracy of the simulations are required in order to perform such tracking.
- Scripts for producing 3D field maps in CST Studio and for reading them into the Matlab workspace were written.
- The code for voltage distribution calculation was also created. The voltage value was obtained by integrating the electric field in the median plane of the resonant cavity.



Voltage distribution along the radius

Conclusion

- Codes for beam dynamics simulations were upgraded by new algorithms for calculations of the magnetic field outside the median plane.
- Scripts for producing 3D magnetic field maps, 3D electric and magnetic field maps from RF cavity simulations in CST Studio and for reading them into the Matlab workspace were written.
- High accuracy and high efficiency of simulations will help on commissioning stage during shimming of the magnet.