Proton 3D tomography:
TMD gluon densities in a spectator model

Gluon content @ NICA

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HAS QCD
HADRONIC STRUCTURE AND QUANTUM CHROMODYNAMICS

3DSPIN
MAPPING THE PROTON IN 3D

INFN
Istituto Nazionale di Fisica Nucleare
Sezione di Pavia

UNIVERSITÀ DI PAVIA

Gluon TMDs: a largely unexplored territory

* ***Theory***: different **gauge-link** structures…

  …more diversified kind of **modified universality**!

* ***Pheno***: golden channels for extraction

  of quark TMDs are subleading for gluon TMDs

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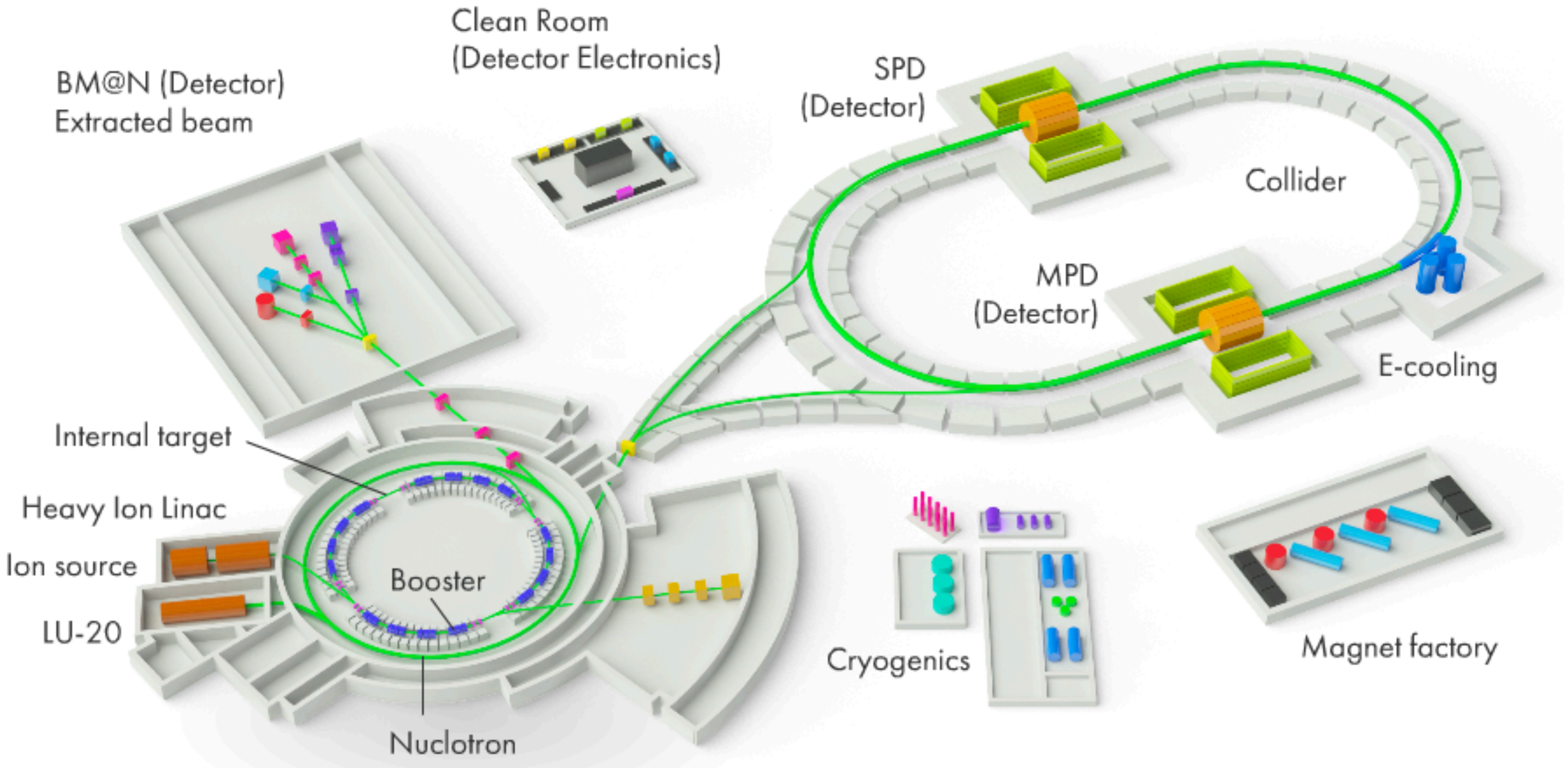
Motivation

- Gluon-TMD PDFs: *core* sector of **EIC** studies
- Need for a *flexible* model, suited to *pheno*
- **Unpolarized** and **polarized gluon TMDs**
- *Consistent* framework for quark TMDs



T-even gluon TMDs at twist-2

gluon pol.

nucleon pol.

	U	circ.	lin.
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g,\ h_{1T}^{\perp g}$

T-even

T-odd

T-even gluon TMDs at twist-2

unpolarized TMD

Boer-Mulders

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T-even

T-odd

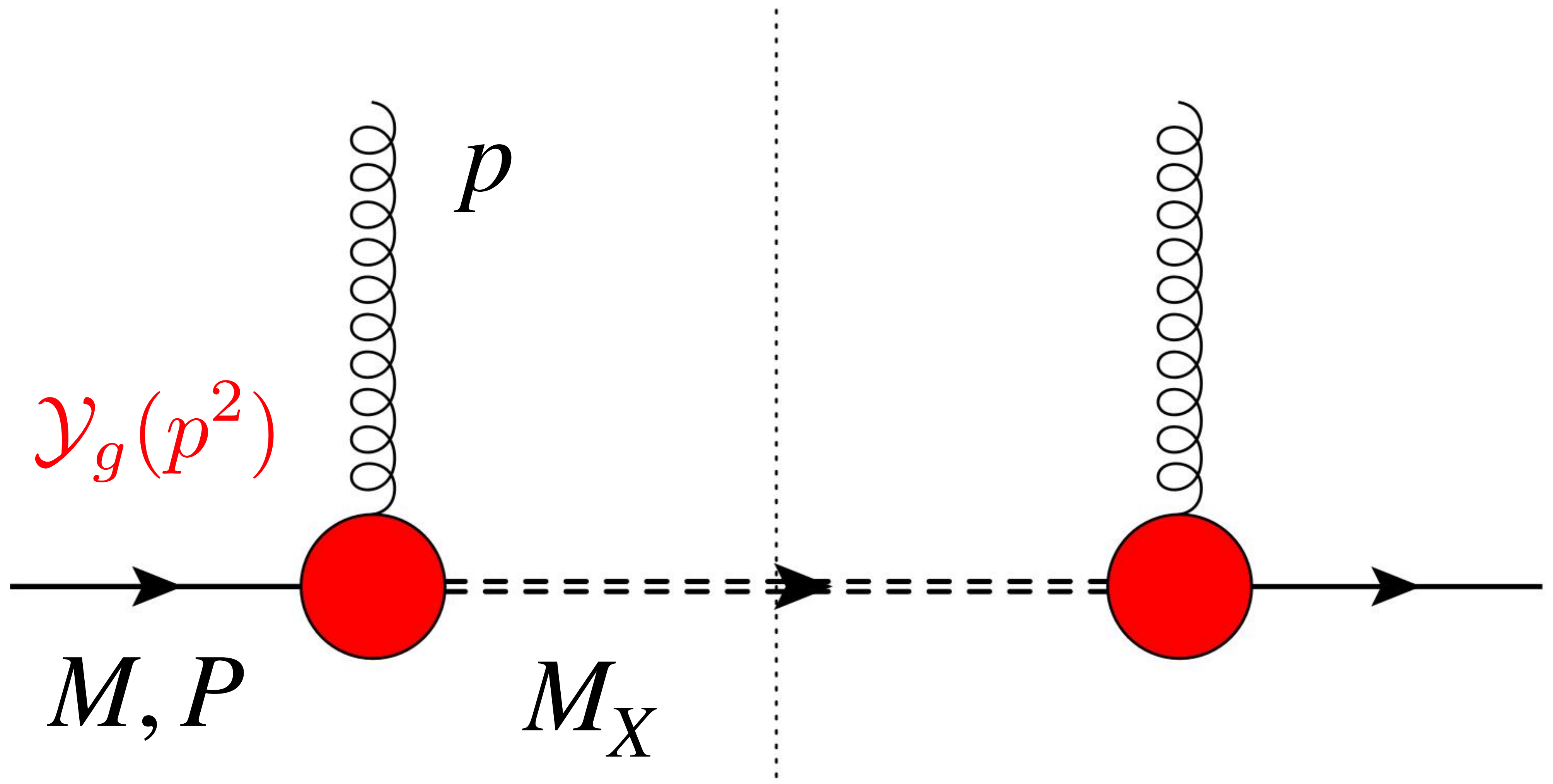
helicity TMD

worm-gear

Assumptions of the model

Effective vertex

Lowest Fock state:
tri-quark spectator
on-shell and
with mass M_X



Spin-1/2 spectator (gluon)

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} Tr\left[(\not P + M)\frac{1+\gamma^5 \not S}{2} G^*_{\mu\rho}(p) G^{\nu\sigma}(p) \mathcal{Y}_g^{\rho *} \mathcal{Y}_{g\sigma}(\not P - \not p + M)\right]$$

$$\mathcal{Y}_g^{\mu} = g_1(p^2)\gamma^{\mu} + i\frac{g_2(p^2)}{2M}\sigma^{\mu\nu}p_{\nu}$$

mimics proton form factors
(conserved EM current
of a free nucleon)

Assumptions of the model

Link with collinear factorization

p_T-integrated TMDs **have to** reproduce PDFs
at the lowest scale (Q_0) *before* evolution

Dipolar form factor(s)

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$

1. Cancels singularity of gluon propagator
2. Suppresses effects of high p_T
3. Compensates log divergences arising from p_T-integration
4. Adds three more parameters: $\kappa_{1,2}$ and Λ_X

Assumptions of the model

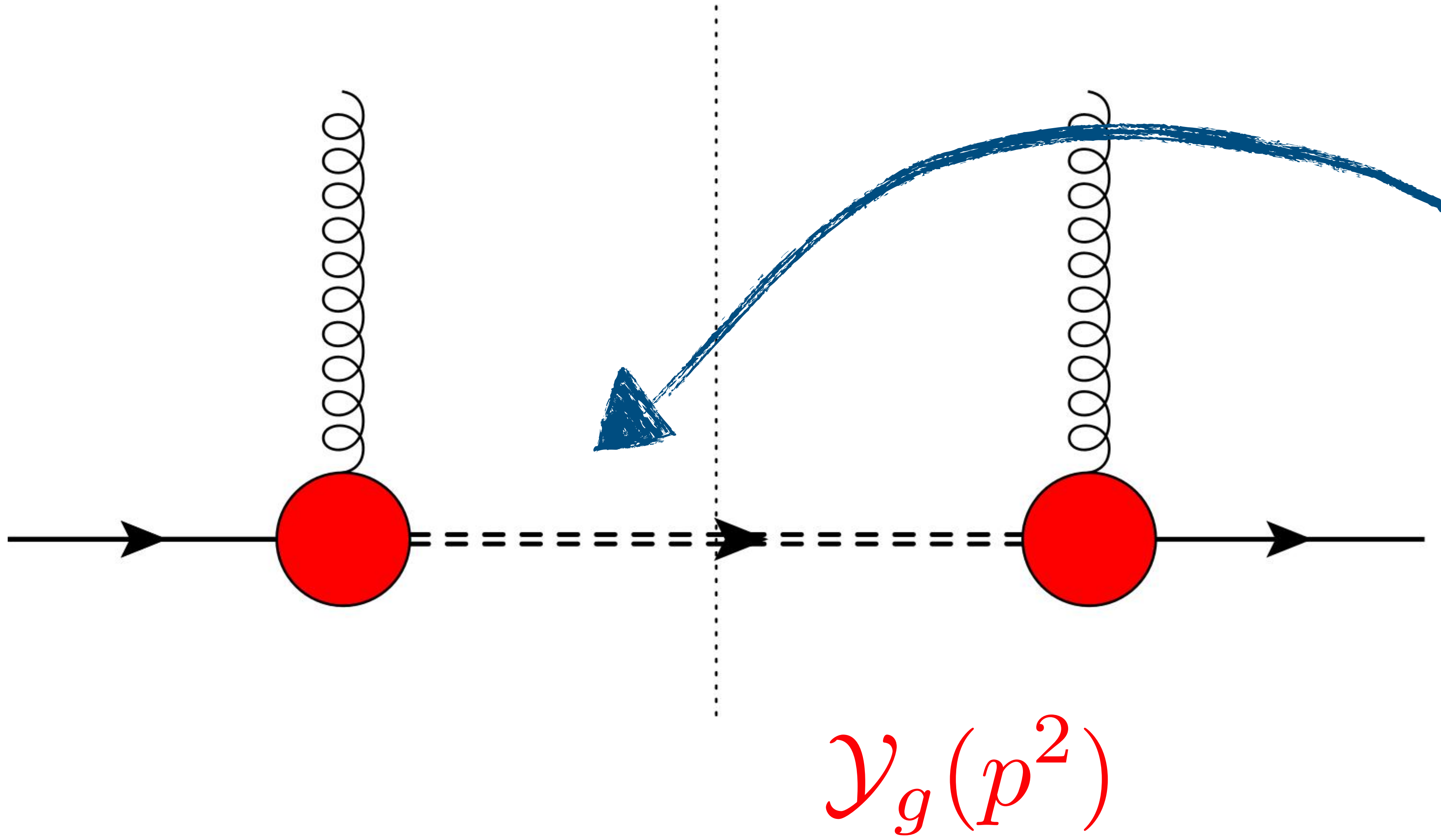
Spectator-system spectral-mass function

$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$

spectral-mass function

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



Instead of a single on-shell spectator, a continuum of spectators

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$$\rho_X\left(M_X; \{X^{(\mathrm{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\}\right) = \mu^{2a}\left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}}\right]$$

low-x (high-μ^2) tail $\propto (a - b)$

$q\bar{q}$ contributions energetically available at large M_X

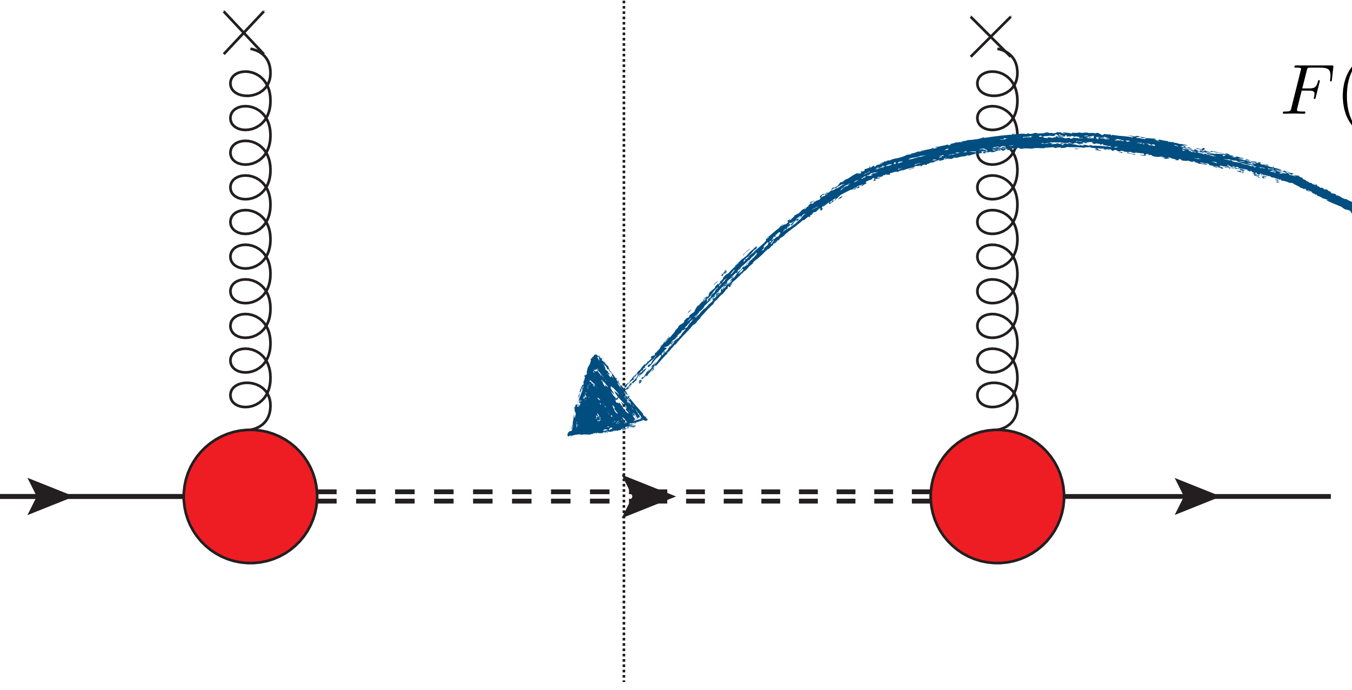
$$\mu^2 = M_X^2 - M^2$$

moderate-x trend

pure tri-quark contribution at low M_X

Our model

Spectator-system spectral-mass function



$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$

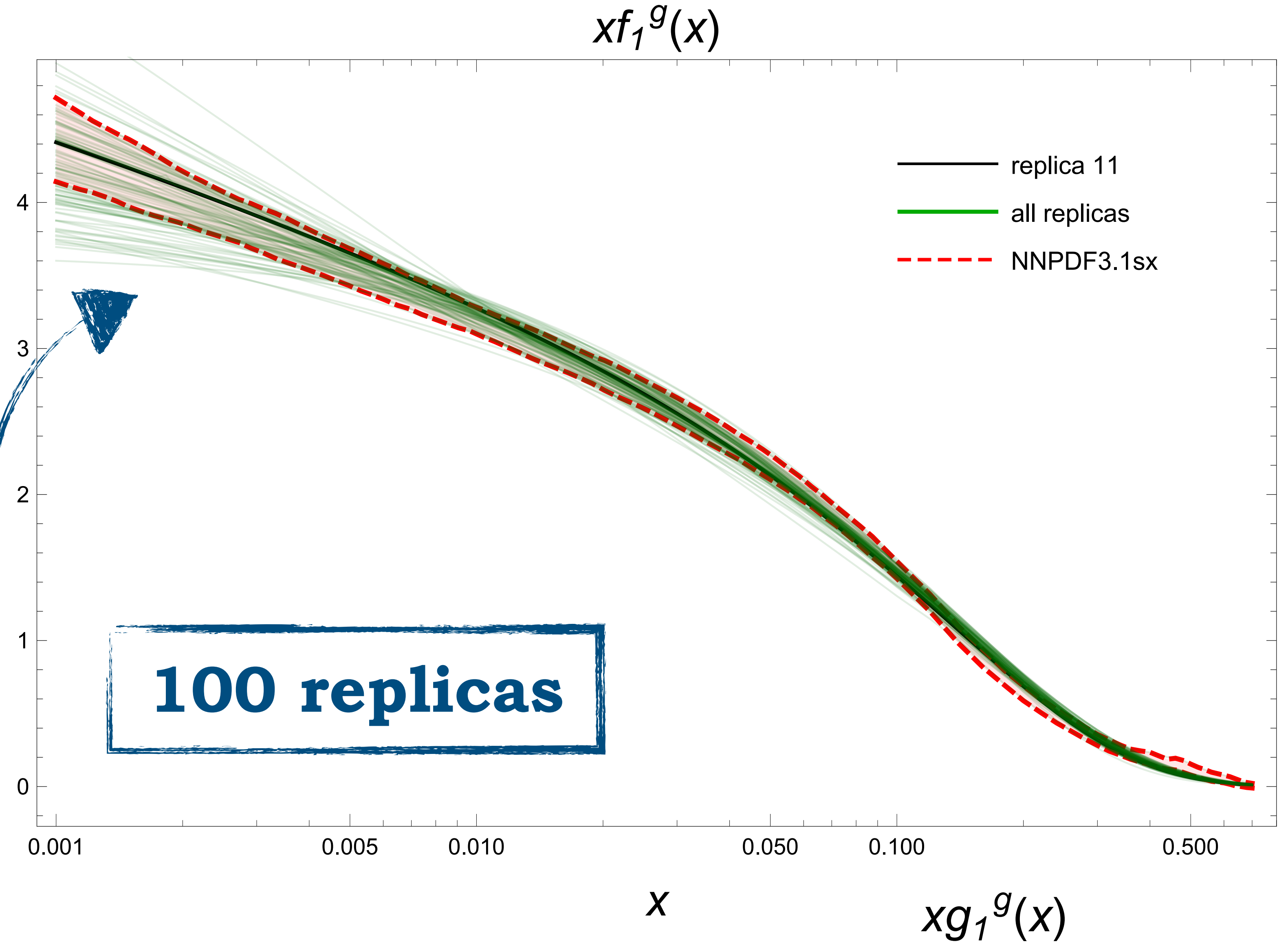
Instead of a single on-shell spectator, a continuum of spectators

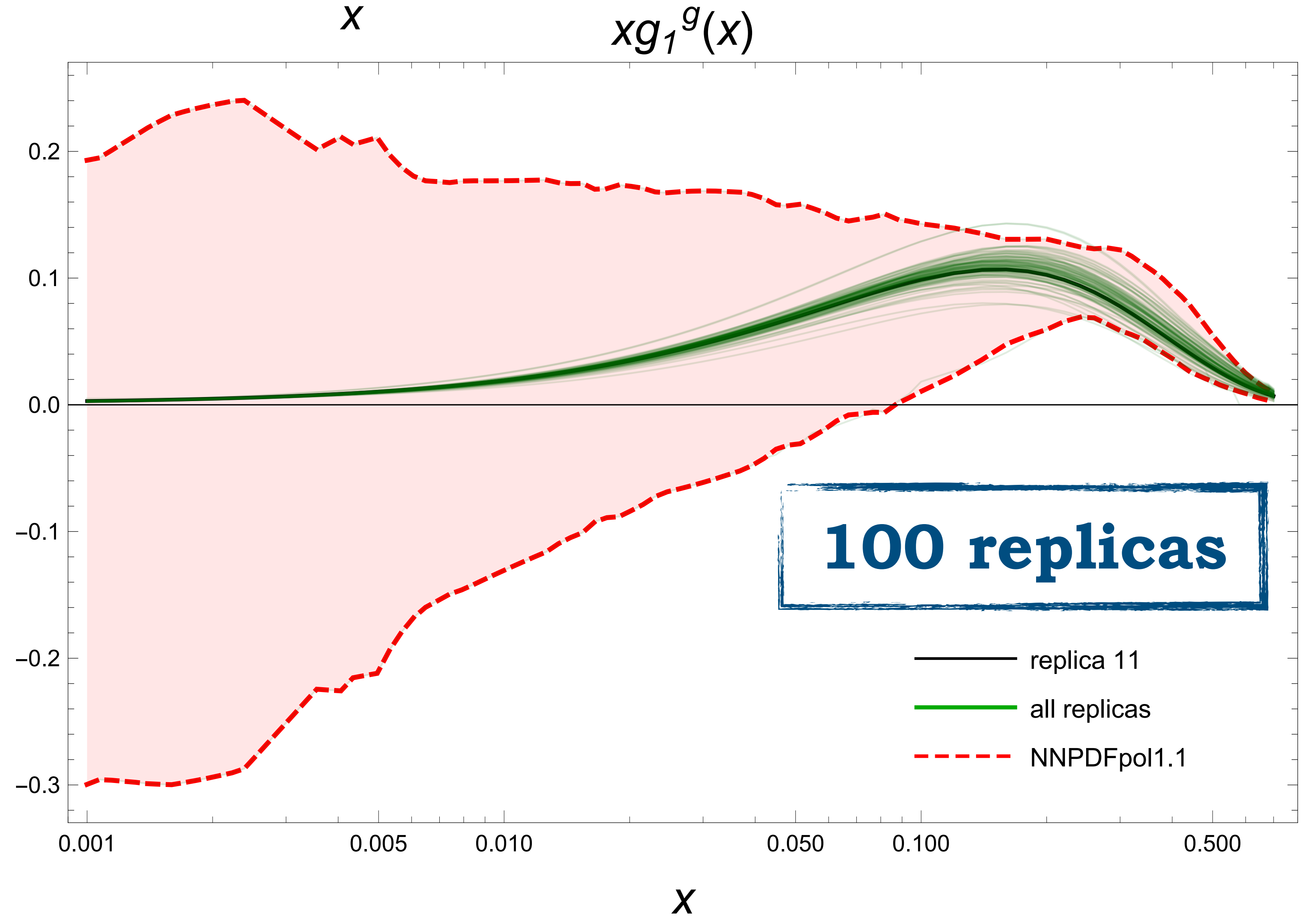
Spectral function **learns** small- and moderate-x info encoded in **NNPDF** collinear parametrizations (`NNPDF3.1sx + NNPDFpol1.1`)

- ☑ **Simultaneous fit** of f_1 and g_1 PDFs
- ☑ Inclusion of small-x resummation effects (**BFKL**)
- ☑ Calculation of all twist-2 T-even gluon TMDs

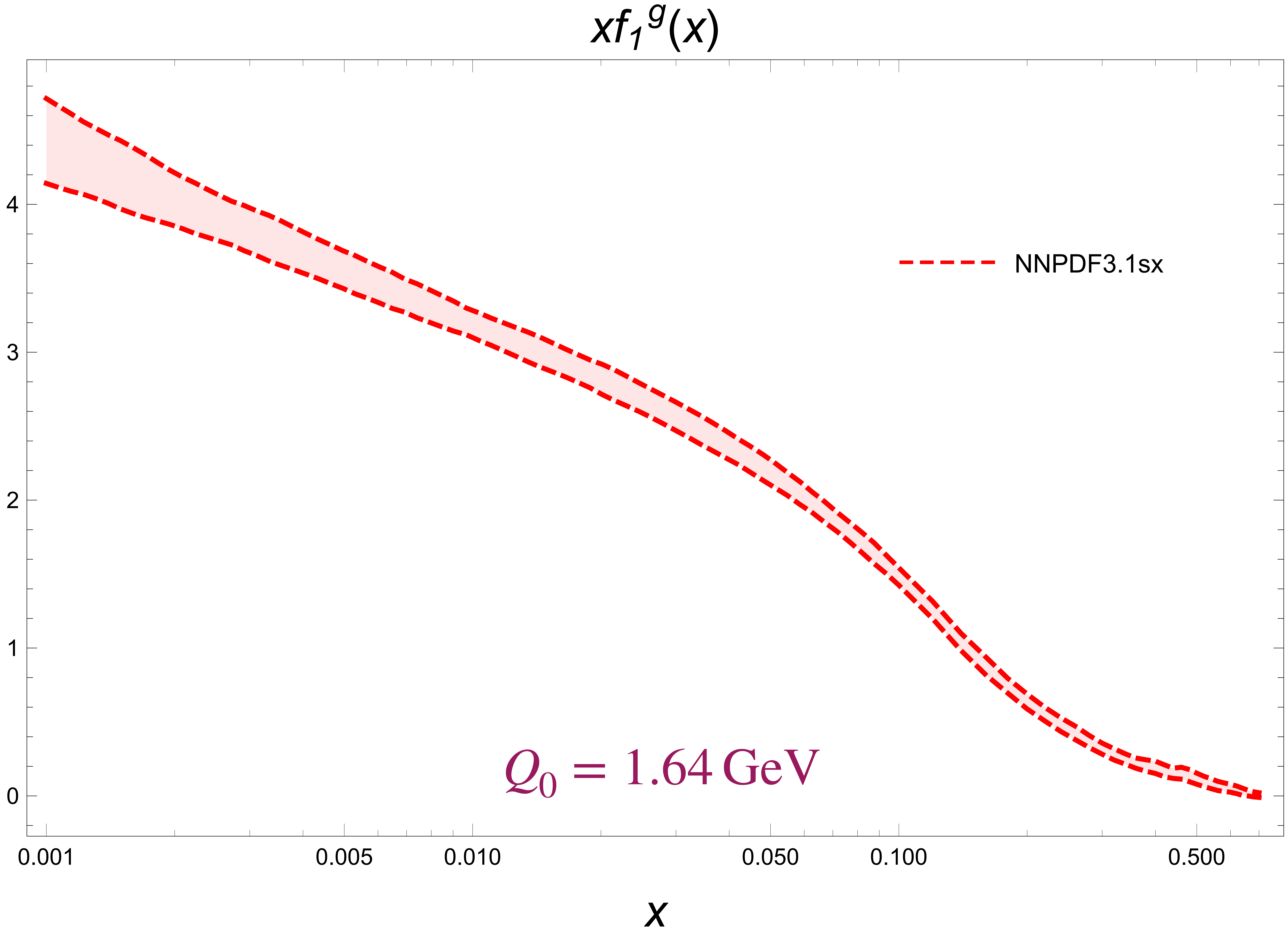
Link with collinear factorization

p_T-integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution





Unpolarized gluon PDF



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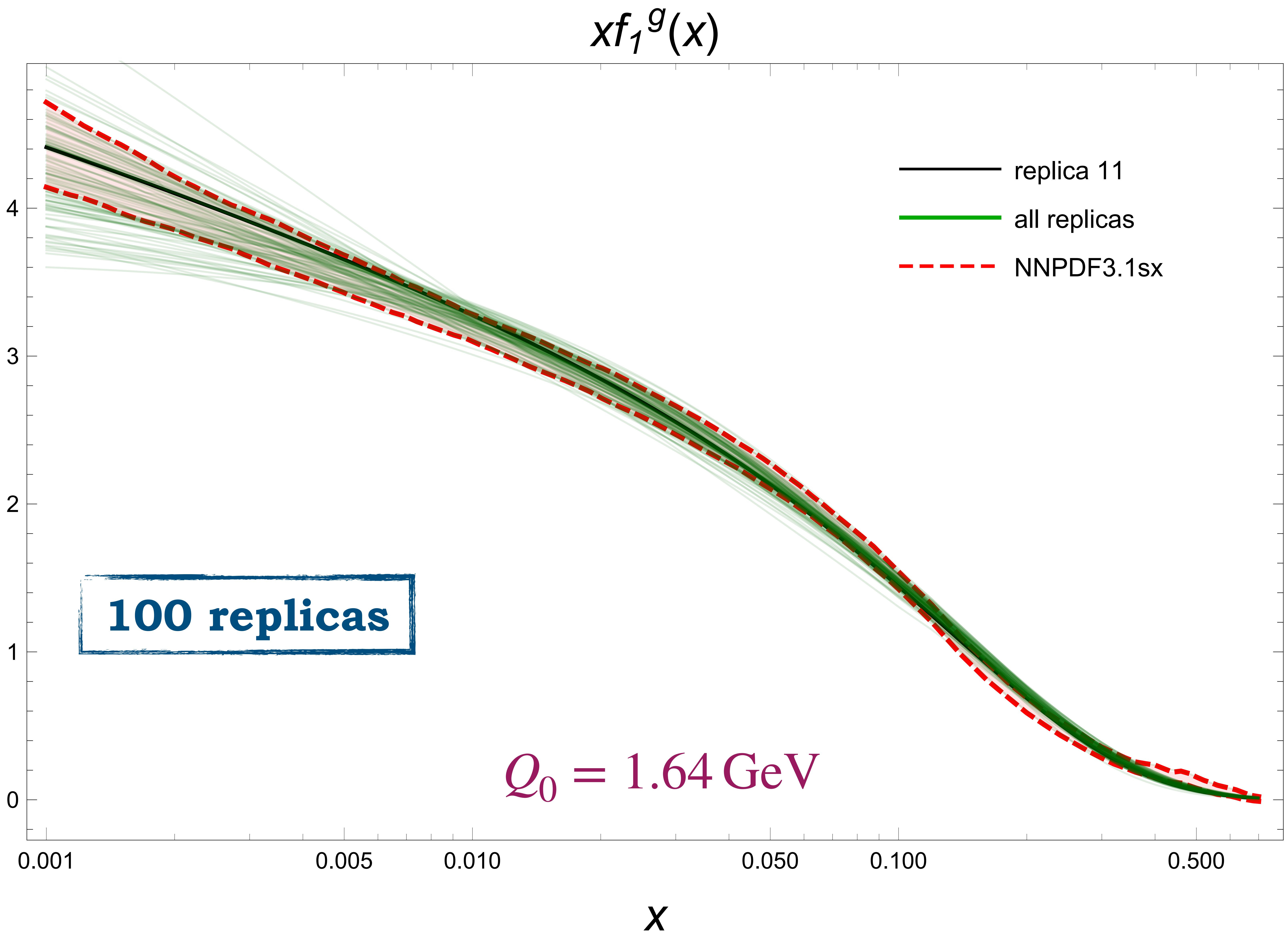
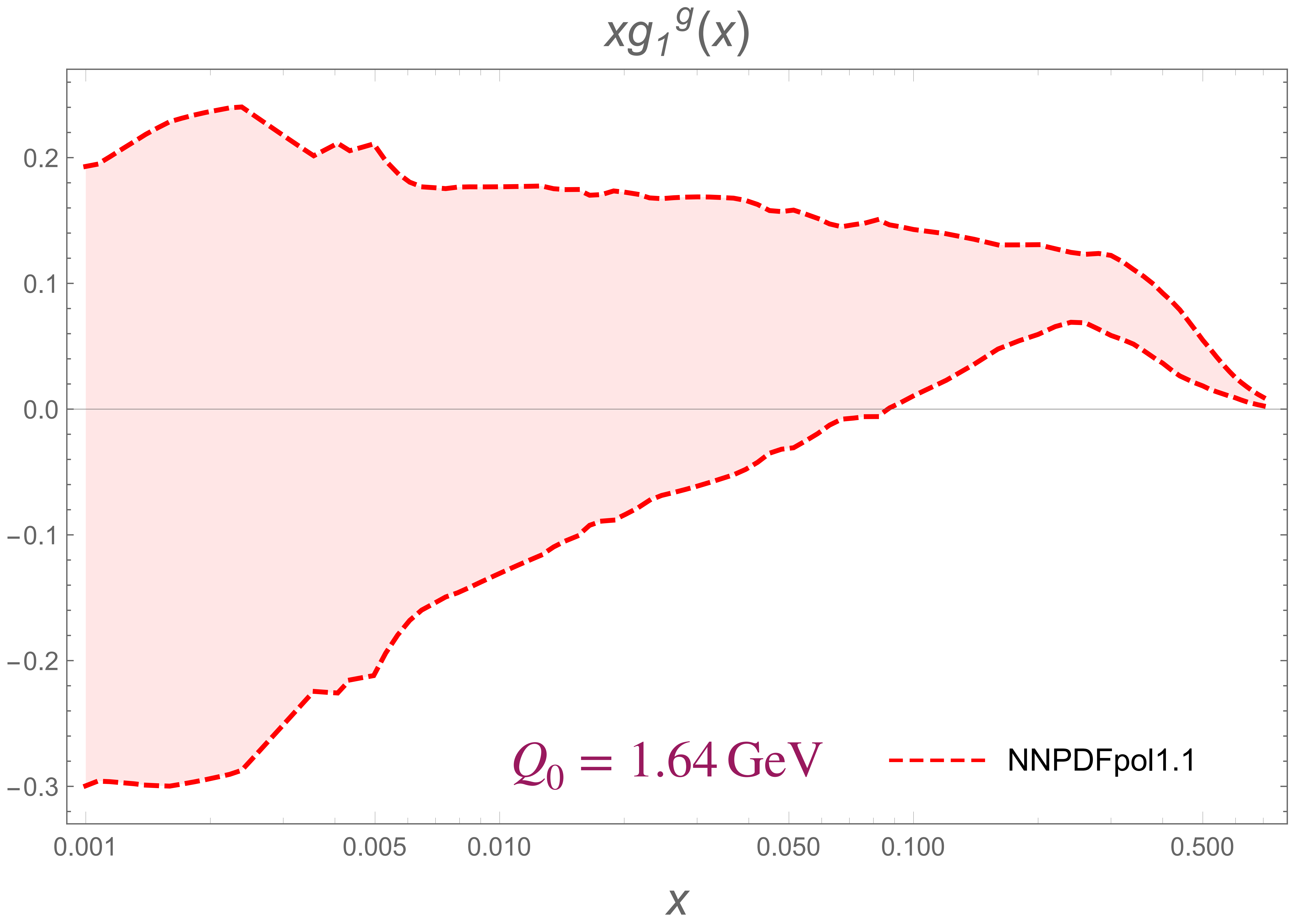


Helicity gluon PDF



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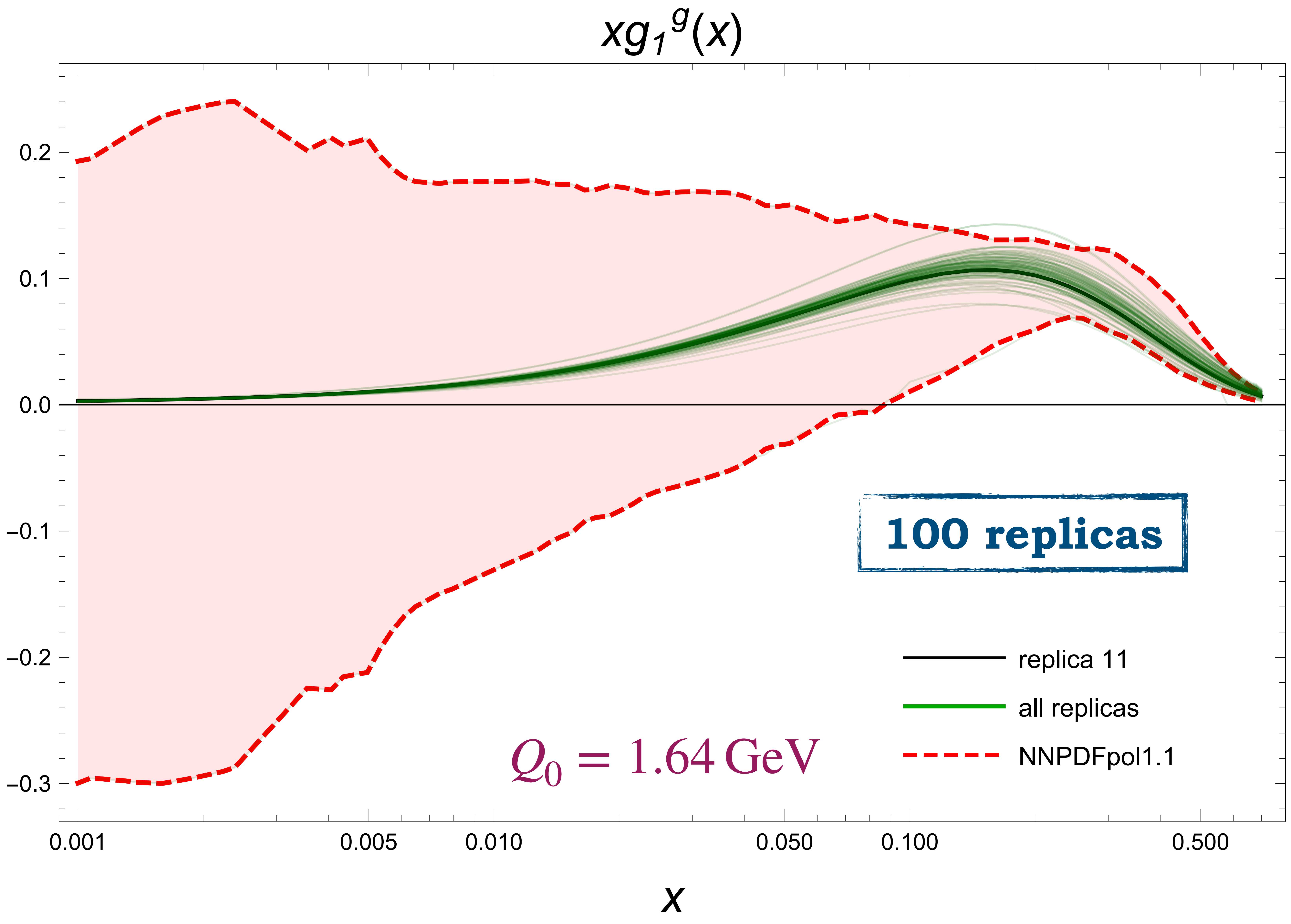

Fit specifics

$$\chi^2/\text{d.o.f.} = 0.54 \pm 0.38$$

no **overlearning**, just large errors for g_1

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$$\langle x \rangle_g = \int_0^1 dx\, x\, f_1^g(x, Q_0) \qquad S_g = \frac{1}{2}\langle 1 \rangle_{\Delta g} = \int_0^1 dx\, g_1^g(x, Q_0)$$

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Our model @ $Q_0 = 1.64$ GeV

$$\langle x \rangle_g = 0.424(9)$$

$$\langle S \rangle_g = 0.159(11)$$

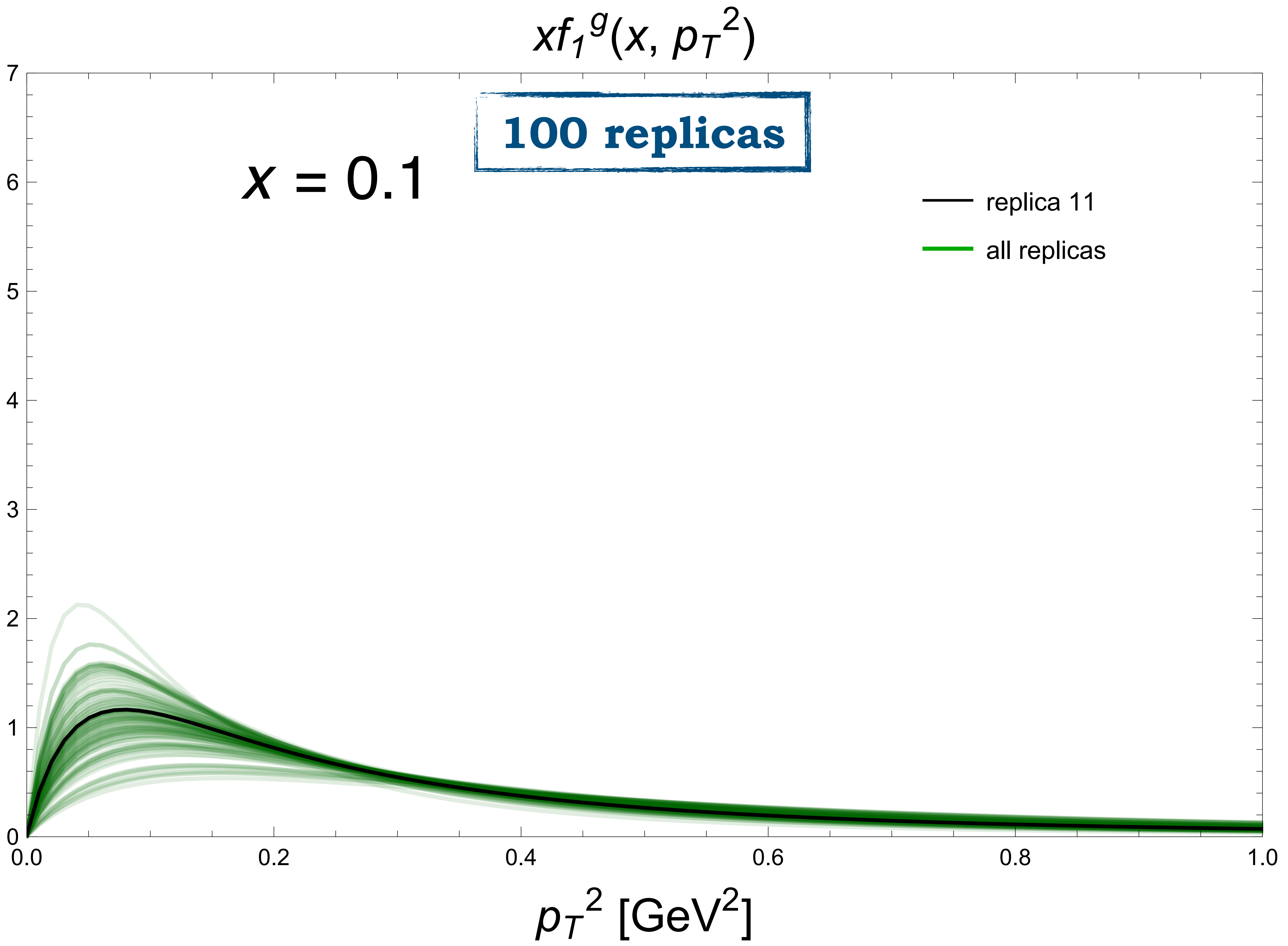
Lattice @ $Q_0 = 2$ GeV

$$\langle x \rangle_g = 0.427(92)$$

$$\langle J \rangle_g = 0.187(46)$$

[C. Alexandrou *et al.* (2020)]

Unpolarized gluon TMD



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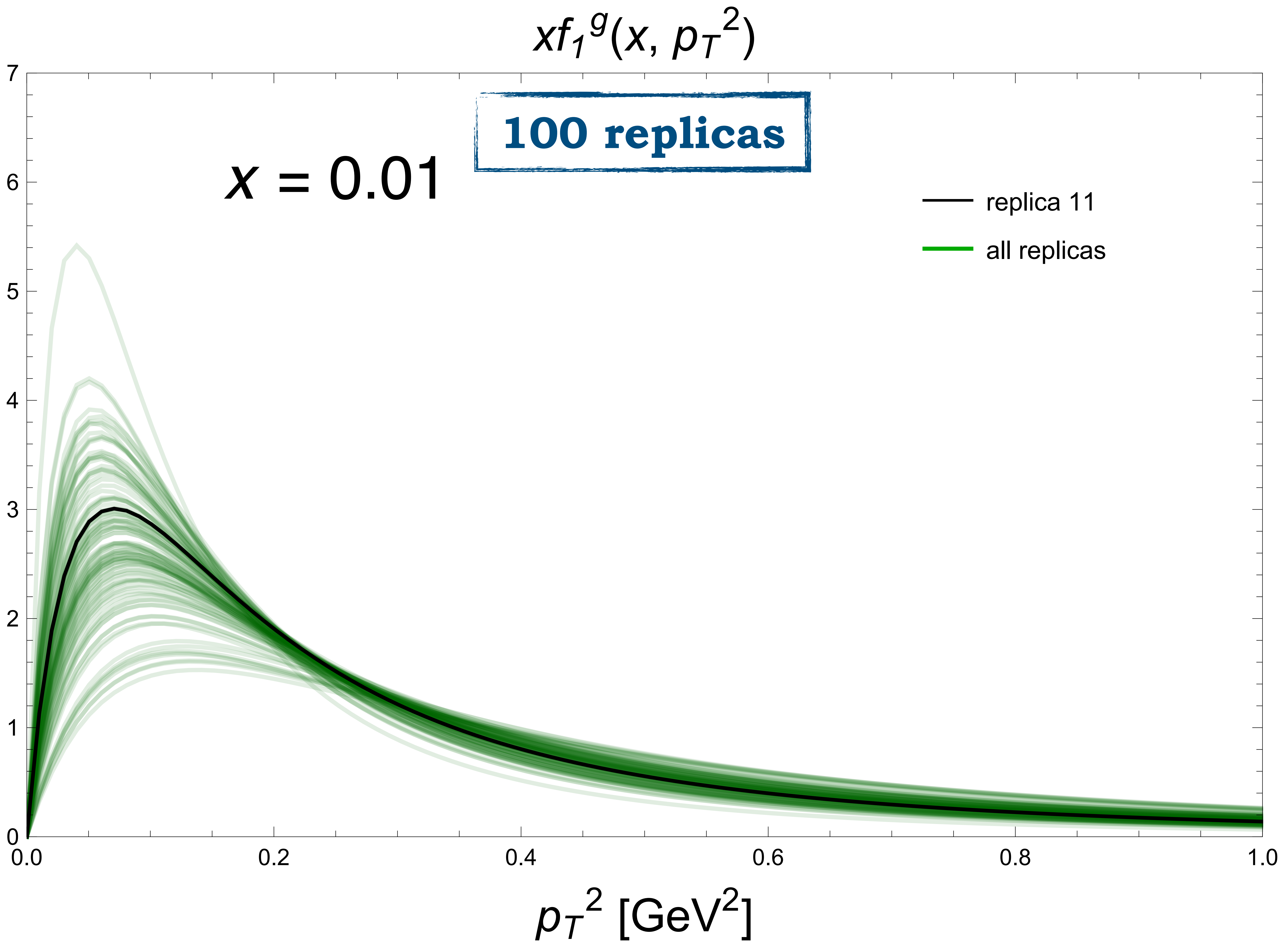

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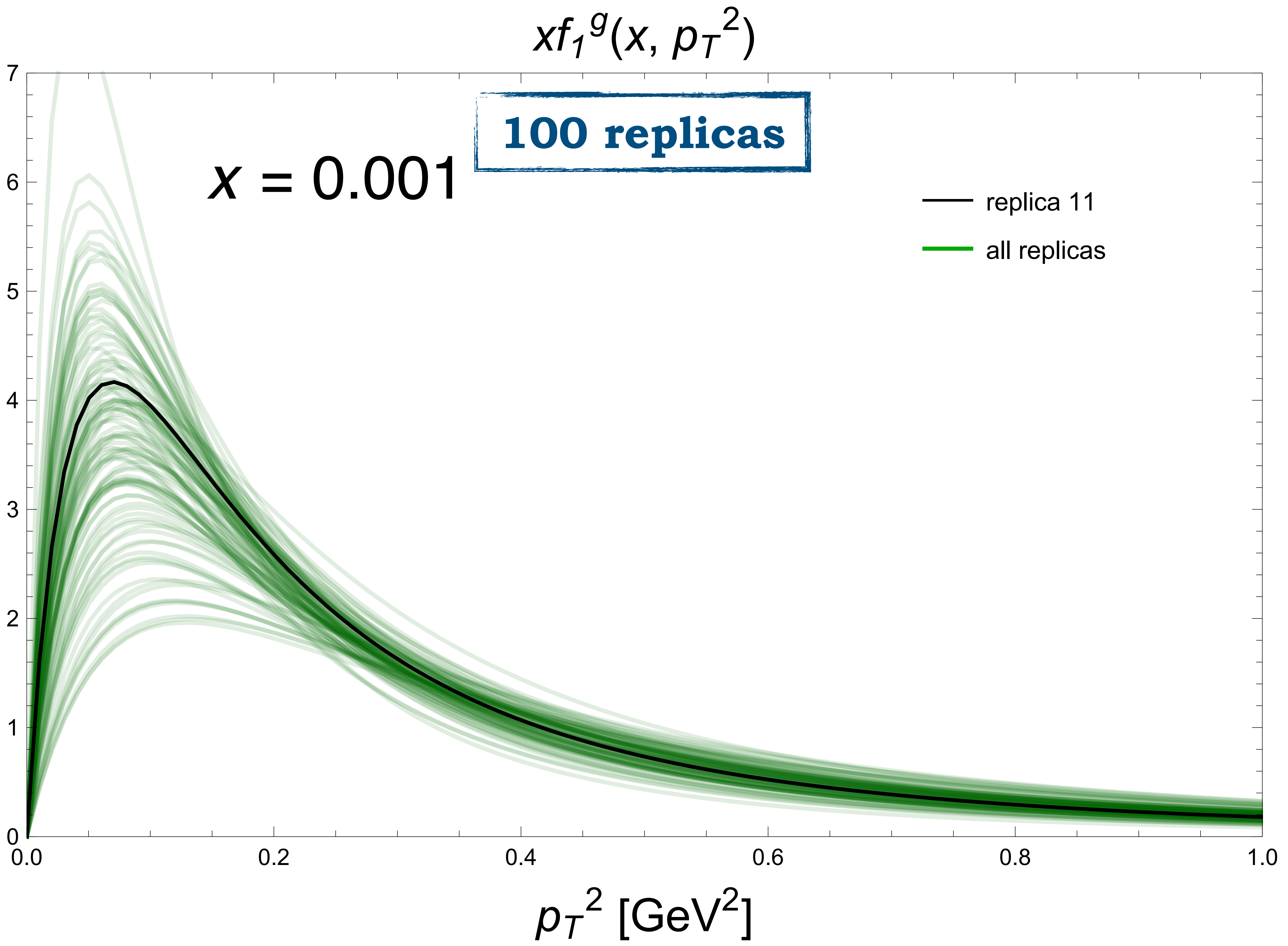

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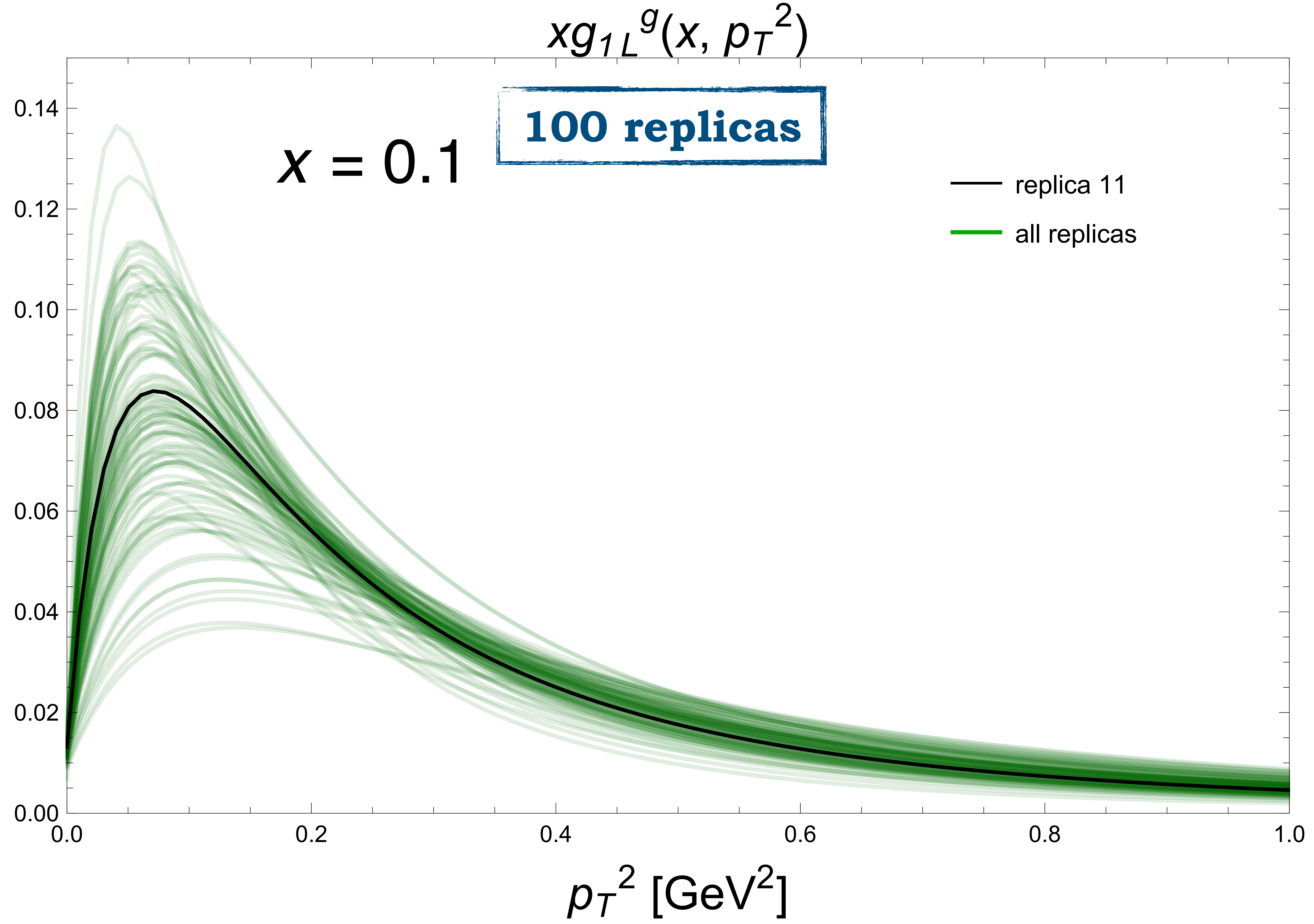

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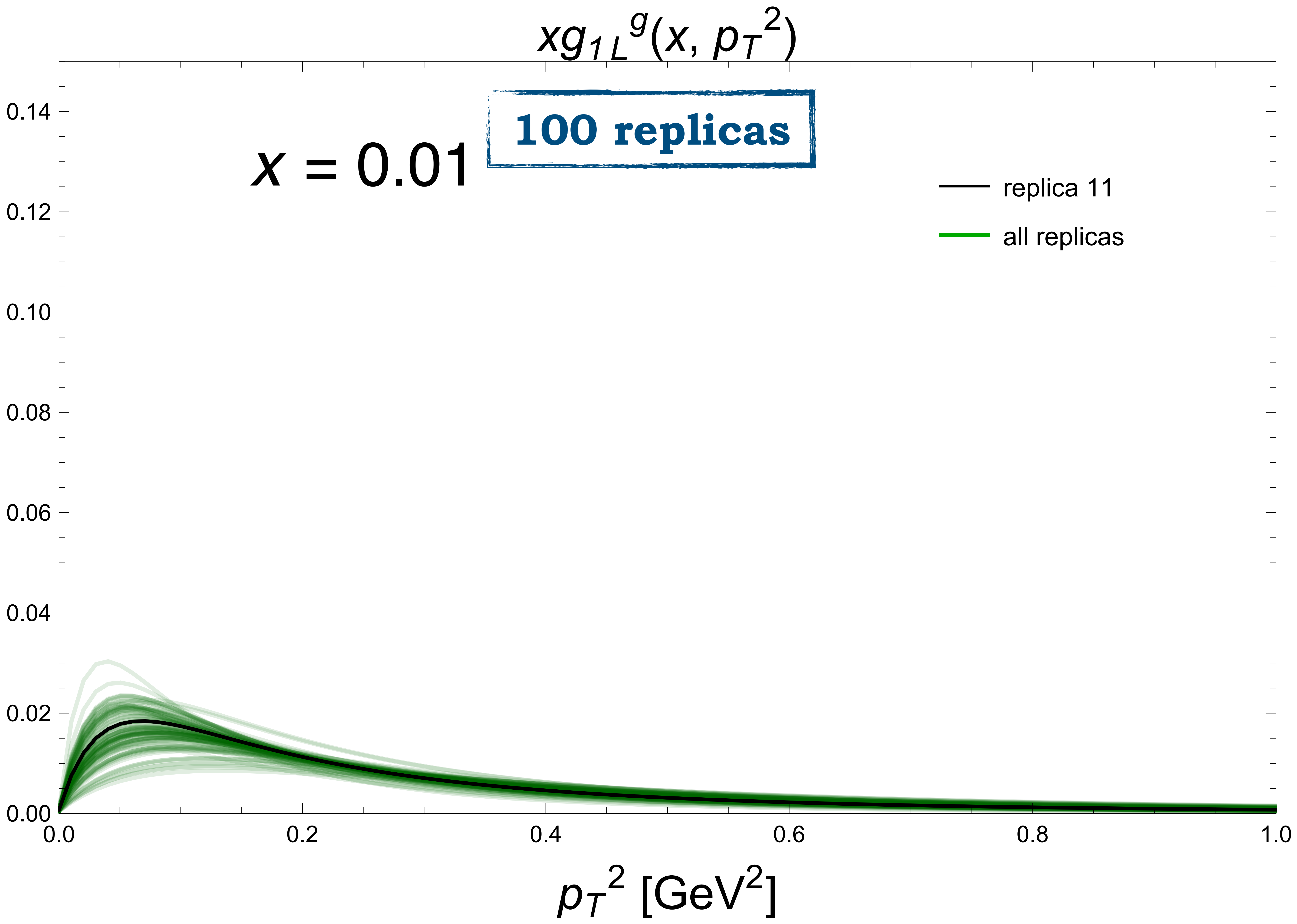

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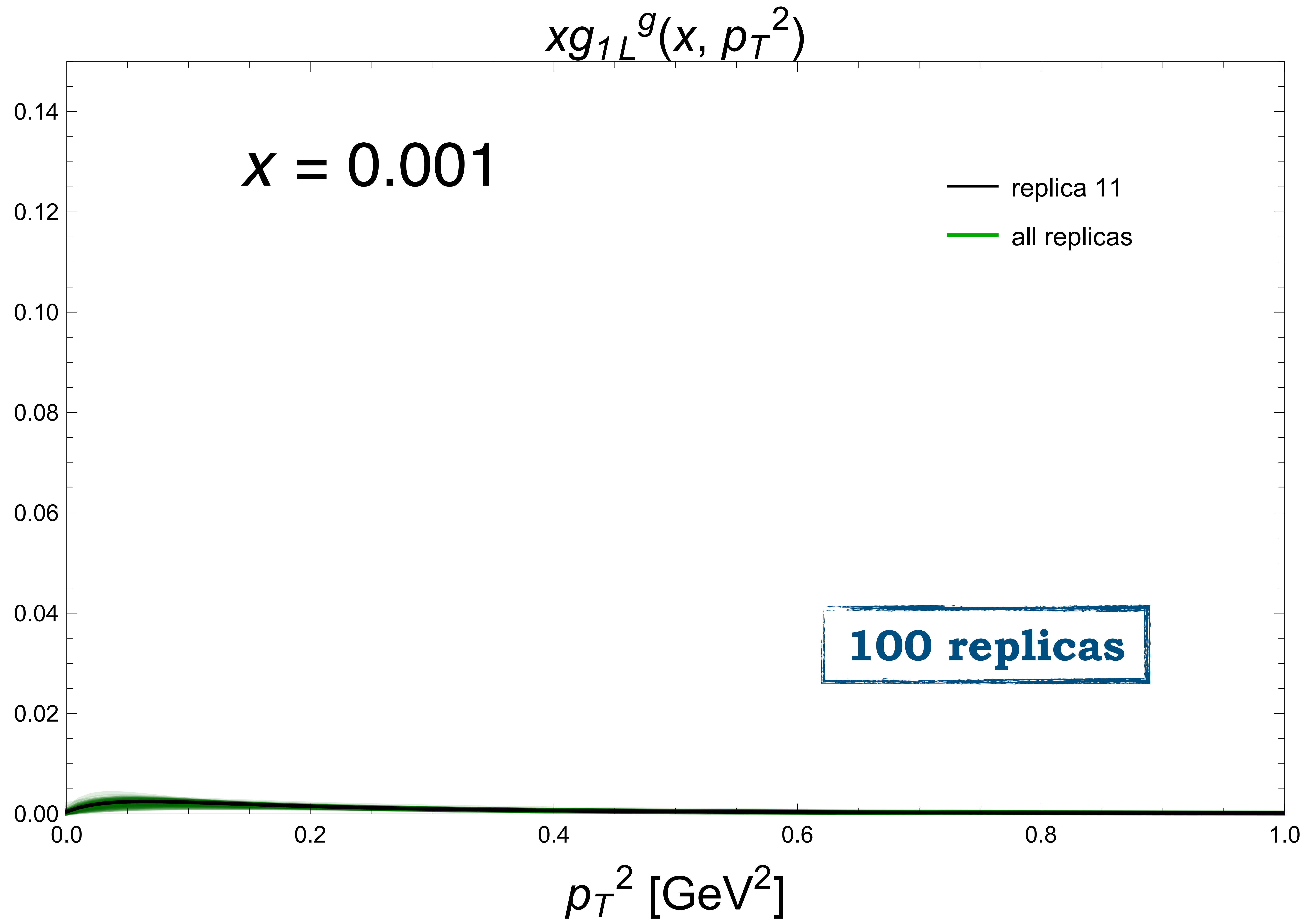
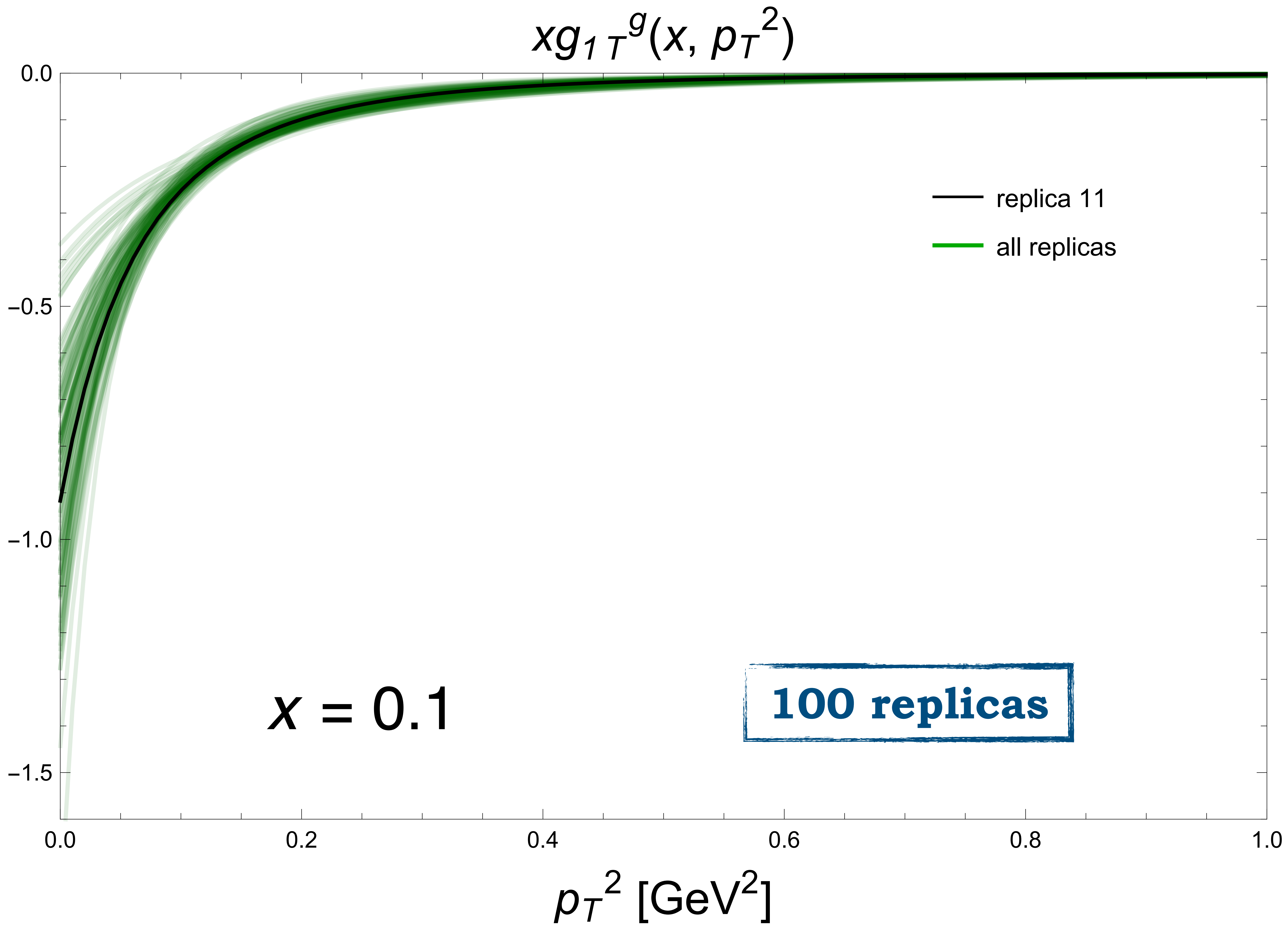


Worm-gear gluon TMD



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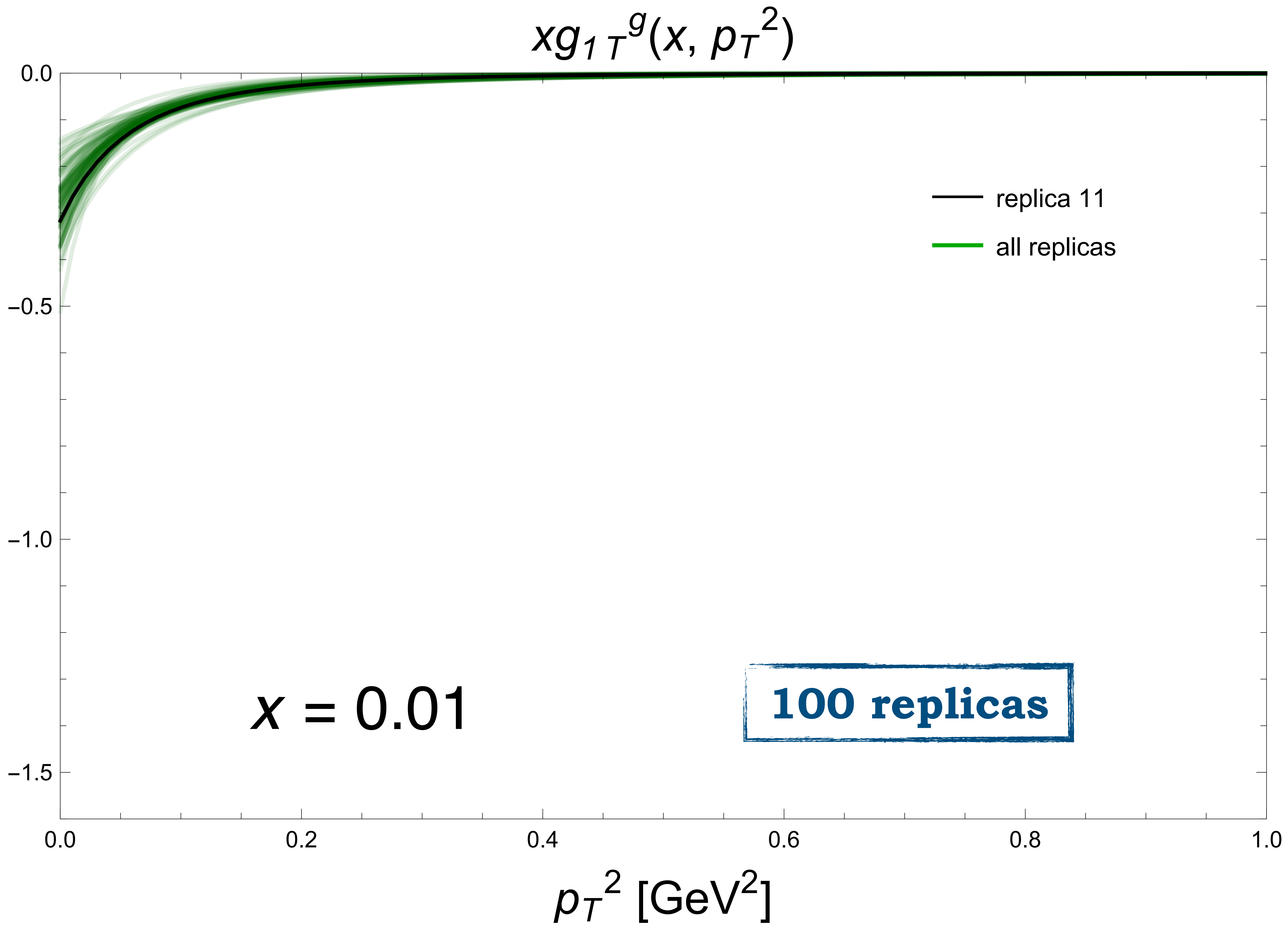

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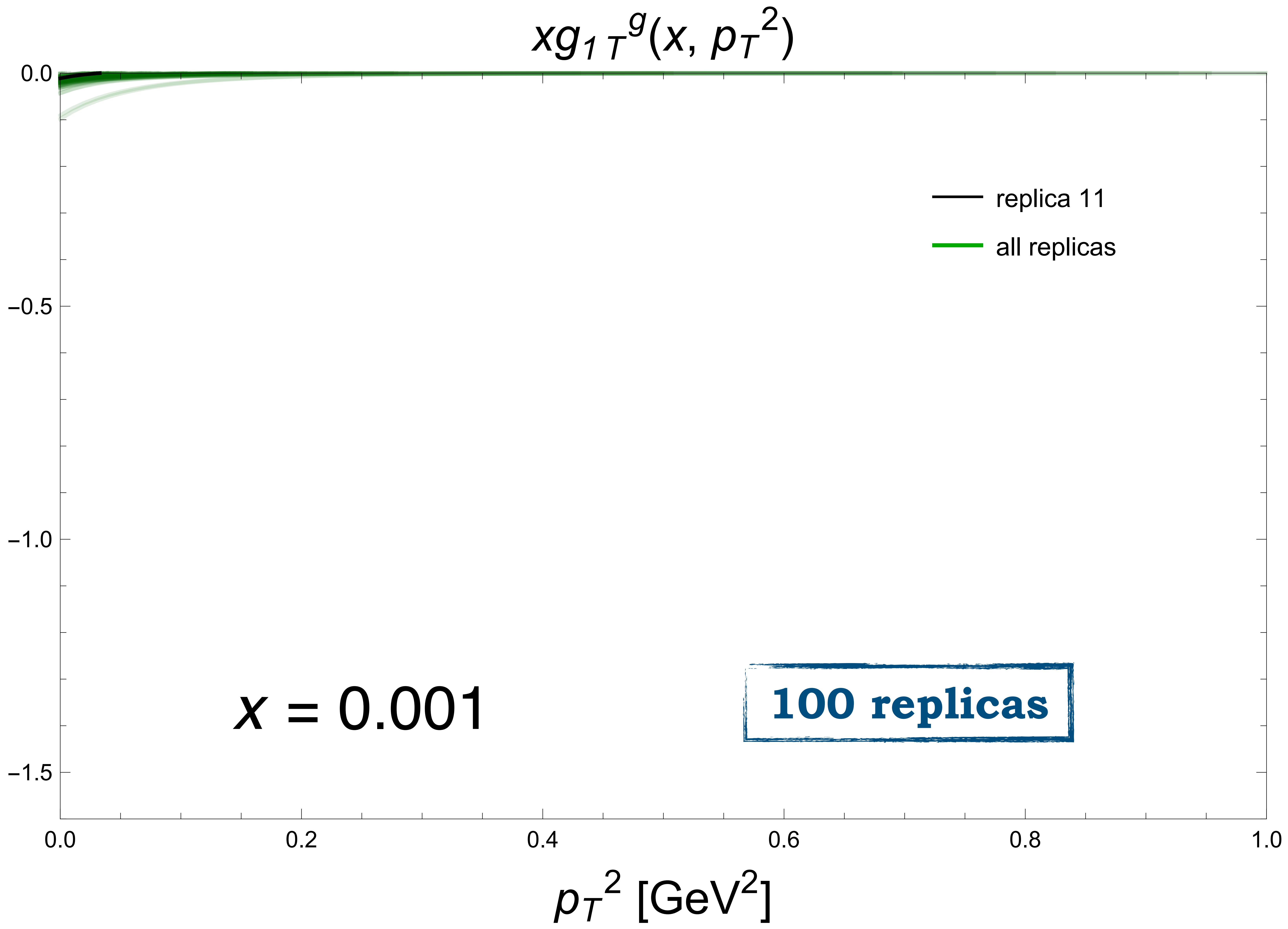
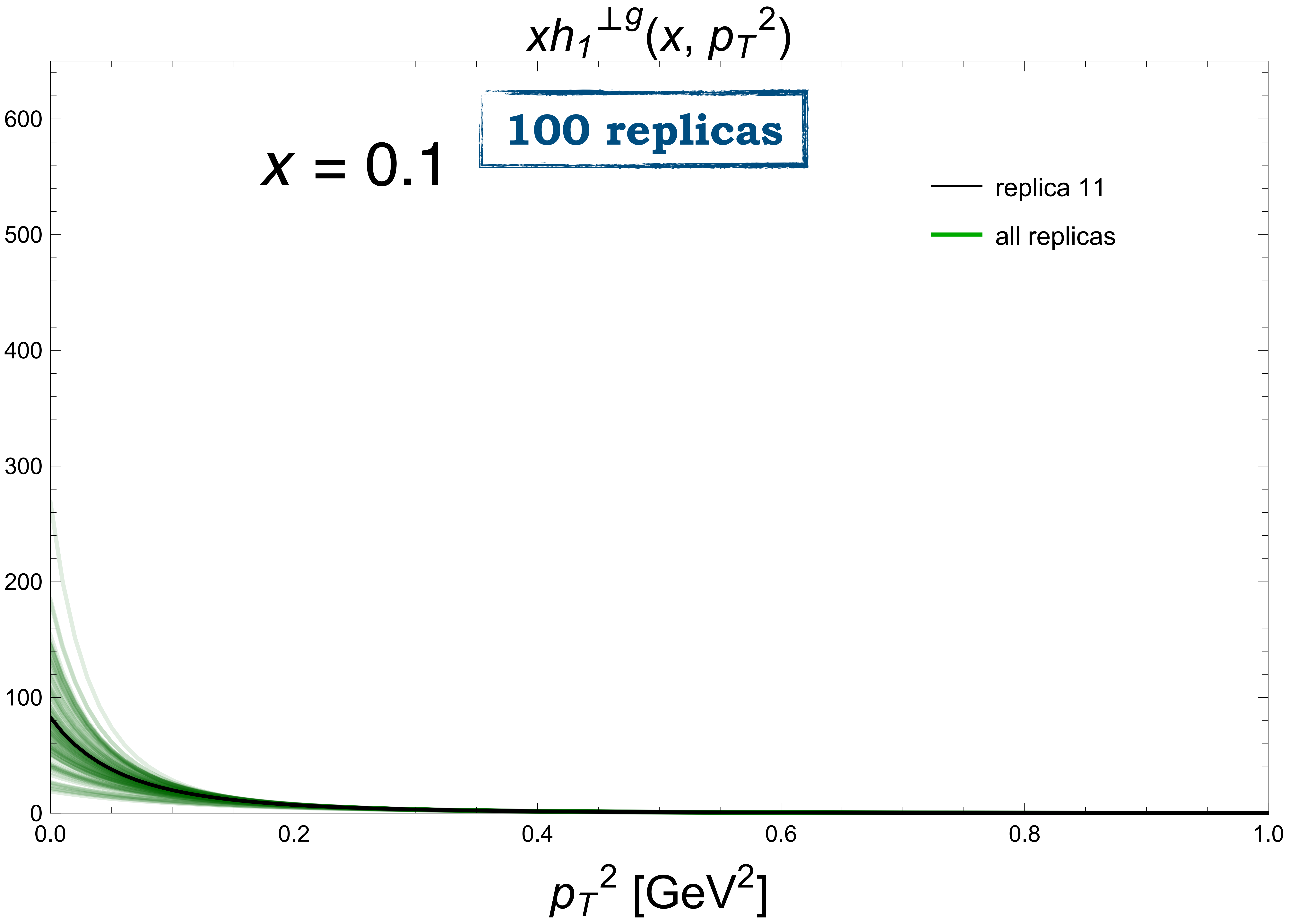


Boer-Mulders gluon TMD



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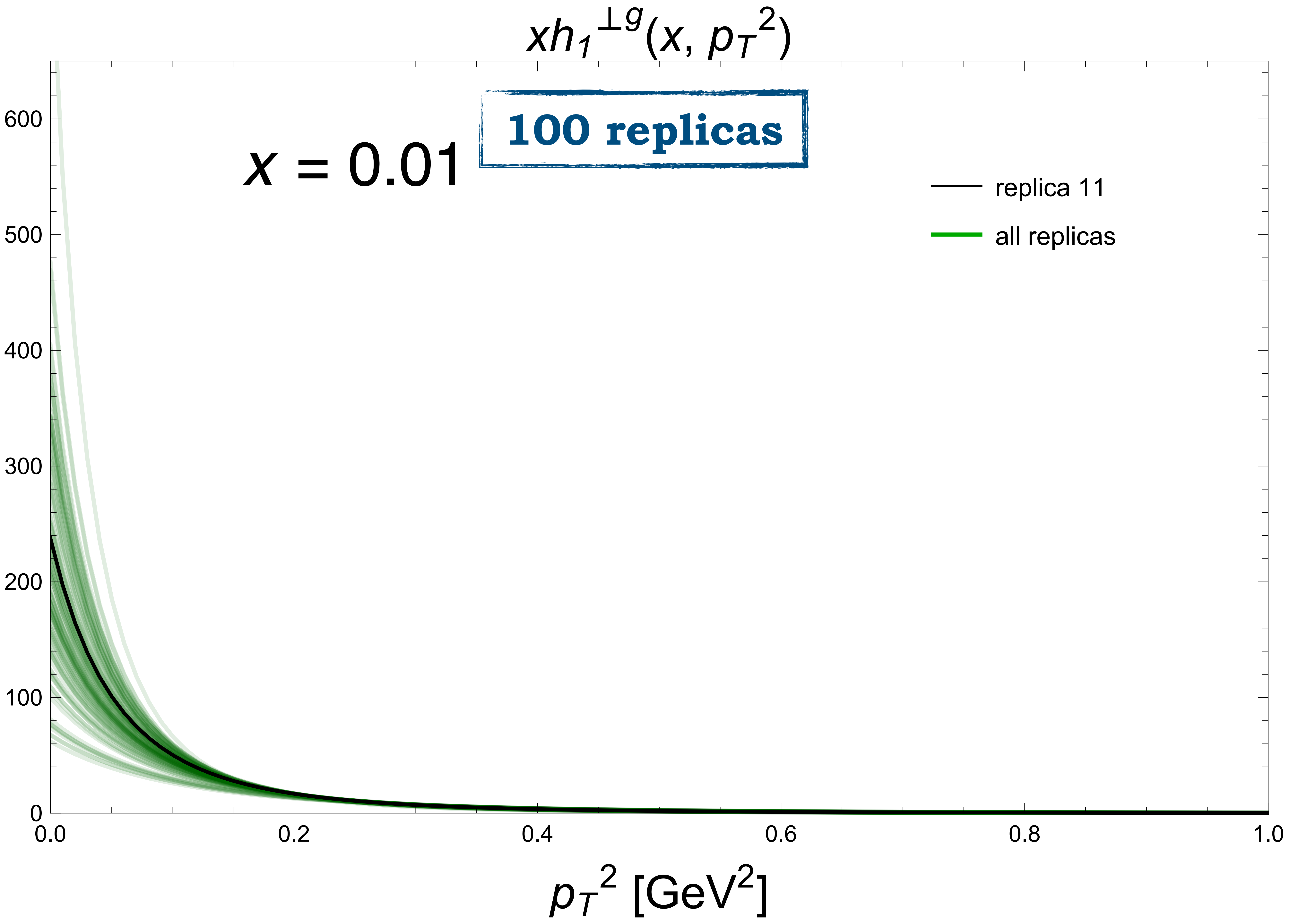

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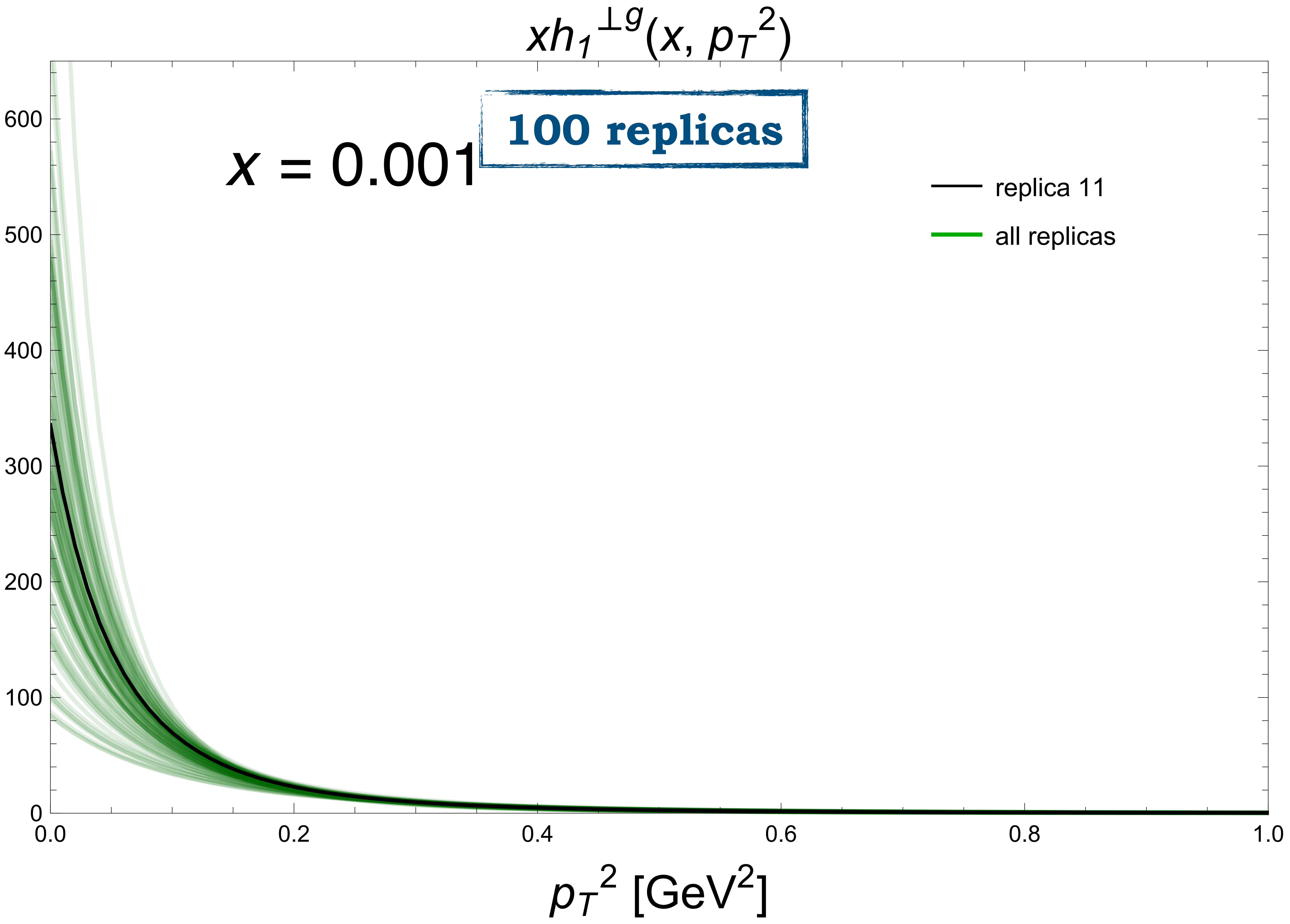


Bottom line

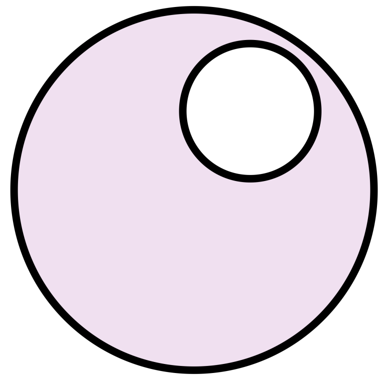
- Each TMD shows a distinctive x- and p_T-behavior

- Data on gluon TMDs will exclude many replicas and constrain parameters not yet so well constrained by collinear PDFs

- Simultaneous fit on two distinct PDFs provides with *corroborating evidence* of reliability of our model

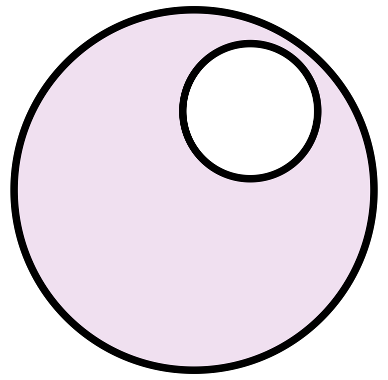
- Standard CSS 🔗🔗 *evolution* can be turned on

ρ-densities



Unpolarized [u/u] $f_1(x, p_x, p_y)$

ρ-densities



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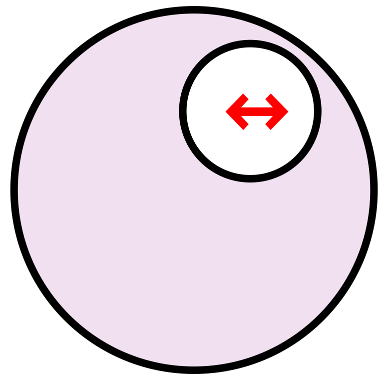
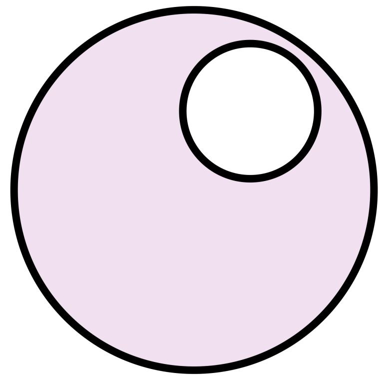


Boer-Mulders [↔/u] $f_1(x, p_x, p_y) + \dfrac{p_x^2 - p_y^2}{2M^2} h_1^{\perp}(x, p_x, p_y)$

ρ-densities



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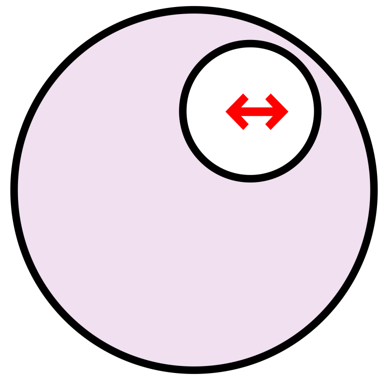

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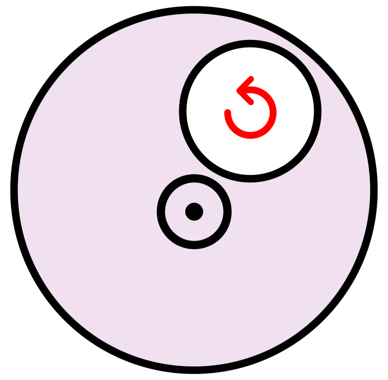
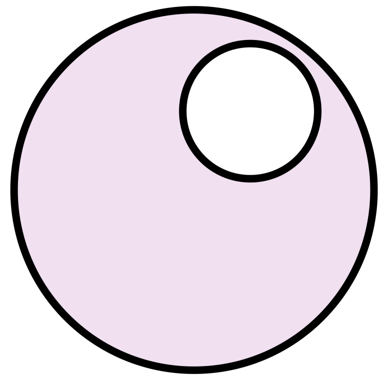


Helicity [↺/+]

$$\frac{1}{2}\left[f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y)\right]$$

ρ-densities



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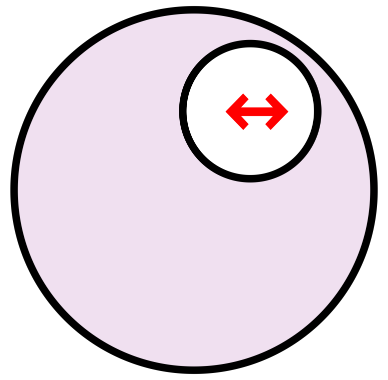

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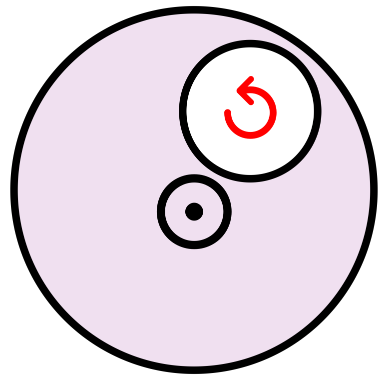

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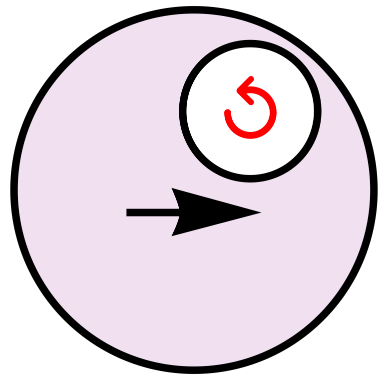
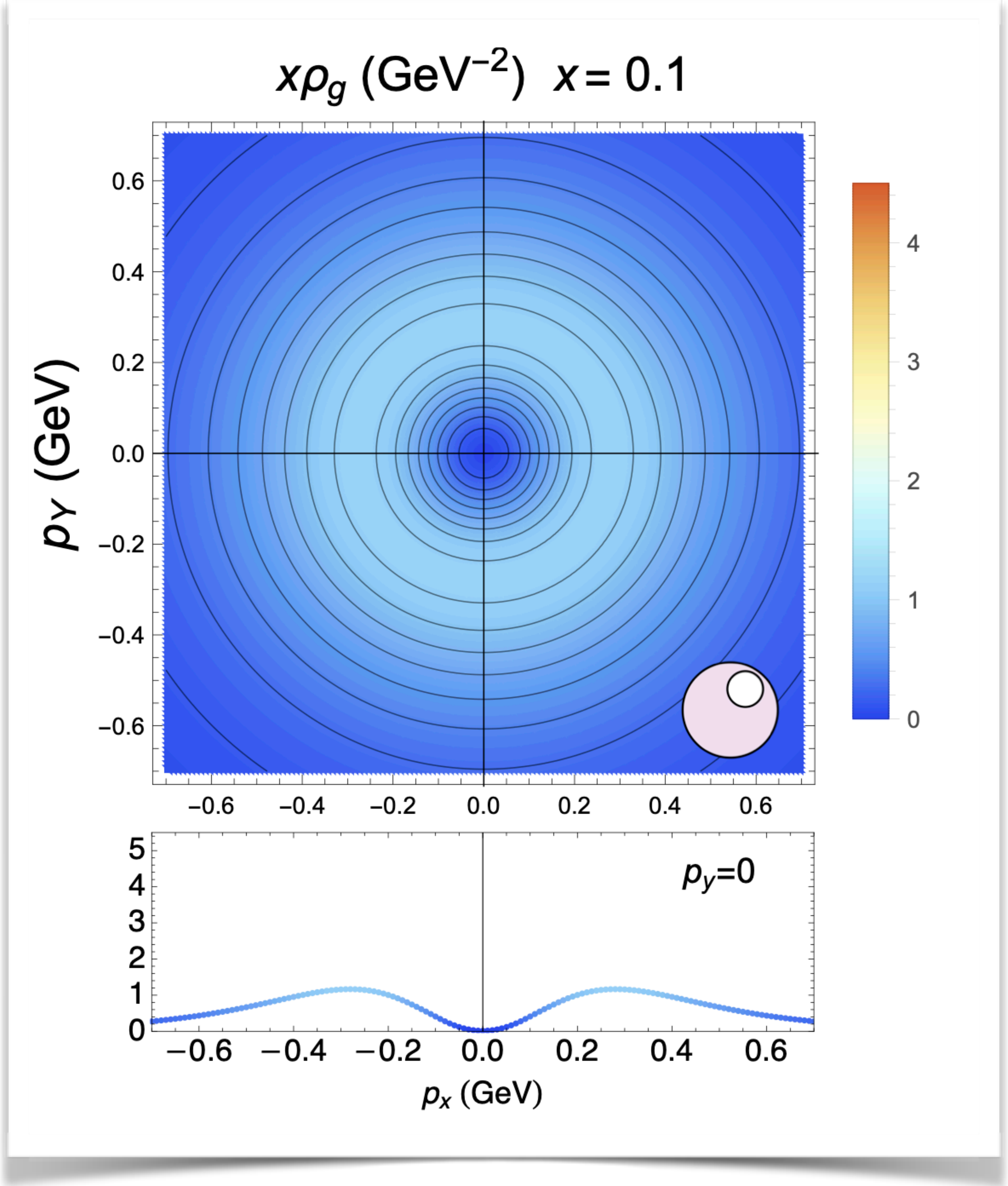


Worm-gear [↺/→]

$$f_1(x, p_x, p_y) - \frac{p_x}{M} g_{1T}(x, p_x, p_y)$$

3D tomography: the gluon content in the proton

unpolarized TMD



🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels, *Eur. Phys. J. C* **80** (2020) no.8 [arXiv:2005.02288]]

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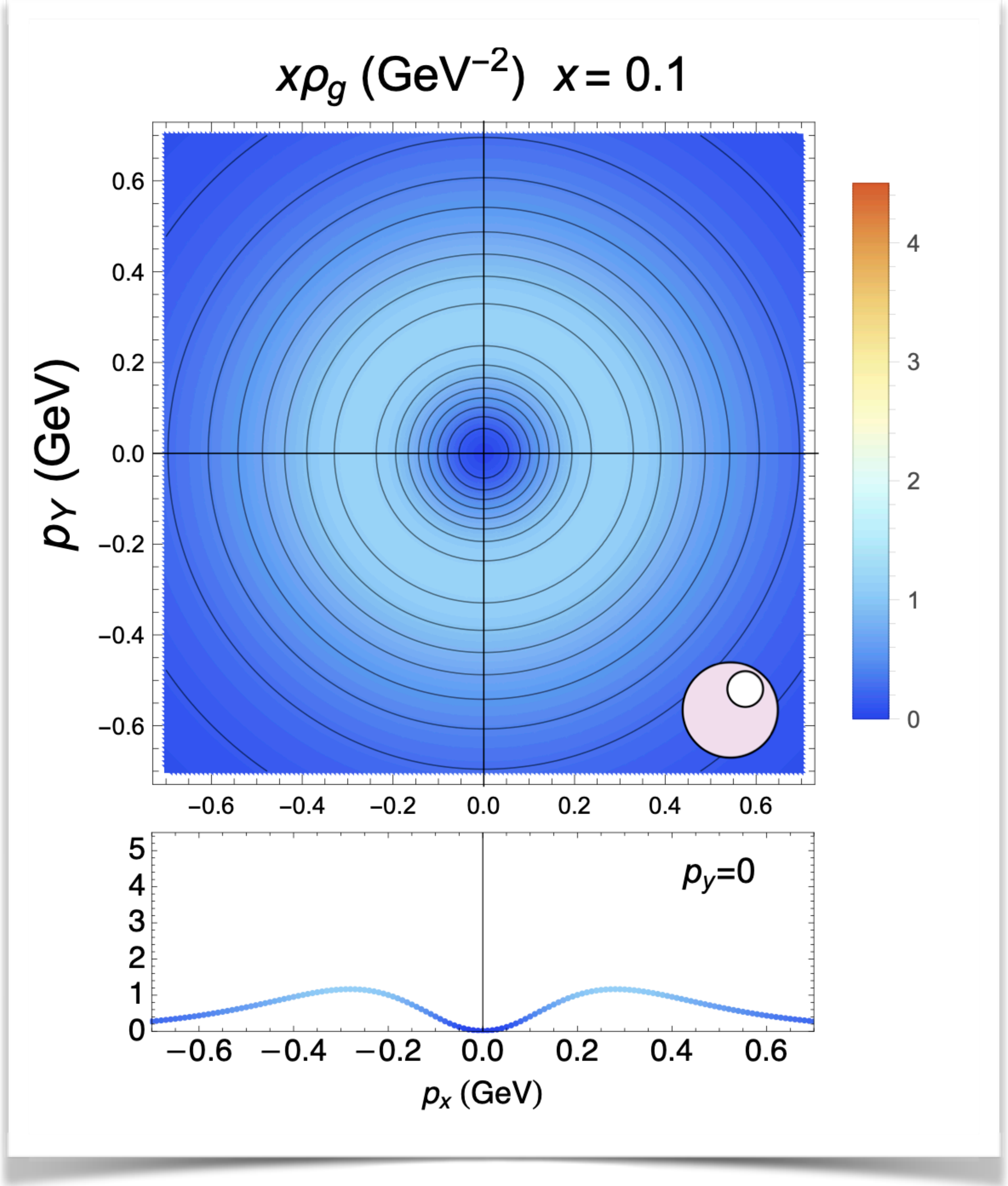
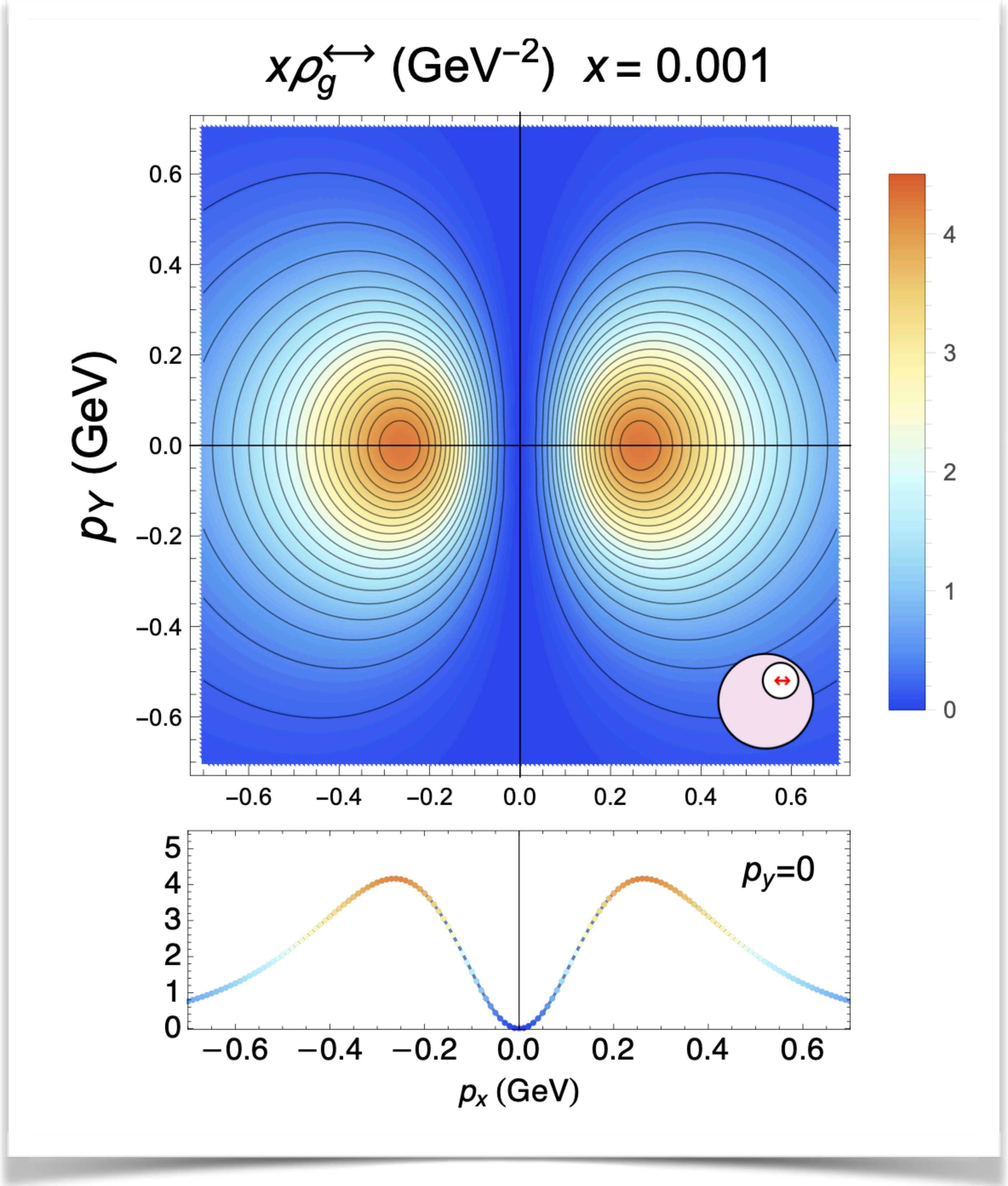


Boer-Mulders



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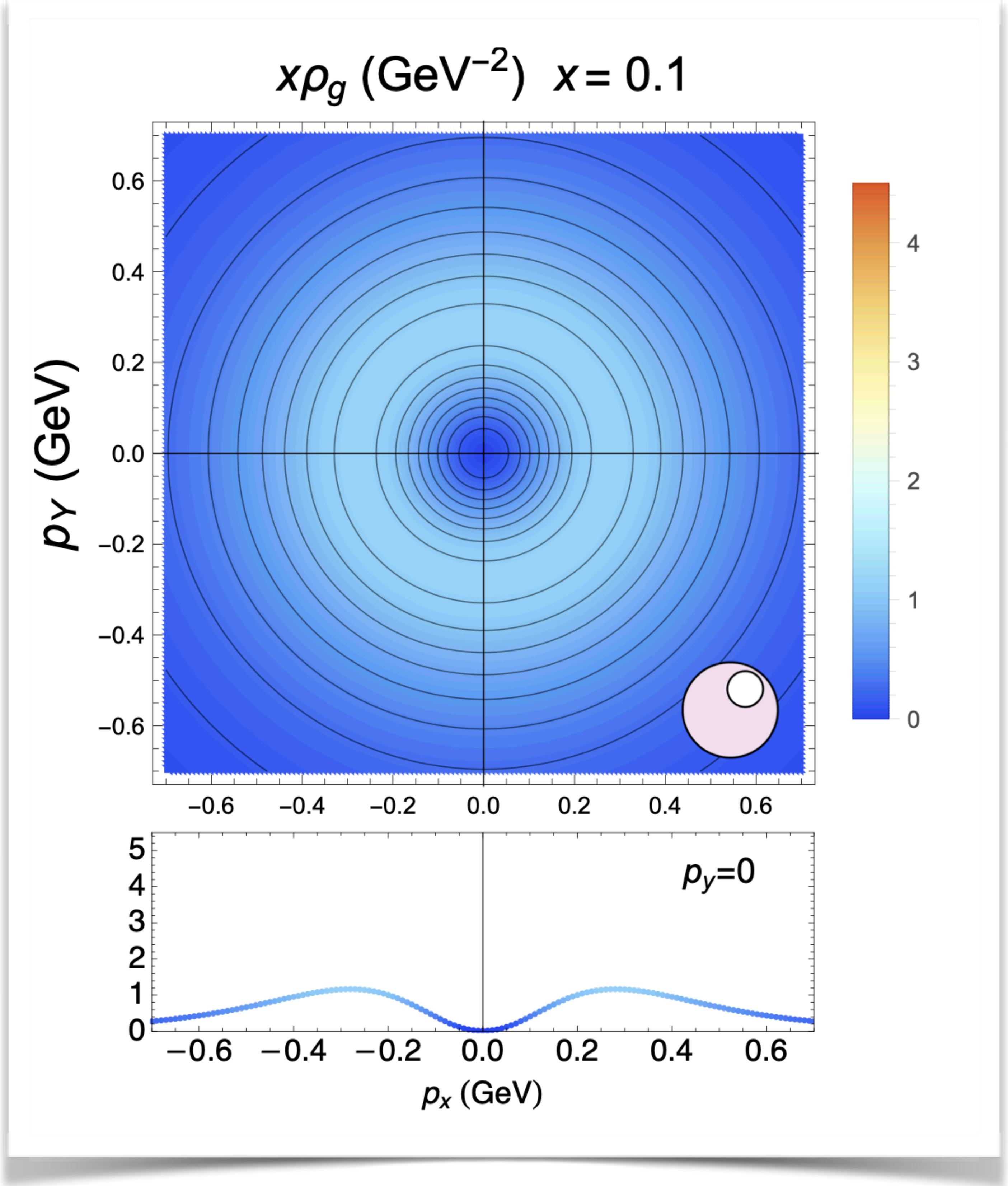

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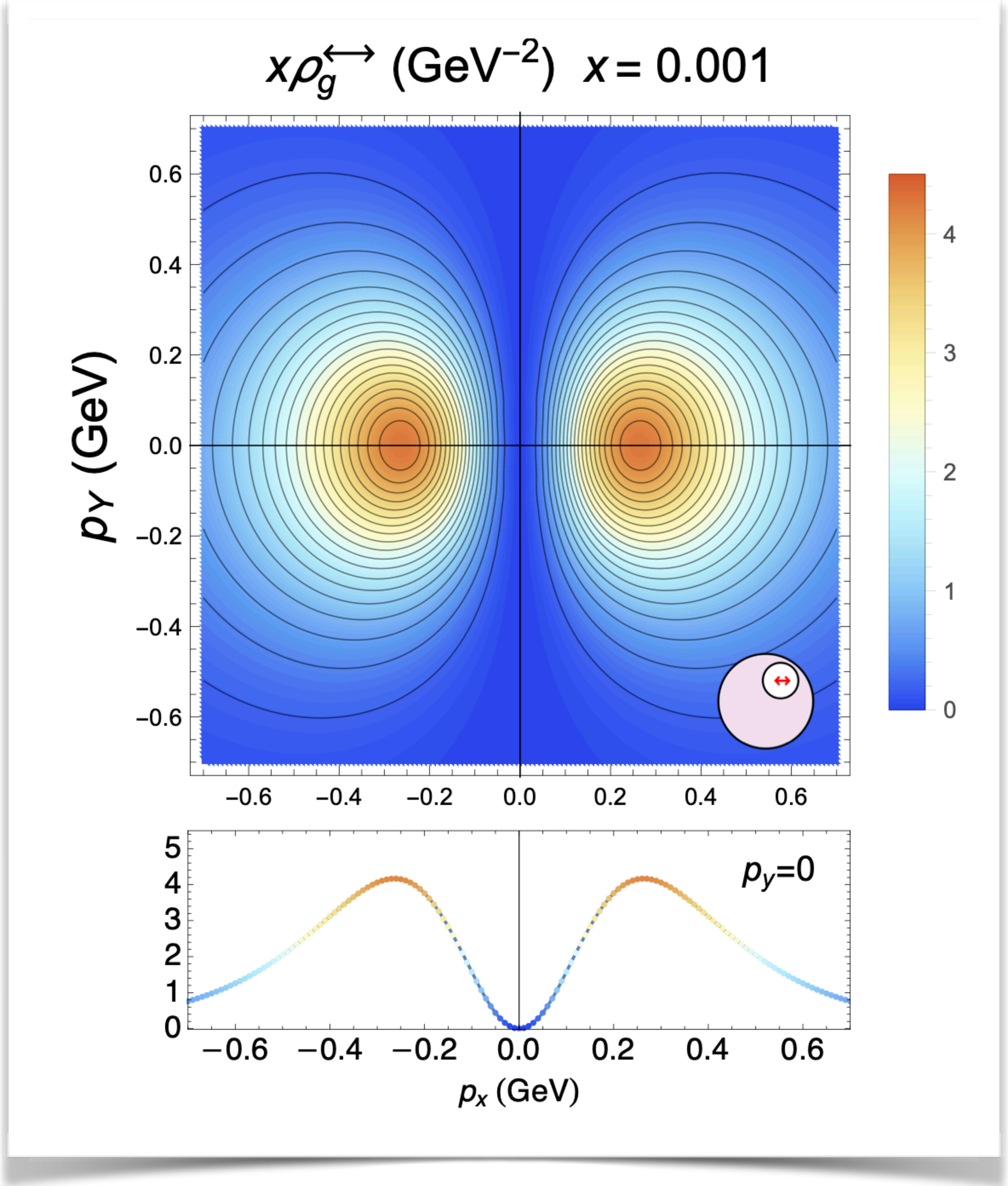
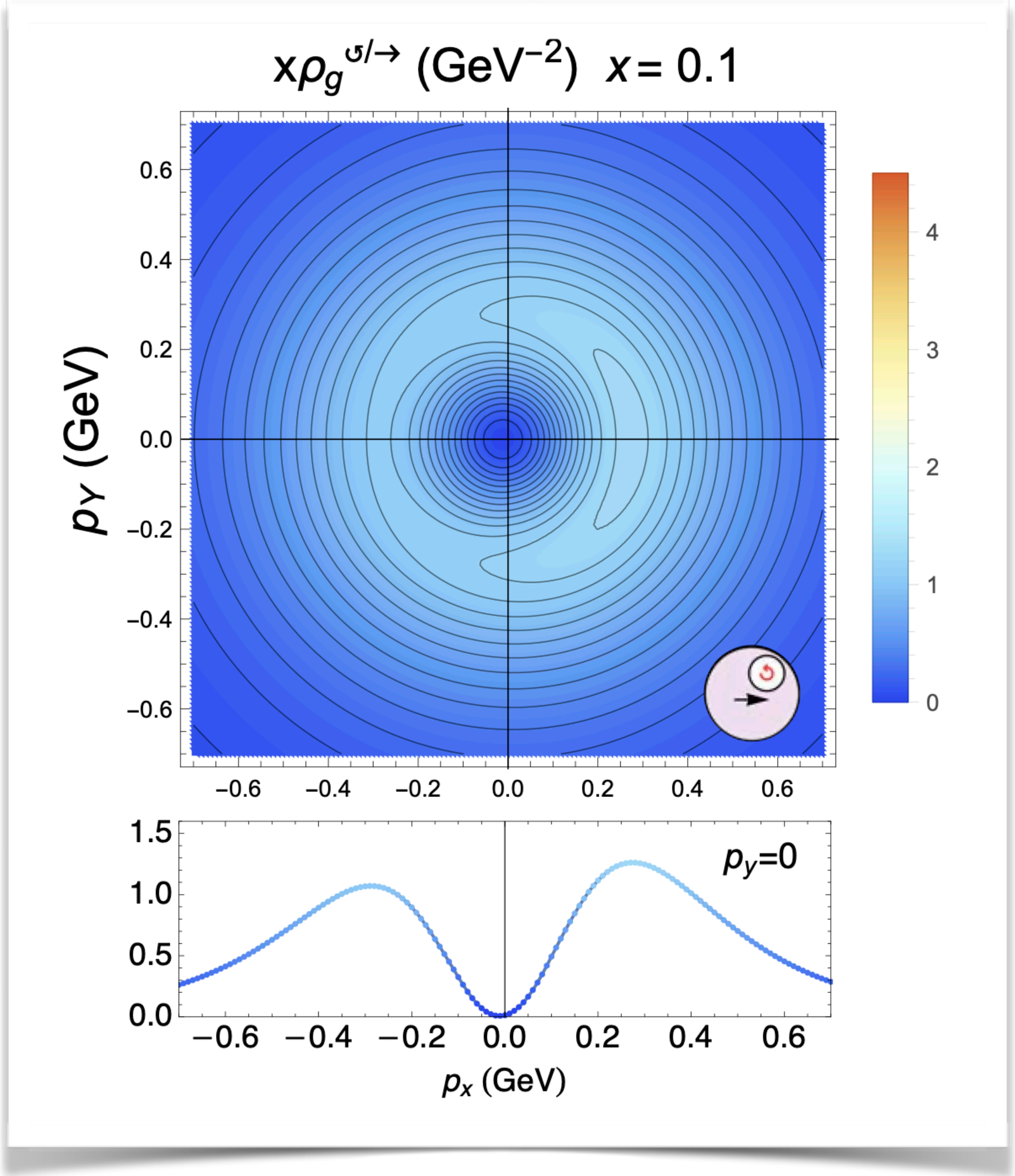


worm-gear



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...towards twist-2 *T*-odd gluon TMDs

T-odd gluon TMDs at twist-2

gluon pol. / nucleon pol.

	U	circ.	lin.
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g,\ h_{1T}^{\perp g}$

T-even

T-odd

T-odd gluon TMDs at twist-2

worm-gear

gluon pol.

nucleon pol.	U	circ.	lin.
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

T-even

T-odd

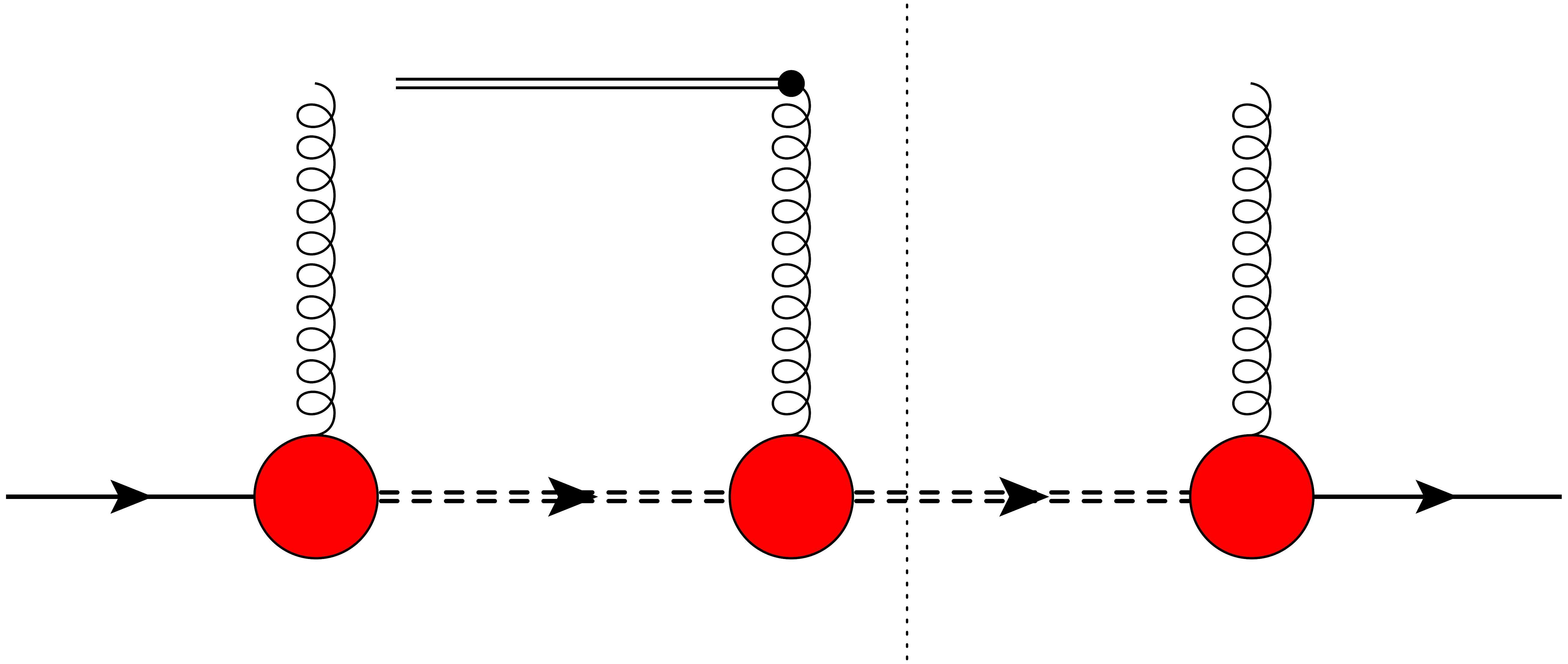
Sivers

linearity TMD

pretzelosity

T-odd gluon TMDs in a spectator model

- No residual gluon-spectator interaction at tree level
- *Interference* with one-gluon exchange (*eikonal*)

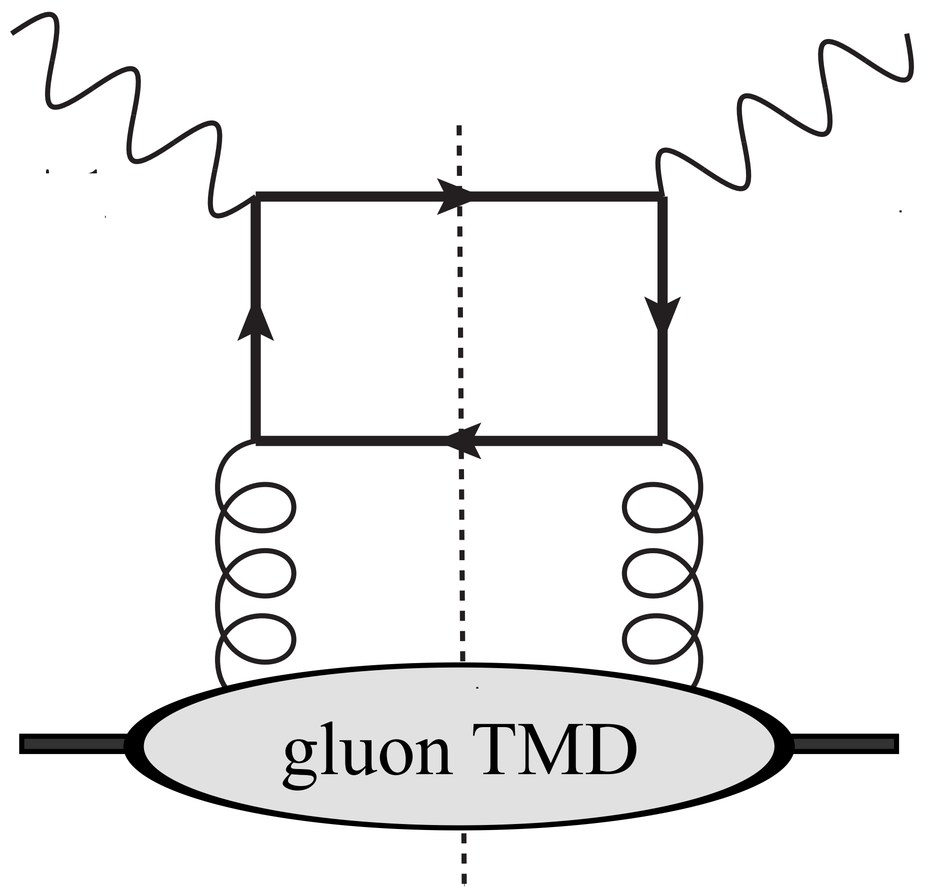


- Calculation of **Sivers** function *underway*!

T-odd gluon TMDs and semi-inclusive reactions

1. Almost back-to-back di-jet production
2. Open-charm (heavy-light meson) states
3. Almost back-to-back J/Ψ-plus-jet production
4. Inclusive J/Ψ production at low p_T

$ep \to e + \text{jet} + \text{jet} + X$



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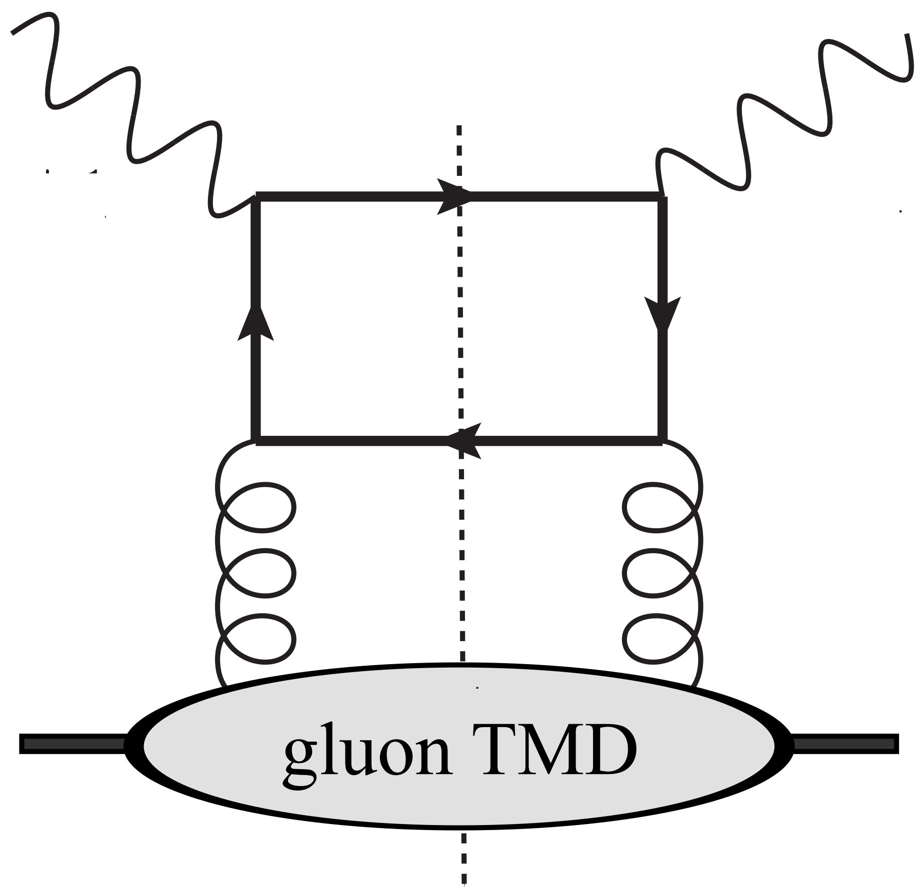


- Gluon-induced processes
- **Spin-asymmetry** studies feasible
- Small- and **large**-x physics supported

Closing statements

- ☑ Systematic calculation of all twist-2 T-even gluon TMDs
- ☑ Spectral mass to catch small- and large-x effects
- ☑ **Simultaneous fit** of f_1 and g_1 PDFs via **replica method**

Closing statements

- [x] Systematic calculation of all twist-2 T-even gluon TMDs
- [x] Spectral mass to catch small- and large-x effects
- [x] **Simultaneous fit** of f_1 and g_1 PDFs via **replica method**
- [ ] Twist-2 T-odd TMDs (**Sivers**, etc.) soon available!
- [ ] Relevant **spin asymmetries** to be identified
- [ ] **Pseudodata** and **impact studies**
- [ ] Extension to quark TMDs in the same framework

Backup slides

Parton densities: an incomplete family tree

Wigner distributions

$\rho\,(x, \mathbf{k}_T, \mathbf{b}_T)$

5D

$\int d^2\mathbf{k}_T$

$\int d^2\mathbf{b}_T$

Generalized Parton Distributions

GPDs

Transverse Momentum Distributions

TMDs

3D

$\int d^2\mathbf{k}_T$

Parton Distribution Functions

PDFs

1D

State of the art

- First calculation of leading-twist T-even quark TMDs with scalar and axial-vector di-quarks

  [R. Jakob, P. J. Mulders, J. Rodrigues (1997)]

- Gluon TMD PDFs and FFs

  [P.J. Mulders, J. Rodrigues (2001)]
  [J. Rodrigues, PhD thesis (2001)]

- Complete calculation of all the leading-twist TMDs with scalar di-quarks

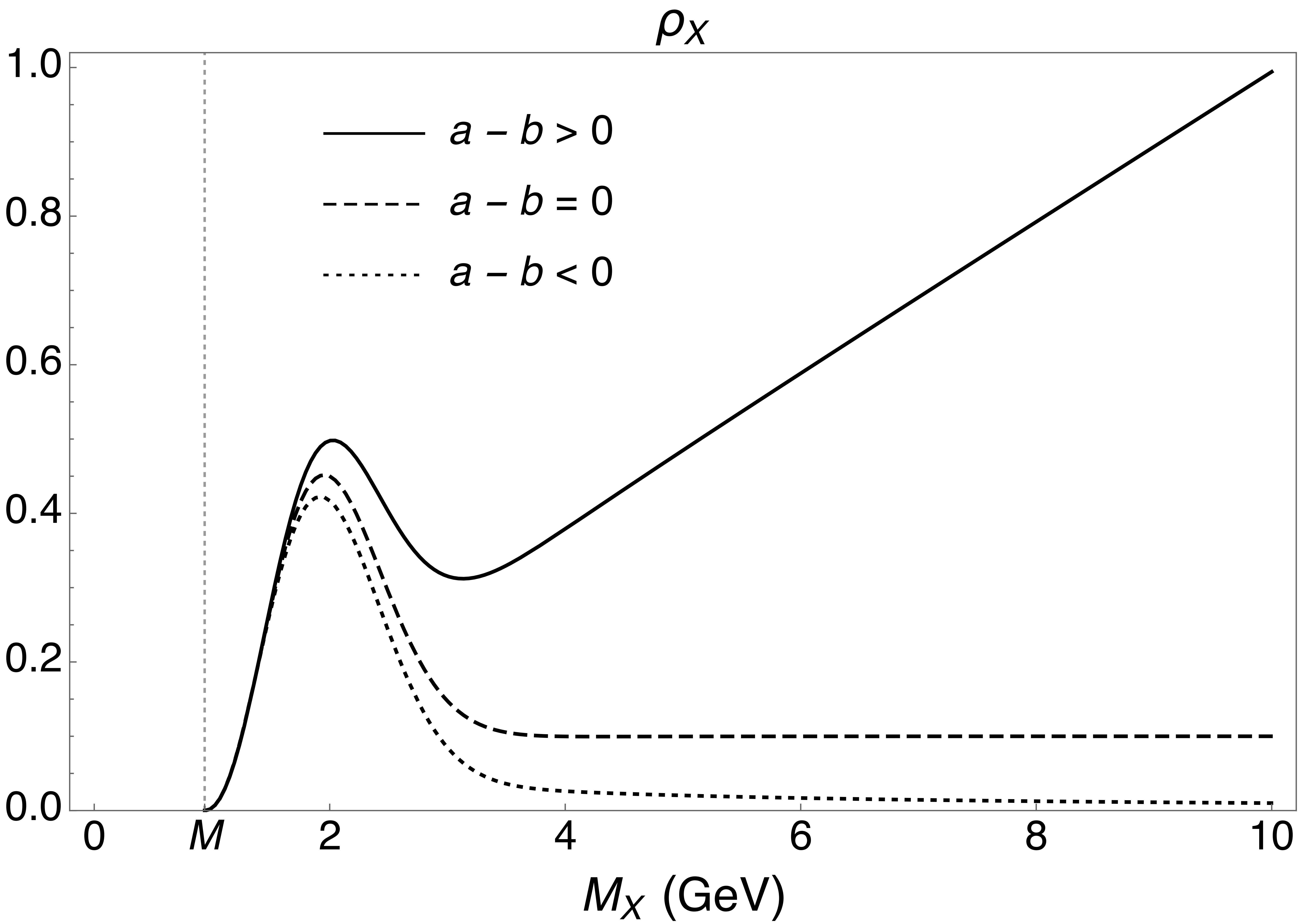
  [S. Meissner, A. Metz, K. Goeke (2007)]

- Inclusion of different axial-vector di-quark polarization states and nucleon-parton-spectator form factors

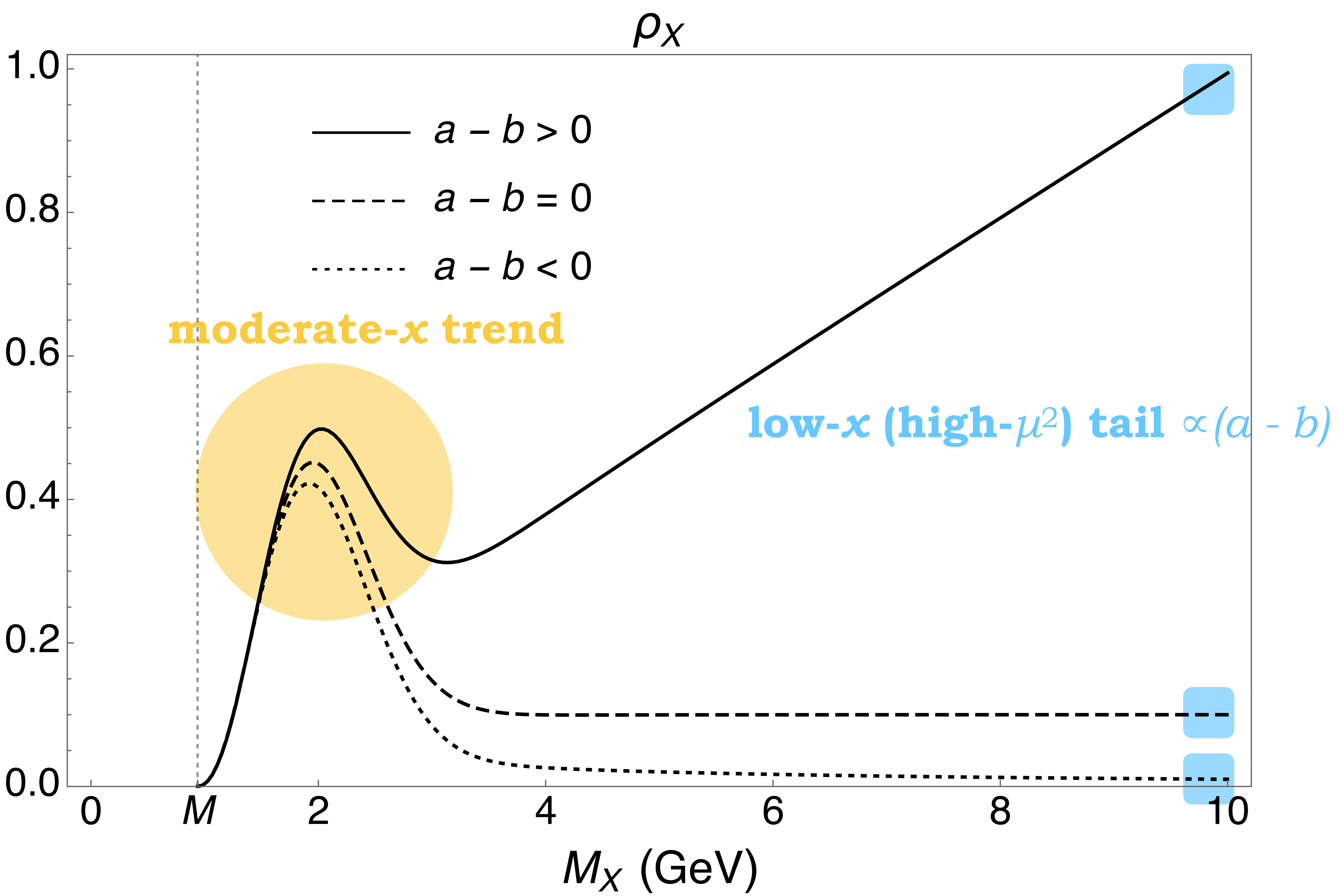
  (fit to PDF parametrizations) [A. Bacchetta, F. Conti, M. Radici (2008)]
  (application on azimuthal asymmetries) [A. Bacchetta, M. Radici, F. Conti, M. Guagnelli (2010)]

How to improve the description?

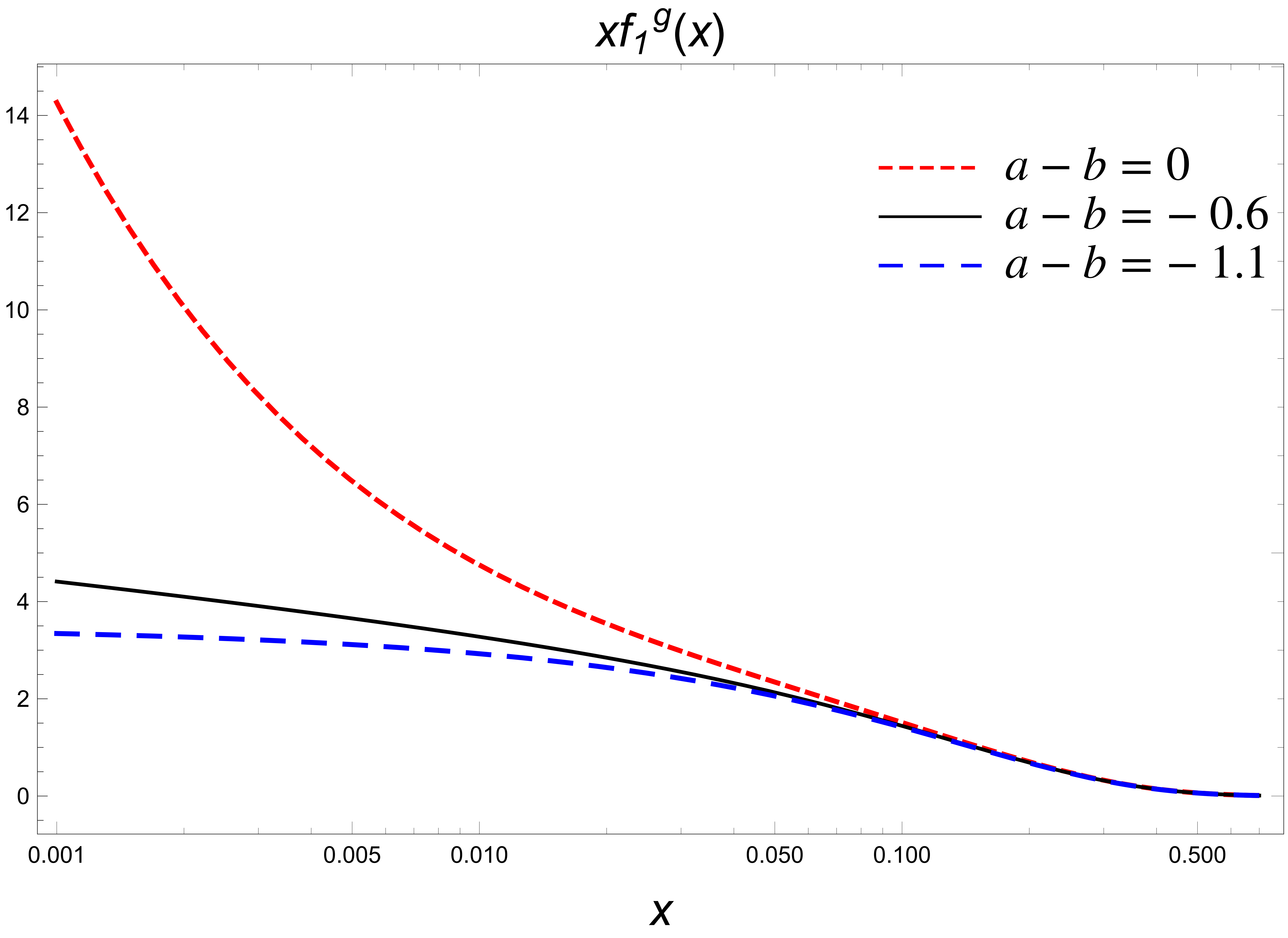
Spectral function vs *(a - b)*



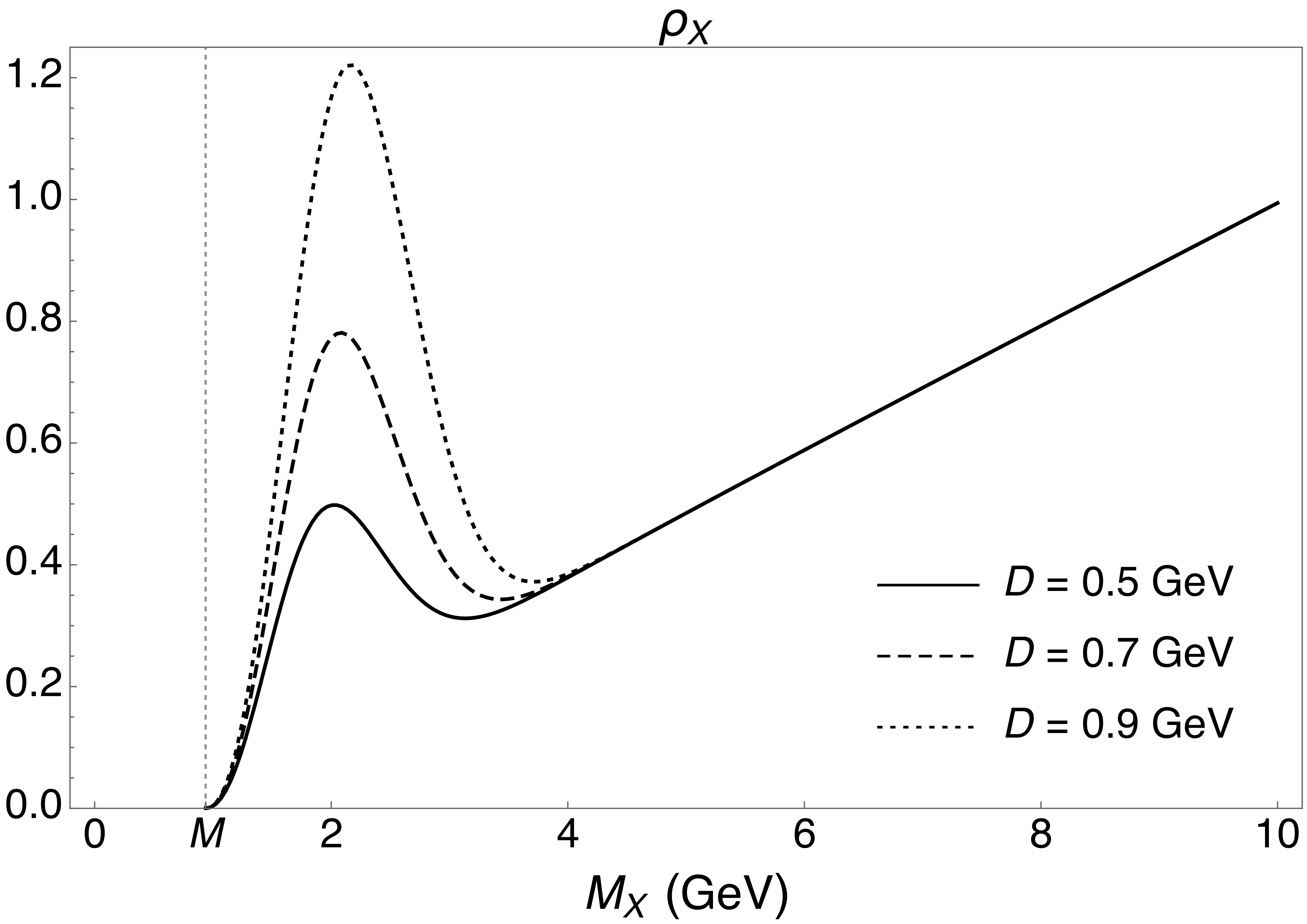
Spectral function vs $(a - b)$



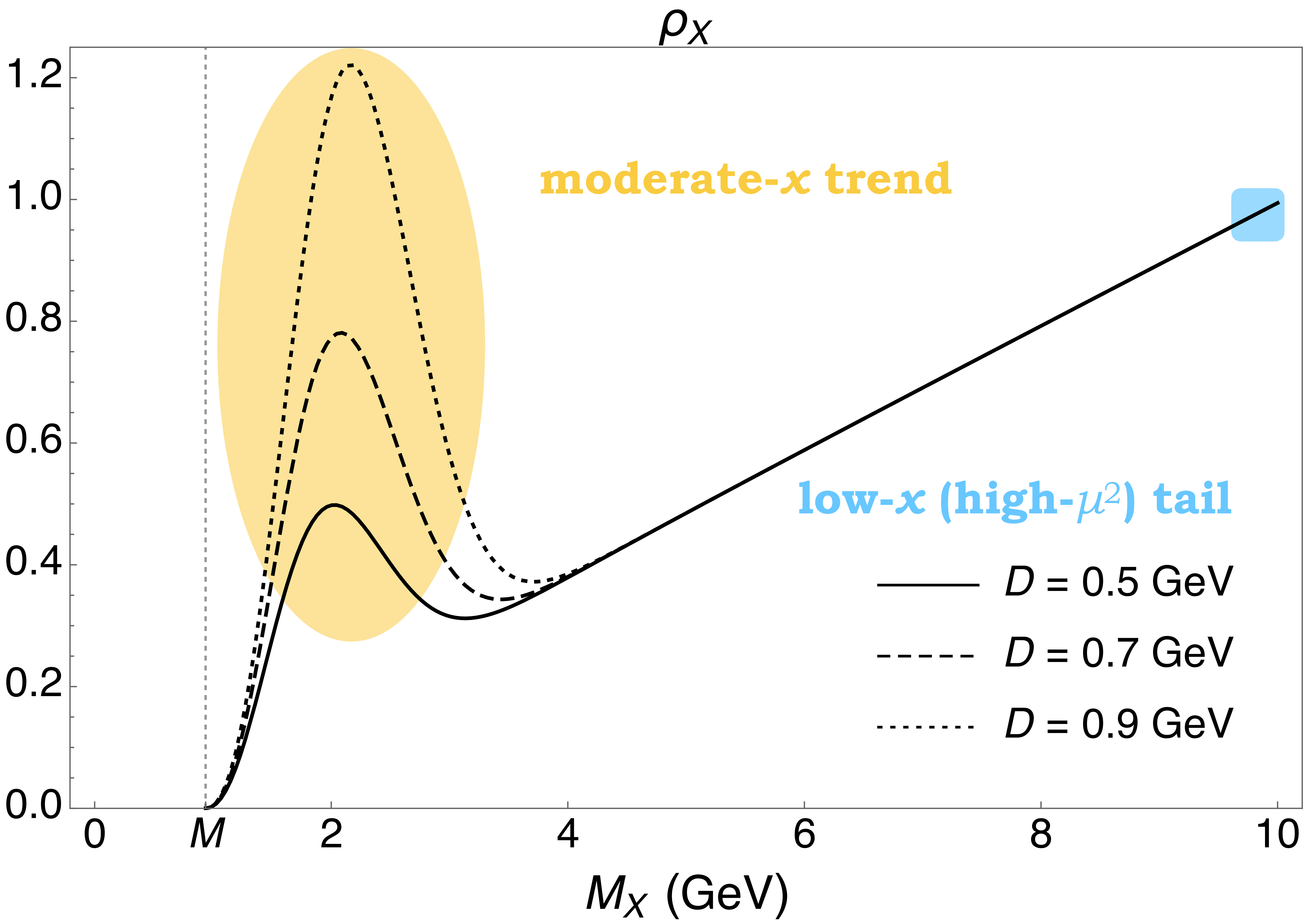
xf_1 collinear PDF vs $(a - b)$



Spectral function vs D



Spectral function vs D

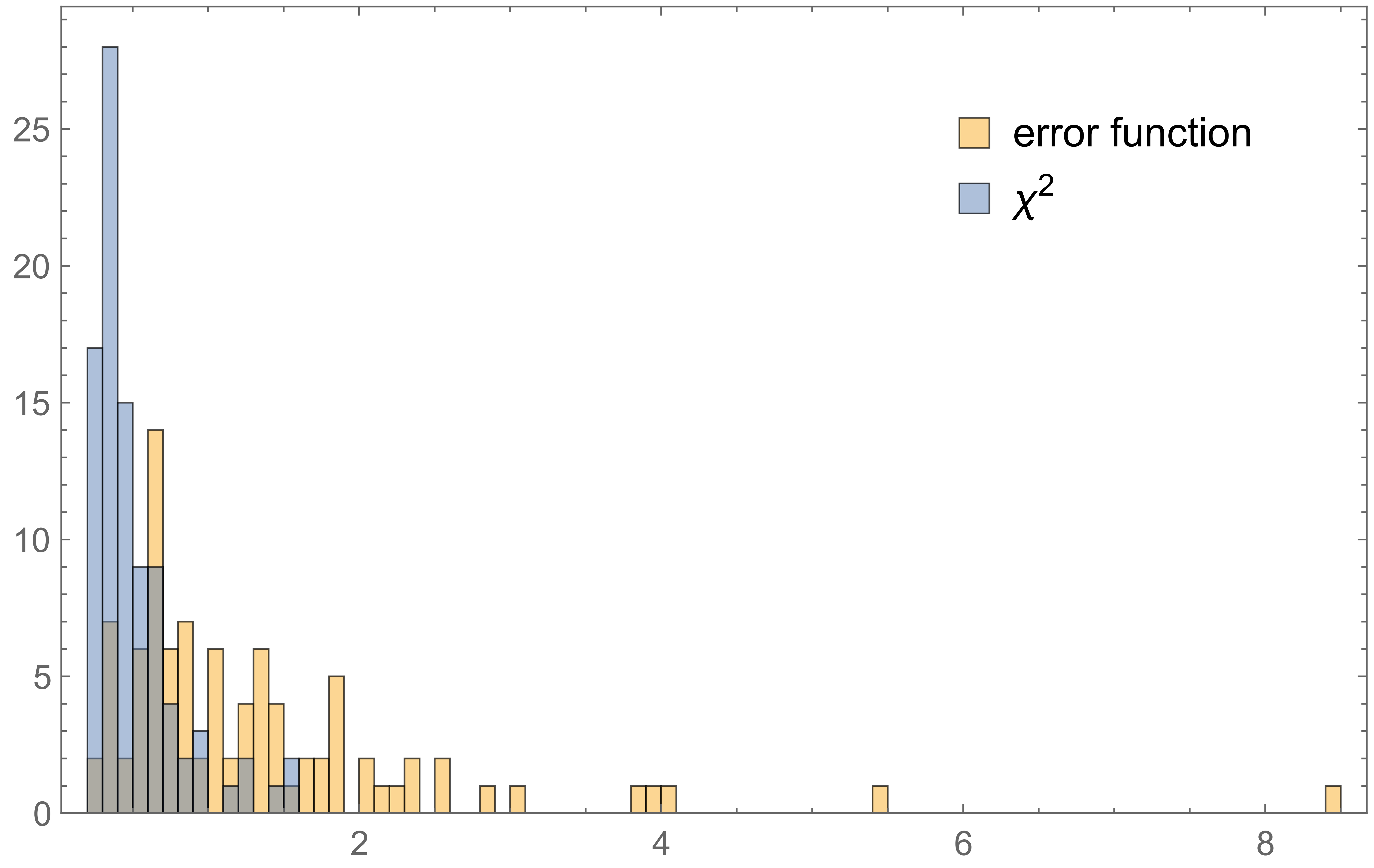


Fit parameters

parameter	mean	replica 11
A	6.1 ± 2.3	6.0
a	0.82 ± 0.21	0.78
b	1.43 ± 0.23	1.38
C	371 ± 58	346
D (GeV)	0.548 ± 0.081	0.548
σ (GeV)	0.52 ± 0.14	0.50
Λ_X (GeV)	0.472 ± 0.058	0.448
κ_1 (GeV^2)	1.51 ± 0.16	1.46
κ_2 (GeV^2)	0.414 ± 0.036	0.414

B fixed at 2.1

χ^2-analisys



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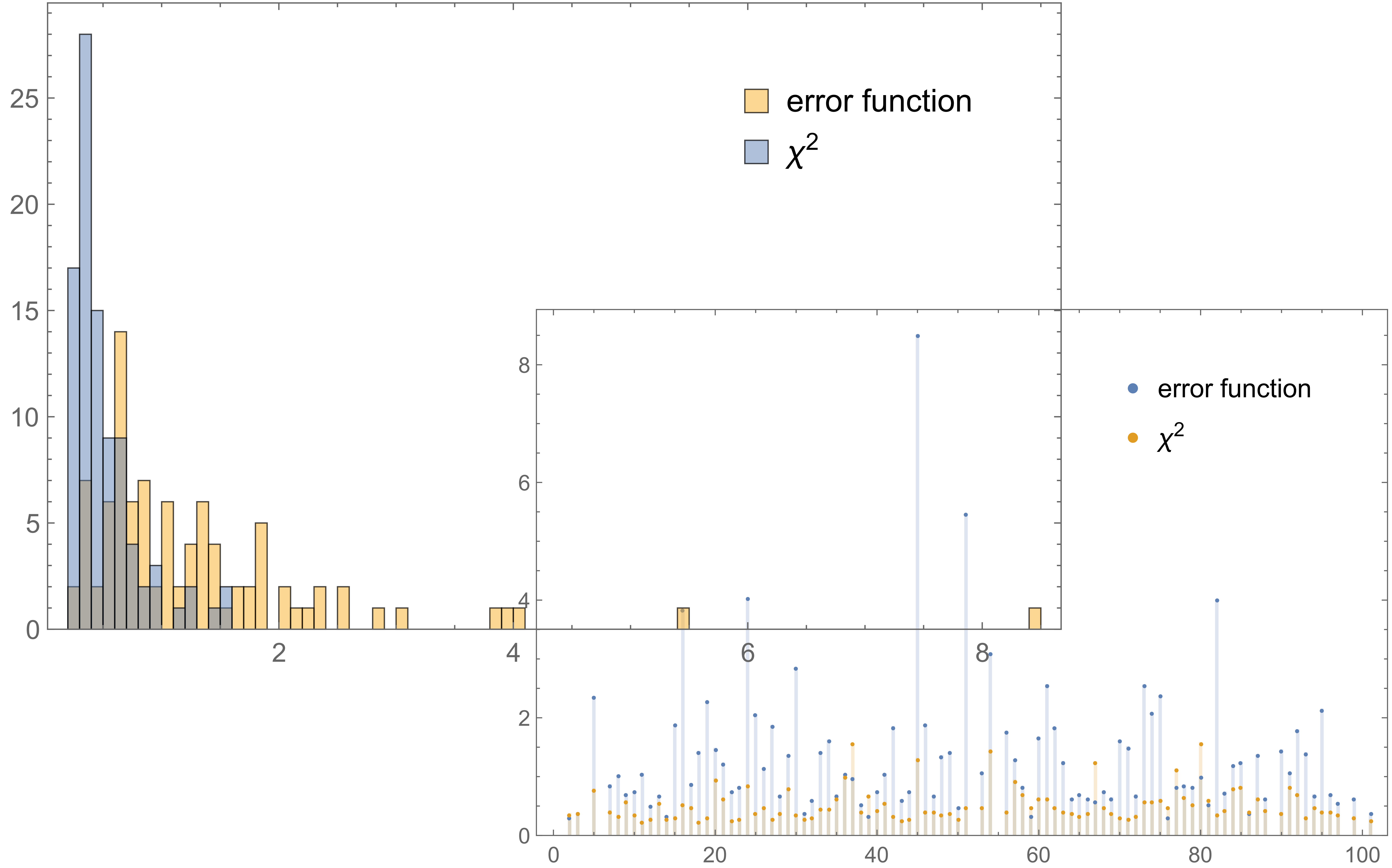

TMD factorization

$$\frac{d\sigma}{dQdydq_T} = \frac{16\pi\alpha^2 q_T \mathcal{P}}{9Q^3} H(Q,\mu) \sum_q c_q(Q) \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} x_1 F_q(x_1,\mathbf{b};\mu,\zeta_1) x_2 F_{\bar{q}}(x_2,\mathbf{b};\mu,\zeta_2)$$

$$F_f(x,\mathbf{b};\mu,\zeta) = \sum_j \left(C_{f/j} \otimes f^j\right)(x,b_*;\mu_b) e^{R(b_*;\mu_b,\mu)} f_{\mathrm{NP}}(x,b,\zeta)$$

scales

$\mu = Q$ $\qquad$ $\zeta_1 = \zeta_2 = Q^2$

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Logarithmic accuracy

$$\frac{d\sigma}{dq_T} \propto H(Q,\mu) \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} F_q(x_1,\mathbf{b};\mu) F_{\bar{q}}(x_2,\mathbf{b};\mu)$$

hard factor

matching coefficients

collinear PDF

$$F(x,\mathbf{b};\mu) = \sum_j \left(C_j \otimes f^j\right)(x, b_*; \mu_b) e^{R(b_*;\mu_b,\mu)} f_{\mathrm{NP}}(x,b)$$

Sudakov form factor

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Sudakov form factor

perturbative expansion in $\alpha_s(\mu)$

resummation of $L = \ln \frac{Q^2}{\mu_b^2}$

define logarithmic ordering

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TMDs

factorizes as hard
and longitudinal non-perturbative

matching to the
collinear region

$b_T \ll 1/\Lambda_{QCD}$

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b)$$

$$\times \exp\left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\}$$

CS and RGE
evolution

$$\times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\}$$

non perturbative
transverse content

parametrized
and **fitted to data**

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