Quark and gluon PDFs, TMDs and GPDs in HQCD

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based on

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Gluon content of proton and deuteron with the Spin Physics Detector at the NICA collider, 30.09.2020

Plan

- Motivation:
  We explicitly demonstrate how to correctly define the hadronic parton distributions (PDFs, TMDs, GPDs) in soft-wall AdS/QCD approach

- Framework (SW AdS/QCD)
  - AdS/QCD = Holographic QCD - novel approach based on correspondence between 5D theories including gravity and gauge 4D theories living on the boundary of AdS space
  - Soft-wall AdS/QCD based on the use of a quadratic dilaton field, providing confinement and breaking of conformal and chiral symmetries

- Advantages in study of hadron structure
  - Mass spectrum of hadrons and exotic states (Regge behavior)
  - Power behavior of hadron FF at large Q^2 and parton distributions at large x consistent with quark counting rules, identities/inequalities

- Application: PDFs, TMDs, GPDs of quarks and gluons

- Summary

Introduction

- 1993 ’t Hooft   Holographic Principle

  Information about string theory contained in some region of space can be represented as “Hologram” (theory which lives on the boundary of that region)

- 1997-1998 Maldacena, Polyakov, Witten et al   AdS/CFT correspondence

  Duality of 4D conformal supersymmetric Yang-Mills and supersting theories

  - Matching partition functions gives relation between parameters

  Strings g_s – coupling, l_s – length, R – AdS radius

  SU(N) YM g_{YM} – coupling, ’t Hooft coupling $\lambda = g_{YM}^2 N$

  $2\pi g_s = g_{YM}^2\,, \qquad \frac{R^4}{l_s^4} = 2\, g_{YM}^2\, N$

  - Symmetry arguments: Conformal group acting in boundary theory isomorphic to $SO(4,2)$ – the isometry group of AdS$_5$ space

Holographic QCD

- ’t Hooft limit (large N at λ fixed) $g_{YM} = \frac{\lambda}{N} \ll 1$
  corresponds $g_s \ll 1$ (tree-level perturbative string threory)
  “Conformal Field side” of duality at work

- Strong coupling limit $\lambda \gg 1$ means $l_s \ll R$ small curvature $\mathcal{R} = -\frac{20}{R^2}$
  Supergravity limit (closed strings shrink to point-like particles)
  “String Theory side” of duality at work

- AdS/CFT $\rightarrow$ AdS/QCD upon breaking conformal invariance

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD:
  attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space

- HQCD models reproduce main features of QCD at low and high energies:
  chiral symmetry, confinement, power scaling of hadron form factors

- Physical interpretation of extra 5th dimension as Scale

Holographic QCD

- AdS metric   Poincaré form

  $ds^2 = g_{MN}(z)\, dx^M dx^N = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)$ R – AdS radius

- Metric Tensor $g_{MN}(z) = \epsilon^a_M(z)\epsilon^b_N(z)\eta_{ab}$

- Vielbein $\epsilon^a_M(z) = \frac{R}{z}\,\delta^a_M$ (relates AdS and Lorentz metric)

- Manifestly scale-invariant $x \to \lambda x,\ z \to \lambda z$.

- z – extra dimensional (holographic) coordinate;
  $z = 0$ is UV boundary, $z = \infty$ is IR boundary

- Five Dimensions: L = Length, W = Width, H = Height, T = Time, S = Scale

Holographic QCD

- Action for scalar field

$$S_\Phi = \frac{1}{2} \int d^d x dz \, \sqrt{g} \, e^{-\varphi(z)} \left(\partial_M \Phi(x,z) \partial^M \Phi(x,z) - m^2 \, \Phi^2(x,z) \right)$$

- Dilaton field $\varphi(z) = \kappa^2 z^2$
- $g = |\mathrm{det} g_{MN}|$
- m – 5d mass, $m^2 R^2 = \Delta(\Delta - 4)$, $\Delta = 3$ conformal dimension
- Kaluza-Klein (KK) expansion $\Phi(x,z) = \sum\limits_n \phi_n(x) \, \Phi_n(z)$
- Tower of KK modes $\phi_n(x)$ dual to 4-dimensional fields describing hadrons
- Bulk profiles $\Phi_n(z)$ dual to hadronic wave functions

Holographic QCD

- Use $-\partial_\mu \partial^\mu \phi_n(x) = M_n^2 \phi_n(x)$
- Substitute $\Phi_n(z) = \left(\frac{R}{z}\right)^{1-d} \phi_n(z)$
- Identify $\Delta = \tau = N + L$ (here $N = 2$ – number of partons in meson)

  Hete τ is twist = Canonical dimension - Sum of spins

  Examples: mesons $\tau = 2 \times 3/2 - 2 \times 1/2 = 2$
  baryons $\tau = 3 \times 3/2 - 3 \times 1/2 = 3$

$$\left[-\frac{d^2}{dz^2} + \frac{4L^2 - 1}{4z^2} + \kappa^4 z^2 - 2\kappa^2 \right] \phi_n(z) = M_n^2 \phi_n(z)$$

- Solutions:
  $\phi_{nL}(z) = \phi_{n,\tau-2}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+1)}}\, \kappa^{L+1}\, z^{L+1/2}\, e^{-\kappa^2 z^2/2}\, L_n^L(\kappa^2 z^2)$
- $M_{nL}^2 = 4\kappa^2 \left(n + \frac{L}{2}\right) = 4\kappa^2 \left(n + \frac{\tau}{2} - 1\right)$
- Massless pion $M_\pi^2 = 0$ for $n = L = 0$ Brodsky, Téramond

Holographic QCD

- Extension to AdS fermions (baryons)

$$S_\psi = \int d^d x dz \sqrt{g}\, \bar{\Psi}(x,z) \Big(\not{D} - \mu - \varphi(z)/R \Big) \Psi(x,z)$$

- Field decomposition (left/right) and KK expansion

$$\Psi(x,z) = \Psi_L(x,z) + \Psi_R(x,z) \quad \Psi_{L/R} = \frac{1 \mp \gamma^5}{2} \Psi$$

$$\Psi_{L/R}(x,z) = \sum_n \Psi^n_{L/R}(x)\, F^n_{L/R}(z)$$

- EOM

$$\left[-\partial_z^2 + \kappa^4 z^2 + 2\kappa^2 \Big(\mu R \mp \tfrac{1}{2} \Big) + \frac{\mu R(\mu R \pm 1)}{z^2} \right] F^n_{L/R}(z) = M_n^2\, F^n_{L/R}(z)$$

Solutions (for $d = 4$ and $\mu R = L + 3/2$)

- Bulk profiles

$$F^n_L(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+3)}}\, \kappa^{L+3}\, z^{L+9/2}\, e^{-\kappa^2 z^2/2}\, L_n^{L+2}(\kappa^2 z^2)$$

$$F^n_R(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+2)}}\, \kappa^{L+2}\, z^{L+7/2}\, e^{-\kappa^2 z^2/2}\, L_n^{L+1}(\kappa^2 z^2)$$

- Mass spectrum: $M^2_{nL} = 4\kappa^2 \Big(n + L + 2 \Big)$

Holographic QCD

- Extension to higher-spin AdS boson (mesons)

  Vasilev, Buchbinder, Metzaev, Pashnev, . . .

  Fields $\Phi \;\to\; \Phi_{M_1 M_2 \cdots M_J}$

  5d mass $m^2 R^2 \;\to\; m_J^2 R^2 = (\Delta - J)(\Delta + J - 4)$

  Dilaton potential

$$U_J(z) = \frac{z^2}{R^2}\left(\varphi''(z) + \frac{1 + 2J - d}{z}\varphi'(z)\right)$$

  Solutions

- $\phi_{nL}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L)}}\;\kappa^{L+1}\;z^{L+1/2}\;e^{-\kappa^2 z^2/2}\;L_n^L(\kappa^2 z^2)$

- $M_{nLJ}^2 = 4\kappa^2\left(n + \frac{L+J}{2}\right) \to 4\kappa^2\left(n + J\right)$ at large J

Introduction

- Scattering problem for AdS field gives information about propagation of external field from z to the boundary $z = 0$ — bulk-to-boundary propagator $\Phi_{\rm ext}(q,z)$ [Fourier-trasform of AdS field $\Phi_{\rm ext}(x,z)$]:
  $\Phi_{\rm ext}(q,z) = \int d^d x e^{-iqx}\, \Phi_{\rm ext}(x,z)$

- Vector field as example
  $$\partial_z \biggl(\frac{e^{-\varphi(z)}}{z} \partial_z V(q,z) \biggr) + q^2 \, \frac{e^{-\varphi(z)}}{z} V(q,z) = 0\,.$$
  $$V(Q,z) \ = \ \Gamma\biggl(1 + \frac{Q^2}{4\kappa^2}\biggr) U\biggl(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\biggr)$$
  Consistent with GI, fulfills UV and IR boundary conditions :
  $V(Q,0) = 1\,, \ \ V(Q,\infty) = 0$

- Hadron form factors

$$F_\tau(Q^2) = \langle \phi_\tau | \hat V(Q) | \phi_\tau \rangle = \int\limits_0^\infty dz\, \phi_\tau^2(z)\, V(Q,z) = \frac{\Gamma(\tau)\,\Gamma(a+1)}{\Gamma(a+\tau)}$$

is implemented by a nontrivial dependence of AdS fields on 5-th coordinate

Introduction

- Power scaling at large Q^2

$$F_\tau(Q^2) \sim \frac{1}{(Q^2)^{\tau-1}}$$

Quark counting rules: Matveev-Muradyan-Tavhelidze-Brodsky-Farrar 1973

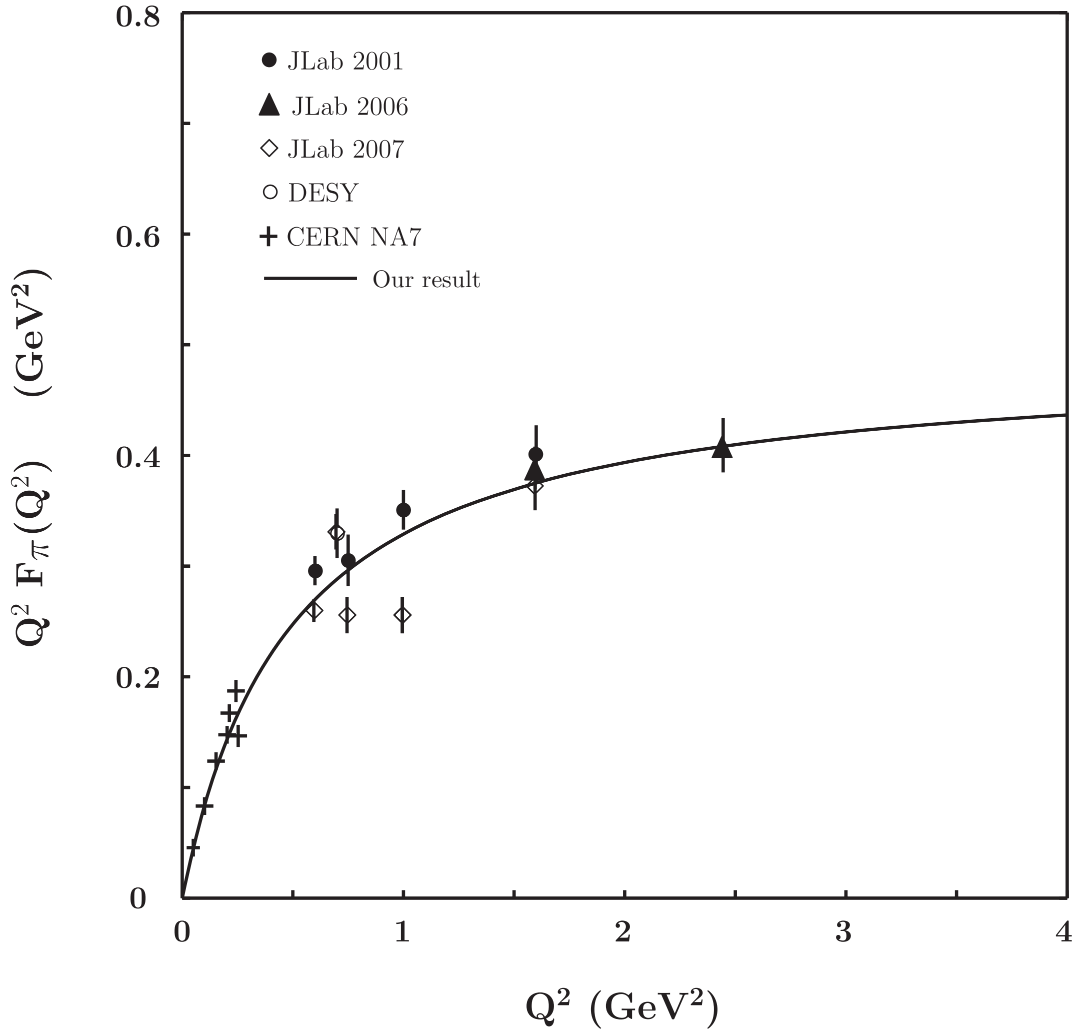
$$\mathrm{Pion}: \quad \frac{1}{Q^2}$$

$$\mathrm{Nucleon(Dirac)}: \quad \frac{1}{Q^4}$$

$$\mathrm{Nucleon(Pauli)}: \quad \frac{1}{Q^6}$$

$$\mathrm{Deuteron(Charge)}: \quad \frac{1}{Q^{10}}$$

Mesons: pion form factor



LFWFs motivated by holographic QCD

- Matching matrix elements (e.g. form factors) in HQCD and LF QCD
- Drell-Yan-West formula

$$F_\tau(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\, \psi_\tau^\dagger(x, \mathbf{k}'_\perp)\, \psi_\tau(x, \mathbf{k}_\perp)\,,$$

where $\psi(x, \mathbf{k}_\perp) \equiv \psi(x, \mathbf{k}_\perp; \mu_0)$, $\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$, and $Q^2 = \mathbf{q}_\perp^2$

- HQCD

$$F_\tau(Q^2) = \int_0^\infty dz\, V(Q, z)\, \varphi_\tau^2(z) = \frac{\Gamma(\frac{Q^2}{4\kappa^2} + 1)\,\Gamma(\tau)}{\Gamma(\frac{Q^2}{4\kappa^2} + \tau)}\,.$$

- Result for effective LFWF at the initial scale μ_0

$$\psi_\tau(x, \mathbf{k}_\perp) = \sqrt{\tau - 1}\, \frac{4\pi}{\kappa}\, \sqrt{\log(1/x)}\, (1-x)^{\frac{\tau-4}{2}}\, \exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\, \frac{\log(1/x)}{(1-x)^2}\right]$$

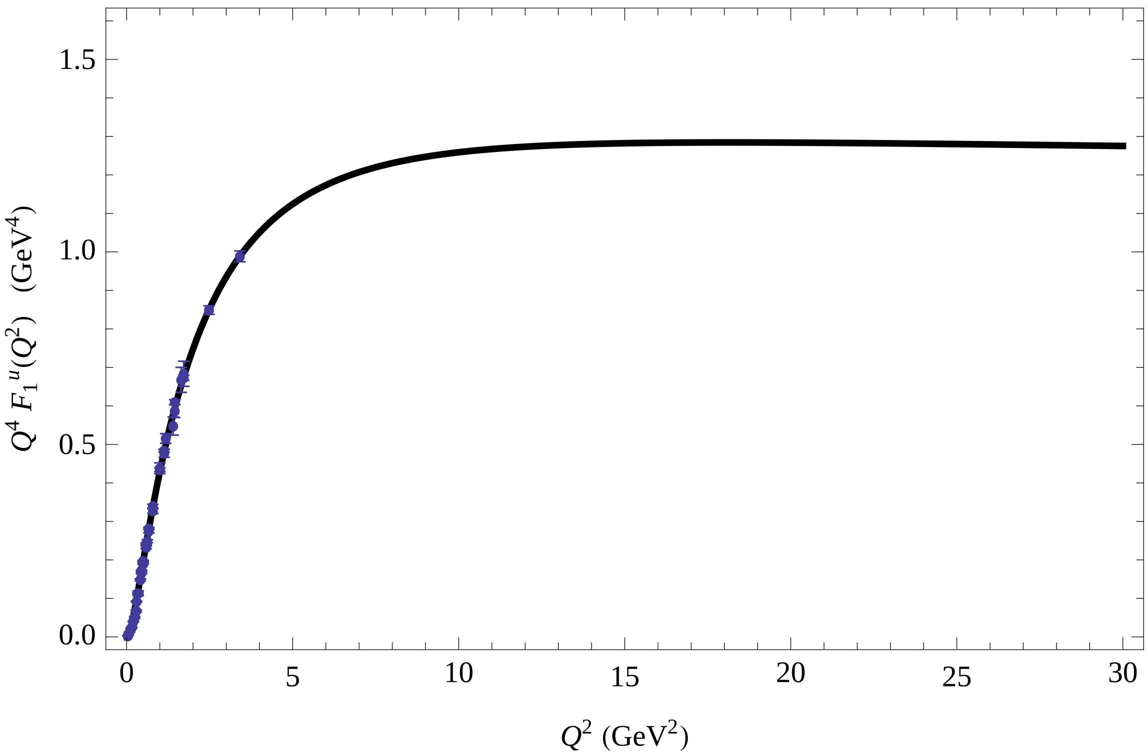
Electromagnetic stucture of nucleons

Scale parameter $\kappa = 383$ MeV

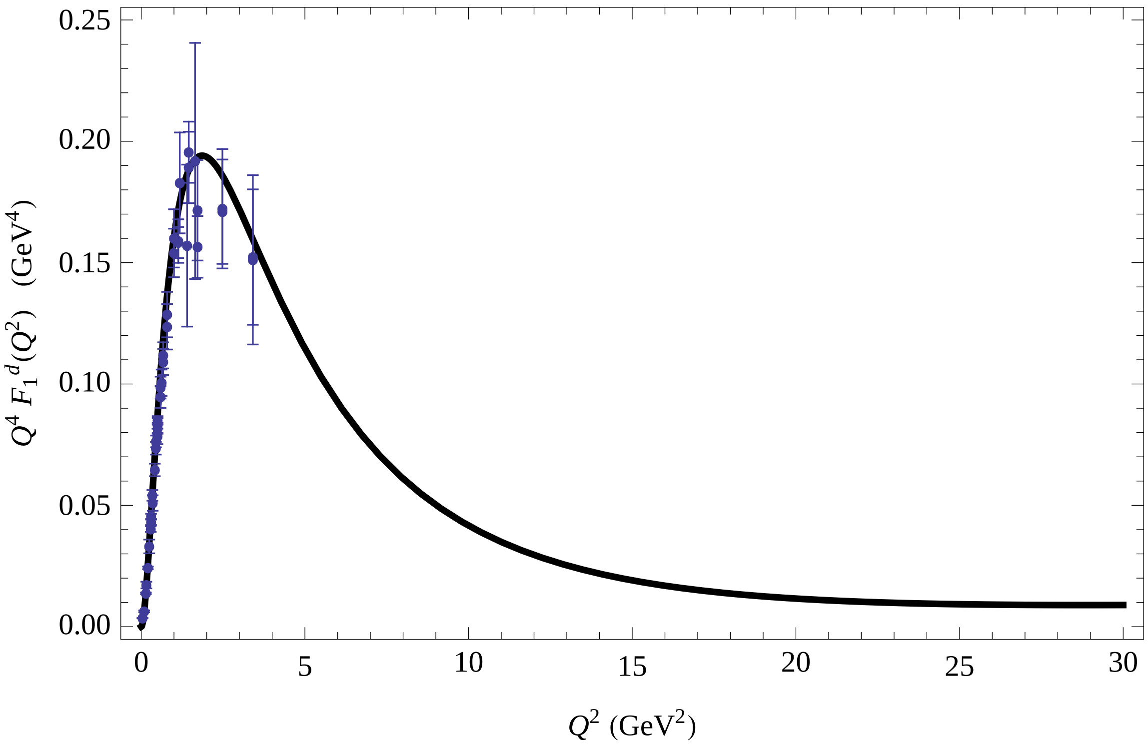
Mass and electromagnetic properties of nucleons

Quantity	Our results	Data
m_p (GeV)	0.93827	0.93827
μ_p (in n.m.)	2.793	2.793
μ_n (in n.m.)	-1.913	-1.913
r_E^p (fm)	0.840	0.8768 $\pm$ 0.0069
$\langle r_E^2 \rangle^n$ (fm^2)	-0.117	-0.1161 $\pm$ 0.0022
r_M^p (fm)	0.785	0.777 $\pm$ 0.013 $\pm$ 0.010
r_M^n (fm)	0.792	$0.862^{+0.009}_{-0.008}$
r_A (fm)	0.667	0.67$\pm$0.01

Electromagnetic stucture of nucleons



Electromagnetic stucture of nucleons



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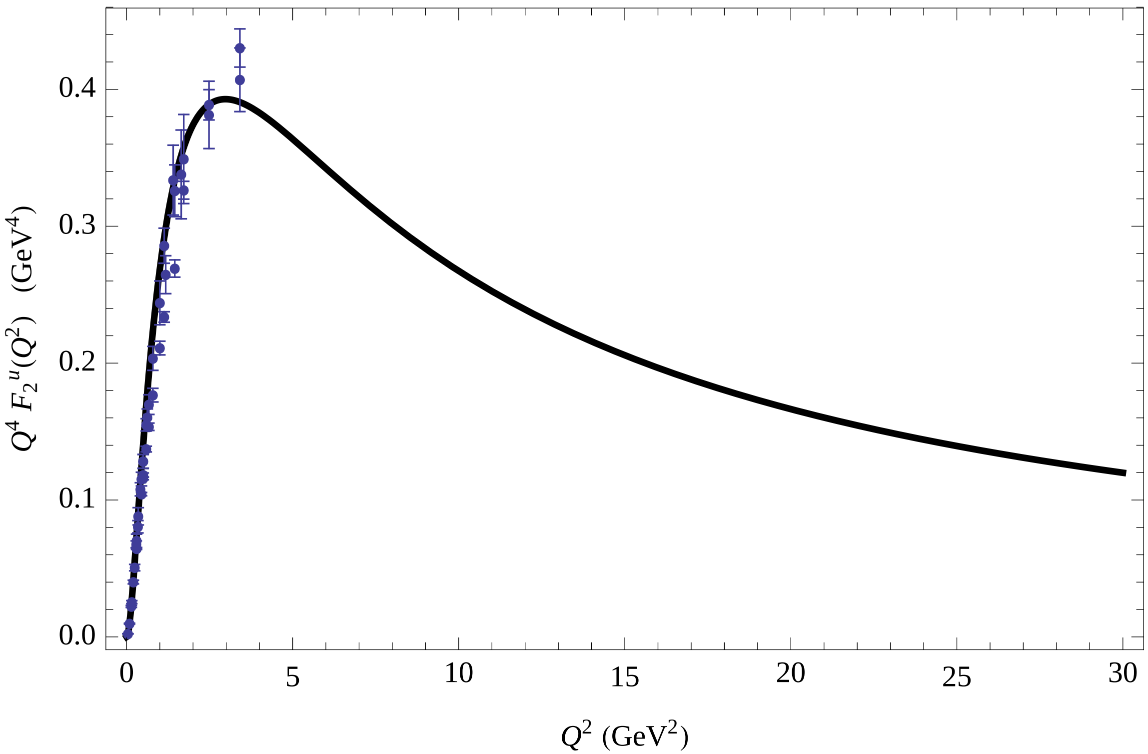

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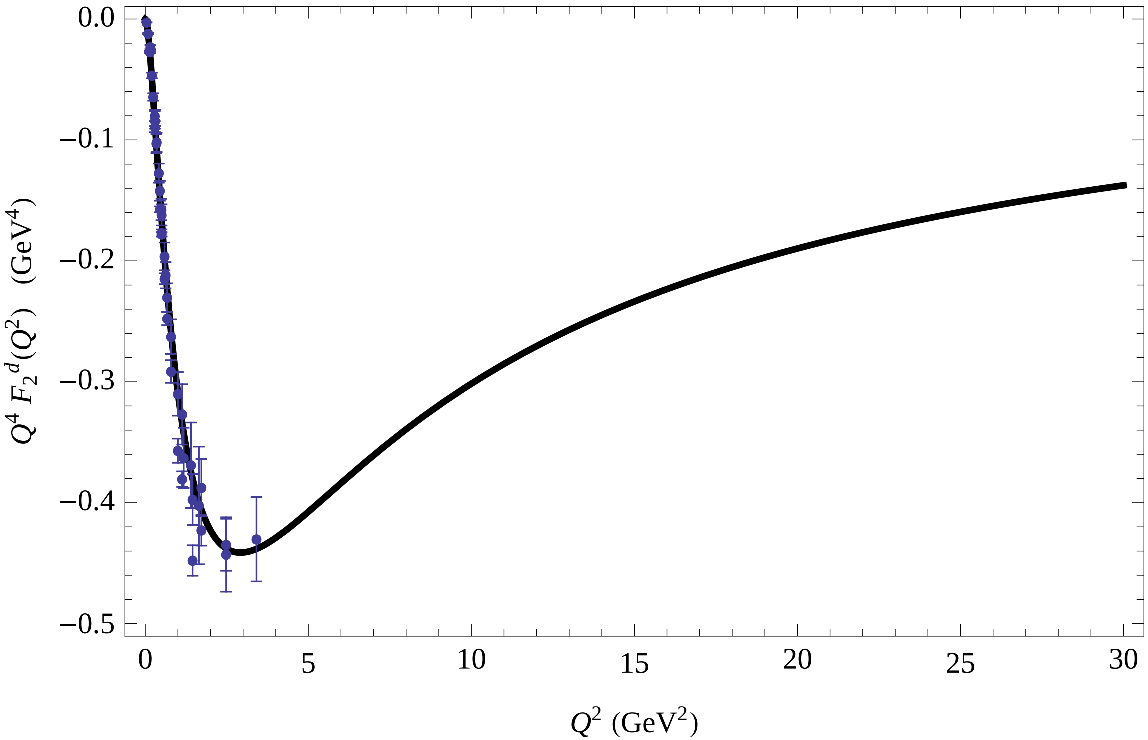

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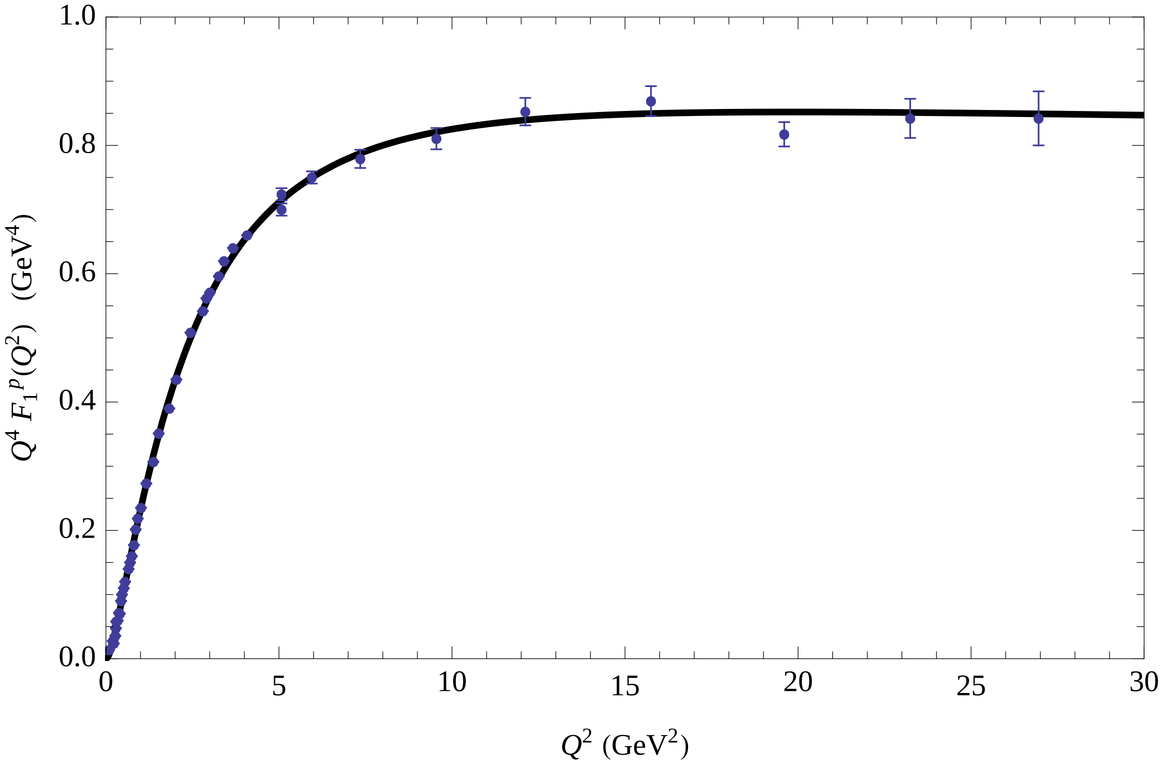

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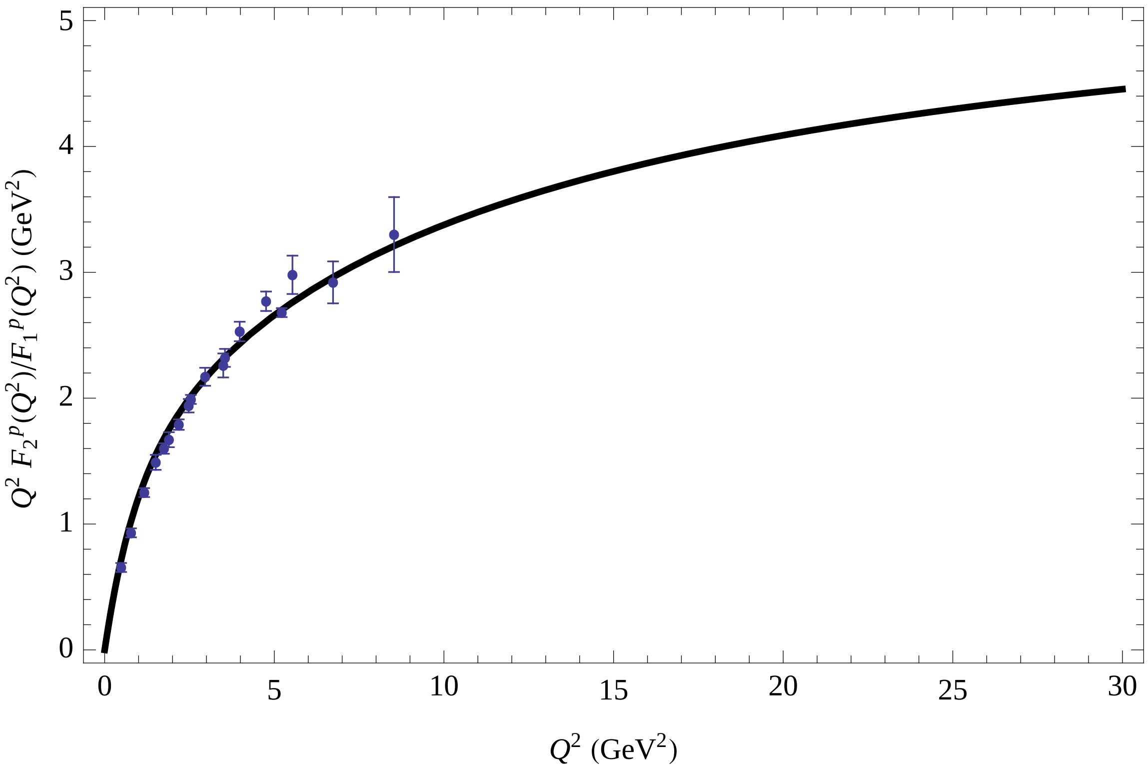

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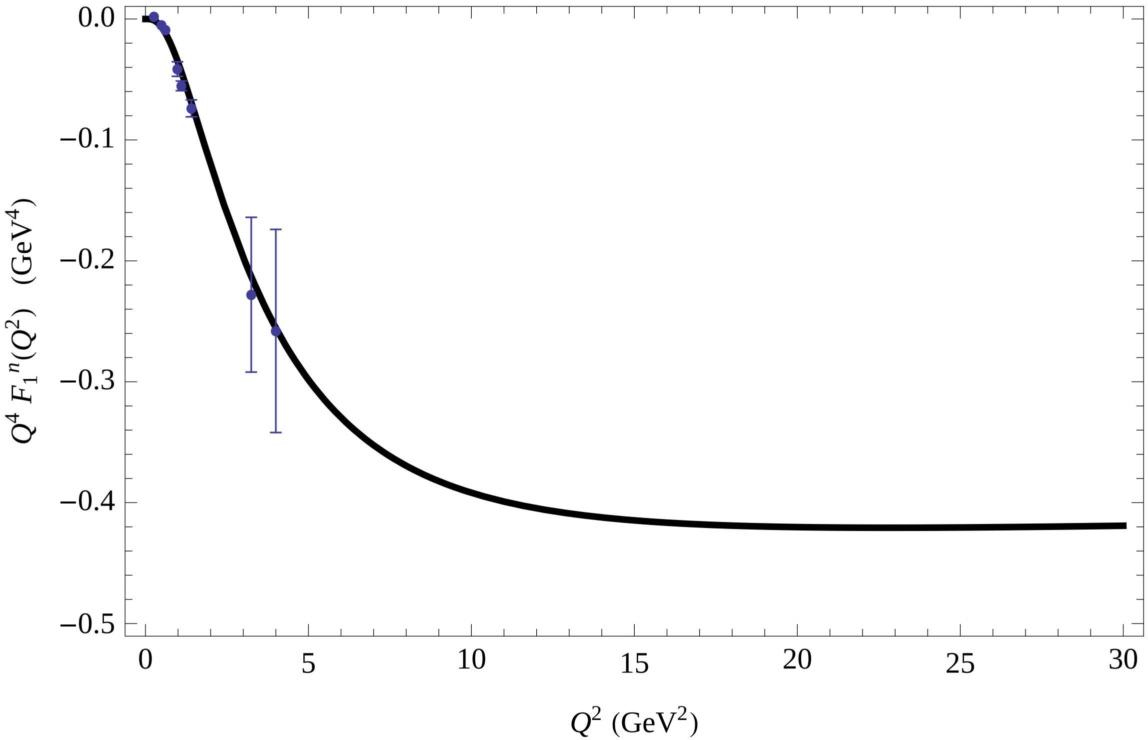

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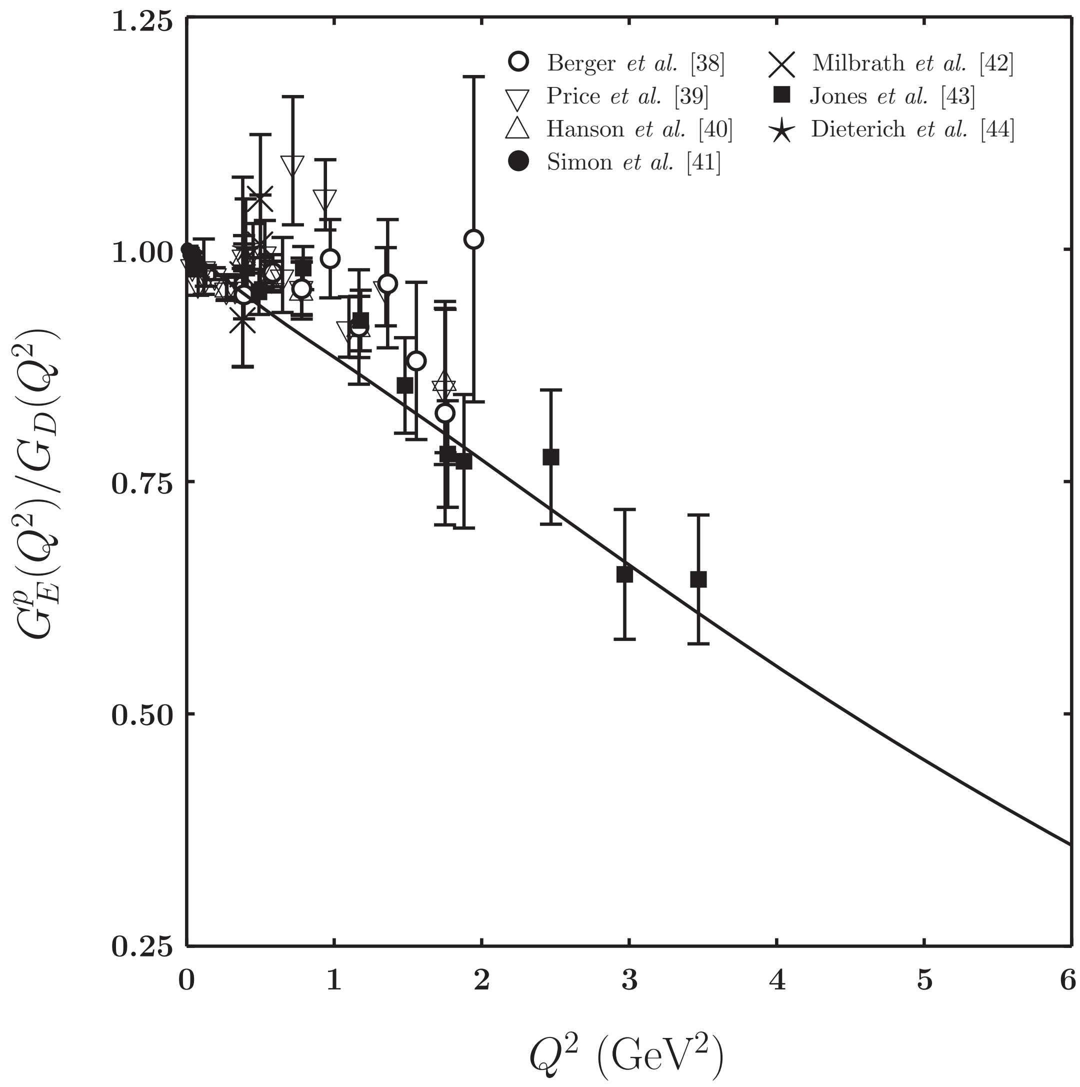

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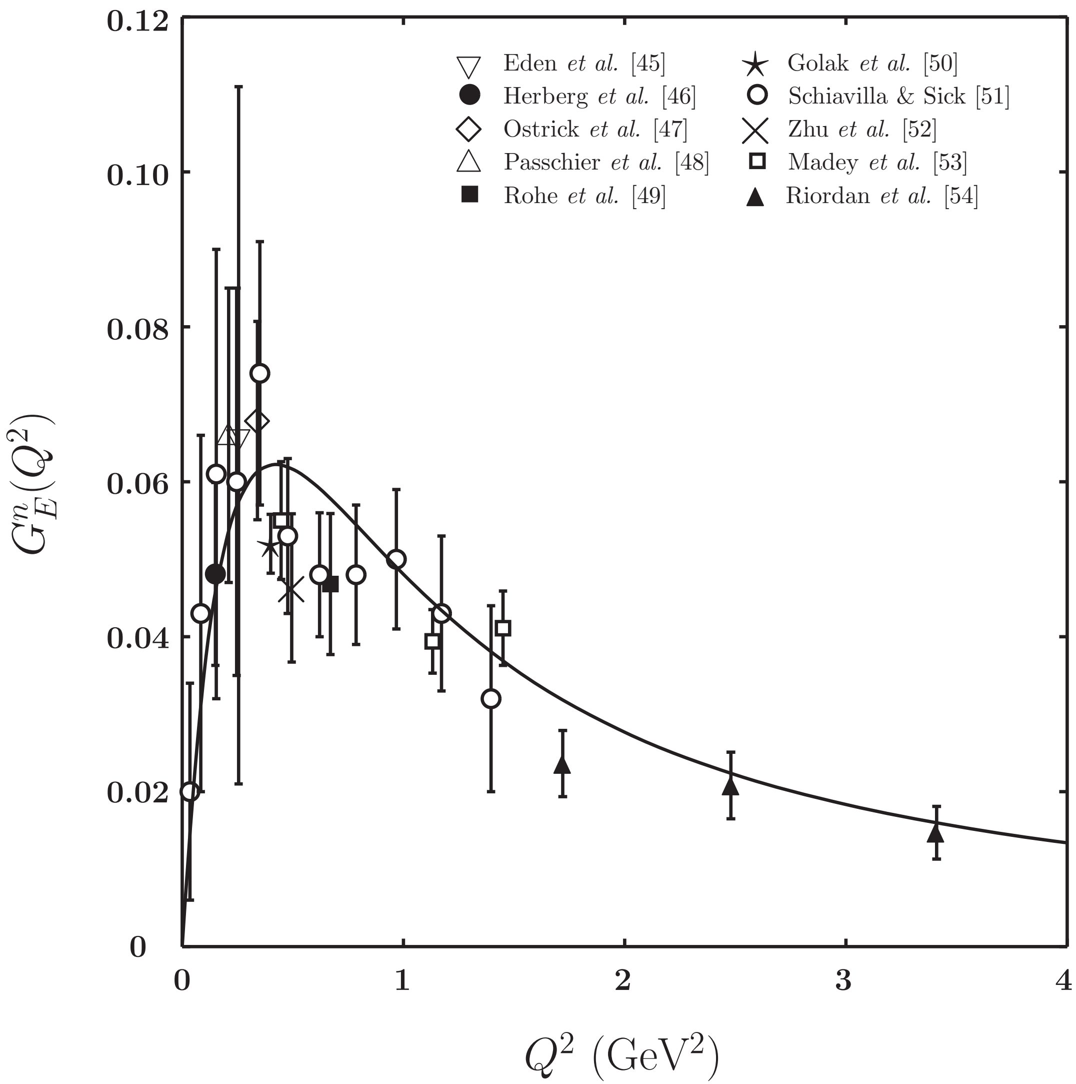

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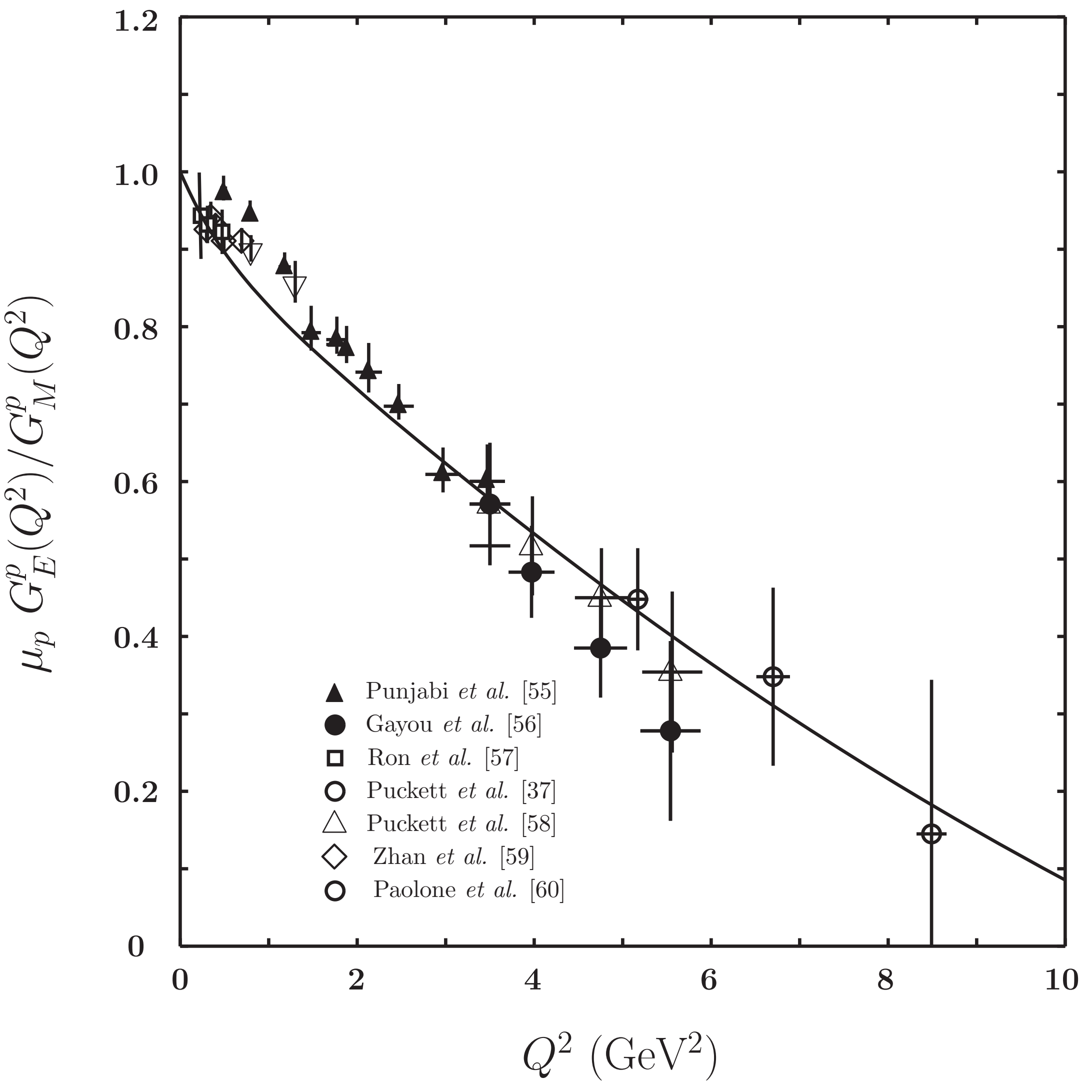
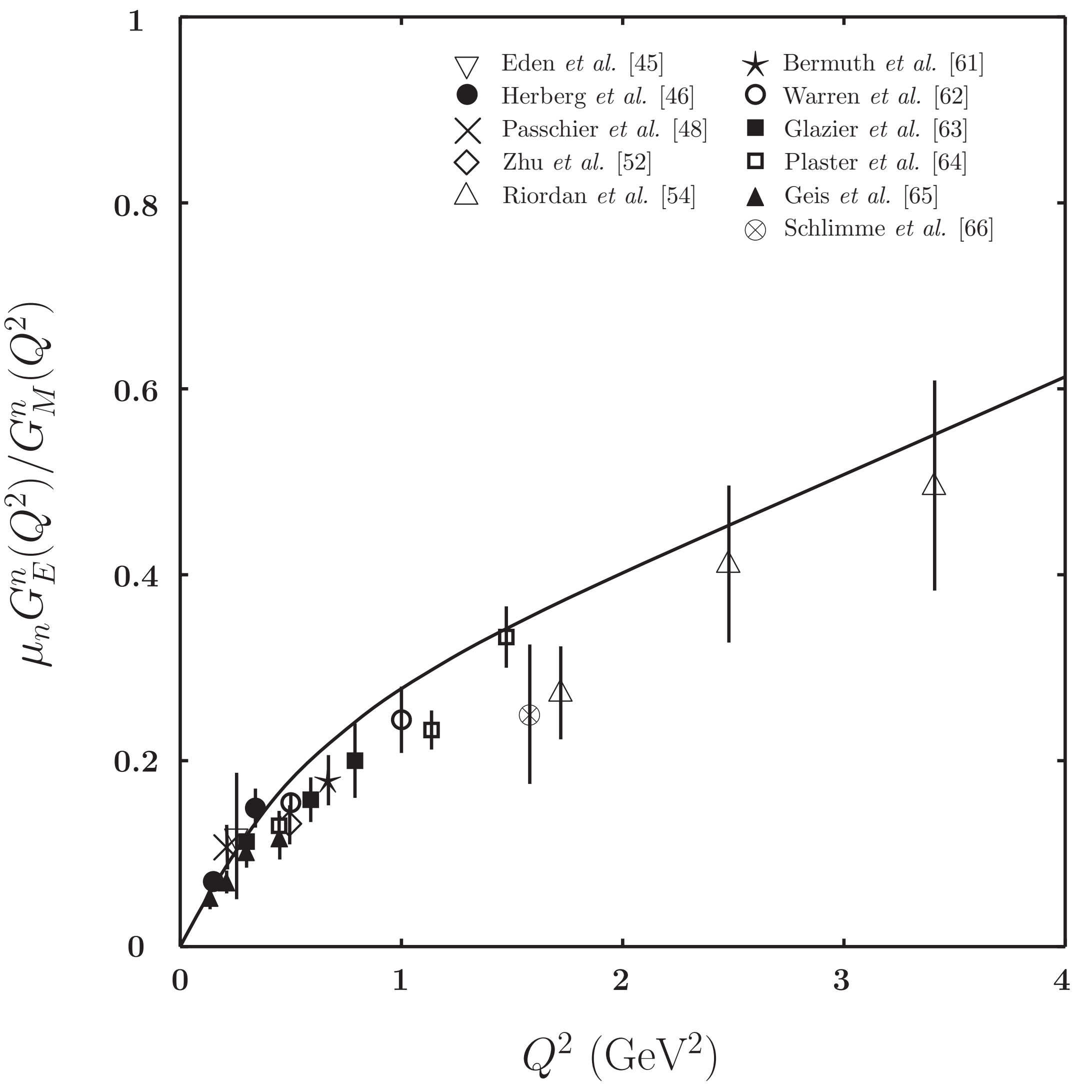
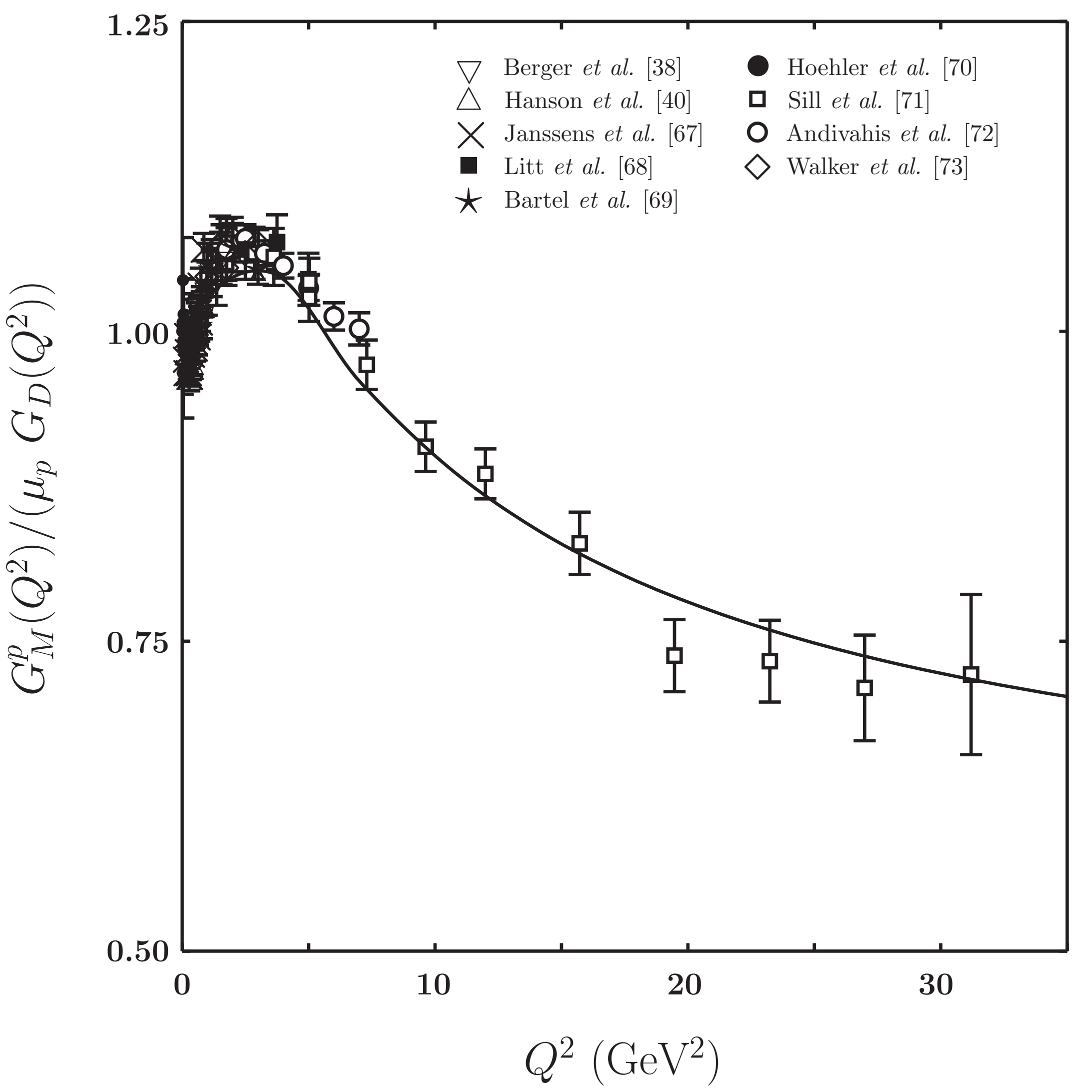


Electromagnetic stucture of nucleons

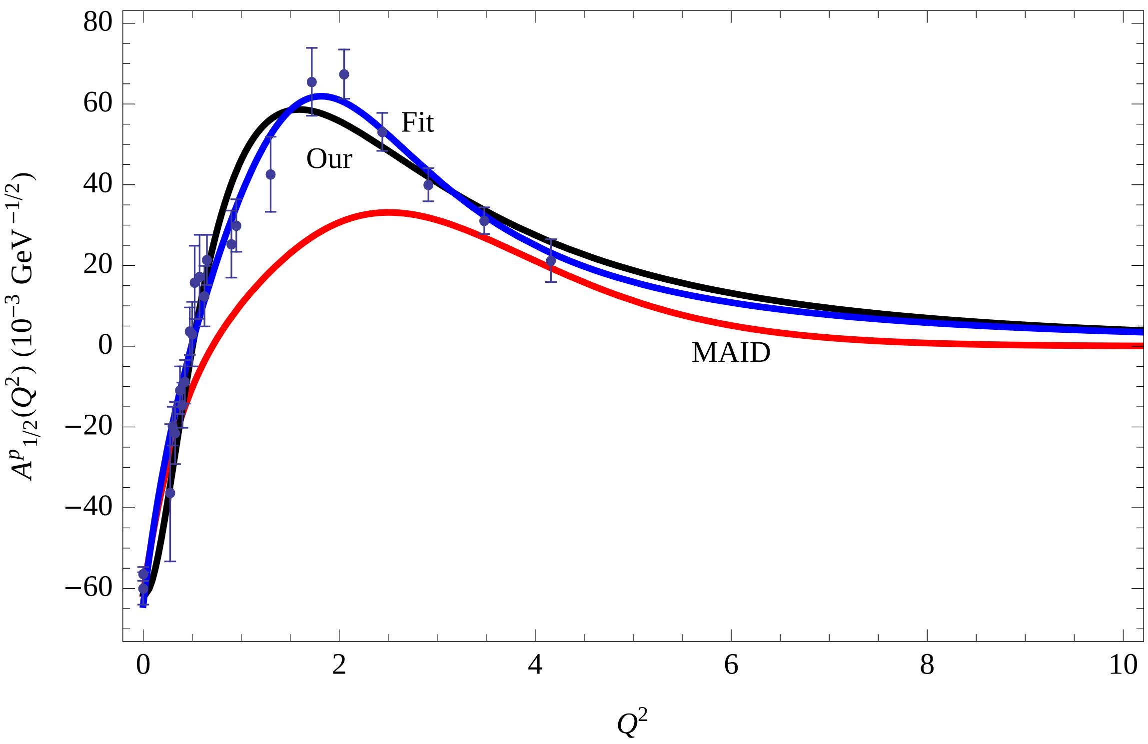


Electromagnetic stucture of nucleons



Electromagnetic stucture of Roper $N(1440)$ with $J^P = \frac{1}{2}^+$

Helicity amplitude $A^p_{1/2}(Q^2)$



Data: CLAS Coll at JLab, Mokeev et al, PRC86 (2012) 035203

Parton Distrubutions of Quarks

- Relation of quark PDF in nucleon $q_v(x)$ at large x with the scaling of the proton Dirac FF $F_1^p(Q^2) \sim 1/(Q^2)^{(p+1)/2}$ at large Q^2
- Parameter p is related to the number of constituents in the proton (or twist τ) as $p = 2\tau - 3$ [Drell-Yan (1869), Blankenbecler-Brodsky (1974)]
- pQCD prediction for GPDs [Yuan (2003)] at arge x and finite Q^2:
$\mathcal{H}_q^\pi(x,Q^2) \sim (1-x)^2\,, \quad \mathcal{H}_q^N(x,Q^2) \sim (1-x)^3\,, \quad \mathcal{E}_q^N(x,Q^2) \sim (1-x)^5$
- pQCD for pion PDF $q_\pi(x) \sim (1-x)^2$ at large x was supported by the updated analysis of the E615 data [Convay et al (1989)] on the cross section of the DY process $\pi^- N \to \mu^+\mu^- X$, including NLL threshold resummation effects [Aicher-Schafer-Vogelsang (2010)]:
$q_\pi(x) \sim (1-x)^{2.03}$ at the initial scale $\mu_0 = 0.63$ GeV

Parton Distrubutions of Quarks

- In Refs. Brodsky-Teramond PRD77 (2008) 056007, Gutsche-Lyubovitskij-Vega-Schmidt PRD83 (2011) 036001, PRD83 (2012) 076003 integral representation for hadronic FF with twist τ has been derived.
- It can be written in closed form as the beta function $B(\alpha, \beta)$

$$F_\tau(Q^2) = \int_0^1 dy\,(\tau-1)\,(1-y)^{\tau-2}\,y^a = (\tau-1)\,B(\tau-1, a+1)$$

- Using identification of the y variable with the light-cone momentum fraction x both PDFs $q_\tau(x)$ and GPDs $\mathcal{H}_\tau(x, Q^2)$ have been extracted

$$q_\tau(x) = (\tau-1)\,(1-x)^{\tau-2}\,, \quad \mathcal{H}_\tau(x, Q^2) = q_\tau(x)\,x^a$$

- Such x dependence of PDF and GPD contradicts model-independent results: the DY inclusive counting rule for $q_\tau(x)$ at $x \to 1$ and the prediction of pQCD for GPDs

Parton Distrubutions of Quarks

- First noticed (Lyubovitskij, LC 2013, FBS55 (2014) 447) that the interpretation of the variable y in the integral representation as LC variable is not truly correct and that one can think about a *generalized light-cone variable* $y(x)$
- One can obtain power behavior of PDFs and GPDs at large x consistent with QCD provided that an appropriate choice of x dependence of $y(x)$ is made
- Simplest choice the function $y(x)$ was found as

$$y_N(x) = \exp\Big[- \log(1/x)(1-x)^{2/(N-1)} \Big]$$

leading to the correct large-x scaling of PDFs and GPDs in mesons

$$q_\tau^M(x) \sim \mathcal{H}_\tau^M(x,Q^2) \sim (1-x)^{2\tau-2}\,, \quad \text{at } N = 2\tau - 2$$

and in baryons

$$q_\tau^B(x) \sim \mathcal{H}_\tau^B(x,Q^2) \sim (1-x)^{2\tau-3}\,, \quad \text{at } N = 2\tau - 3$$

$y_\tau(x)$ obeys the boundary conditions $y_\tau(0) = 0$ and $y_\tau(1) = 1$

Parton Distrubutions of Quarks

- Similar idea was recently considered in LFHQCD [Brodsky, Teramond et al, PRL120 (2018) 182001, hep-ph/2004.07756]:

$$F_\tau(Q^2) = \frac{1}{N_\tau} \int_0^1 dx\, w'(x)\, [w(x)]^{Q^2/4\lambda - 1/2}\, [1 - w(x)]^{\tau - 2}$$

- Both mathematical extensions considered by two groups (Lyubovitskij et al. and Brodsky et al.) are equivalent upon substitution $y(x) = w(x)$.
- Difference: LFQCD included extra power $-1/2$ in the $[w(x)]^{Q^2/4\lambda - 1/2}$, while in SW AdS/QCD the factor is $[w(x)]^{Q^2/4\lambda}$
- Slightly different analytical expressions for the hadronic form factors but with the same asymptotics:

$$\mathrm{AdS/QCD}: \qquad F_\tau(Q^2) \sim B(\tau - 1, 1 + Q^2/4\lambda) \sim \frac{1}{(Q^2)^{\tau-1}}$$

$$\mathrm{LFQCD}: \qquad F_\tau(Q^2) \sim B(\tau - 1, 1/2 + Q^2/4\lambda) \sim \frac{1}{(Q^2)^{\tau-1}}$$

Parton Distrubutions of Quarks

- Start with the hadronic WF normalization condition (integral over z)

$$1 = \int\limits_0^1 dz\, \phi_\tau^2(z)$$

where

$$\phi_\tau(z) \quad = \quad \sqrt{\frac{2}{\Gamma(\tau-1)}}\, \kappa^{\tau-1}\, z^{\tau-3/2}\, e^{-\kappa^2 z^2/2}$$

is the AdS bulk profile function (for simplicity we restrict here to the bosonic case and extension on fermion case is straightforward)

- Next we use the integral representation for unity

$$\begin{aligned} 1 \quad &= \quad -e^{\kappa^2 z^2} \int\limits_0^1 d\Big[f_\tau(x)\, e^{-\kappa^2 z^2/(1-x)^2}\Big] \\ &= \quad e^{\kappa^2 z^2} \int\limits_0^1 dx \left[\frac{2 f_\tau(x)\, \kappa^2 z^2}{(1-x)^3} - f_\tau'(x)\right] e^{-\kappa^2 z^2/(1-x)^2}, \quad f_\tau(0) = 1 \end{aligned}$$

Parton Distrubutions of Quarks

- Functions $f_\tau(x)$ and $y_\tau(x)$ are related as:

$$\Big(1 - y_\tau(x)\Big)^{\tau-1} = f_\tau(x)\,(1-x)^{2(\tau-1)}$$

  or

$$y_\tau(x) = 1 - \Big[f_\tau(x)\Big]^{\frac{1}{\tau-1}}\,(1-x)^2\,.$$

- We remind that at $x = 0$ the functions $y_\tau(x)$ and $f_\tau(x)$ obey the boundary conditions $y_\tau(0) = 0$ and $f_\tau(0) = 1$. At $x = 1$ function f_τ is finite and its value depends on the specific choice of twist τ (see below), while $y_\tau(1) = 1$ is independent on twist.
- After integration over the variable z we get

$$1 = \int\limits_0^1 dx\,(1-x)^{2\tau-3}\Big[2f_\tau(x)(\tau-1) - f'_\tau(x)(1-x)\Big]\,.$$

- Here and in the following the superscript $(')$ means derivative with respect to x

Parton Distrubutions of Quarks

- Using a general definition for the hadronic PDF $q_\tau(x)$, in the form of the integral representation (first moment) over x

$$1 = \int_0^1 dx\, q_\tau(x)$$

we get:

$$q_\tau(x) = (1-x)^{2\tau-3}\left[2f_\tau(x)(\tau-1) - f'_\tau(x)(1-x)\right] = \left[-f_\tau(x)(1-x)^{2\tau-2}\right]'.$$

- We require that the hadronic PDF $q_\tau(x)$ must have the correct scaling at large x and this behavior is governed by the profile function $f_\tau(x)$.

Pion PDF

- Pion PDF at leading twist $\tau = 2$:

$$q_\pi(x) = (1-x)^2 \left[\frac{2f_\pi(x)}{1-x} - f'_\pi(x)\right] = [-f_\pi(x)(1-x)^2]'$$

- Following pQCD (ASV2010) we consider parametrization at $\mu_0 = 0.63$ GeV

$$q_\pi(x, \mu_0) = N_\pi x^{\alpha - 1} (1-x)^\beta (1 + \gamma x^\delta) ,$$

  where N_π - normalization constant, $\alpha = 0.70$, $\beta = 2.03$, $\gamma = 13.8$, $\delta = 2$

- In JPG42 (2015) 095005 we derived LF wave function which produces this PDF. Now we fix profile function $f_\pi(x)$ matching our result to pQCD
- Restricting to leading twist and $\beta \simeq 2$ in (pQCD result) and using boundary condition $f_\pi(0) = 1$ we fix $f_\pi(x)$:

$$f_\pi(x)(1-x)^2 = 1 - N_\pi\, x^\alpha \left[\frac{1}{\alpha} - \frac{2x}{\alpha+1} + \frac{x^2}{\alpha+2} + \gamma x^\delta \left(\frac{1}{\alpha+\delta} - \frac{2x}{\alpha+\delta+1} + \frac{x^2}{\alpha+\delta+2}\right)\right]$$

- f_π obeys the boundary conditions $f_\pi(0) = 1$ and $f_\pi(1) = 0$. At large x we have $f_\pi(x) \sim (1-x)$ and $q_\pi(x) \sim (1-x)^2$

Pion PDF

- Relation of $f_\pi(x)$ with $y_\pi(x) \equiv y_2(x)$ is

$$y_\pi(x) = 1 - f_\pi(x)(1-x)^2 \,,$$

which for large x due to $f_\pi(x) \sim (1-x)$ simplifies to

$$y_\pi(x) = 1 - (1-x)^3 = x\,(3 - 3x + x^2)\,.$$

In PRD86 (2012) 036007 we proposed a formalism for inclusion of high-Fock states in SW AdS/QCD. In case of PDF it is given by the sum:

$$q_\pi(x) = \sum_{\tau=2,4,\ldots} c_\tau\, q_\tau(x) \,,$$

where c_τ - mixing coefficients defining the partial contributions to the pion PDF, from specific twists $\tau = 2, 4, \ldots$, which obey the normalization condition:

$$1 = \int\limits_0^1 dx\, q_\pi(x) = \sum_{\tau=2,4,\ldots} c_\tau \int\limits_0^1 dx\, q_\tau(x) = \sum_{\tau=2,4,\ldots} c_\tau \,.$$

Nucleon PDFs

- In nucleon case: two holographic functions dual to its right- (f_τ^R) and left-chirality (f_τ^L) wave functions
- Normalization conditions for u and d quark WF, equivalent to the normalization conditions for their valence PDFs [$u_v(x)$ and $d_v(x)$], read

$$u - \text{quark}: \; 2 = \int_0^1 dx\, u_v(x) = \int_0^\infty dz \left[2\Phi^+(z) + \eta_u\, \partial_z \left[z\, \Phi^-(z) \right] \right],$$

$$d - \text{quark}: \; 1 = \int_0^1 dx\, d_v(x) = \int_0^\infty dz \left[\Phi^+(z) + \eta_d\, \partial_z \left[z\, \Phi^-(z) \right] \right]$$

where

$$\Phi^\pm = \frac{1}{2} \left[\left(f_\tau^R \right)^2 \pm \left(f_\tau^L \right)^2 \right],$$

are the combinations of right and left holographic wave functions, $\eta_u = 2\eta_p + \eta_n$ and $\eta_d = 2\eta_n + \eta_p$ are the linear combinations of the nucleon couplings with vector field related to nucleon anomalous magnetic moments k_N and fixed as: $\eta_N = k_N \kappa/(2M_N\sqrt{2})$, where M_N is the nucleon mass

Nucleon PDFs

- Contribution of “nonminimal” terms vanish in the normalization condition for wave functions and PDFs due gauge invariance, but they contribute to the x-dependence of PDFs. “Nonminimal contributions” to the quark PDFs are sufficient to violate the symmetry condition $u_v(x)/d_v(x) = 2$, which occurs at $\eta_p = \eta_n = 0$
- For leading twist $\tau = 3$, the results for $u_v(x)$ and $d_v(x)$ read

$$\begin{aligned}
u_v(x) &= \Big[-f_u(x)(1-x)^4\Big(1 + 2\eta_u + (1-x)^2(1-4\eta_u) + 2\eta_u(1-x)^4\Big)\Big]' \\
d_v(x) &= \Big[-f_d(x)(1-x)^4\Big(\frac{1}{2} + 2\eta_d + (1-x)^2\Big(\frac{1}{2} - 4\eta_d\Big) + 2\eta_d(1-x)^4\Big)\Big]'
\end{aligned}$$

At large x both PDFs scale as $(1-x)^3$, as dictated by the counting rules, when the $f_u(x)$ and $f_d(x)$ go to constants independent on x

- Taylor expansion for $f_q(x)$, $q = u, d$ has the generic form

$$f_q(x) = \sum_n c_n (1-x)^n \,,$$

with $\sum\limits_n c_n = 1$, due to the boundary condition $f_q(0) = 1$

Nucleon PDFs

- World data analysis (e.g., MSTW 2008) supports the $(1-x)^3$ scaling of the u_v, while the extracted d_v has softer behavior $(1-x)^5$. Other groups give either similar fits or softer behavior of the d quark PDF, such as e.g. $(1-x)^{4.47\pm 0.55}$ (Alekhin et al, 2017), or introduce into the d quark PDF a nontrivial polynomial depending on $\sqrt{x}$ (CTEQ Coll., Hou et al, 2019).
- We can resolve this puzzle. The solution is based on a suppression of $(1-x)^3$ term in d_v, which can occur when the following constraint on the η_d coupling holds:

$$\frac{1}{2} + 2\eta_d = 0\,.$$

- From the latter condition it follows that the dilaton scale parameter κ is related to the nucleon mass as $\kappa = 0.348\,M_N = 326$ MeV, which is very close to the value $\kappa = 350$ MeV used in our calculations.
- Adopting above condition and restricting to the leading order in the $(1-x)$ expansion, we get the following expressions for the quark PDFs in the nucleon:

$$u_v(x) = \Big[- f_u(x)(1-x)^4 \Big]^\prime\,, \quad d_v(x) = \Big[- f_d(x)(1-x)^6 \Big]^\prime\,.$$

Nucleon PDFs

- Now we fix the u and d profile functions $f_u(x)$ and $f_d(x)$, using predictions for the valence PDFs $u_v(x)$ and $d_v(x)$ extracted from world data analysis. As an example, we use the results of the MSTW 2008 LO global analysis:

$$\begin{aligned} u_v(x,\mu_0) &= A_u\, x^{\alpha_u - 1}\, (1-x)^{\beta_u}\, (1 + \epsilon_u \sqrt{x} + \gamma_u x)\,, \\ d_v(x,\mu_0) &= A_d\, x^{\alpha_d - 1}\, (1-x)^{\beta_d}\, (1 + \epsilon_d \sqrt{x} + \gamma_d x)\,, \end{aligned}$$

where $\mu_0 = 1$ GeV is the initial scale. The normalization constants A_q and the constants α_q, β_q, ϵ_q, γ_q were fixed as

$$\begin{aligned} &A_u = 1.4335\,, \quad A_d = 5.0903\,, \\ &\alpha_u = 0.45232\,, \quad \alpha_d = 0.71978\,, \\ &\beta_u = 3.0409 \simeq 3\,, \quad \beta_d = 5.1244 \simeq 5\,, \\ &\epsilon_u = -2.3737\,, \quad \epsilon_d = -4.3654\,, \\ &\gamma_u = 8.9924\,, \quad \gamma_d = 7.4730\,. \end{aligned}$$

Nucleon PDFs

- Solving the differential equations for profile functions $f_q(x)$ with the boundary condition $f_q(0) = 1$ and using $\beta_u = 2$, $\beta_d = 5$ we get:

$$\begin{aligned} f_u(x)\,(1-x)^4 &= 1 - A_u x^{\alpha_u}\left[B_u(x,0) + \epsilon_u\sqrt{x}B_u(x,1/2) + \gamma_u x B_u(x,1)\right], \\ f_d(x)\,(1-x)^6 &= 1 - A_d x^{\alpha_d}\left[B_d(x,0) + \epsilon_d\sqrt{x}D_d(x,1/2) + \gamma_d x B_d(x,1)\right], \end{aligned}$$

  where

$$B_u(x,n) = \sum_{k=0}^{3}\frac{C_3^k\,(-x)^k}{\delta_u + n + k}\,, \qquad B_d(x,n) = \sum_{k=0}^{5}\frac{C_5^k\,(-x)^k}{\delta_d + n + k}$$

  Here $C_m^k = \frac{m!}{k!(m-k)!}$ are the binomial coefficients

- As in π case we derive relations between nucleon functions $y_q(x)$ and $f_q(x)$:

$$y_u(x) = 1 - \sqrt{f_u(x)}\,(1-x)^2\,, \qquad y_d(x) = 1 - \sqrt{f_d(x)}\,(1-x)^3$$

Nucleon PDFs

- For large $x \to 1$ the expressions for $f_q(x)$ and $y_q(x)$ are simplified:

$$\begin{aligned} f_u(x) &= f_d(x) \, = \, 1 \,, \\ y_u(x) &= 1-(1-x)^2 \, = \, x(2-x) \,, \\ y_d(x) &= 1-(1-x)^3 \, = \, x(3-3x+x^2) \end{aligned}$$

  It is clear that in this limit quark PDFs in nucleon obey correct large x scaling:

$$u_v(x) \, = \, 8\,(1-x)^3 \,, \qquad d_v(x) \, = \, 6\,(1-x)^5$$

- We use the MSTW 2008 LO global analysis as an example of application of our framework. We can choose any other and match the profile functions f_q accordingly. The universality of our approach is that the profile functions f_q appear in other parton distributions like TMDs and GPDs. Therefore, as soon as the profile functions f_q are fixed from PDFs, one can have predictions for the other parton densities.

Nucleon PDFs

- Now we turn to a discussion of the magnetization PDFs in nucleons $\mathcal{E}_v^u(x)$ and $\mathcal{E}_v^d(x)$. The idea of their derivation is similar to the case of the charged PDFs $u_v(x)$ and $d_v(x)$. We start with expressions for the contribution to the anomalous magnetic moments k^q of u and d quarks in soft-wall AdS/QCD model, given as integrals over left- and right-chirality nucleon wave functions with specific twist τ:

$$k_\tau^q = 2M_N \eta_q \int\limits_0^\infty dz\, z\, \phi_\tau^L(z)\, \phi_\tau^R(z) = \frac{2M_N}{\kappa}\, \eta_q \sqrt{\tau - 1}\,.$$

- Next we use integral representation for unity. After integration over z we get the magnetization PDFs in nucleon for leading twist $\tau = 3$:

$$\mathcal{E}_v^q(x) = k^q \Big[- f_q(x)\,(1-x)^6 \Big]'\,.$$

In principle, the $f_q(x)$ profile functions can be different in charged and magnetization PDFs. In the case when they are the same we derive the following relation:

$$\mathcal{E}_v^d(x)/d_v(x) = 4\eta_d\, M_N/\kappa$$

TMDs

- TMD arise in SW AdS/QCD by analogy with PDF, using generalized integral representation for unity, including integration over x and $\mathbf{k}_\perp$ variables:

$$
\begin{aligned}
1 &= -e^{\kappa^2 z^2} \int\limits_0^1 d\Big[f_\tau(x) e^{-\kappa^2 z^2/(1-x)^2}\Big] \int d^2\mathbf{k}_\perp \, \frac{D_\tau(x)}{\pi\kappa^2} e^{-\mathbf{k}_\perp^2 D_\tau(x)/\kappa^2} \\
&= \frac{e^{\kappa^2 z^2}}{\pi\kappa^2} \int\limits_0^1 dx \int d^2\mathbf{k}_\perp \left[\frac{2 f_\tau(x)\kappa^2 z^2}{(1-x)^3} - f'_\tau(x)\right] D_\tau(x) e^{-\kappa^2 z^2/(1-x)^2} e^{-\mathbf{k}_\perp^2 D_\tau(x)/\kappa^2}
\end{aligned}
$$

  where $D_\tau(x)$ is the factor derived in (Gutsche, Lyubovitskij, Schmidt, EPJC77 (2016) 86), which was fixed from data on the nucleon electromagnetic form factors

- Purpose of $D_\tau(x)$ is to include a running scale in TMD, i.e. scale parameter, which accompanies $\mathbf{k}_\perp$ dependence in TMDs. In our case the running scale parameter is $\Lambda_\tau(x) = \kappa/\sqrt{D_\tau(x)}$

- Such choice of $\Lambda_\tau(x)$ is a generalization of the Gaussian ansatz for TMD with constant scale $\Lambda^2 = \langle \mathbf{k}_\perp^2 \rangle$ in the exponential, proposed by Turin group (Anselmino et al., PRD67 (2003) 074010): $F(x, \mathbf{k}_\perp) = F(x)\, e^{-\mathbf{k}_\perp^2/\langle \mathbf{k}_\perp^2 \rangle}$

TMDs

- This Gaussian ansatz is simple and very useful in practical calculations and analysis of data. However, it is known (see e.g., Bacchetta et el., PRD100 (2019) 014018), that it presents difficulties in the description of data on DY processes in some kinematical regions (e.g. at $Q_\perp \leq Q$)
- Therefore, the ansatz for the TMD can be crucially checked. In this vein, one can mention results of AdS/QCD and light-front quark models motivated by AdS/QCD where it was shown that the hadronic light-front wave functions, PDFs, and TMDs contain scale parameter depending on the light-cone variable x, i.e. they can be considered as x-dependent scale quantities
- We found that x-dependent scale is crucial for a successful description of data on electromagnetic form factors of nucleons and electroexcitation of nucleon resonances. Also we can see below that our result for the unpolarazed quark TMD in nucleon will contain two terms multiplied with a Gaussian: constant term and term proportional to $\mathbf{k}_\perp^2$. It is consistent with the form of TMD used by the Pavia group (Bacchetta et al., JHEP 06 (2017) 081). Next we will show that function $D_\tau(x)$ can be fixed from expression for the electromagnetic form factor and related to functions $f_\tau(x)$ and $y_\tau(x)$.

TMDs

- Using the same calculation technique as for the case of PDFs, we insert the integral representation into the normalization condition for the holographic wave function and integrate over the z variable. After that we arrive at the normalization condition for the TMD $F_\tau(x, \mathbf{k}_\perp)$, from which the latter can be extracted and expressed through PDF as:

$$1 = \int_0^1 dx \int d^2\mathbf{k}_\perp \, F_\tau(x, \mathbf{k}_\perp) \,, \qquad F_\tau(x, \mathbf{k}_\perp) = q_\tau(x) \, \frac{D_\tau(x)}{\pi\kappa^2} \, e^{-\mathbf{k}_\perp^2 \, D_\tau(x)/\kappa^2}$$

- Also it is important to stress that from the results for generic PDFs and TMDs derived in present paper one can set up LF quark model in analogy with our previous analysis. In particular, the LF wave function for generic hadron with twist τ reads:

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi}{\kappa} \sqrt{q_\tau(x) \, D_\tau(x)} \, \exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2} \, D_\tau(x)\right] .$$

TMDs

- Note that generic TMD and PDF are expressed in term of LF wave function as:

$$F_\tau(x,\mathbf{k}_\perp) = \frac{1}{16\pi^3}\,|\psi(x,\mathbf{k}_\perp)|^2\,, \quad q_\tau(x) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\,|\psi(x,\mathbf{k}_\perp)|^2 = \int d^2\mathbf{k}_\perp\, F_\tau(x,\mathbf{k}_\perp)$$

- As example we consider unpolarized quark TMD in nucleon $f_1^{q_v}(x,\mathbf{k}_\perp)$. As in case of PDF it is contributed by two wave functions $\phi^R(z)$ and $\phi^L(z)$
- $\phi^R(z)$ function generates the contribution to TMD $f_{1,R}^{q_v}(x,\mathbf{k}_\perp)$, while the $\phi^L(z)$ gives the contribution $f_{1,L}^{q_v}(x,\mathbf{k}_\perp)$ proportional to $\mathbf{k}_\perp^2$:

$$f_1^{q_v}(x,\mathbf{k}_\perp) = f_{1,R}^{q_v}(x,\mathbf{k}_\perp) + f_{1,L}^{q_v}(x,\mathbf{k}_\perp)\,,$$

where

$$f_{1,R}^{q_v}(x,\mathbf{k}_\perp) = q_v^+(x)\,\frac{D_q(x)}{2\pi\kappa^2}\,e^{-\frac{\mathbf{k}_\perp^2\,D_q(x)}{\kappa^2}}\,, \quad f_{1,L}^{q_v}(x,\mathbf{k}_\perp) = q_v^-(x)\,\frac{\mathbf{k}_\perp^2\,D_q^2(x)}{2\pi\kappa^4}\,e^{-\frac{\mathbf{k}_\perp^2\,D_q(x)}{\kappa^2}}$$

Here $q_v^\pm(x) = q_v(x) \pm \delta q_v(x)$, $q_v(x)$ and $\delta q_v(x)$ are the helicity-independent and helicity-dependent valence quark parton distributions.

TMDs

- As we mentioned before, the form of our expression for TMD

$$f_1^{q_v}(x, \mathbf{k}_\perp) \quad = \quad \left[q_v^+(x) + q_v^-(x)\, \frac{\mathbf{k}_\perp^2\, D_q(x)}{\kappa^2}\right] \frac{D_q(x)}{2\pi\kappa^2}\, e^{-\mathbf{k}_\perp^2 D_q(x)/\kappa^2}$$

  is very similar to the parametrization used by Pavia group:

$$f_1^a(x, \mathbf{k}_\perp) = \frac{1}{\pi g_{1a}} \frac{1 + \lambda \mathbf{k}_\perp^2}{1 + \lambda g_{1a}}\, e^{-\mathbf{k}_\perp^2/g_{1a}}$$

- Using expressions for nucleon PDFs and TMDs one can set up the LF wave functions for the nucleon following our findings:

$$\psi^{\pm}_{\pm q}(x, \mathbf{k}_\perp) = \varphi_q^{(1)}(x, \mathbf{k}_\perp)\,, \qquad \psi^{\pm}_{\mp q}(x, \mathbf{k}_\perp) = \mp \frac{k^1 \pm i k^2}{M_N}\, \varphi_q^{(2)}(x, \mathbf{k}_\perp)\,,$$

$$\varphi_q^{(1)}(x, \mathbf{k}_\perp) = \frac{2\pi\sqrt{2}}{\kappa} \sqrt{q_v^+(x)\, D_q(x)}\, \exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\, D_q(x)\right],$$

$$\varphi_q^{(2)}(x, \mathbf{k}_\perp) = \frac{2\pi c_q \sqrt{2} M_N}{\kappa^2} \sqrt{q_v^-(x)}\, D_q(x)\, \exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\, D_q(x)\right].$$

TMDs

- Here $c_u = 1$, $c_d = -1$, $\psi^{\lambda_N}_{\lambda_q q}(x, \mathbf{k}_\perp)$ are the LFWFs at the initial scale μ_0 with specific helicities for the nucleon $\lambda_N = \pm$ and for the struck quark $\lambda_q = \pm$, where plus and minus correspond to $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively. Note, in terms LF wave functions the unpolarized quark TMD in nucleon reads (Bacchetta et al, PRD78 (2008) 074010):

$$
\begin{aligned}
f_1^{q_v}(x, \mathbf{k}_\perp) &= \frac{1}{16\pi^3}\left[|\psi^{+}_{+q}(x, \mathbf{k}_\perp)|^2 + |\psi^{+}_{-q}(x, \mathbf{k}_\perp)|^2\right] \\
&= \frac{1}{16\pi^3}\left[\left(\varphi_q^{(1)}(x, \mathbf{k}_\perp)\right)^2 + \frac{\mathbf{k}_\perp^2}{M_N^2}\left(\varphi_q^{(2)}(x, \mathbf{k}_\perp)\right)^2\right]
\end{aligned}
$$

Note $q_v^{\pm}(x)$ and $\mathcal{E}_v^q(x)$ PDFs are related as (Lyubovitskij et al., 2017):

$$
\mathcal{E}_v^q(x) = c_q \sqrt{q_v^{+}(x)\, q_v^{-}(x)\, D_q(x)}\,(1 - x)\,.
$$

The full set of the valence T-even TMDs generated by LF wave functions derived above is listed below.

TMDs

- Here we list the T-even TMDs of nucleon using derived LF decomposition discussed in (Bacchetta et al., 2008) and (Lyubovitskij et al., 2017)

$$f_1^{q_v}(x,\mathbf{k}_\perp) \equiv h_{1T}^{q_v}(x,\mathbf{k}_\perp) = \frac{1}{16\pi^3}\left[\left(\varphi_q^{(1)}(x,\mathbf{k}_\perp)\right)^2 + \frac{\mathbf{k}_\perp^2}{M_N^2}\left(\varphi_q^{(2)}(x,\mathbf{k}_\perp)\right)^2\right],$$

$$g_{1L}^{q_v}(x,\mathbf{k}_\perp) = \frac{1}{16\pi^3}\left[\left(\varphi_q^{(1)}(x,\mathbf{k}_\perp)\right)^2 - \frac{\mathbf{k}_\perp^2}{M_N^2}\left(\varphi_q^{(2)}(x,\mathbf{k}_\perp)\right)^2\right],$$

$$g_{1T}^{q_v}(x,\mathbf{k}_\perp) \equiv -h_{1L}^{\perp q_v}(x,\mathbf{k}_\perp) = \frac{1}{8\pi^3}\,\varphi_q^{(1)}(x,\mathbf{k}_\perp)\,\varphi_q^{(2)}(x,\mathbf{k}_\perp)\,,$$

$$h_1^{q_v}(x,\mathbf{k}_\perp) \equiv h_{1T}^{q_v}(x,\mathbf{k}_\perp) + \frac{\mathbf{k}_\perp^2}{2M_N^2}h_{1T}^{\perp q_v}(x,\mathbf{k}_\perp) = \frac{1}{16\pi^3}\left(\varphi_q^{(1)}(x,\mathbf{k}_\perp)\right)^2,$$

$$\frac{\mathbf{k}_\perp^2}{2M_N^2}h_{1T}^{\perp q_v}(x,\mathbf{k}_\perp) \equiv \frac{1}{2}\left[g_{1L}^{q_v}(x,\mathbf{k}_\perp) - f_1^{q_v}(x,\mathbf{k}_\perp)\right] \equiv g_{1L}^{q_v}(x,\mathbf{k}_\perp) - h_1^{q_v}(x,\mathbf{k}_\perp)$$

$$= -\frac{\mathbf{k}_\perp^2}{16\pi^3 M_N^2}\left(\varphi_q^{(2)}(x,\mathbf{k}_\perp)\right)^2.$$

TMDs

- Using our LF WF we express TMDs through the PDFs

$$
\begin{aligned}
f_1^{q_v}(x,\mathbf{k}_\perp) &\equiv h_{1T}^{q_v}(x,\mathbf{k}_\perp) = \mathcal{F}_1(x,\mathbf{k}_\perp) + \mathcal{F}_2(x,\mathbf{k}_\perp)\,, \\
g_{1L}^{q_v}(x,\mathbf{k}_\perp) &= \mathcal{F}_1(x,\mathbf{k}_\perp) - \mathcal{F}_2(x,\mathbf{k}_\perp)\,, \\
g_{1T}^{q_v}(x,\mathbf{k}_\perp) &\equiv -h_{1L}^{\perp q_v}(x,\mathbf{k}_\perp) = \mathcal{F}_3(x,\mathbf{k}_\perp)\,, \\
h_1^{q_v}(x,\mathbf{k}_\perp) &= \mathcal{F}_1(x,\mathbf{k}_\perp)\,, \\
\frac{\mathbf{k}_\perp^2}{2M_N^2} h_{1T}^{\perp q_v}(x,\mathbf{k}_\perp) &= -\mathcal{F}_2(x,\mathbf{k}_\perp)\,,
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{F}_1(x,\mathbf{k}_\perp) &= q_v^+(x)\,\frac{D_q(x)}{2\pi\kappa^2}\,e^{-\frac{\mathbf{k}_\perp^2}{\kappa^2}D_q(x)}\,, \\
\mathcal{F}_2(x,\mathbf{k}_\perp) &= q_v^-(x)\,\frac{\mathbf{k}_\perp^2\,D_q^2(x)}{2\pi\kappa^4}\,e^{-\frac{\mathbf{k}_\perp^2}{\kappa^2}D_q(x)}\,, \\
\mathcal{F}_3(x,\mathbf{k}_\perp) &= c_q\sqrt{\frac{4\kappa^2}{\mathbf{k}_\perp^2}\,\mathcal{F}_1(x,\mathbf{k}_\perp)\,\mathcal{F}_2(x,\mathbf{k}_\perp)} = \sqrt{q_v^+(x)\,q_v^-(x)}\,\frac{c_q\,D_q^{3/2}(x)}{\pi\kappa^2}\,e^{-\frac{\mathbf{k}_\perp^2}{\kappa^2}D_q(x)}
\end{aligned}
$$

TMDs

- Performing the $\mathbf{k}_\perp$-integration over the TMDs with

$$\mathrm{TMD}(x) = \int d^2\mathbf{k}_\perp\, \mathrm{TMD}(x, \mathbf{k}_\perp)\,, \qquad \overline{\mathrm{TMD}}(x) = \int d^2\mathbf{k}_\perp\, \frac{\mathbf{k}_\perp^2}{2M_N^2}\, \mathrm{TMD}(x, \mathbf{k}_\perp)$$

gives the identities

$$f_1^{q_v}(x) \equiv h_{1T}^{q_v}(x) \;=\; q_v(x)\,, \qquad g_{1L}^{q_v}(x) = \delta q_v(x)\,, \qquad g_{1T}^{q_v}(x) \equiv -h_{1L}^{\perp q_v}(x) = \frac{\mathcal{E}^q(x)}{1-x}\,,$$
$$h_1^{q_v}(x) = \frac{q_v(x) + \delta q_v(x)}{2}\,, \qquad \overline{h_{1T}^{\perp q_v}}(x) = -\frac{q_v(x) - \delta q_v(x)}{2}\,.$$

The integration over x leads to the normalization conditions

$$\int\limits_0^1 dx f_1^{q_v}(x) = \int\limits_0^1 dx h_{1T}^{q_v}(x) = n_q\,, \quad \int\limits_0^1 dx g_{1L}^{q_v}(x) = g_A^q\,, \quad \int\limits_0^1 dx h_1^{q_v}(x) = g_T^q$$

where n_q is the number of u or d valence quarks in the proton, g_A^q is the axial charge of a quark with flavor $q = u$ or d, and g_T^q is the tensor charge. Our TMDs satisfy all relations and inequalities found before in theoretical approaches.

GPDs

- As we mentioned before, the nucleon GPDs were calculated for the first time in soft-wall AdS/QCD in (Vega, Schmidt, Gutsche, Lyubovitskij, PRD83 (2011) 036011). These quantities were expressed in terms of *generalized light-cone variable* $y_\tau(x)$, which has direct relation to the profile function $f_\tau(x)$. Function $f_\tau(x)$ is more convenient for displaying power behavior of hadronic parton distributions (PDFs, TMDs, GPDs). In particular, for arbitrary twist τ, a generic GPD in hadron reads

$$\mathcal{H}_\tau(y_\tau(x), Q^2) = (\tau - 1)\,(1 - y_\tau(x))^{\tau-2}\,\Big[y_\tau(x)\Big]^a\,, \quad a = \frac{Q^2}{4\kappa^2}\,.$$

It can be written in more convenient form in terms of PDF:

$$\mathcal{H}_\tau(x, Q^2) = q_\tau(x)\,\Big[y_\tau(x)\Big]^a = q_\tau(x)\,\exp\Big(- a\log\Big[1/y_\tau(x)\Big]\Big)\,,$$

where the PDF $q_\tau(x)$ and light-cone function $y_\tau(x)$ are expressed through profile function $f_\tau(x)$ according formulas discussed before.
Next we constrain function $D_\tau(x)$ and relate it to functions $y_\tau(x)$ and $f_\tau(x)$ matching the expression for the hadronic form factors in two approaches — soft-wall AdS/QCD and LF QCD

GPDs

- The LF QCD result for the hadron form factor is given by the Drell-Yan-West formula

$$F_\tau(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \, \psi^\dagger_\tau(x, \mathbf{k}'_\perp) \, \psi_\tau(x, \mathbf{k}_\perp) \, ,$$

where $\psi(x, \mathbf{k}_\perp) \equiv \psi(x, \mathbf{k}_\perp; \mu_0)$ is wave function derived in DYW formula, $\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$, and $Q^2 = \mathbf{q}^2_\perp$.
We get:

$$F_\tau(Q^2) = \int_0^1 dx \, q_\tau(x) \, \exp\Big[- a \log[1/y_\tau(x)] \Big] = \int_0^1 dx \, q_\tau(x) \, \exp\Big[- a D_\tau(x)(1-x)^2 \Big]$$

or

$$D_\tau(x) = \frac{1}{(1-x)^2} \, \log[1/y_\tau(x)] = \frac{1}{(1-x)^2} \, \log\Big[1 - \Big(f_\tau(x)\Big)^{\frac{1}{\tau-1}} (1-x)^2 \Big]^{-1} .$$

GPDs

- For large x function $D_\tau(x)$ behaves as

$$D_\tau(x) = \Big(f_\tau(x)\Big)^{\frac{1}{\tau-1}} ,$$

where $f_\pi(x) = 1 - x$, $f_u(x) = f_d(x) = 1$ and therefore $D_\pi(x) = 1 - x$, $D_u(x) = D_d(x) = 1$. It leads to the following scaling of the TMDs at large x:

$$f_1^\pi(x, \mathbf{k}_\perp) = q_\pi(x)\,(1-x)\,\frac{e^{-\mathbf{k}_\perp^2(1-x)/\kappa^2}}{\pi\kappa^2}$$

for pion,

$$f_1^{q_v}(x, \mathbf{k}_\perp) = \left[q_v^+(x) + q_v^-(x)\,\frac{\mathbf{k}_\perp^2}{\kappa^2}\right]\frac{e^{-\mathbf{k}_\perp^2/\kappa^2}}{2\pi\kappa^2}$$

for nucleon.

GPDs

- Now we consider specific cases for GPDs. In the pion case we have $\tau = 2$ and $y_\pi(x) = 1 - f_\pi(x)\,(1-x)^2$, where the pion profile function $f_\pi(x)$ is fixed from pion PDF
- Pion PDF $q_\pi(x)$ is fixed from data. Therefore, we give the pion GPD prediction at the initial scale $\mu_0 = 1$ GeV in terms of the pion PDF, or more precisely in terms of constants parametrizing PDF (N_π, α, β, γ, δ) fixed in by (Aicher-Schafer-Vogelsang, 2010)
- At large x the profile functions $f_\pi(x) \to (1-x)$ and $y_\pi(x) \to 1$, and the scaling of our result for the pion GPD $(1-x)^2$ is consistent with the pQCD prediction (Yuan, 2003): it coincides with the leading-order result for the pion PDF and is independent on Q^2:

$$\mathcal{H}_\pi(x, Q^2) = q_\pi(x) = 3\,(1-x)^2$$

GPDs

- In the nucleon case we have $\tau = 3$, $y_u(x) = 1 - \sqrt{f_u(x)}\,(1-x)^2$, and $y_d(x) = 1 - \sqrt{f_d(x)}\,(1-x)^3$. The quark profile functions $f_u(x)$ and $f_d(x)$ are fixed from the corresponding nucleon PDFs extracted from global data analysis at the initial scale $\mu_0 = 1$ GeV (MSTW 2009). The four (charged and magnetization) nucleon GPDs at the initial scale $\mu_0 = 1$ GeV are defined as:

$$\mathcal{H}_v^q(x,Q^2) = q_v(x)\,\Big[y_q(x)\Big]^a\,, \qquad \mathcal{E}_v^q(x,Q^2) = \mathcal{E}_v^q(x)\,\Big[y_q(x)\Big]^a\,.$$

Finally we consider the limit of large x. In this case the profile functions $f_q(x)$ and functions $y_q(x)$ approach 1: $f_u(x) = f_d(x) = 1$ and $y_u(x) = y_d(x) = 1$. The scaling of the nucleon charge and magnetization GPDs are also (as in case of pion) consistent with the pQCD predictions:

$$\mathcal{H}_v^u(x,Q^2) = u_v(x) = 8\,(1-x)^3\,, \qquad \mathcal{E}_v^q(x,Q^2) = \mathcal{E}_v^q(x) = 6\,\mathcal{E}_v^q\,(1-x)^5\,.$$

In case of$d$ quark charge GPD $\mathcal{H}_v^d(x,Q^2)$ we have two possibilities at large x. In general it scales as $(1-x)^3$ in agreement with pQCD. On the other hand, if we suppress LO term $(1-x)^3$ in the d quark PDF, then $d_v(x)$ has softer $(1-x)^5$ behavior consistent with result of world data analysis (MSTW 2009). In this vein, we also get $(1-x)^5$ scaling of the $\mathcal{H}_v^d(x,Q^2)$

Gluon Parton Distributions

- Unpolarized $G(x)$ and upolarized $\Delta G(x)$ gluon PDFs in QCD (Brodsky-Schmidt):

$$G(x) = G^+(x) + G^-(x)\,, \quad \Delta G(x) = G^+(x) - G^-(x)$$
$$G^+(x) = G_{g\uparrow/N\uparrow}(x)\,, \quad G^-(x) = G_{g\downarrow/N\uparrow}(x)$$

helicity-aligned and anti-aligned gluon distributions

$$G^+(x) = \frac{N}{x}\,[5(1-x)^4 - 4(1-x)^5]\,, \qquad G^-(x) = \frac{N}{x}\,(1-x)^6$$

Hence

$$G(x) = 2N(1-x)^4\,(1/x + 1 + x/2)\,, \qquad \Delta G(x) = 6N(1-x)^4\,(1-x/6)$$

Gluon Parton Distributions

- Limits

$$\lim_{x\to 0} \frac{\Delta G(x)}{G(x)} = N_q\, x\,, \quad N_q = 3 \text{ number of val. quarks}$$
$$\lim_{x\to 1} \frac{G^-(x)}{G^+(x)} = \frac{1}{5}\,(1-x)^2$$

- Momentum $\langle x_g \rangle$ and helicity ΔG fractions carried by intrinsic gluons in nucleon

$$\langle x_g \rangle = \int\limits_0^1 dx x G(x) = \frac{10}{21}\,N\,, \quad \Delta G = \int\limits_0^1 dx \Delta G(x) = \frac{7}{6}\,N\,, \quad \frac{\Delta G}{\langle x_g \rangle} = \frac{49}{20}$$

- Lattice: $\langle x_g \rangle = 0.427 \pm 0.092$ at $Q_0 = 2$ GeV
(Alexandrou et al., PRD101 (2020) 094513)
- Spectator Model: $\langle x_g \rangle = 0.424 \pm 0.009$ at $Q_0 = 1.64$ GeV
(Bacchetta et al., EPJC80 (2020) 733)

Gluon Parton Distributions

- Start with the hadronic WF normalization condition (integral over z) $1 = \int_0^1 dz\, \phi_\tau^2(z)$
- Next we use the integral representation for unity

$$1 = -\frac{1}{\langle x_g \rangle}\, e^{\kappa^2 z^2} \int_0^1 d\Big[f_\tau^g(x)\, e^{-\kappa^2 z^2/(1-x)^2}\Big], \quad f_\tau^g(0) = \langle x_g \rangle$$

- Profile function $f_\tau^g(x)$ is fixed from condition

$$[-f_\tau^g(x)(1-x)^{2\tau-3}]' = \frac{1}{\langle x_g \rangle}\, xG(x)$$

with boundary condition $f_\tau^g(0) = \langle x_g \rangle$. Using Brodsky-Schmidt $G(x)$ one gets

$$f^g(x)(1-x)^6 = \langle x_g \rangle \Big[1 - \frac{21x}{10}\Big(2 - 3x + \frac{5x^2}{3} - \frac{x^5}{3} + -\frac{x^6}{7}\Big)\Big]$$

Gluon Parton Distributions

- TMDs

$$
\begin{aligned}
f_1^g(x,\mathbf{k}_\perp) &= \left[G^+(x)\ +\ G^-(x)\frac{\mathbf{k}_\perp^2\,D_q(x)}{\kappa^2}\right]\frac{D_g(x)}{\pi\kappa^2}\,e^{-\mathbf{k}_\perp^2\,D_g(x)/\kappa^2} \\
g_{1L}^g(x,\mathbf{k}_\perp) &= \left[G^+(x)\ -\ G^-(x)\frac{\mathbf{k}_\perp^2\,D_q(x)}{\kappa^2}\right]\frac{D_g(x)}{\pi\kappa^2}\,e^{-\mathbf{k}_\perp^2\,D_g(x)/\kappa^2} \\
g_{1T}^g(x,\mathbf{k}_\perp) &= \sqrt{G^+(x)\,G^-(x)}\,\frac{2D_q(x)}{\kappa^2}\,e^{-\mathbf{k}_\perp^2\,D_g(x)/\kappa^2} \\
h_1^g(x,\mathbf{k}_\perp) &= G^+(x)\,G^+(x)\,\frac{D_q(x)}{\kappa^2}\,e^{-\mathbf{k}_\perp^2\,D_g(x)/\kappa^2}
\end{aligned}
$$

where $D_g(x) = [f_g(x)]^{1/3}$

Gluon Parton Distributions

- All TMDs have corret connection to PDFs

$$f_1^g(x) = \int d^2\mathbf{k}_\perp f_1^g(x,\mathbf{k}_\perp) = G^+(x) + G^-(x) = G(x)\,,$$

$$g_{1L}^g(x) = \int d^2\mathbf{k}_\perp g_{1L}^g(x,\mathbf{k}_\perp) = G^+(x) - G^-(x) = \Delta G(x)\,,$$

$$g_{1T}^g(x) = \int d^2\mathbf{k}_\perp g_{1T}^g(x,\mathbf{k}_\perp) = 2\sqrt{G^+(x)\,G^-(x)} = \frac{1}{2}\sqrt{G^2(x) - \Delta G^2(x)}\,,$$

$$h_1^g(x) = \int d^2\mathbf{k}_\perp h_1^g(x,\mathbf{k}_\perp) = G^+(x) = \frac{1}{2}\left(G(x) - \Delta G(x)\right)$$

- GPDs

$$H_f(x,Q^2) = f(x)\left[y_g(x)\right]^{Q^2/(4\kappa^2)}\,, \quad y_g(x) = 1 - f_g(x)(1-x)^4$$

Summary

- It was explicitly demonstrated how to correctly define the hadronic parton distributions (PDFs, TMDs, and GPDs) in the soft-wall AdS/QCD approach based on the use of quadratic dilaton.
- The large x behavior of PDFs and GPDs is consistent with model-independent counting rules. For the first time, we derive results for the large x behavior of TMDs.
- All parton distributions are defined in terms of profile functions $f_\tau(x)$ depending on the light-cone coordinate.
- The functions $f_\tau(x)$ are related to the PDFs and obey the boundary condition $f_\tau(0) = 1$ (quarks) and $f_\tau^g(x) = \langle x_g \rangle$ (gluon).
- Profile functions are fixed from data analysis on PDFs and can then be tested in the phenomenology of TMDs and GPDs.