Gluon structure of nucleons and nuclei from lattice QCD

Phiala Shanahan, MIT

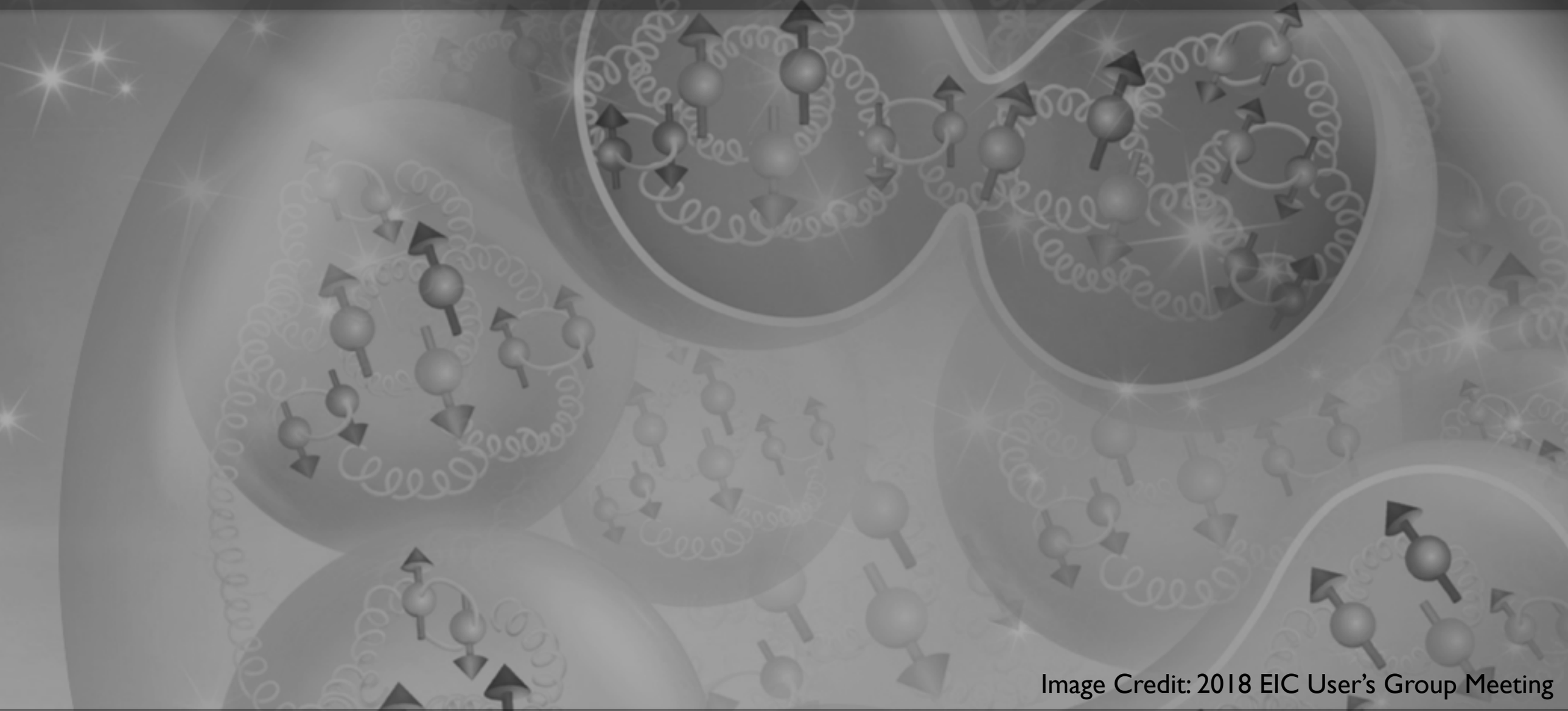

Image Credit: 2018 EIC User's Group Meeting

Massachusetts Institute of Technology

The gluon structure of the nucleon

How much do gluons contribute to the proton's

- Momentum
- Spin
- Mass
- D-term

[see Keh-Fei Liu's talk]

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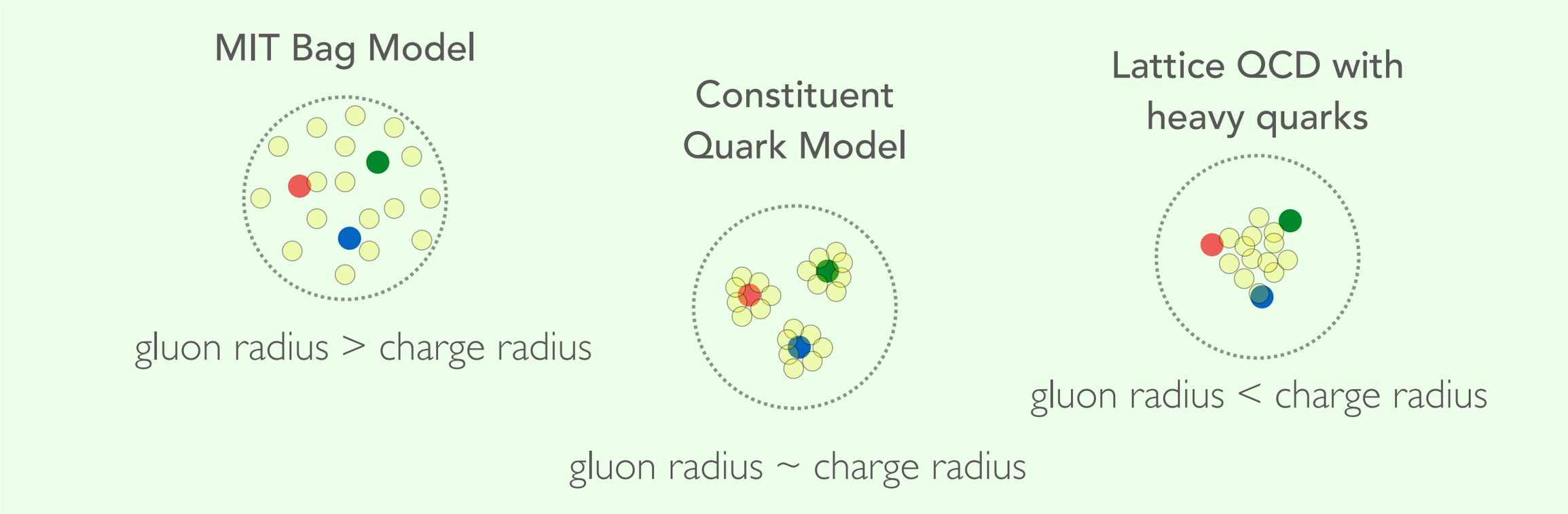
What is the 3D gluon distribution of a proton

- PDFs
- GPDs
- TMDs
- Pressure distribution

The gluon structure of the nucleon

How does the gluon radius of a proton compare to the quark/charge radius?



Charge radius:
slope of electric form factor with respect to momentum transfer

Gluon radius:
[see Maxim Polyakov's talk]

The gluon structure of the nucleon

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[see Keh-Fei Liu's talk]

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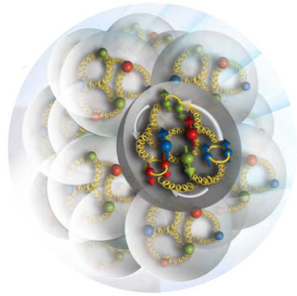
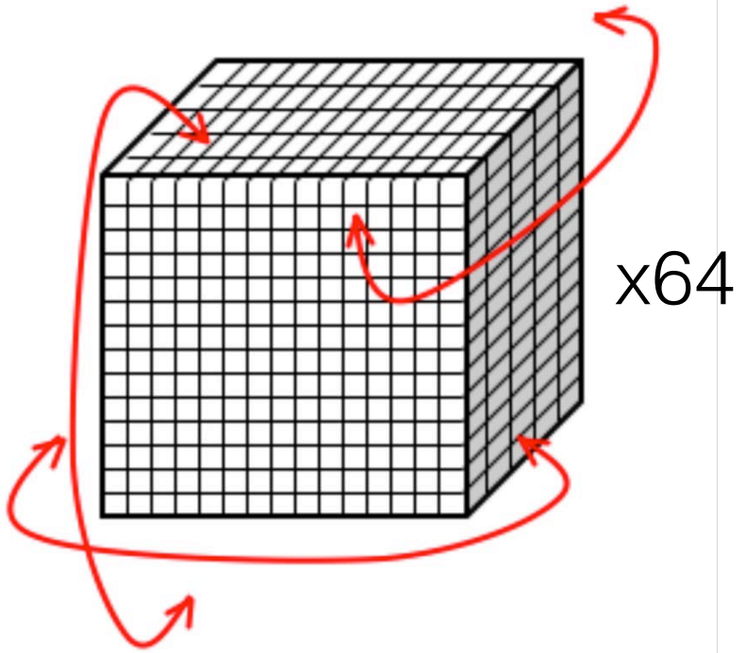
- Gluon 'EMC' effect
- Exotic glue



Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Euclidean space-time
  - Finite lattice spacing a
  - Volume $L^3 \times T \approx 32^3 \times 64$
- Some calculations use larger-than-physical quark masses (cheaper)



Approximate the QCD path integral by **Monte Carlo**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\overline{\psi} \mathcal{D}\psi \mathcal{O}[A, \overline{\psi}\psi] e^{-S[A,\overline{\psi}\psi]} \rightarrow \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\rm conf}} \sum_i^{N_{\rm conf}} \mathcal{O}([U^i])$$

with field configurations U^i distributed according to $e^{-S[U]}$

Lattice QCD

Numerical first-principles approach to non-perturbative QCD

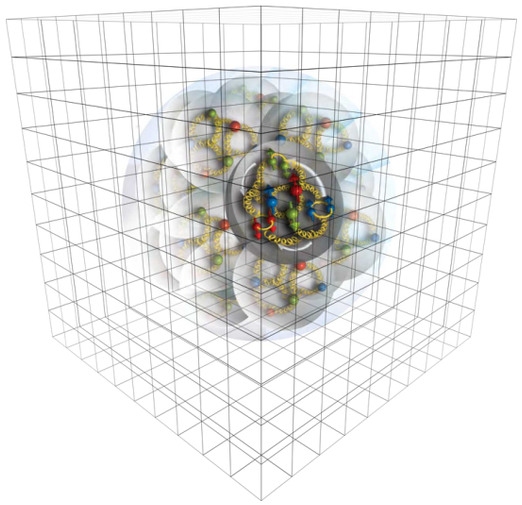
INPUT

Lattice QCD action has same free parameters as QCD: quark masses, α_S

- Fix quark masses by matching to measured hadron masses, e.g., π, K, D_s, B_s for u, d, s, c, b
- One experimental input to fix lattice spacing in GeV (and also α_S), e.g., $2S$-$1S$ splitting in Υ, or f_π or Ω mass

OUTPUT

Calculations of all other quantities are QCD predictions



Energy Momentum Tensor

- Generalised form factors encode e.g., Energy-Momentum Tensor
- Matrix elements of traceless gluon EMT for spin-half nucleon:

$$\langle p', s' | G^a_{\{\mu\alpha} G^{a\alpha}{}_{\nu\}} | p, s \rangle = \bar{U}(p', s') \left(A_g(t)\, \gamma_{\{\mu} P_{\nu\}} + B_g(t)\, \frac{i\, P_{\{\mu} \sigma_{\nu\}\rho} \Delta^\rho}{2M_N} + D_g(t)\, \frac{\Delta_{\{\mu} \Delta_{\nu\}}}{4M_N} \right) U(p, s)$$

Gluon field-strength tensor

Generalised gluon form factors

$\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2, \quad t = \Delta^2$

- Sum rules of gluon and quark GFFs in forward limit
  - Momentum fraction $A_a(0) = \langle x \rangle_a$ → $\sum_{a=q,g} A_a(0) = 1$
  - Spin $J_a(t) = \frac{1}{2}(A_a(t) + B_a(t))$ → $\sum_{a=q,g} J_a(0) = \frac{1}{2}$
  - D-terms $D_a(0)$ unknown but equally fundamental! [see Maxim Polyakov's talk]
    - $D_a(t)$ GFFs encodes pressure and shear distributions

Generalised Parton Distributions

- Quark GPDs: constraints from JLab, HERA, COMPASS, by DVCS, DVMP, future improvements from JLab 12GeV, EIC

- Gluon GPDs: almost unknown from experiment, future constraints central goal of EIC

Leading twist nucleon gluon GPDs:

Gluon field-strength tensor

$$\Delta_\mu = p'_\mu - p_\mu$$
$$P_\mu = (p_\mu + p'_\mu)/2$$
$$t = \Delta^2$$
$$n^2 = 0$$

$$\int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p', s' | G_a^{\{\mu\alpha}(-\frac{\lambda}{2}n) \left[\mathcal{U}^{(A)}_{[-\frac{\lambda}{2}n, \frac{\lambda}{2}n]}\right]_{ab} G_{b\alpha}^{\ \nu\}}(\frac{\lambda}{2}n) | p, s \rangle$$
$$= \frac{1}{2}\left(H_g(x,\xi,t) \bar{U}(p',s') P^{\{\mu}\gamma^{\nu\}} U(p,s) + E_g(x,\xi,t) \bar{U}(p',s') \frac{P^{\{\mu} i\sigma^{\nu\}\alpha} \Delta_\alpha}{2M} U(p,s) \right) + \dots ,$$

GPDs(Bjorken x, skewness, mom transfer)

- Moments of GPDs: Generalised Form Factors (GFFs)

e.g., $\int_0^1 \mathrm{d}x\ H_g(x,\xi,t) = A_g(t) + \xi^2 D_g(t)\,, \qquad \int_0^1 \mathrm{d}x\ E_g(x,\xi,t) = B_g(t) - \xi^2 D_g(t)$

D-term from JLab DVCS

Experimental determination of DVCS D-term and extraction of proton pressure distribution

[Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)]

$$s(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r)$$

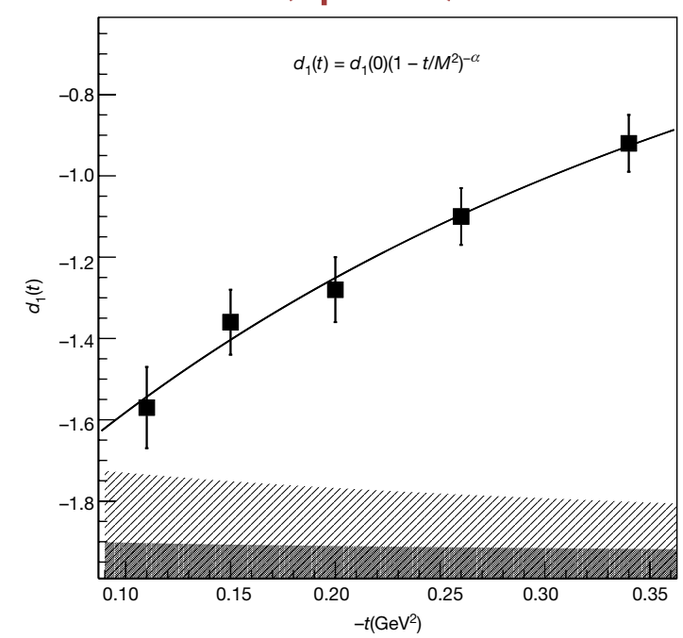
- Peak pressure near centre ~10^{35} Pascal, greater than pressure estimated for neutron stars
- Key assumptions: gluon D-term same as quark term, tripole form factor model, $D_u(t,\mu) = D_d(t,\mu)$

EXP + LQCD

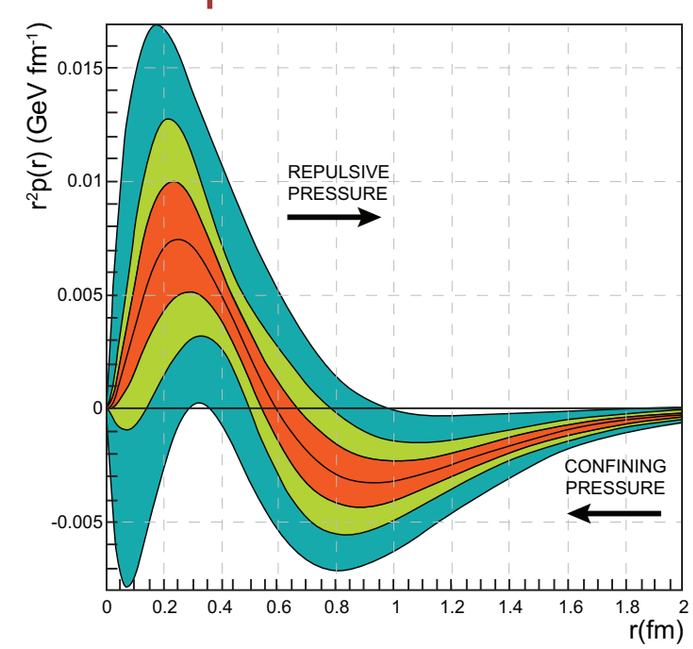
first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]

DVCS (quark) D-term



Radial pressure distribution

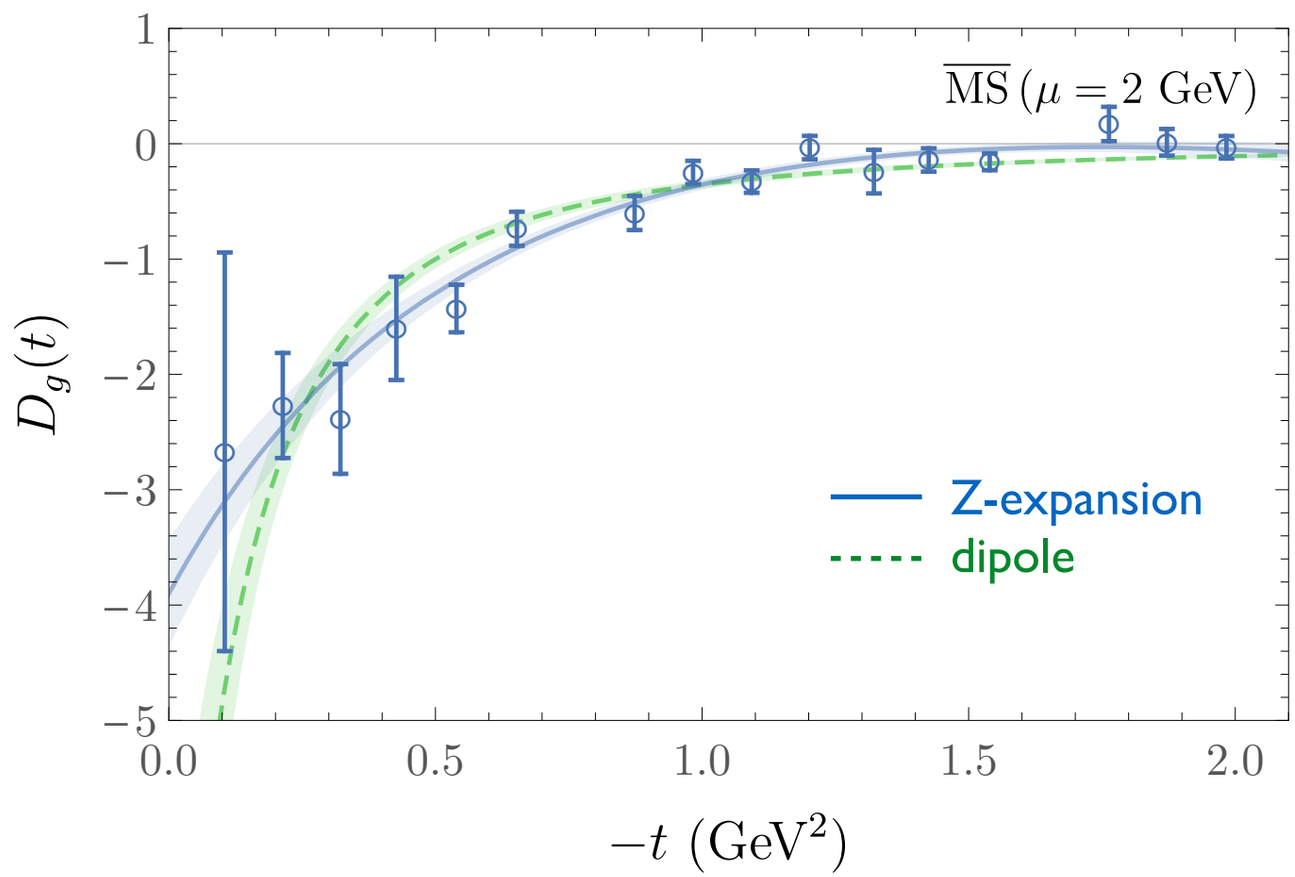


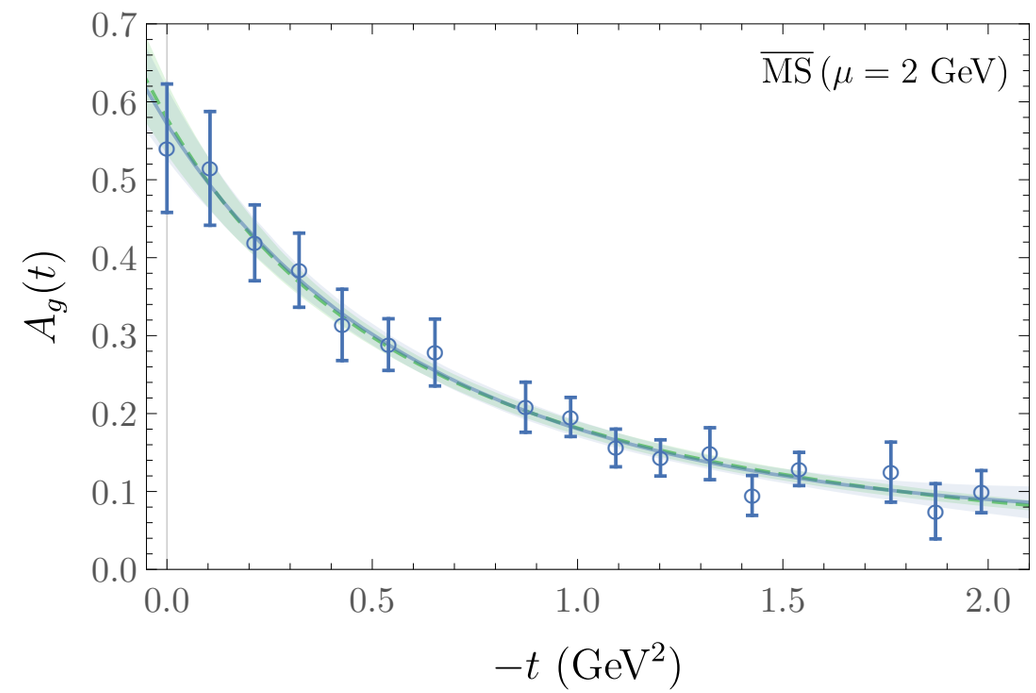
LQCD nucleon GFFs

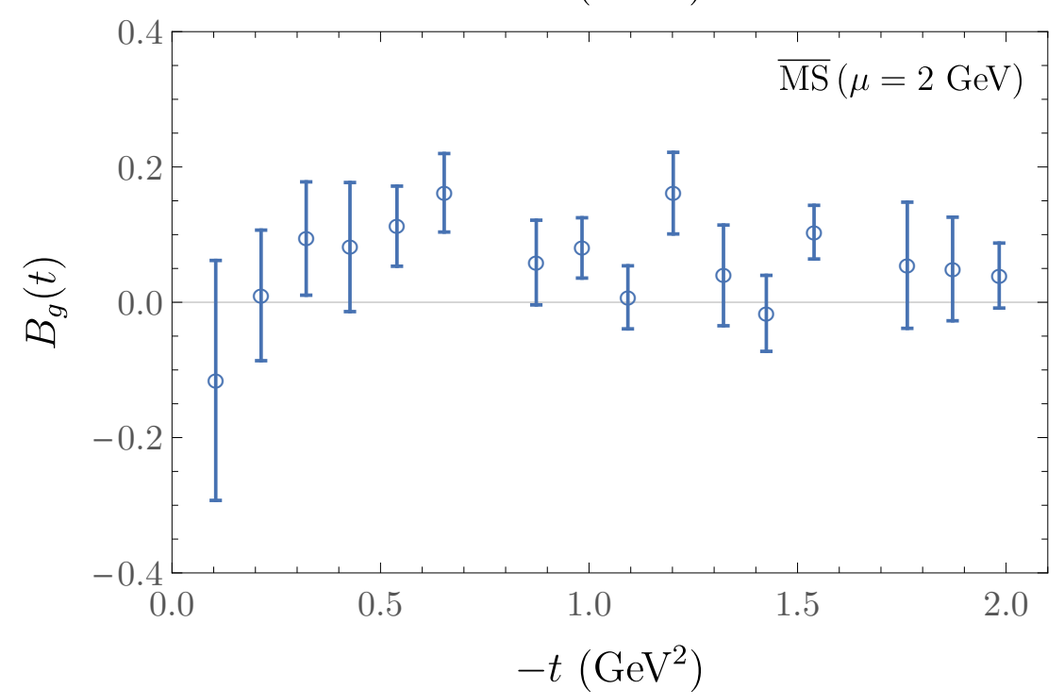
LQCD results for nucleon gluon GFFs

m_π ~450 MeV

Dipole-like fall-off with momentum transfer







[Shanahan, Detmold, PRL122, 072003 (2019) PRD99, 014511 (2019)]

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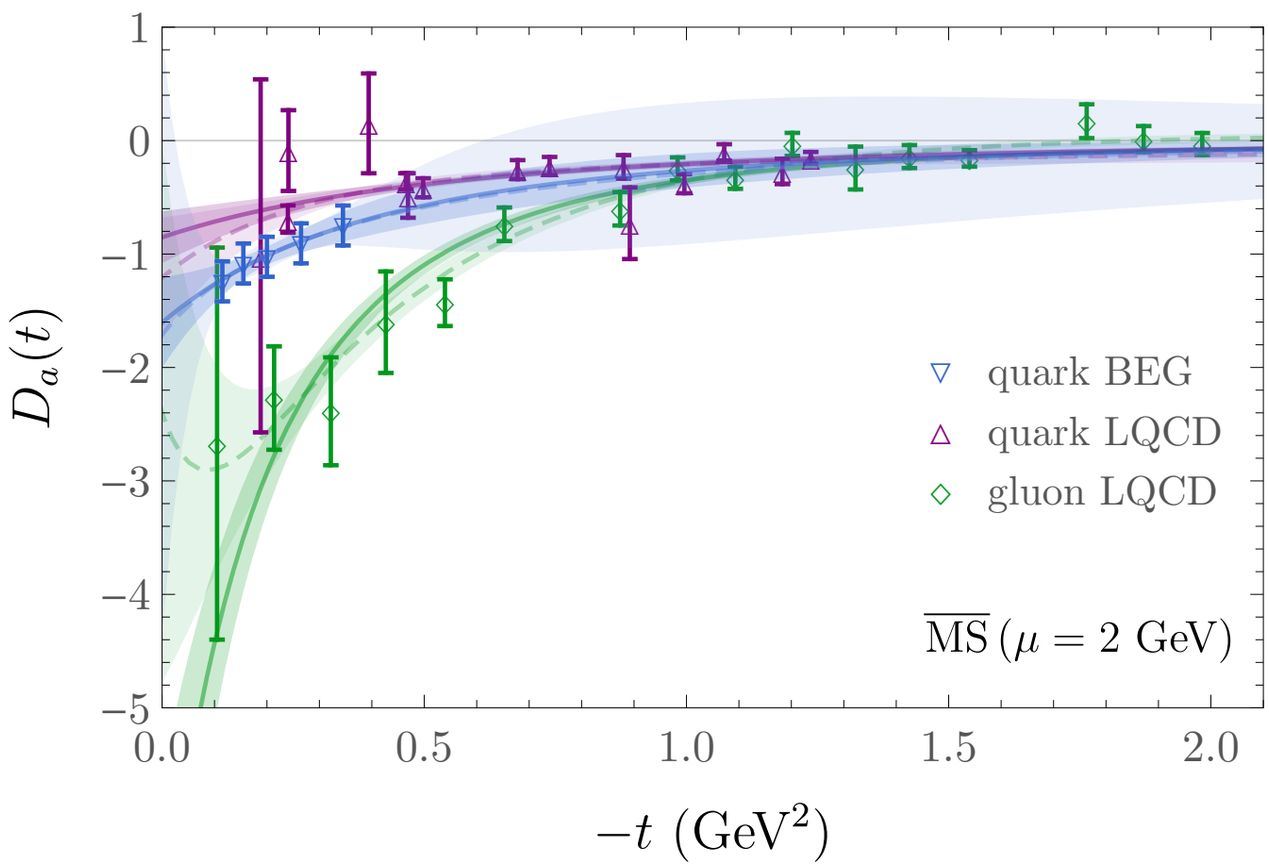
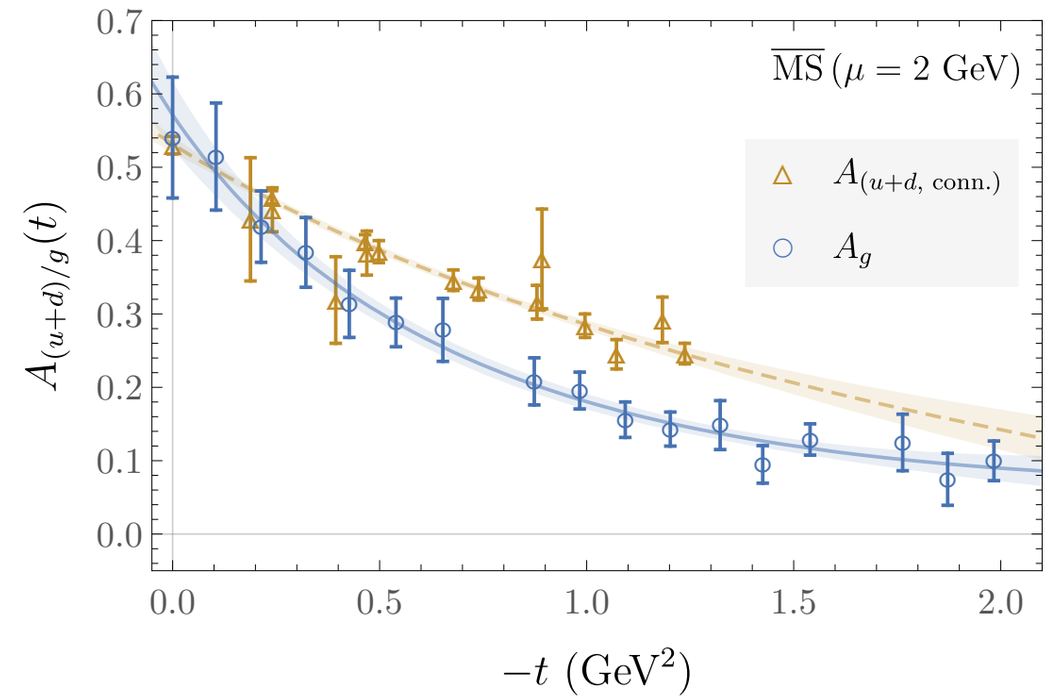
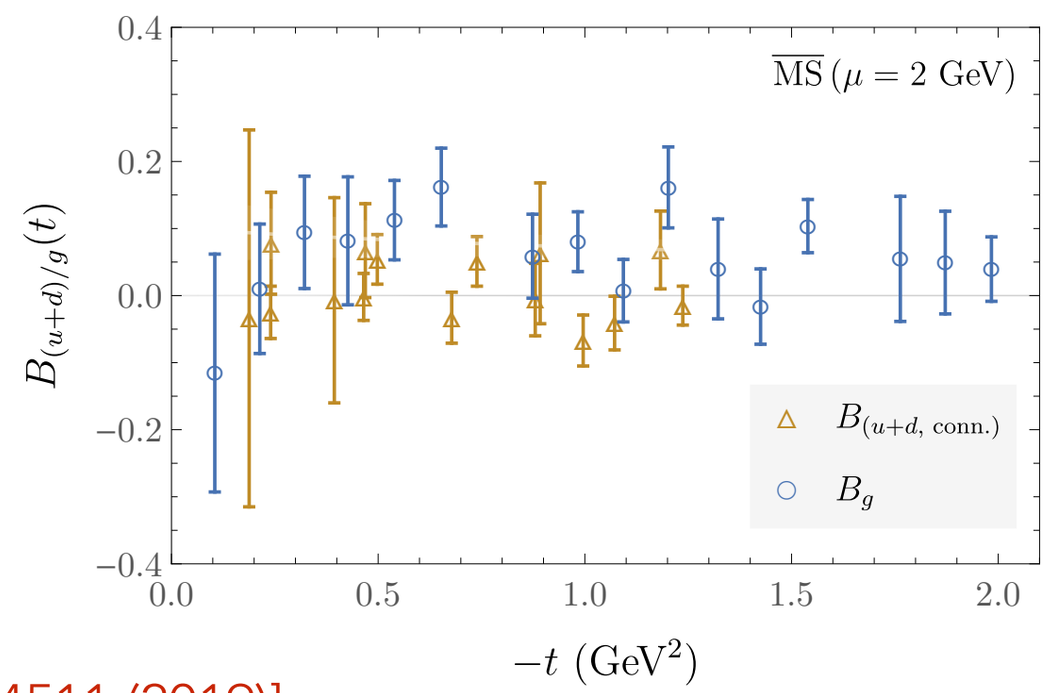






[Shanahan, Detmold, PRL122, 072003 (2019) PRD99, 014511 (2019)]

Nucleon D-term GFFs from LQCD

EXP + LQCD

first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]

Key assumptions in pressure extraction from DVCS

- Gluon D-term same as quark term in magnitude and shape

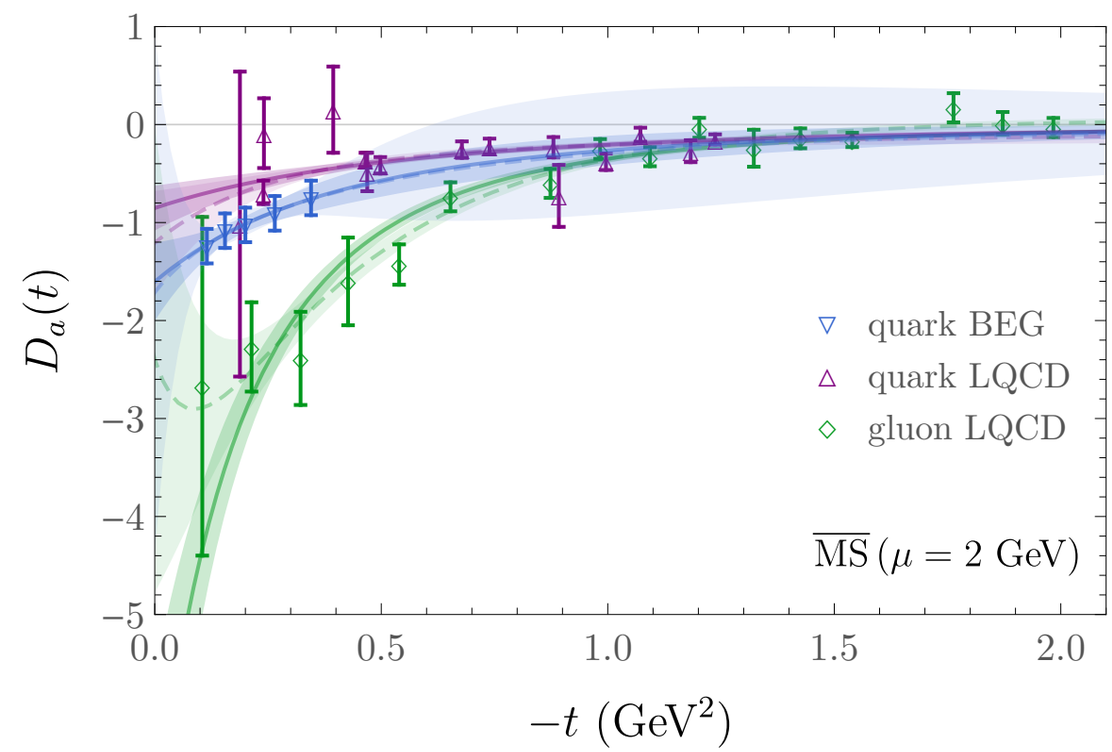
  Factor of ~2 difference in magnitude, somewhat different t-dependence

- Tripole form factor model

  LQCD results consistent with ansatz, but more general form is less well constrained

- Isovector quark D-term vanishes

  $D_{u-d}(t) \sim 0$ *from other LQCD studies*



Gluon GFFs: Shanahan, Detmold, PRD 99, 014511

Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

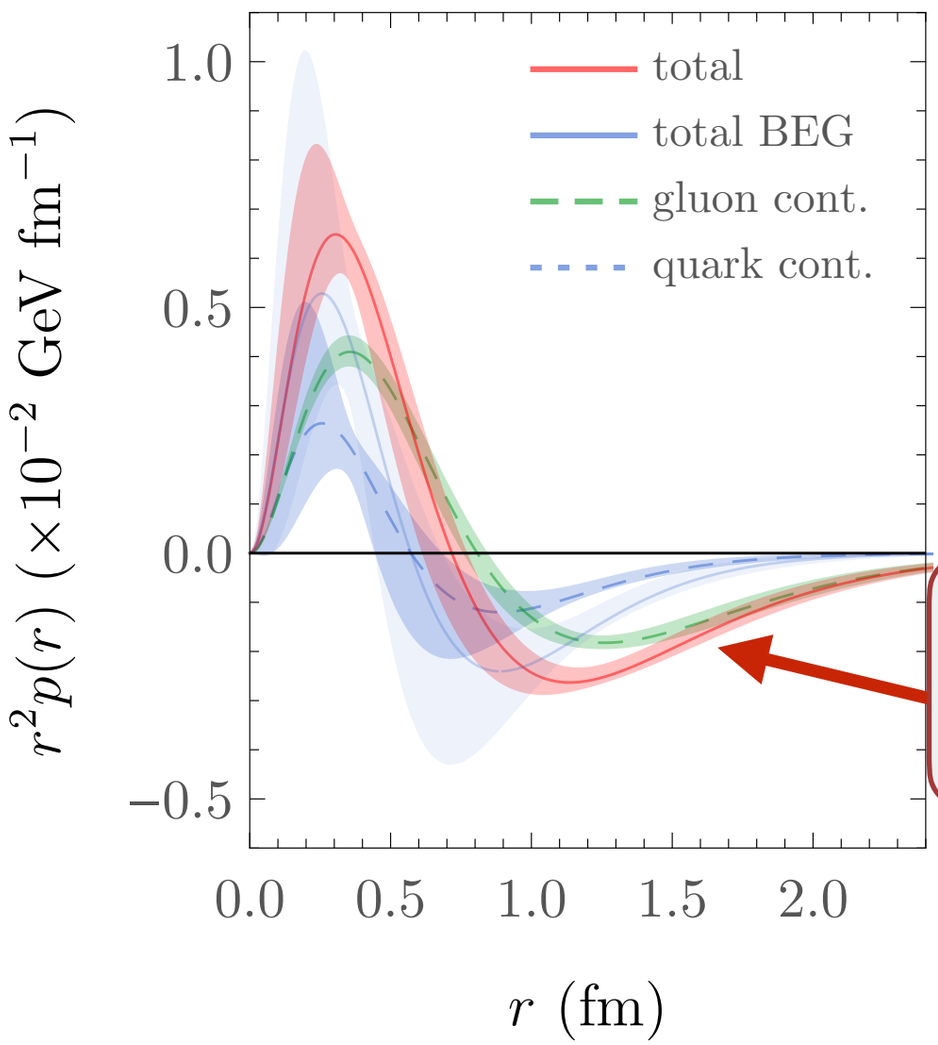
Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

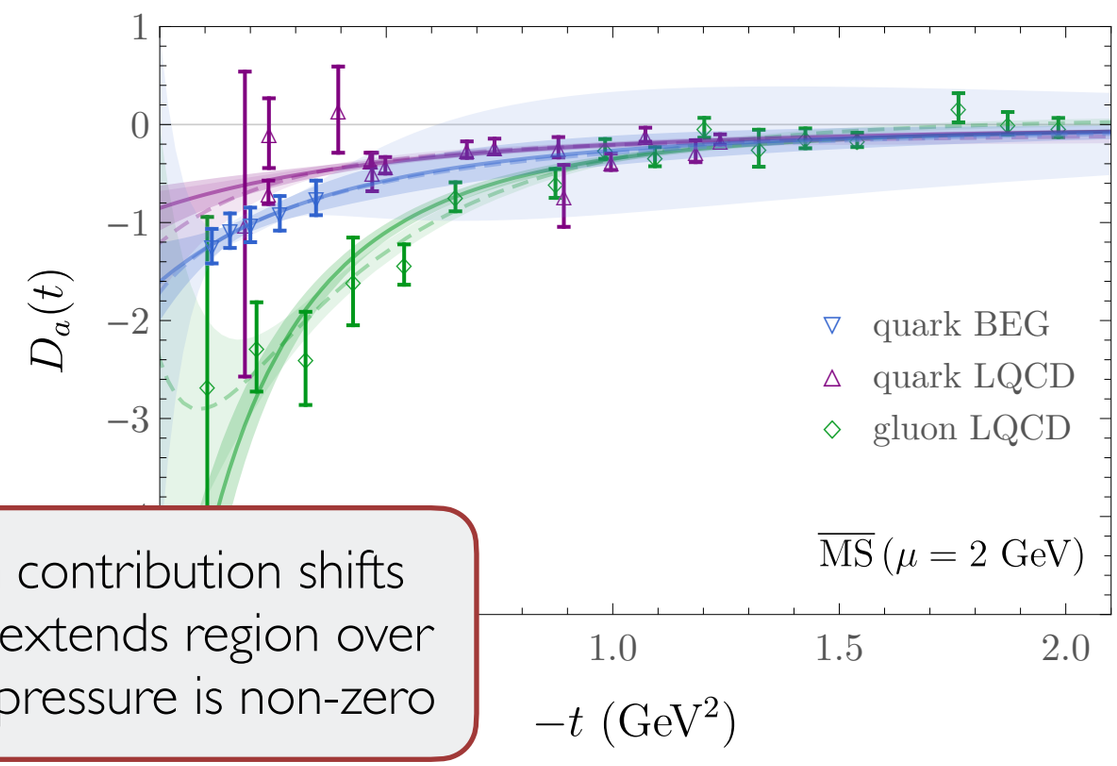
Proton pressure from LQCD

EXP + LQCD

first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]





gluon contribution shifts peaks, extends region over which pressure is non-zero

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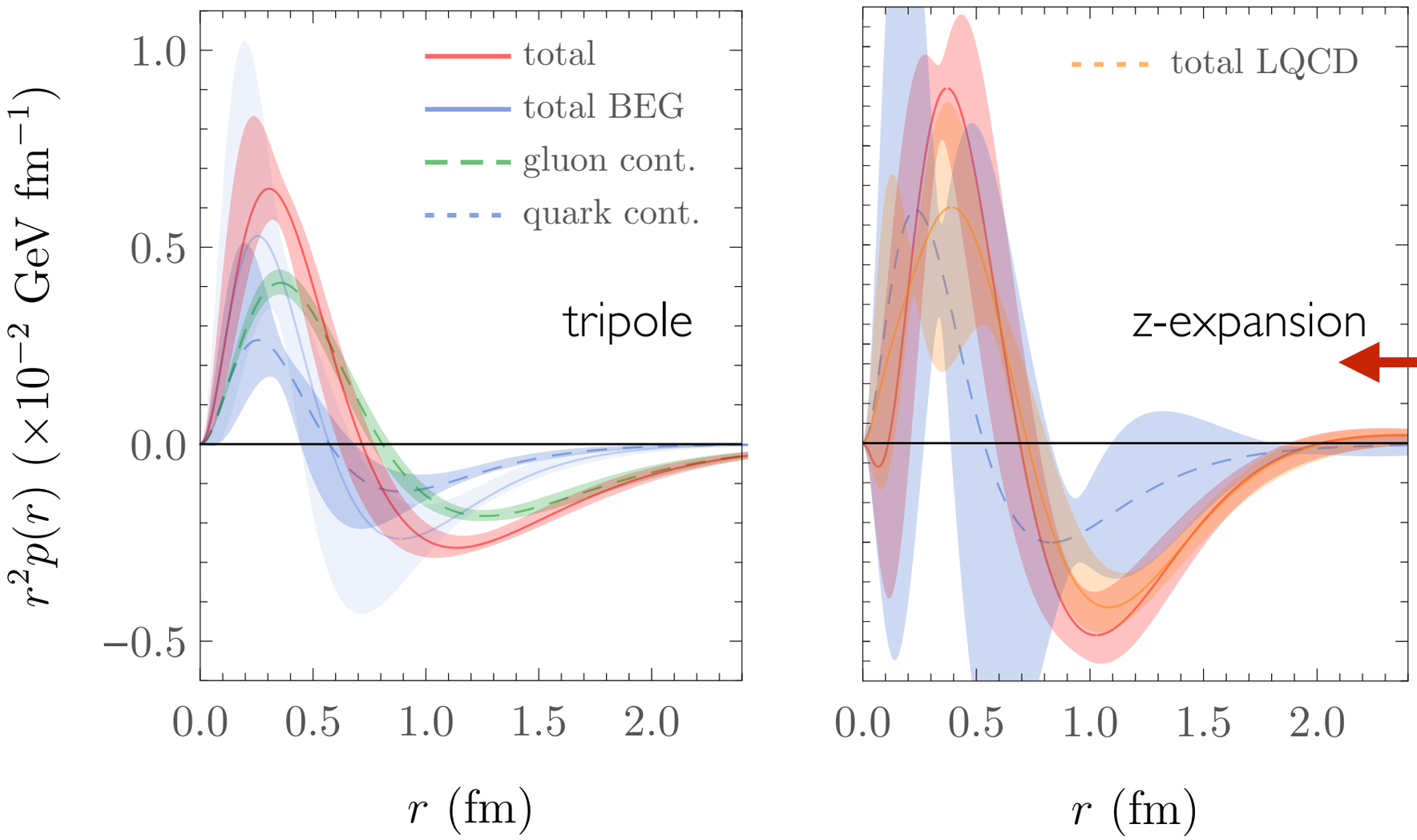
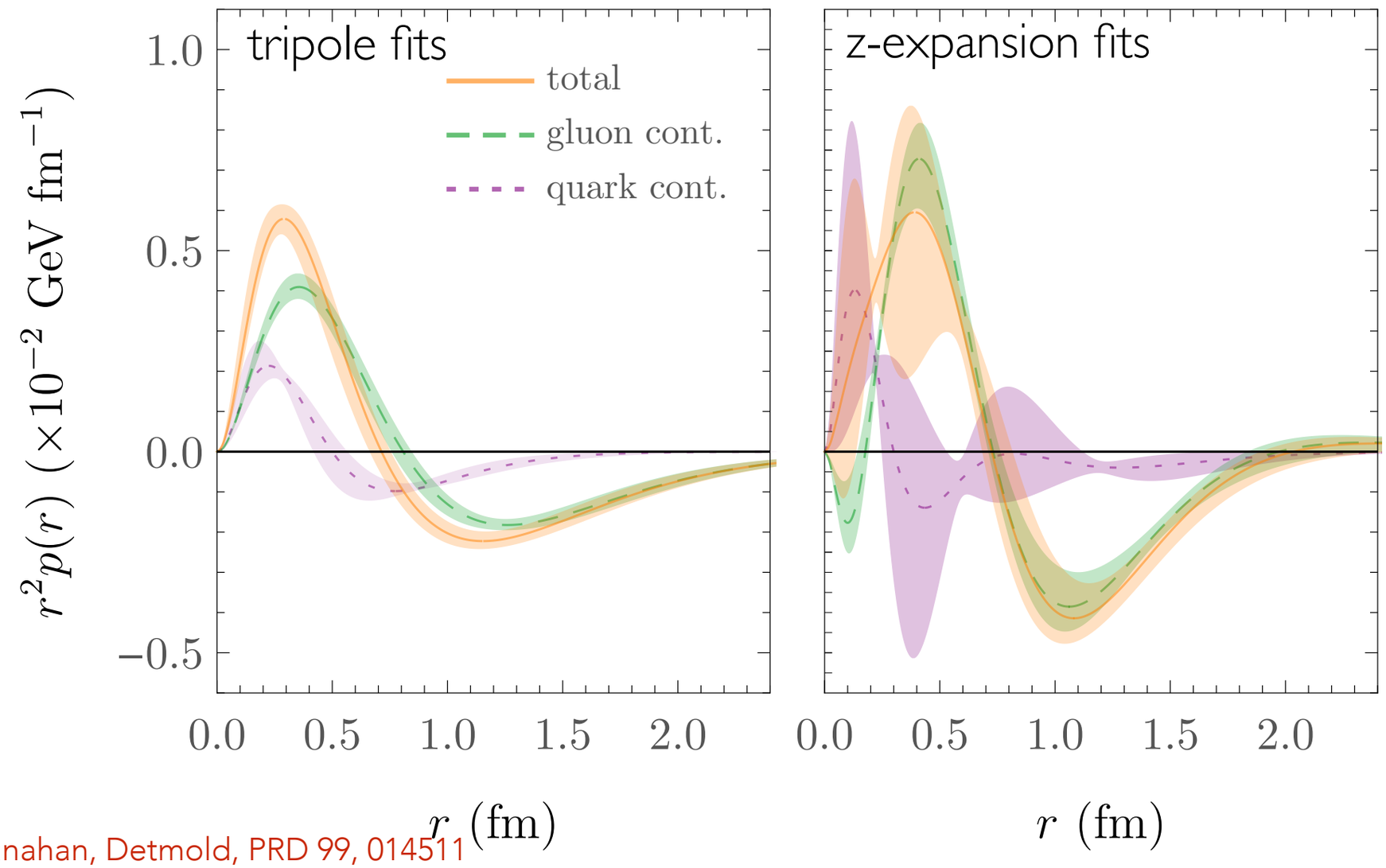


Significant model-dependence in Fourier transform of D-term form factor

Proton pressure from LQCD

LQCD ONLY

pressure determination, m_π ~450 MeV



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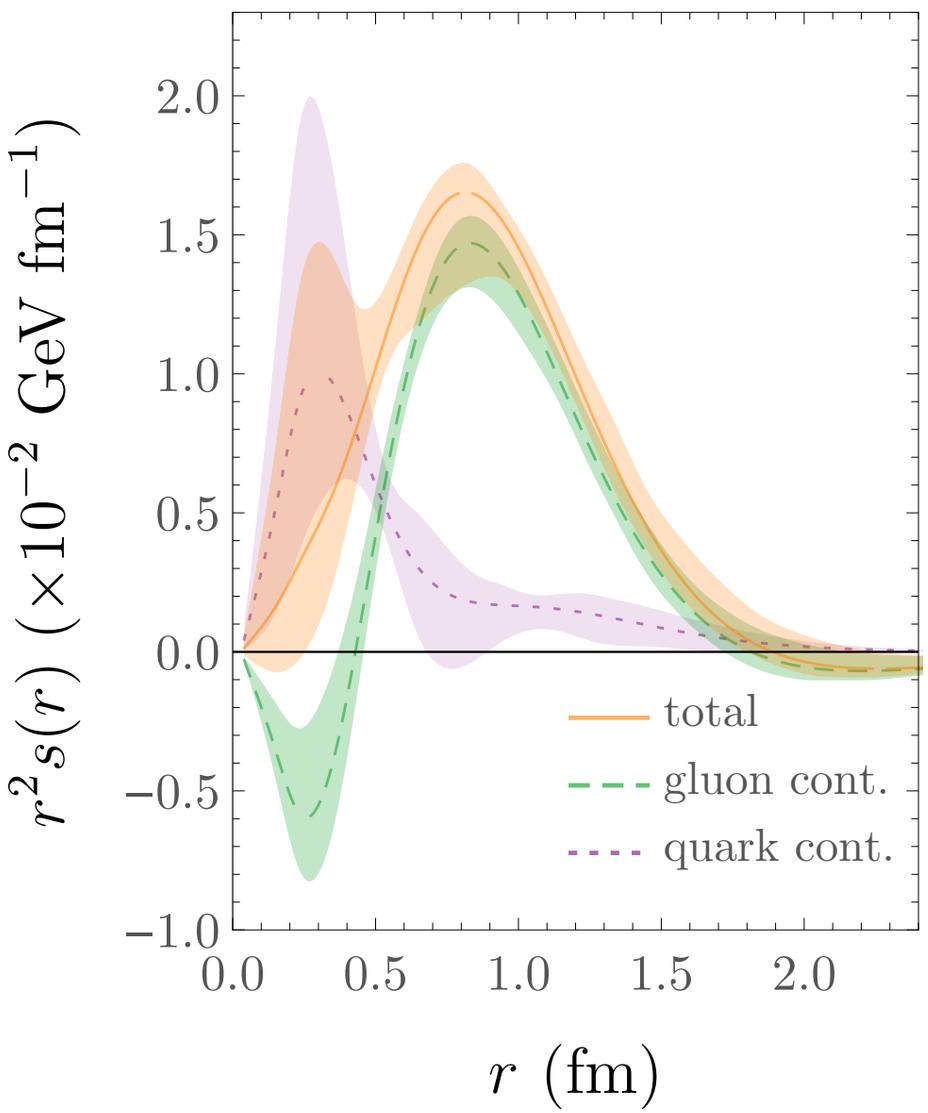
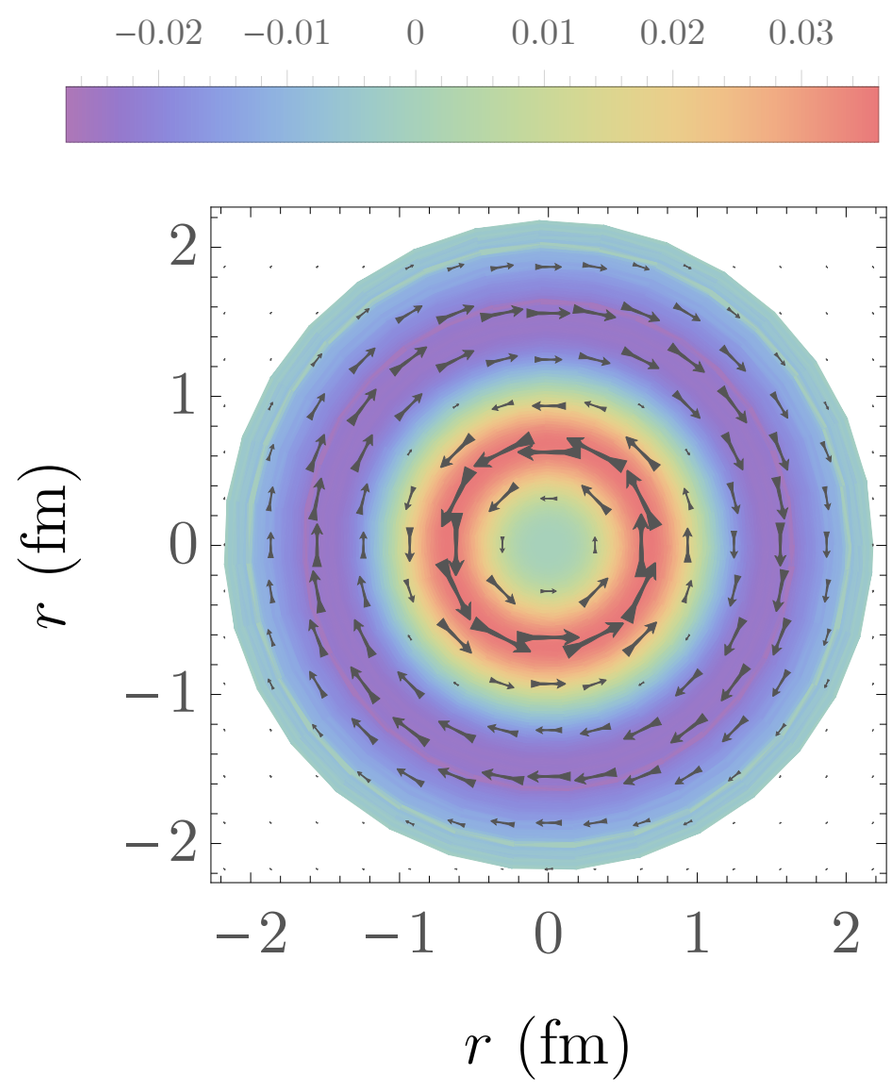
Proton shear from LQCD

z-expansion fits



Tangential shear vector field $4\pi r^2 T_{ij} e_j^{\phi}$



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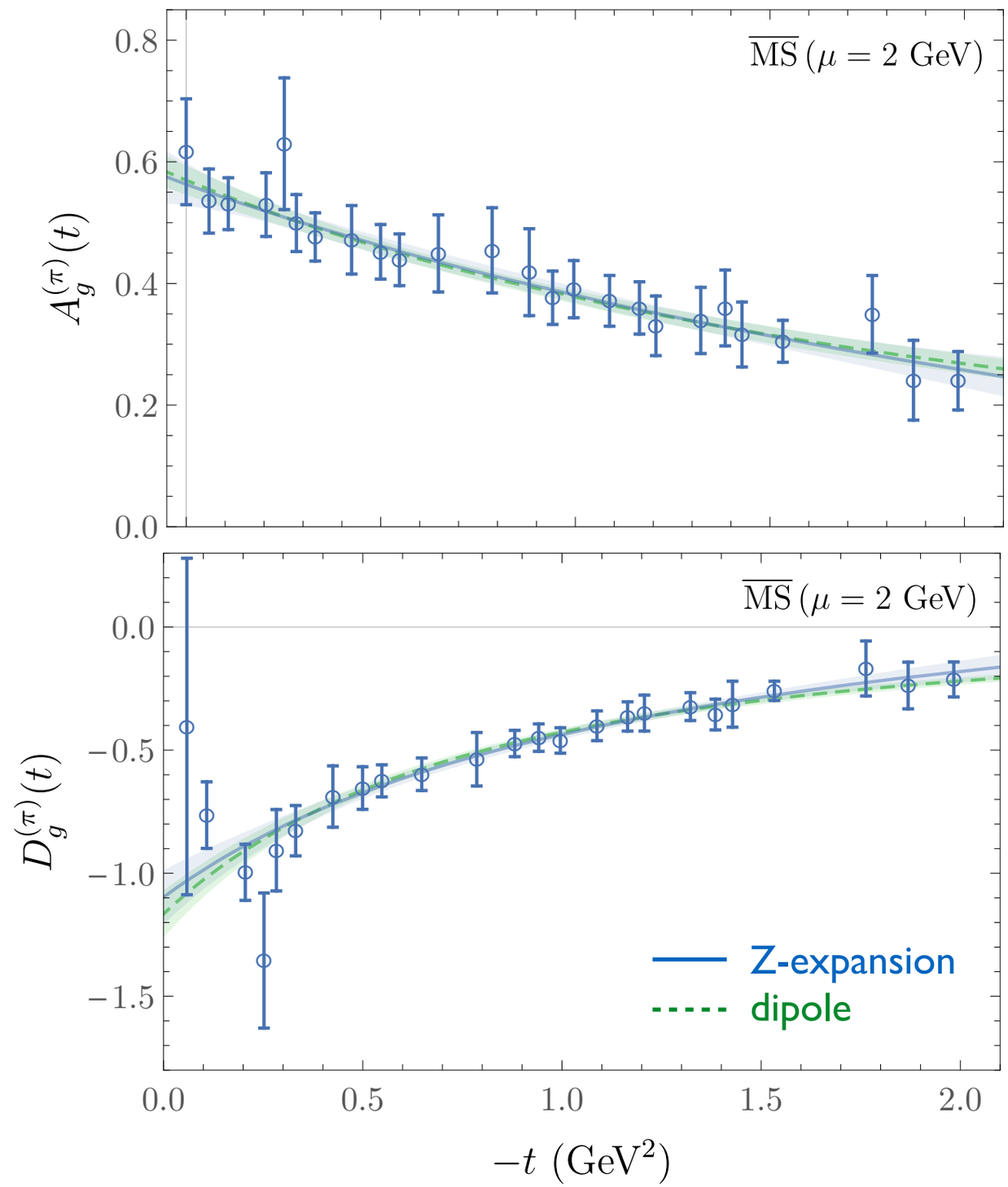
Pion GFFs from LQCD

Pion gluon GFFs m_π ~450 MeV

- Dipole-like fall-ff with momentum transfer
- Momentum fraction $A_a(0) = \langle x \rangle_a$

  → $\sum_{a=q,g} A_a(0) = 1$
- D-terms $D_a(0)$ related to pressure and shear distributions



[Shanahan, Detmold, PRD 99, 014511, PRL 122 072003 (2019)]

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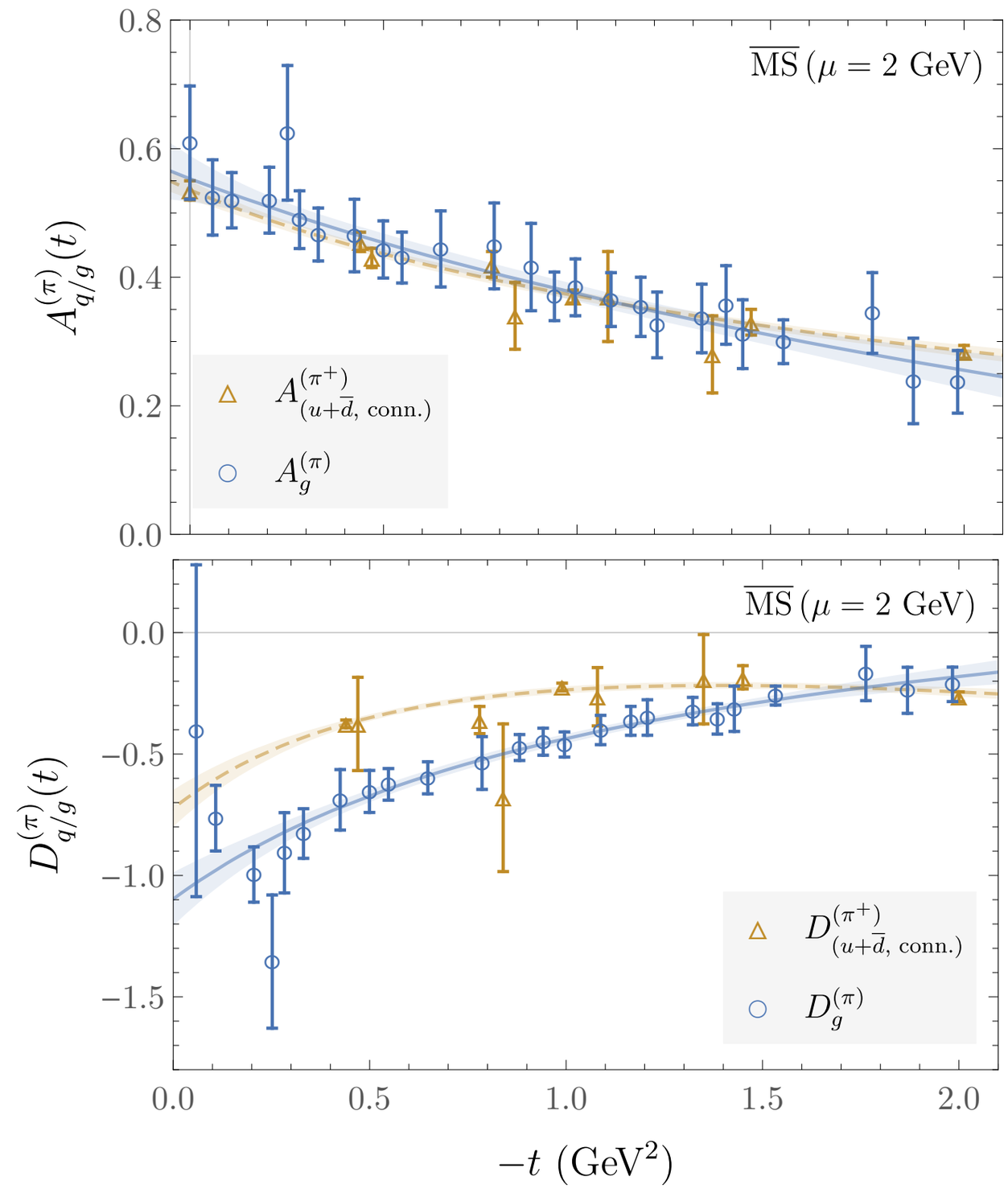


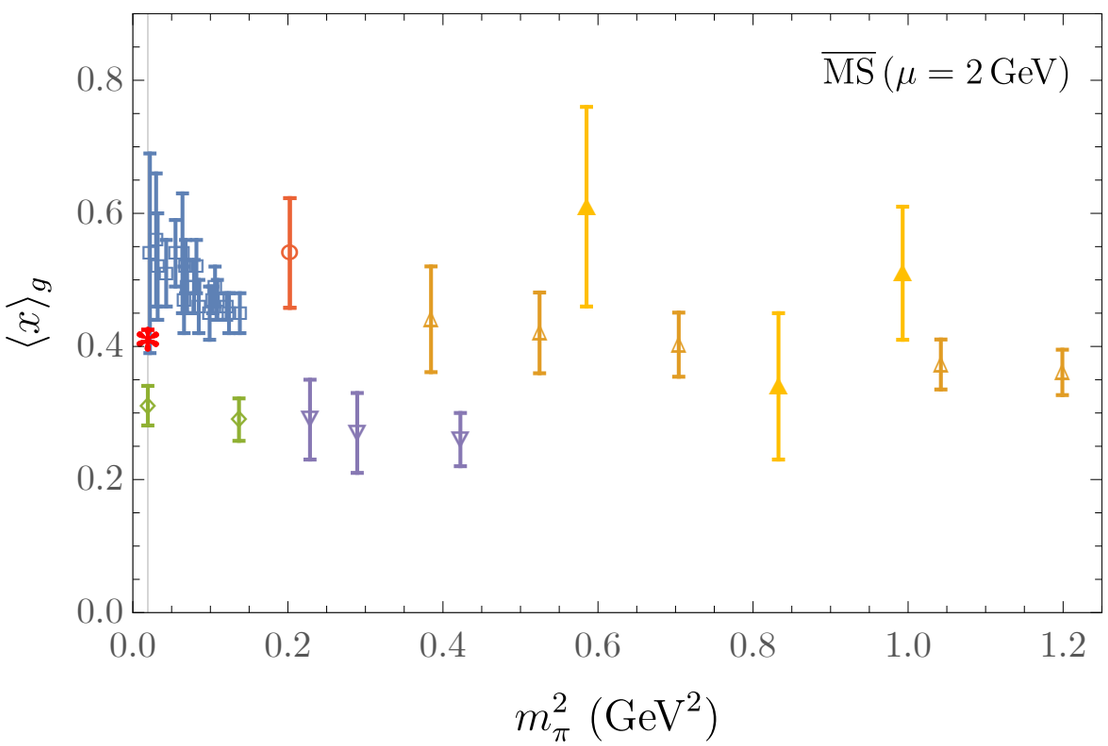
gluon: Shanahan, Detmold, PRD 99, 014511 (2019)

quark: Dirk Brömmel Ph.D. thesis (2007) m_π ~840 MeV

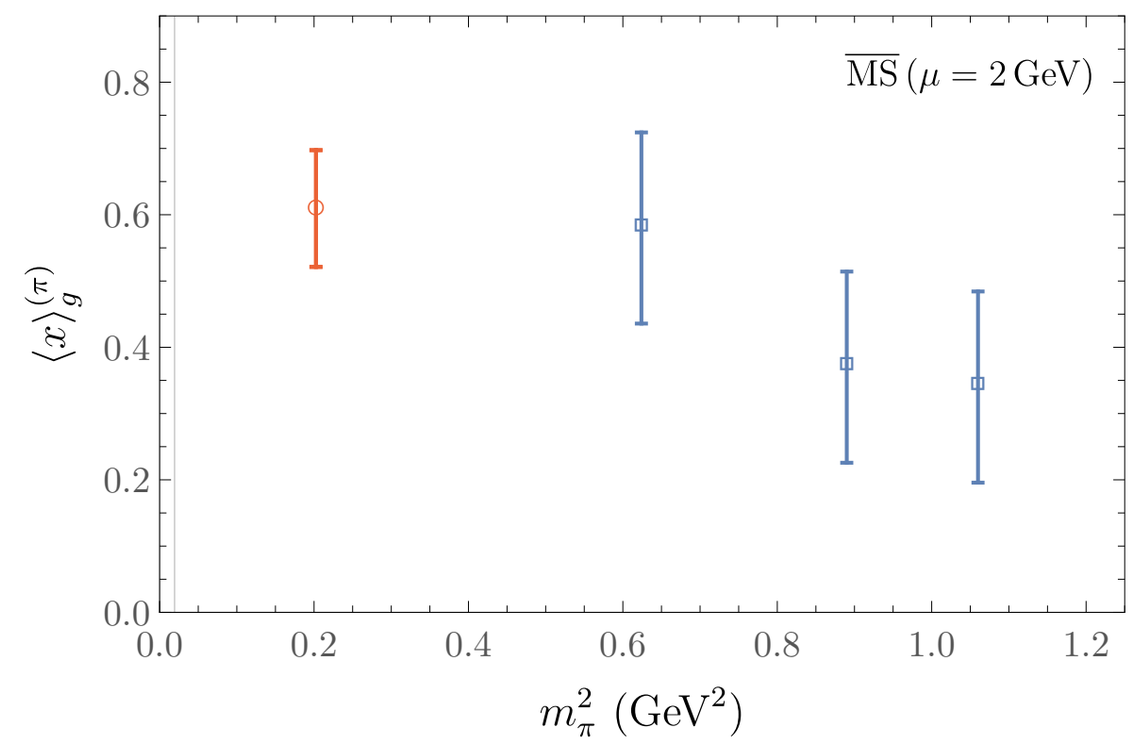
Gluon momentum fraction

Gluon momentum fraction $A_a(0) = \langle x \rangle_a$

Nucleon



Pion



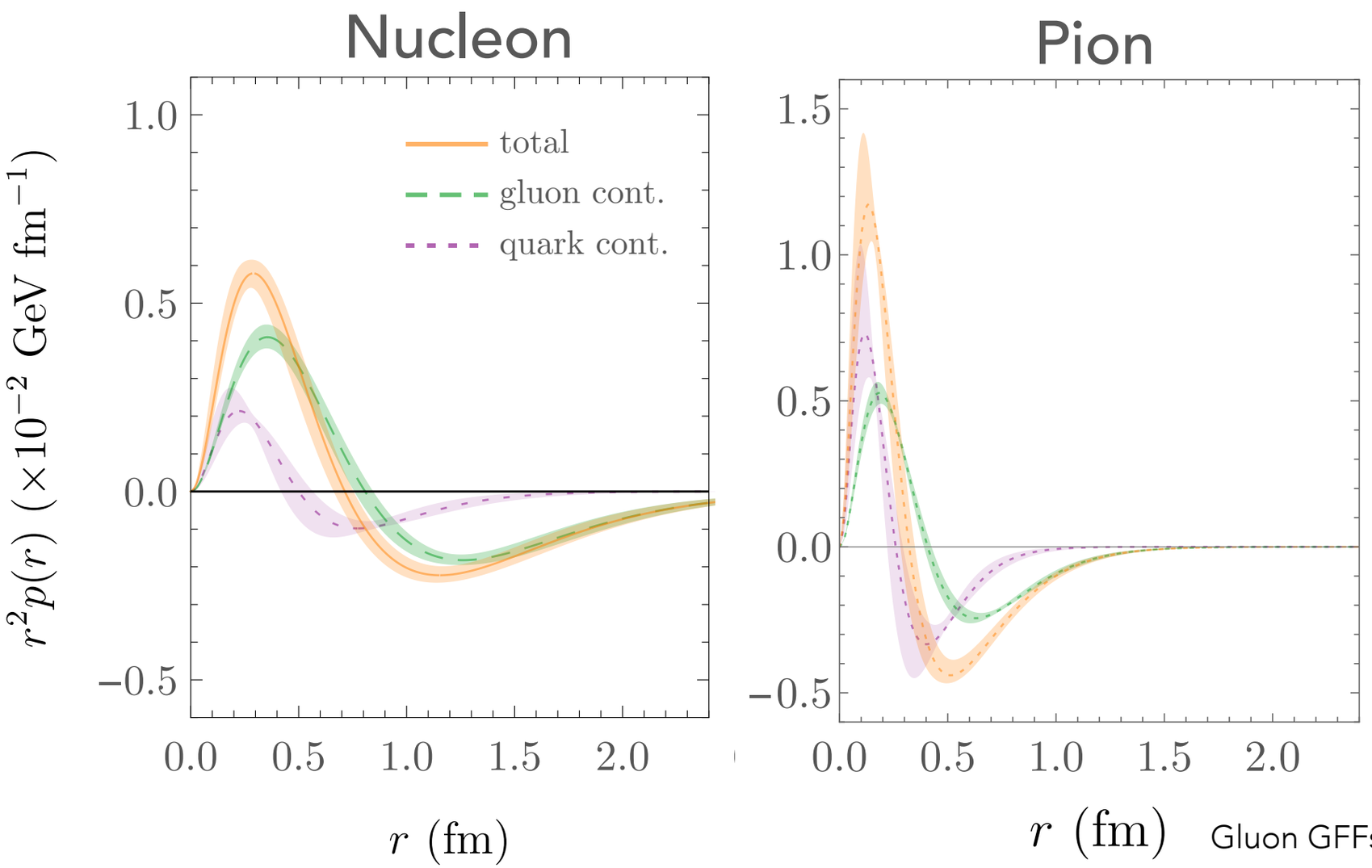
χQCD
ETM
χQCD quenched
QCDSF quenched
Meyer/Negele quenched

Very little pion-mass dependence within each set of calculations

Next: pressure distribution in nuclei

Pressure in light nuclei
c.f. pressure in the nucleon?

New studies of ρ and Δ coming soon! [Pefkou, Hackett, Shanahan]



Pion & Nucleon quark and gluon momentum fractions consistent within uncertainties, but very different pressure distributions!

Gluon GFFs: Shanahan, Detmold, PRD 99, 014511
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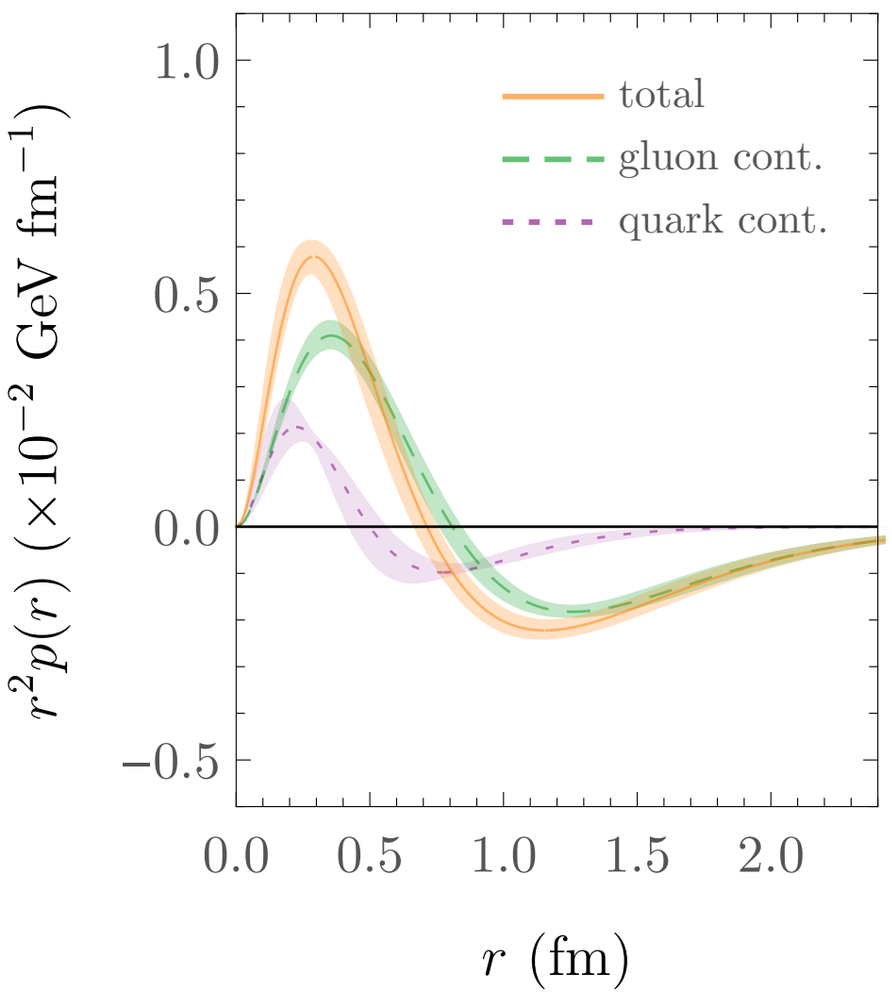
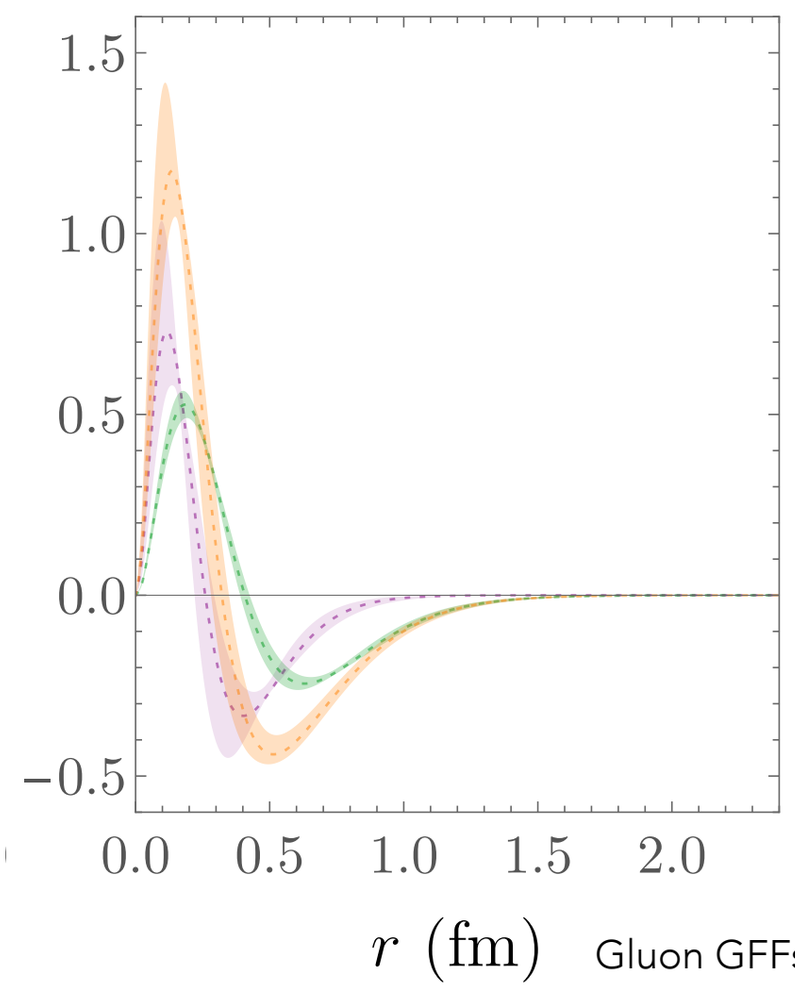
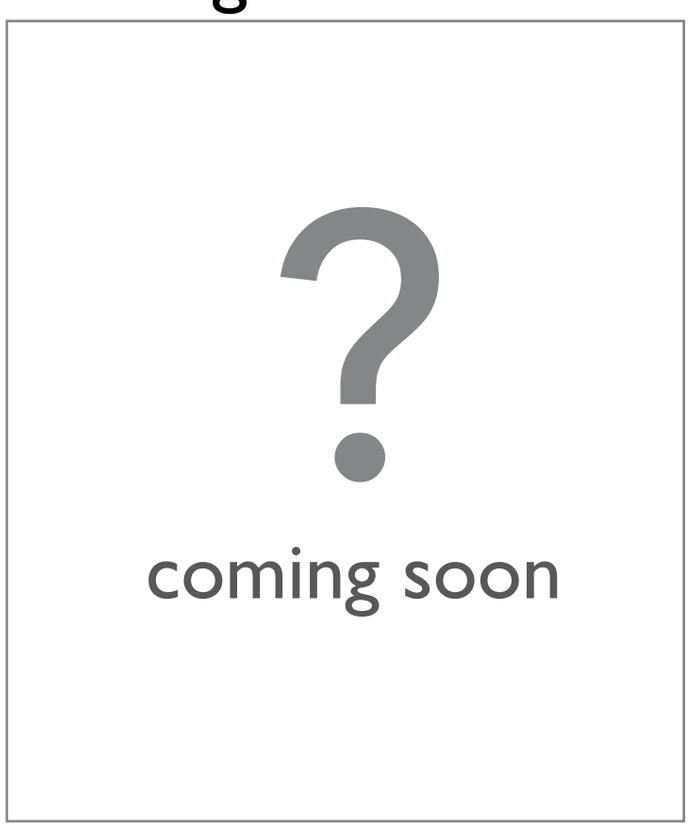


Pion



Light nuclei



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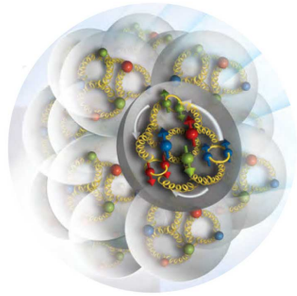
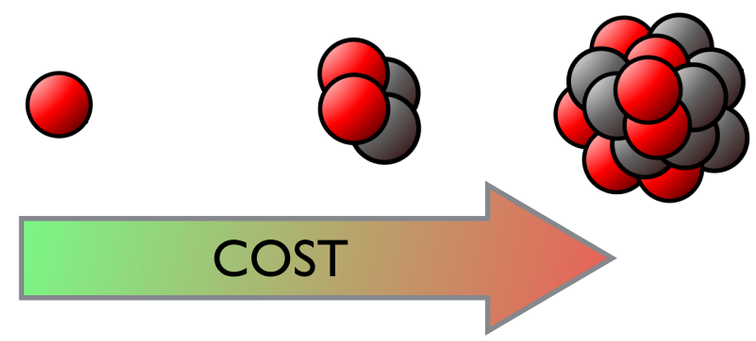


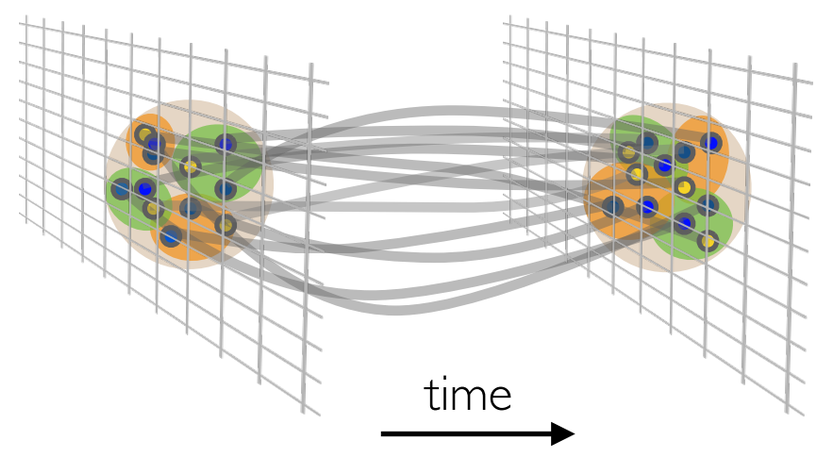
Nuclear physics from lattice QCD

Nuclei on the lattice is HARD

- **Noise:**
  Statistical uncertainty grows exponentially with number of nucleons

- **Complexity:**
  Number of contractions grows factorially





Calculations possible for A<5

NPLQCD effort in lattice QCD for nuclei

- Nuclei with A<5
- QCD with unphysical quark masses

  m_π~800 MeV, m_N~1,600 MeV

  m_π~450 MeV, m_N~1,200 MeV

  m_π~170 MeV, m_N~950 MeV

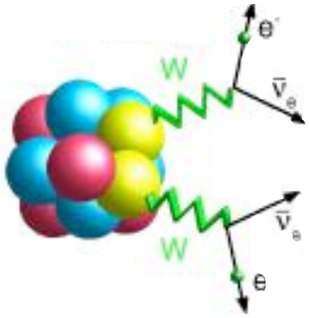
IN PROGRESS

- First calculation of spectrum of light nuclei in 2013

Highlights of 2017-2020

- Proton-proton fusion and tritium β-decay

  [PRL 119, 062002 (2017)]



- Double β-decay

  [PRL 119, 062003 (2017), PRD 96, 054505 (2017)]

- Gluon structure of light nuclei

  [PRD 96 094512 (2017)]

- Scalar, axial, tensor MEs

  [PRL 120 152002 (2018)]

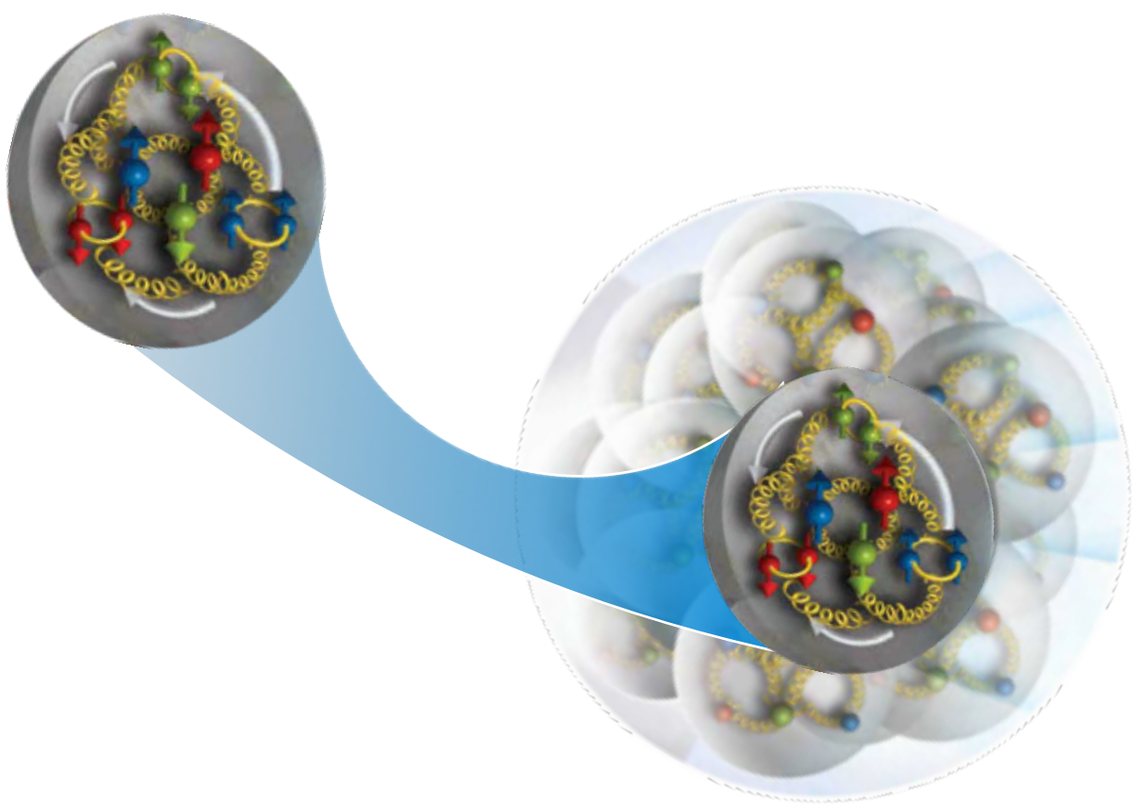
- Isovector EMC effect

  [2009.05522]

Gluon structure of nuclei

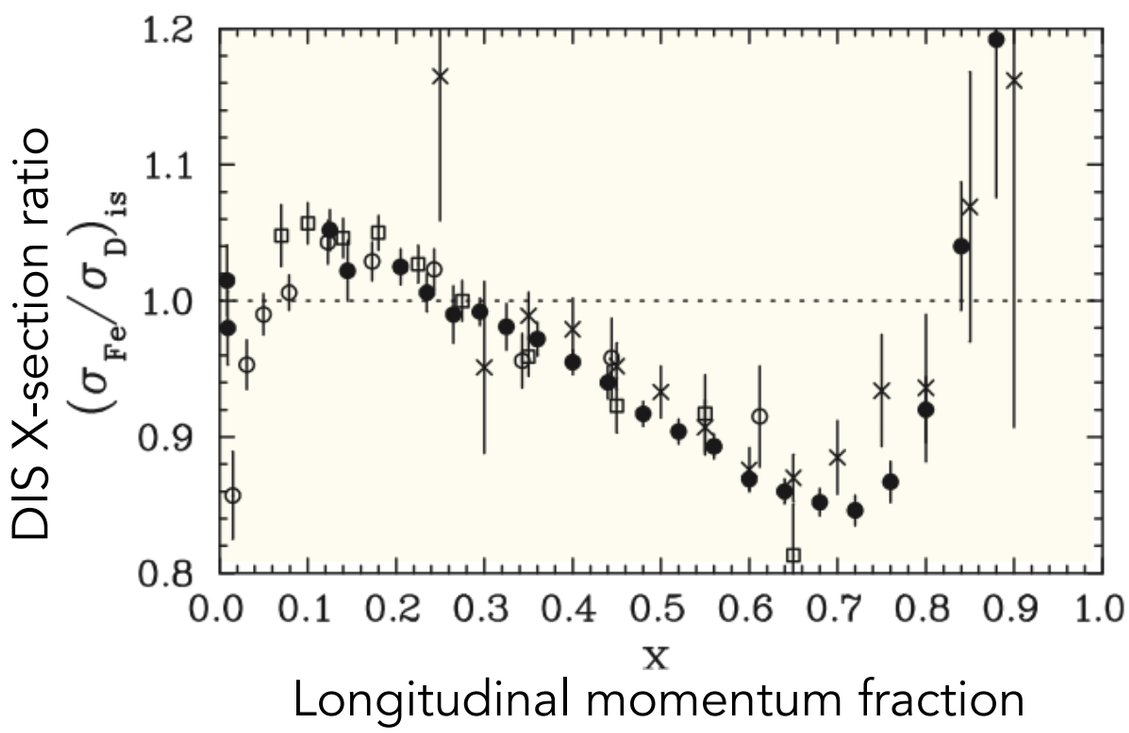
How does the gluon structure of a nucleon change in a nucleus?

Gluonic analogue of EMC effect? [EMC: Aubert et al., 1983]



Ratio of structure function F_2 per nucleon for iron and deuterium

$$F_2(x, Q^2) = \sum_{q=u,d,s\ldots} x\, e_q^2 \left[q(x, Q^2) + \overline{q}(x, Q^2)\right]$$

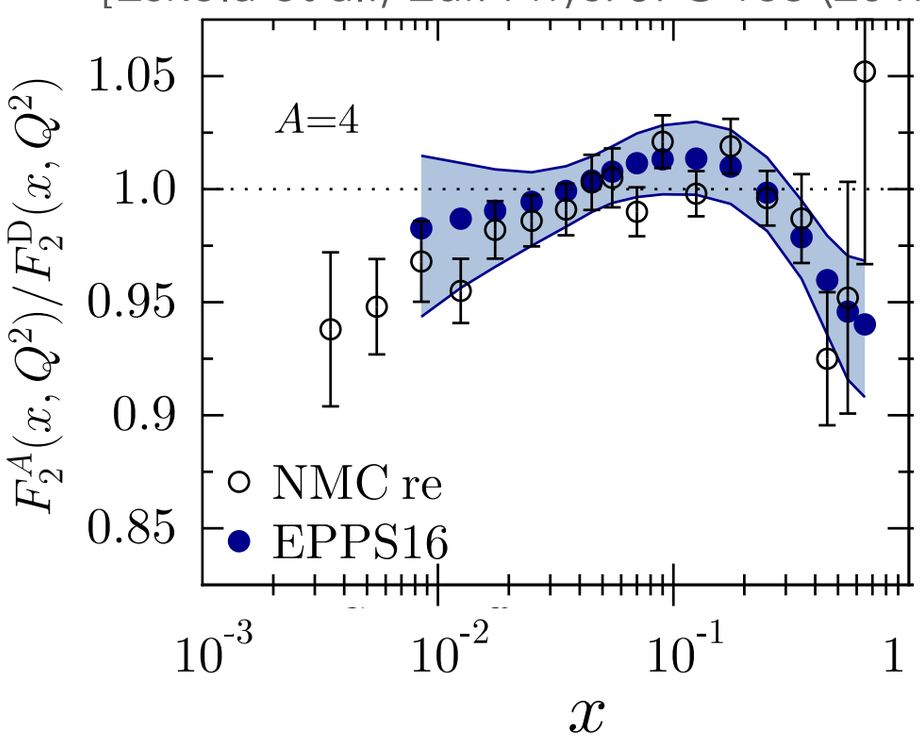


EMC effects in Mellin moments

First investigation of EMC-type effects from LQCD:
Nuclear effects in Mellin moments of PDFs

- Calculable from local operators
- BUT EMC effects in moments are very small

[Eskola et al., Eur. Phys. J. C 163 (2017)]



Classic EMC effect is defined in F_2:

$$F_2(x,Q^2) = \sum_{q=u,d,s\ldots} x\, e_q^2\, [q(x,Q^2) + \bar{q}(x,Q^2)]$$

Number density of partons of flavour q

x-integrals of numerator and denominator $\int_0^1 dx\, x^n q(x,Q^2)$

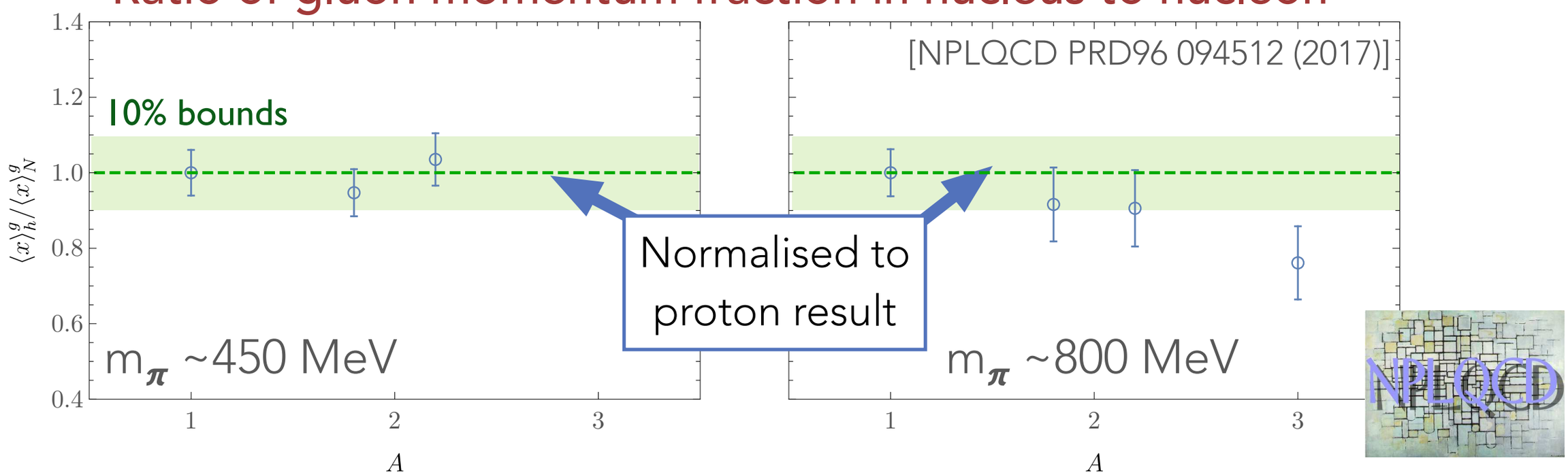
Gluon momentum fraction of nuclei

Matrix elements of the spin-independent gluon operator in nucleon + light nuclei [NPLQCD PRD96 094512 (2017)]

→ **first determination of gluon momentum fraction of nuclei**

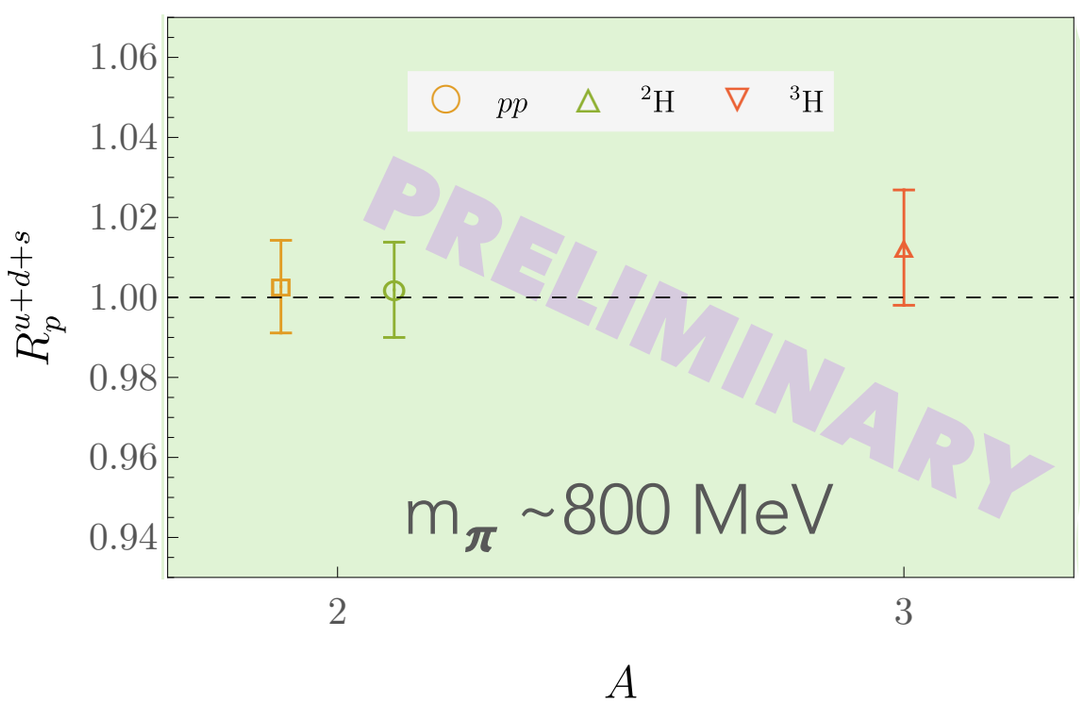
- Constraints at ~10% level on EMC-effect in gluon momentum fraction
- Small mixing with quark EMT operators (neglected)
- Sum rule constraint

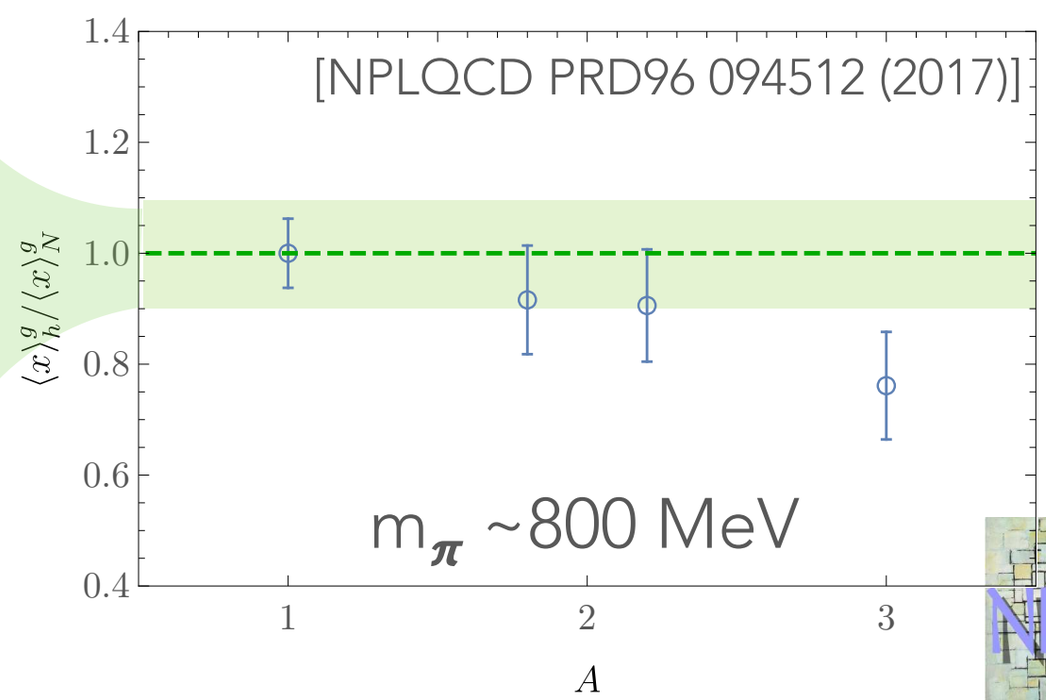
Ratio of gluon momentum fraction in nucleus to nucleon



Momentum fractions of nuclei

- First determination of all components of momentum decomposition of light nuclei
- Small mixing between quark and gluon EMT operators neglected
- Constraint on either quark or gluon EMC in this quantity implies constraint on the other from sum rules:





Exotic glue in the deuteron

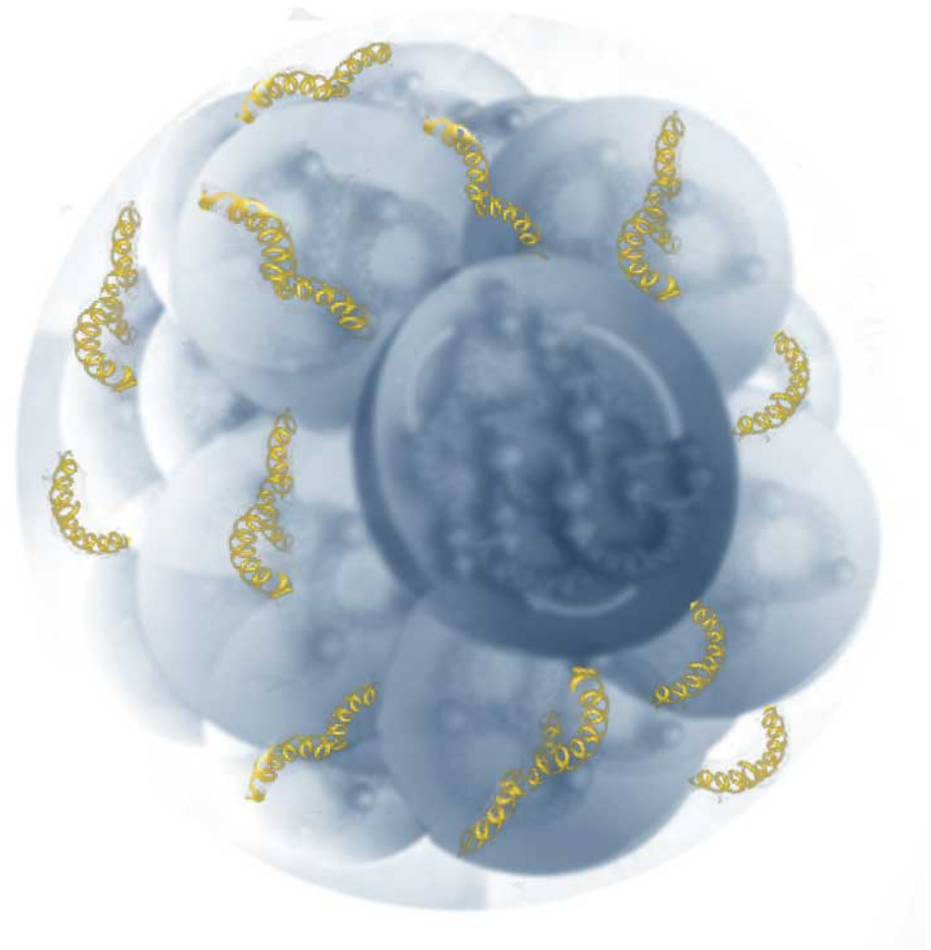
a "pure" EMC-type effect

Contributions to nuclear structure from gluons not associated with individual nucleons in nucleus

Exotic glue operator:

nucleon $\langle p|\mathcal{O}|p\rangle = 0$

nucleus $\langle N, Z|\mathcal{O}|N, Z\rangle \neq 0$

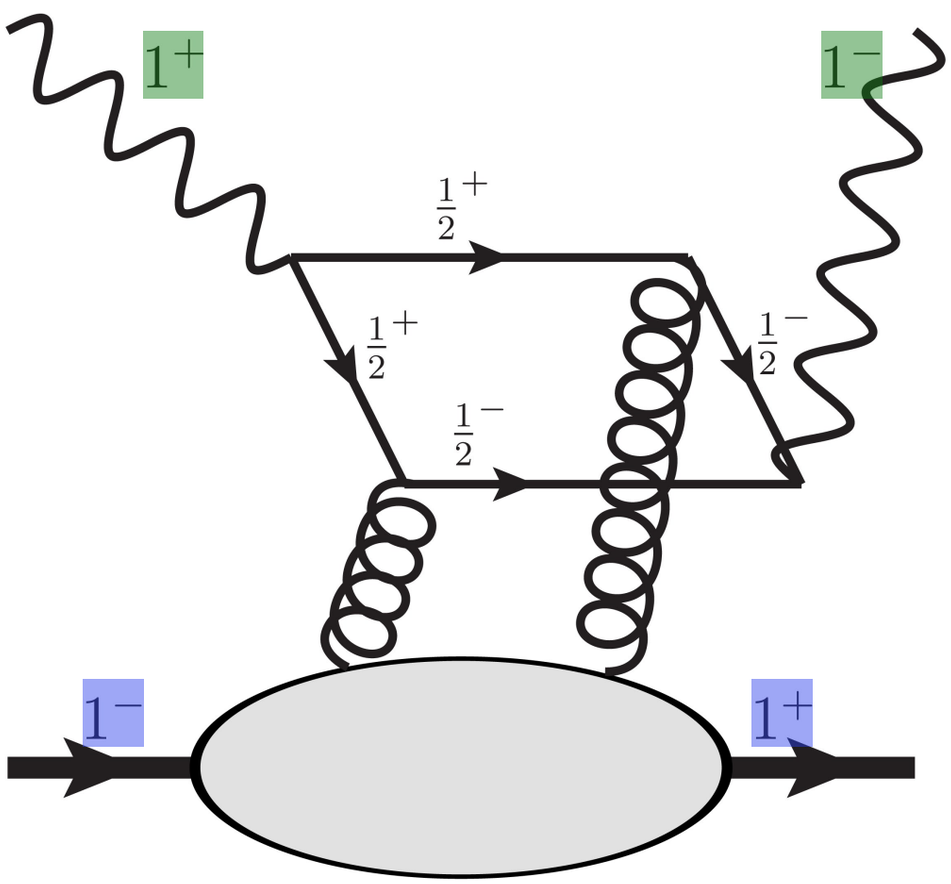


Jaffe and Manohar, "Nuclear Gluonometry"
Phys. Lett. B223 (1989) 218

Exotic glue in the deuteron

a "pure" EMC-type effect

Double helicity flip structure function $\Delta(x,Q^2)$:
changes both photon and target helicity by 2 units



- **Unambiguously gluonic:** no analogous quark PDF at twist-2
- Non-vanishing in forward limit for targets with spin≥1
- **Experimentally measurable** [see Shunzo Kumano's talk]
  - Unpolarised electron DIS on polarised target: JLab LoI 2015
  - Proton-deuteron Drell-Yan at FNAL
  - J/ψ production at NICA
- **Moments calculable in LQCD**

Exotic glue in the deuteron

a "pure" EMC-type effect

Double helicity flip structure function $\Delta(x,Q^2)$:
changes both photon and target helicity by 2 units

Parton model interpretation: gluonic transversity

$$\Delta(x,Q^2) = -\frac{\alpha_s(Q^2)}{2\pi}\mathrm{Tr}\mathcal{Q}^2\, x^2 \int_x^1 \frac{dy}{y^3}\left[g_{\hat{x}}(y,Q^2) - g_{\hat{y}}(x,Q^2)\right]$$

$g_{\hat{x},\hat{y}}(y,Q^2)$: probability of finding a gluon with momentum fraction y linearly polarised in $\hat{x}$, $\hat{y}$ direction

Exotic glue in the deuteron

Contributions to nuclear structure from gluons not associated with individual nucleons in nucleus

nucleon: $\langle p|\mathcal{O}|p\rangle = 0$

nucleus: $\langle N, Z|\mathcal{O}|N, Z\rangle \neq 0$

- First moment of gluon transversity distribution in the deuteron
  [Jaffe, Manohar PLB223 (1989) 218]
- First evidence for non-nucleonic gluon contributions to nuclear structure: LQCD with m_π ~800 MeV [NPLQCD PRD96 (2017)]
- Magnitude relative to momentum fraction as expected from large-N_c

Signal in LQCD data

Gluon structure from LQCD

Future colliders will dramatically alter our knowledge of the gluonic structure of hadrons and nuclei

- Work towards a complete 3D picture of parton structure (moments, x-dependence of PDFs, GPDs, TMDs)
- **First determination of gluon contributions to shear and pressure distributions in the proton**
  - ❖ Supports analysis assumptions in recent experimental determination
  - ❖ Suggests target kinematics for future model-independent extractions at JLab12 and EIC
- Compare quark and gluon distributions in hadrons and nuclei

Lattice QCD calculations in hadrons and light nuclei will complement and extend understanding of fundamental structure of nature

Image Credit: 2018 EIC User's Group Meeting