Probing the linearly polarized gluons in unpolarized hadrons

with heavy flavor production

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Outline:

1. Heavy-quark leptoproduction
   - Perturbatively stable observables in heavy-quark lepto-production:
     - Azimuthal asymmetries;
     - Callan-Gross ratio $R = F_L / F_T$ in 1PI kinematics
   - Search for linearly polarized gluons in unpolarized proton using the heavy-quark pair production:
     - Maximal values for the $\cos\varphi$, $\cos 2\varphi$ and R allowed by unpolarized photon-gluon fusion, f_1^g;
     - Contribution of the linearly polarized gluons, $h_1^{\perp g}$;
     - Recommendations for EIC and LHeC
2. Heavy-quark hadroproduction
   - Outlook for NICA

Our main conclusions:

Probing $h_1^{\perp g}$:

- ❑ cos φ, cos 2φ asymmetries and ratio $R = F_L / F_T$ in heavy-quark pair leptoproduction depend dramatically on the contribution of linearly polarized gluons;
- ❑ Future measurements of these quantities at EIC and LHeC seem to be very promising for determination of $h_1^{\perp g}$, gluonic counterpart of the Boer–Mulders function;

Outlook for COMPASS:

- ❑ cos 2φ asymmety in 1PI charm leptoproduction is predicted to be large (~15%) in the COMPASS kinematics;
- ❑ Extraction of the azimuthal asymmetries from available COMPASS data will provide valuable information about the TMD distribution $f_1^g(\xi, \vec{k}_T^2)$

References

Perturbative stability of cos 2φ and *R* quantities:

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Search for linearly polarized gluons in unpolarized proton:

- D.Boer, S.J.Brodsky, P.J.Mulders, C.Pisano, PRL 106 (2011), 132001
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- A.V.Efremov, N.Ya.Ivanov, O.V.Teryaev, Phys.Lett. **B** 772 (2017), 283
- A.V.Efremov, N.Ya.Ivanov, O.V.Teryaev, Phys.Lett. **B** 777 (2018), 435
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Parton TMD distributions inside the proton

The quark - proton correlator Φ_q and Boer - Mulders function $h_1^{\perp q}$:

$$\Phi_q(\zeta, k_T) = \int \frac{d(P\cdot\lambda)d^2\lambda_T}{(2\pi)^3} e^{ik\cdot\lambda} \langle P|\bar{\psi}(0)\psi(\lambda)|P\rangle_{\mathrm{LF}}$$
$$= \frac{1}{2}\left\{ f_1^q(\zeta, \vec{k}_T^2)\hat{P} + i h_1^{\perp q}(\zeta, \vec{k}_T^2)\frac{[\hat{k}_T, \hat{P}]}{2m_N} \right\}$$

Boer, Mulders, PR D 57 (1998), 5780

The gluon - proton correlator $\Phi_g^{\mu\nu}$ and gluonic analogue of the Boer - Mulders function $h_1^{\perp g}$:

$$\Phi_g^{\mu\nu}(\zeta, k_T) = \frac{n_\rho n_\sigma}{(P\cdot n)^2}\int \frac{d(P\cdot\lambda)d^2\lambda_T}{(2\pi)^3} e^{ik\cdot\lambda} \langle P|\mathrm{Tr}[F^{\rho\mu}(0)F^{\sigma\nu}(\lambda)]|P\rangle_{\mathrm{LF}}$$
$$= \frac{x}{2}\left\{ -g_T^{\mu\nu} f_1^g(\zeta, \vec{k}_T^2) + \left(g_T^{\mu\nu} - 2\frac{k_T^\mu k_T^\nu}{k_T^2} \right) \frac{\vec{k}_T^2}{2m_N^2} h_1^{\perp g}(\zeta, \vec{k}_T^2) \right\}$$

Mulders, Rodrigues PR D 63 (2001), 5780

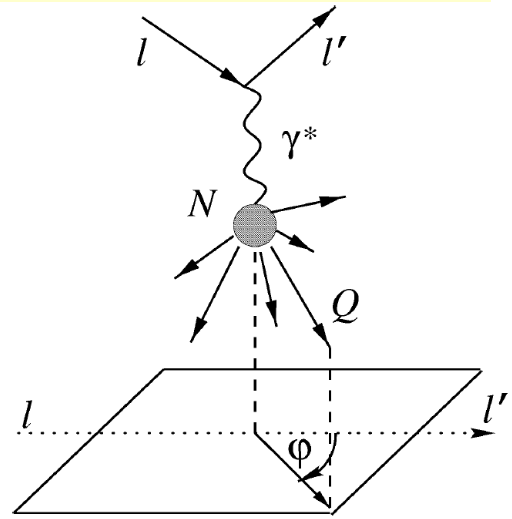
$$\lambda^+ = \lambda\cdot n = n^2 = 0; \quad P = n^+ + \frac{m_N}{2}n^-; \quad k = \zeta P + k_T + k^- n$$

Perturbative stability in charm electroproduction

Definitions and Cross Sections

We consider the Callan-Gross ratio $R = F_L / F_T$ and azimuthal $\cos 2\varphi$ asymmetry, $A = 2xF_A / F_2$, in heavy-quark leptoproduction:

$$l(\ell) + N(p) \to l(\ell - q) + Q(p_Q) + X[\overline{Q}](p_X)$$

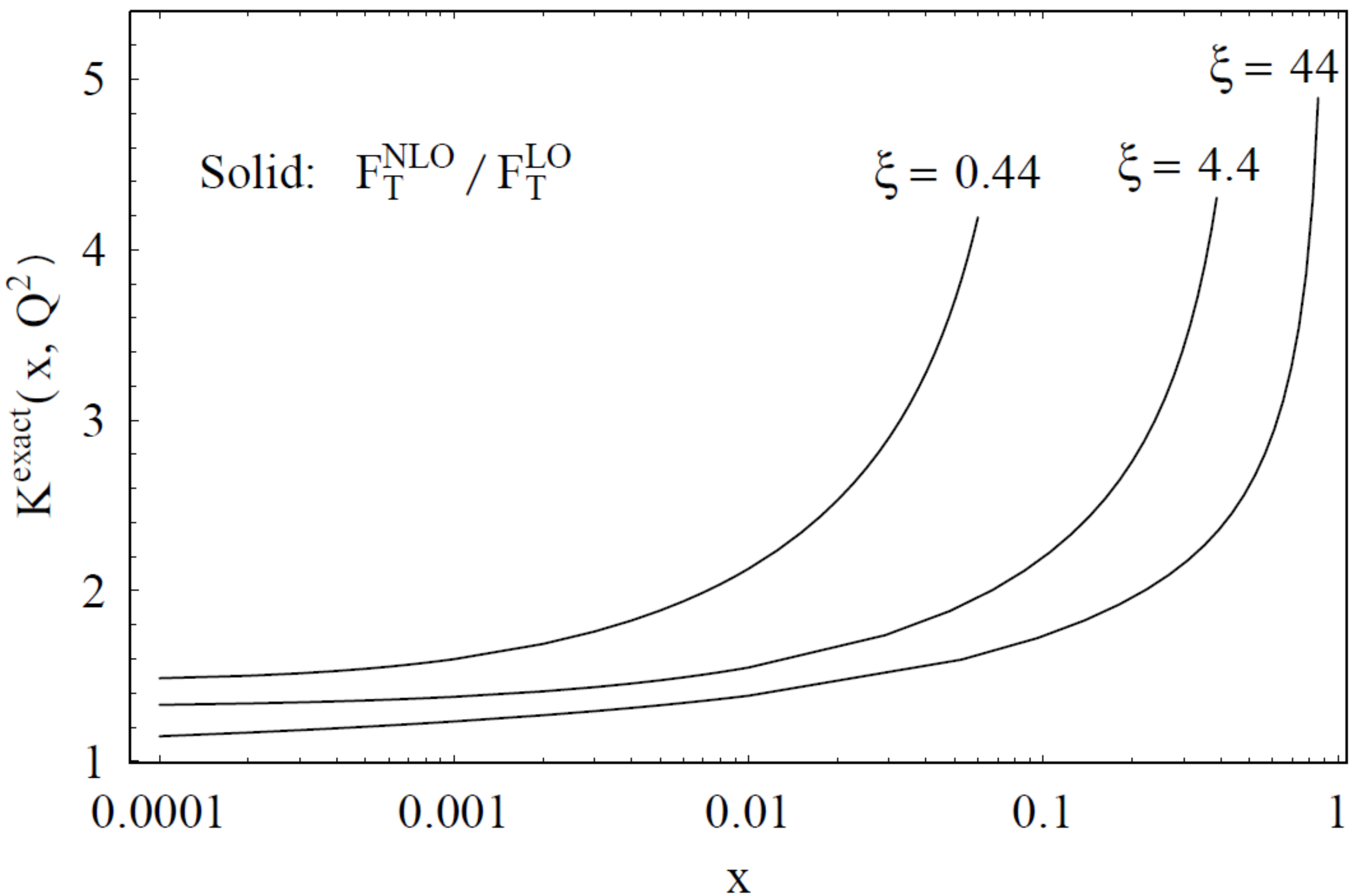


Corresponding cross section in 1PI kinematics is:

$$\frac{\mathrm{d}^3\sigma_{lN}}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}\varphi} = \frac{\alpha_{em}^2}{xQ^4}\Big\{\left[1+(1-y)^2\right]F_2(x,Q^2) - 2xy^2F_L(x,Q^2)$$
$$+4x\,(1-y)\,F_A(x,Q^2)\cos 2\varphi + 4x(2-y)\sqrt{2(1-y)}F_I(x,Q^2)\cos\varphi\Big\}$$

where $F_2(x,Q^2) = 2x(F_T + F_L)$ and x, y, Q^2 are usual DIS observables

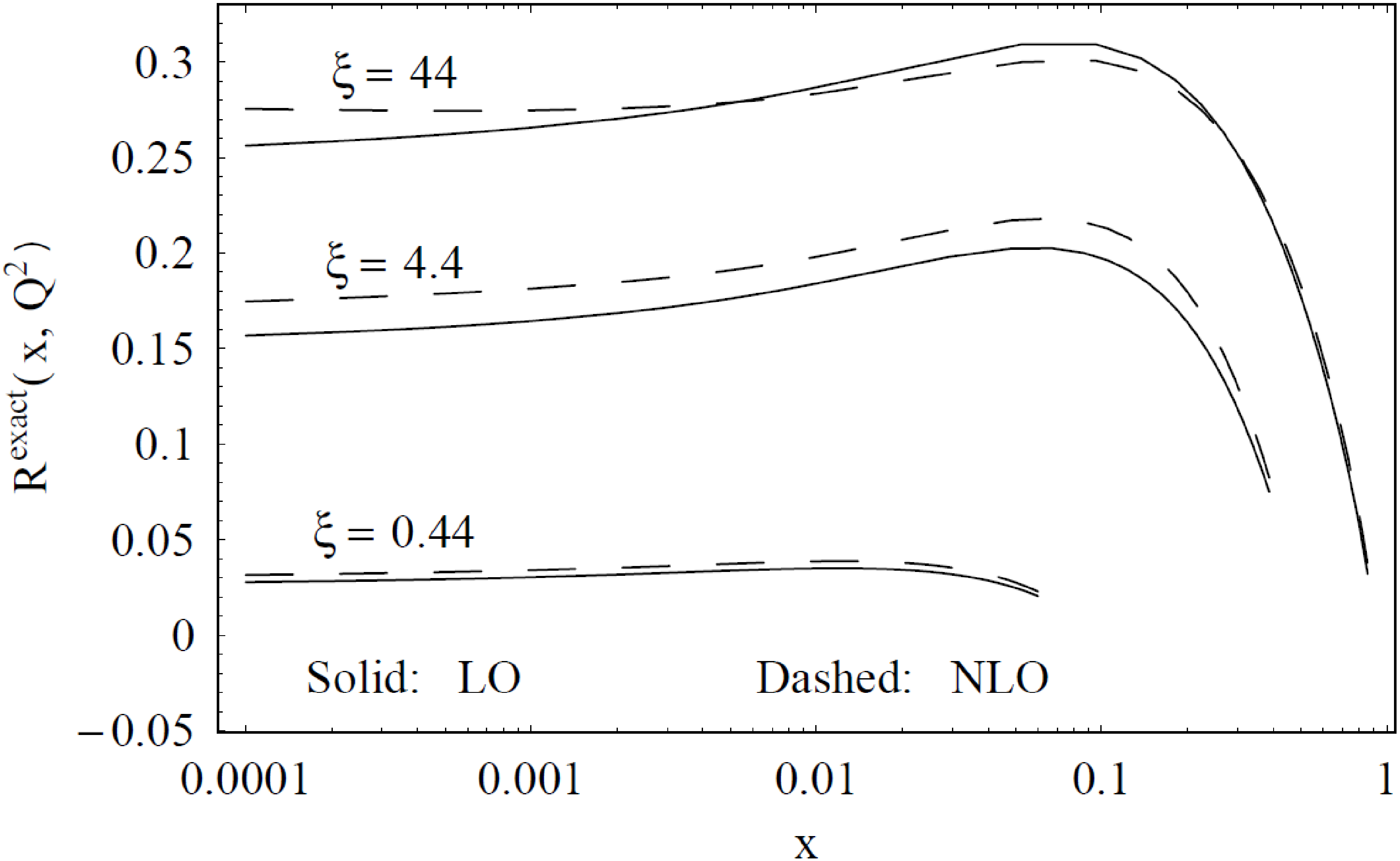
Perturbative instability of the cross section



$$\xi = \frac{Q^2}{m^2}$$

$$K(x, Q^2) = \frac{F_T^{\mathsf{NLO}}}{F_T^{\mathsf{LO}}}$$

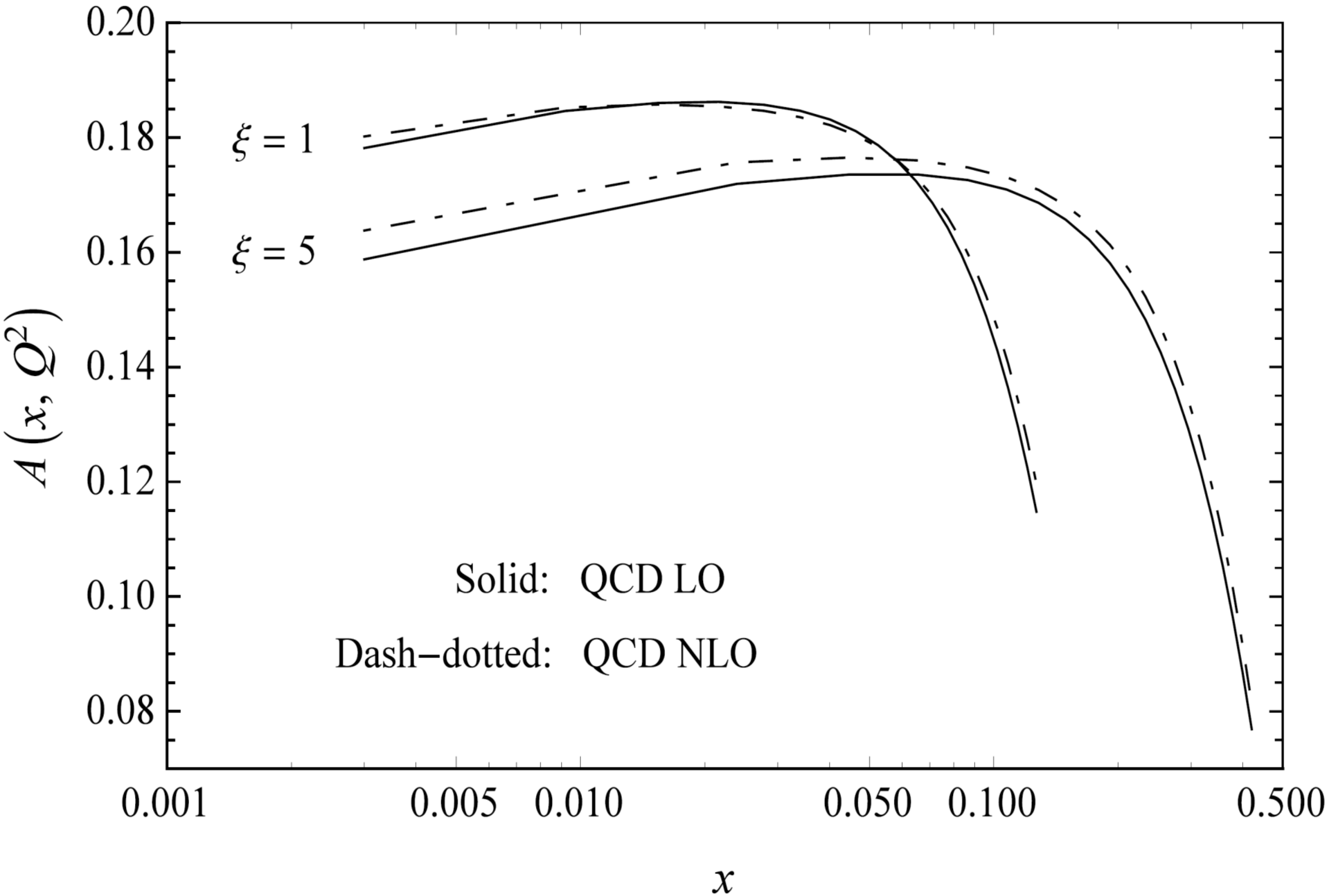
Perturbative stability of $R = F_L / F_T$



$$\xi = \frac{Q^2}{m^2}$$

$$R(x, Q^2) = \frac{F_L}{F_T}$$

Perturbative stability of $A = 2xF_A / F_2$



Laenen ,Moch, PR D 59 (1999) 034027

Ivanov, Kniehl, Eur. Phys. J. C 59 (2009) 647

$$\xi = \frac{Q^2}{m^2}$$

$$A(x, Q^2) = 2x\frac{F_A}{F_2}$$

- The soft-gluon NLO NLL corrections are given

Linearly polarized gluons in unpolarized proton

To probe the TMD distribution $h_1^{\perp g}$, the momenta of both heavy quark and anti-quark should be measured (reconstructed) in the reaction:

$$l(\ell) + N(P) \to l'(\ell - q) + Q(p_Q) + \bar{Q}(p_{\bar{Q}}) + X(p_X)$$

The LO parton - level subprocess is:

$$\gamma^*(q) + g(k_g) \to Q(p_Q) + \bar{Q}(p_{\bar{Q}}) \qquad k_g^\mu \simeq \zeta P^\mu + k_T^\mu$$

Corresponding cross section is:

$$\mathrm{d}\sigma \propto L(\ell, q) \otimes \Phi_g(\zeta, k_T) \otimes \left| H_{\gamma^* g \to Q\bar{Q}X}(q, k_g, p_Q, p_{\bar{Q}}) \right|^2$$

$$\Phi_g^{\mu\nu}(\zeta, k_T) \propto -g_T^{\mu\nu} f_1^g(\zeta, \vec{k}_T^2) + \left(g_T^{\mu\nu} - 2\frac{k_T^\mu k_T^\nu}{k_T^2} \right) \frac{\vec{k}_T^2}{2m_N^2} h_1^{\perp g}(\zeta, \vec{k}_T^2)$$

The positivity bound: $\frac{\vec{k}_T^2}{2m_N^2} \left| h_1^{\perp g}(\zeta, \vec{k}_T^2) \right| \leq f_1^g(\zeta, \vec{k}_T^2)$

The resulting cross section is:

$$
\begin{aligned}
d^7\sigma_{lN} \;\propto\;\; & A_0 + A_1 \cos\phi_\perp + A_2 \cos 2\phi_\perp + \vec{q}_T^{\,2}[B_0 \cos 2(\phi_\perp - \phi_T) \\
& + B_1 \cos(\phi_\perp - 2\phi_T) + B_1' \cos(3\phi_\perp - 2\phi_T) \\
& + B_2 \cos 2\phi_T + B_2' \cos 2(2\phi_\perp - \phi_T)]
\end{aligned}
$$

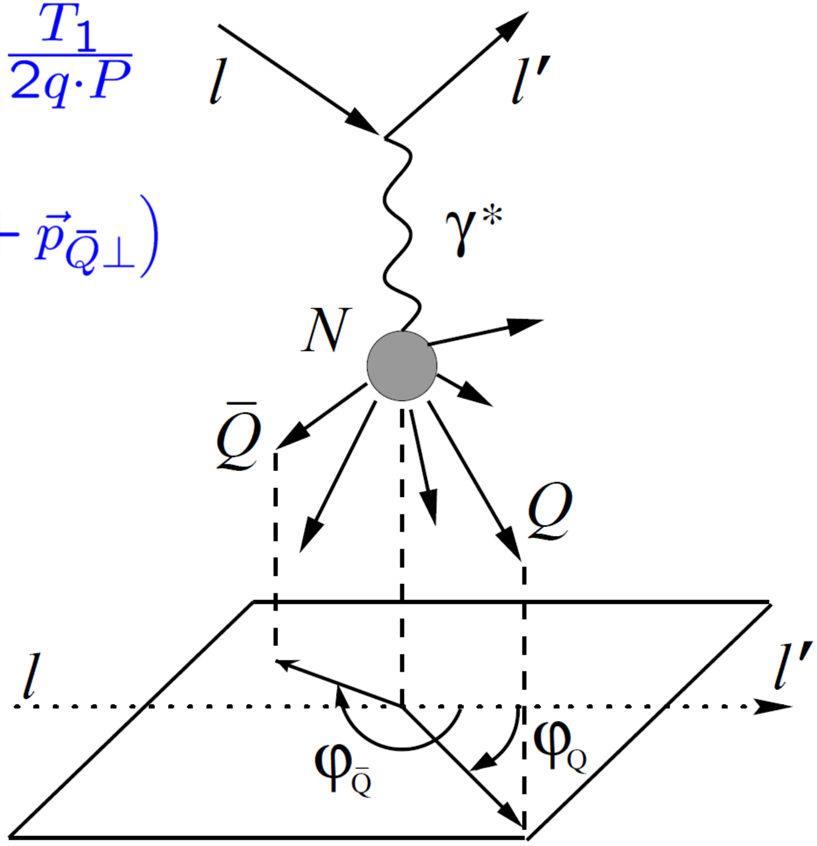
$$\zeta = \frac{-U_1}{y\bar{S} + T_1} = x + \frac{m^2 + \vec{K}_\perp^2}{z\,(1-z)y\,\bar{S}}, \quad z = -\frac{T_1}{2q\cdot P}$$

$$\vec{K}_\perp = \frac{1}{2}\left(\vec{p}_{Q\perp} - \vec{p}_{\bar{Q}\perp}\right) \qquad \vec{q}_T = \frac{1}{2}\left(\vec{p}_{Q\perp} + \vec{p}_{\bar{Q}\perp}\right)$$

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

$\phi_\perp$ $\qquad\qquad\qquad\qquad$ ϕ_T

$$A_i \sim f_1^g, \qquad B_i^{(\prime)} \sim h_1^{\perp g}$$



- Boer, Brodsky, Mulders, Pisano, PRL 106 (2011), 132001
- Pisano, Boer, Brodsky, Mulders, JHEP 1310 (2013) 024

We work in the approximation:

$$|\vec{q}_T| \ll |\vec{K}_\perp|, \; |\varphi_Q - \varphi_{\bar{Q}}| \sim \pi \qquad \Rightarrow \qquad \phi_T \simeq \phi_\perp - \frac{\pi}{2}$$

Integration over ϕ_T gives:

$$\frac{\mathrm{d}^6\sigma^{(\pi)}}{\mathrm{d}y\,\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}\vec{K}_\perp^2\mathrm{d}\vec{q}_T^2\mathrm{d}\varphi} = \frac{e_Q^2\alpha_{em}^2\alpha_s}{8\,\bar{S}^2}\frac{f_1^g(\zeta,\vec{q}_T^2)\hat{B}_2}{y^3x\,\zeta z\,(1-z)}\Big\{\left[1+(1-y)^2\right]\left(1-2r\frac{\hat{B}_2^h}{\hat{B}_2}\right) - y^2\frac{\hat{B}_L}{\hat{B}_2}\left(1-2r\frac{\hat{B}_L^h}{\hat{B}_L}\right)$$
$$+2(1-y)\frac{\hat{B}_A}{\hat{B}_2}\left(1-2r\frac{\hat{B}_A^h}{\hat{B}_A}\right)\cos 2\varphi + (2-y)\sqrt{1-y}\frac{\hat{B}_I}{\hat{B}_2}\left(1-2r\frac{\hat{B}_I^h}{\hat{B}_I}\right)\cos\varphi\Big\}$$

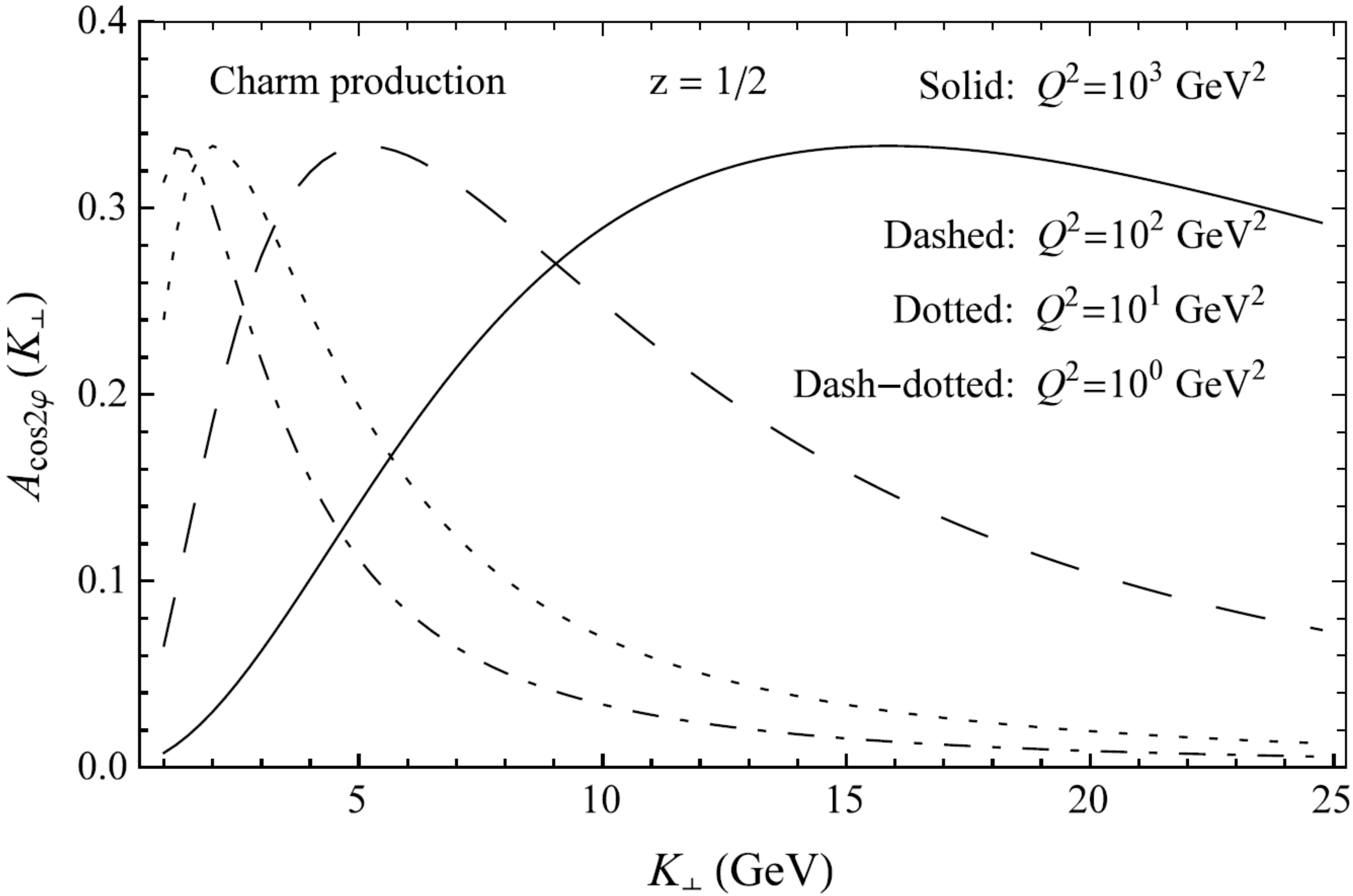
$$\varphi = \varphi_Q$$

$$r \equiv r(\zeta,\vec{q}_T^2) = \frac{\vec{q}_T^2}{2m_N^2}\frac{h_1^{\perp g}(\zeta,\vec{q}_T^2)}{f_1^g(\zeta,\vec{q}_T^2)} \qquad -1 \leq r \leq 1$$

$$\hat{B}_i \sim f_1^g, \qquad \hat{B}_i^h \sim h_1^{\perp g}$$

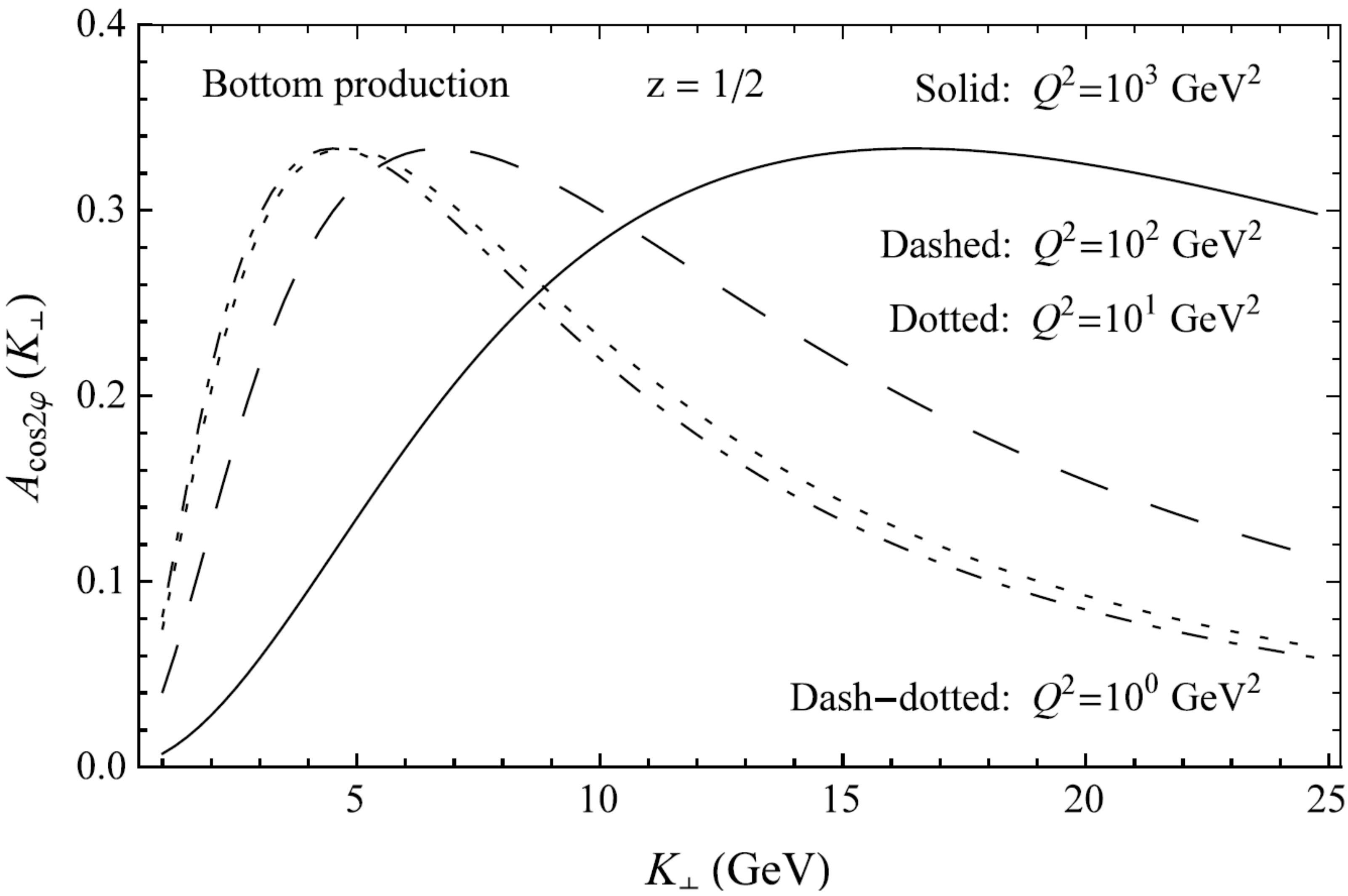
- **Efremov, Ivanov, Teryaev, Phys.Lett. B 772 (2017), 283**
- **Efremov, Ivanov, Teryaev, Phys.Lett. B 777 (2018), 435**

pQCD predictions for cos 2φ asymmetry (r = 0)



$$A_{\cos 2\varphi}\left(z, \vec{K}_\perp^2\right) \simeq \frac{\widehat{B}_A}{\widehat{B}_2}\left(z, \vec{K}_\perp^2\right) \qquad A_{\cos 2\varphi}\left(z = 1/2, \vec{K}_\perp^2 = m^2 + Q^2/4\right) = \frac{1}{3}$$

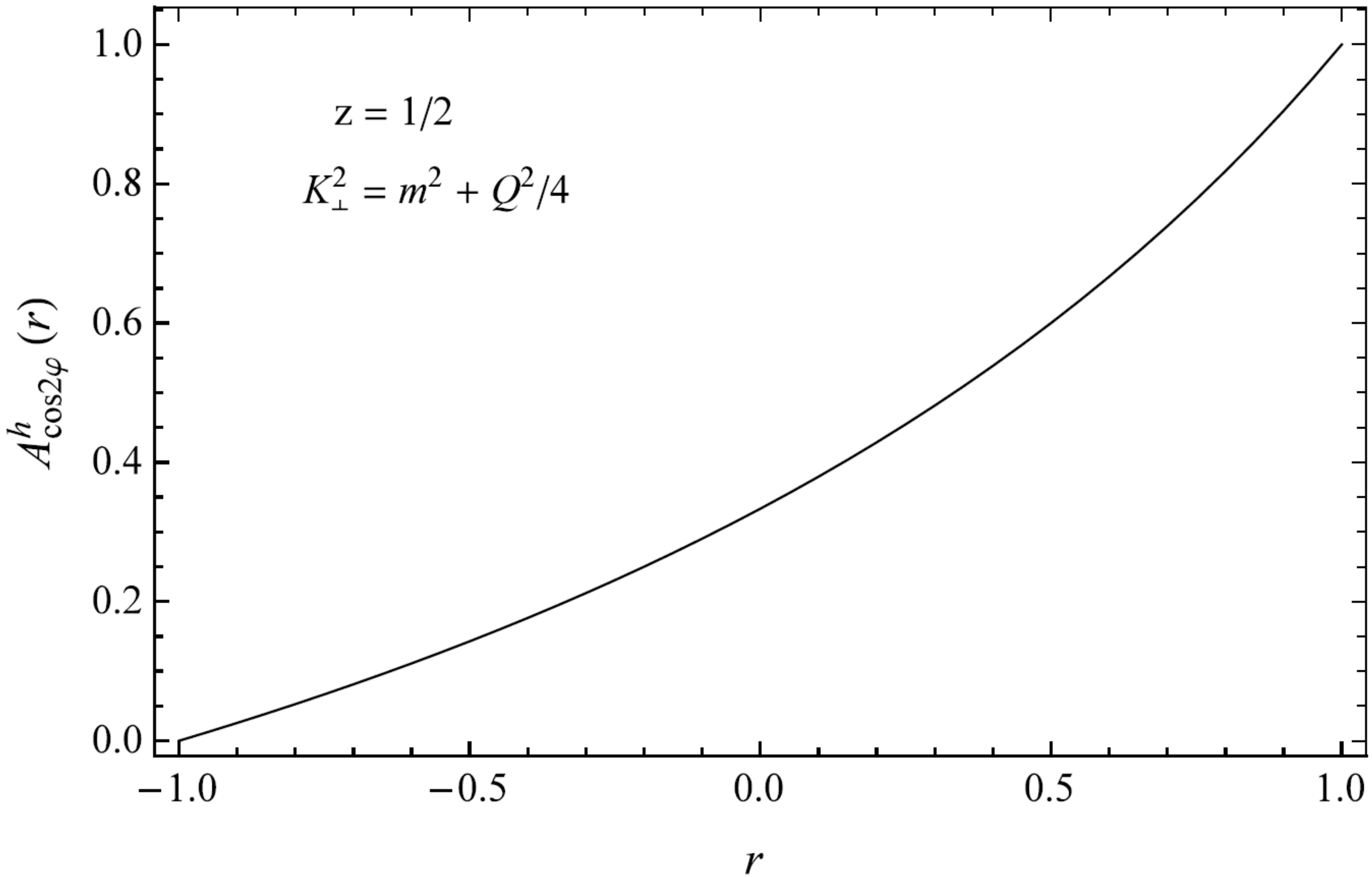
pQCD predictions for cos 2φ asymmetry (r = 0)



pQCD predictions for cos 2φ asymmetry (r ≠ 0)

$$A^h_{\cos 2\varphi}\left(z, \vec{K}_\perp^2, r\right) \simeq \frac{\hat{B}_A}{\hat{B}_2}\frac{1-2r\hat{B}^h_A/\hat{B}_A}{1-2r\hat{B}^h_2/\hat{B}_2} \qquad r = \frac{\vec{q}_T^{\,2}}{2m_N^2}\frac{h_1^{\perp g}}{f_1^g}$$

$$A^h_{\cos 2\varphi}(r) \equiv A^h_{\cos 2\varphi}\left(z = 1/2, \vec{K}_\perp^2 = m^2 + Q^2/4, r\right) = \frac{1+r}{3-r}$$



pQCD predictions for cos φ asymmetry (r = 0)

$$A_{\cos\varphi}\left(z,\vec{K}_\perp^2\right) \simeq \frac{\widehat{B}_I}{\widehat{B}_2}\left(z,\vec{K}_\perp^2\right) \qquad \int \mathrm{d}z\, A_{\cos\varphi}\left(z,\vec{K}_\perp^2\right) = 0$$

$$\begin{cases} A_{\cos\varphi}(z,\vec{K}_\perp^2) = -A_{\cos\varphi}(1-z,\vec{K}_\perp^2) \\ A_{\cos\varphi}(z,\vec{K}_\perp^2) = -A_{\cos\varphi}(z,\,(z(1-z)Q^2+m^2)^2/\vec{K}_\perp^2) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} A_{\cos\varphi}(z=1/2,\vec{K}_\perp^2) = 0 \\ A_{\cos\varphi}(z,\vec{K}_\perp^2 = z\,(1-z)Q^2+m^2) = 0 \end{cases}$$

$$\max A_{\cos\varphi}\left(z,\vec{K}_\perp^2\right) = A_{\cos\varphi}(z=z_\pm,\vec{K}_\perp^2 = Q^2\,\hat{k}_\pm^2)$$

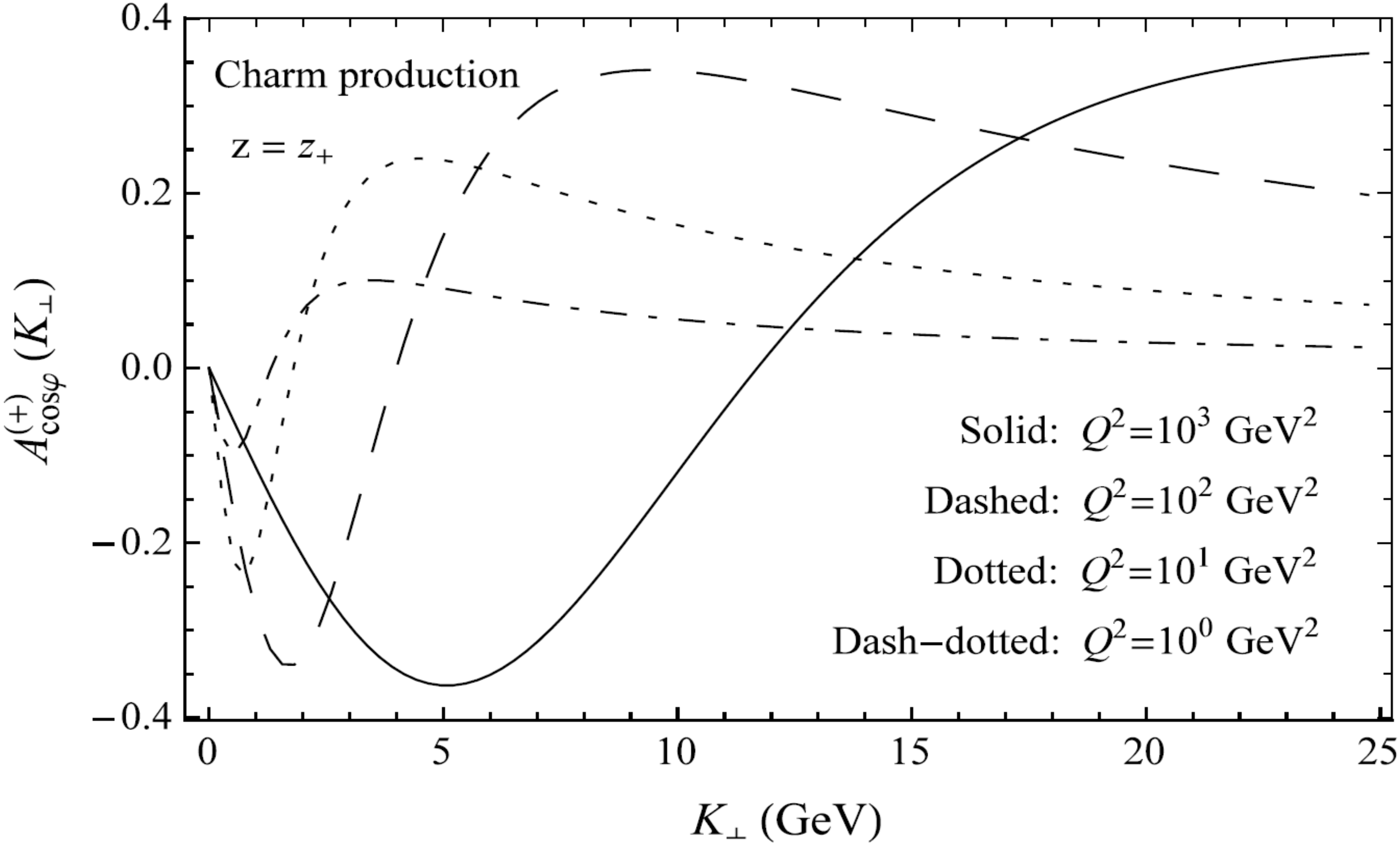
$$\min A_{\cos\varphi}\left(z,\vec{K}_\perp^2\right) = A_{\cos\varphi}(z=z_\mp,\vec{K}_\perp^2 = Q^2\,\hat{k}_\pm^2)$$

$$z_\pm(\lambda\to 0) \simeq \begin{cases} 0.841 \\ 0.159 \end{cases} \qquad \hat{k}_\pm^2(\lambda\to 0) \simeq \begin{cases} 0.707 \\ 0.025 \end{cases} \qquad \lambda = \frac{m^2}{Q^2}$$

$$A^{(\pm)}_{\cos\varphi}(K_\perp) \equiv A_{\cos\varphi}(z=z_\pm,K_\perp), \qquad A^{(-)}_{\cos\varphi} = -A^{(+)}_{\cos\varphi}$$

$$\left|A^{(\pm)}_{\cos\varphi}\right|_{\max} \simeq 0.366$$

pQCD predictions for cos φ asymmetry (r = 0)



$$A^{(\pm)}_{\cos\varphi}(K_\perp) \equiv A_{\cos\varphi}(z = z_\pm, K_\perp), \quad A^{(-)}_{\cos\varphi} = -A^{(+)}_{\cos\varphi}$$

$$\left|A^{(\pm)}_{\cos\varphi}\right|_{\max} \simeq 0.366 \quad \text{at} \quad \lambda = \frac{m^2}{Q^2} \to 0$$

pQCD predictions for cos φ asymmetry ($r \neq 0$)

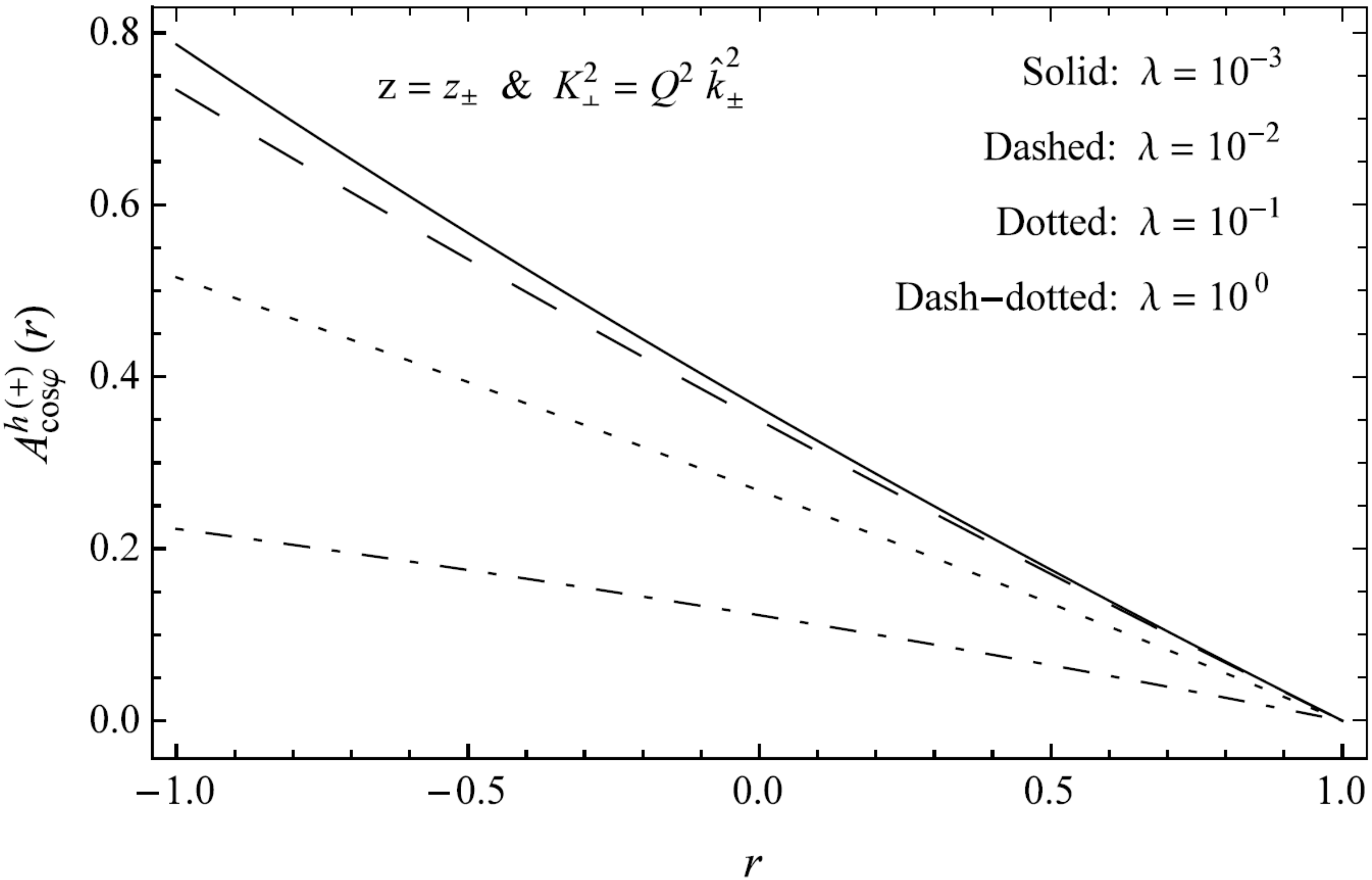
$$A^h_{\cos\varphi}\left(z, \vec{K}_\perp^2, r\right) \simeq \frac{\hat{B}_I}{\hat{B}_2} \frac{1-2r\hat{B}_I^h/\hat{B}_I}{1-2r\hat{B}_2^h/\hat{B}_2}$$

$$A^{h(+)}_{\cos\varphi}(r) \equiv A^h_{\cos\varphi}\left(z = z_\pm, \vec{K}_\perp^2 = Q^2\hat{k}_\pm^2, r\right)$$

$$A^{h(-)}_{\cos\varphi}(r) \equiv A^h_{\cos\varphi}\left(z = z_\pm, \vec{K}_\perp^2 = Q^2\hat{k}_\mp^2, r\right)$$

$$\left|A^{h(\pm)}_{\cos\varphi}\right|_{\max} \simeq 0.793$$

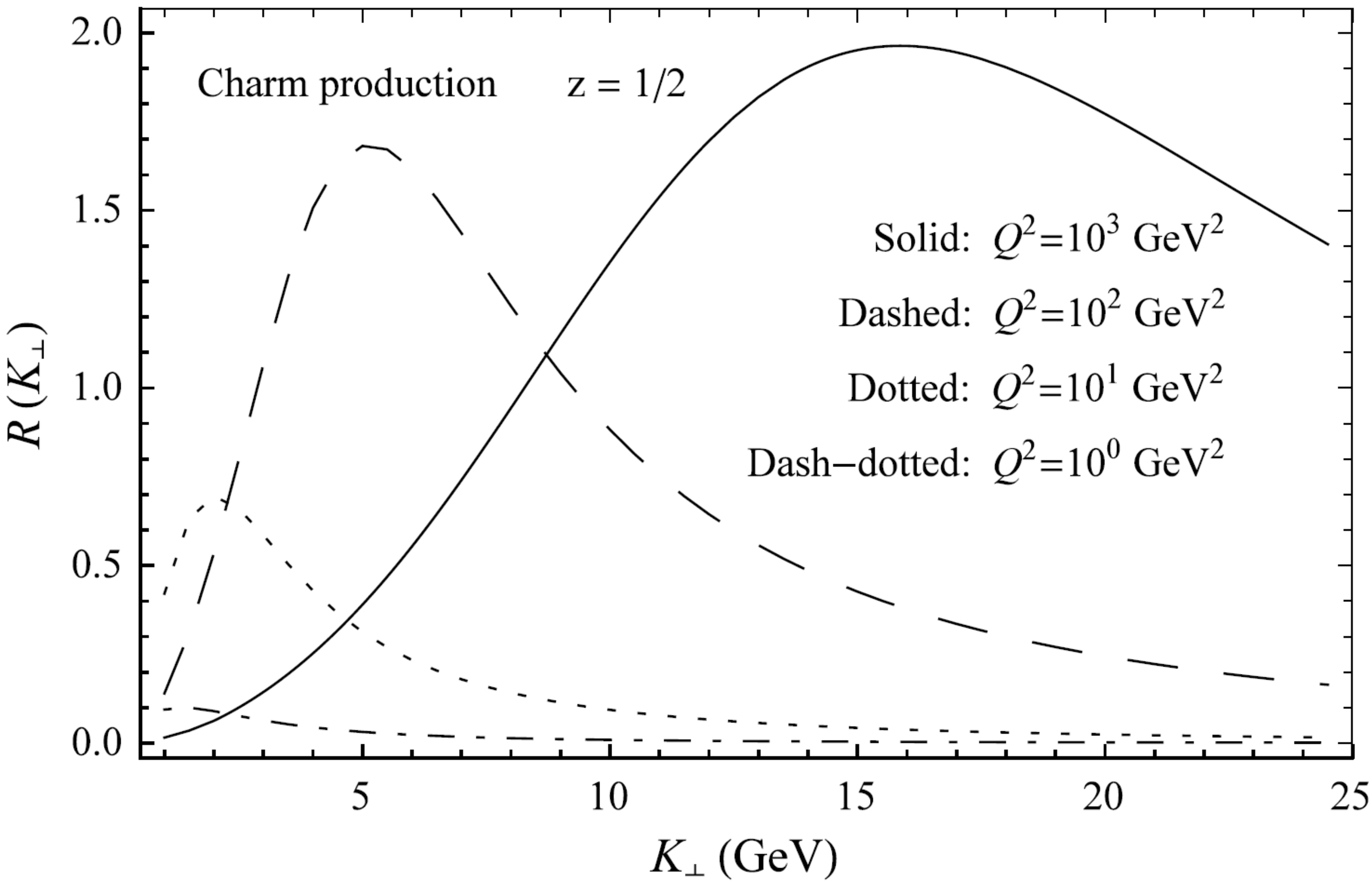
$$A^{h(\pm)}_{\cos\varphi}(r) \overset{\lambda\to 0}{=} \pm\frac{(\sqrt{3}-1)(1-r)}{2-r(1-2/\sqrt{3})}$$



$$\lambda = \frac{m^2}{Q^2}$$

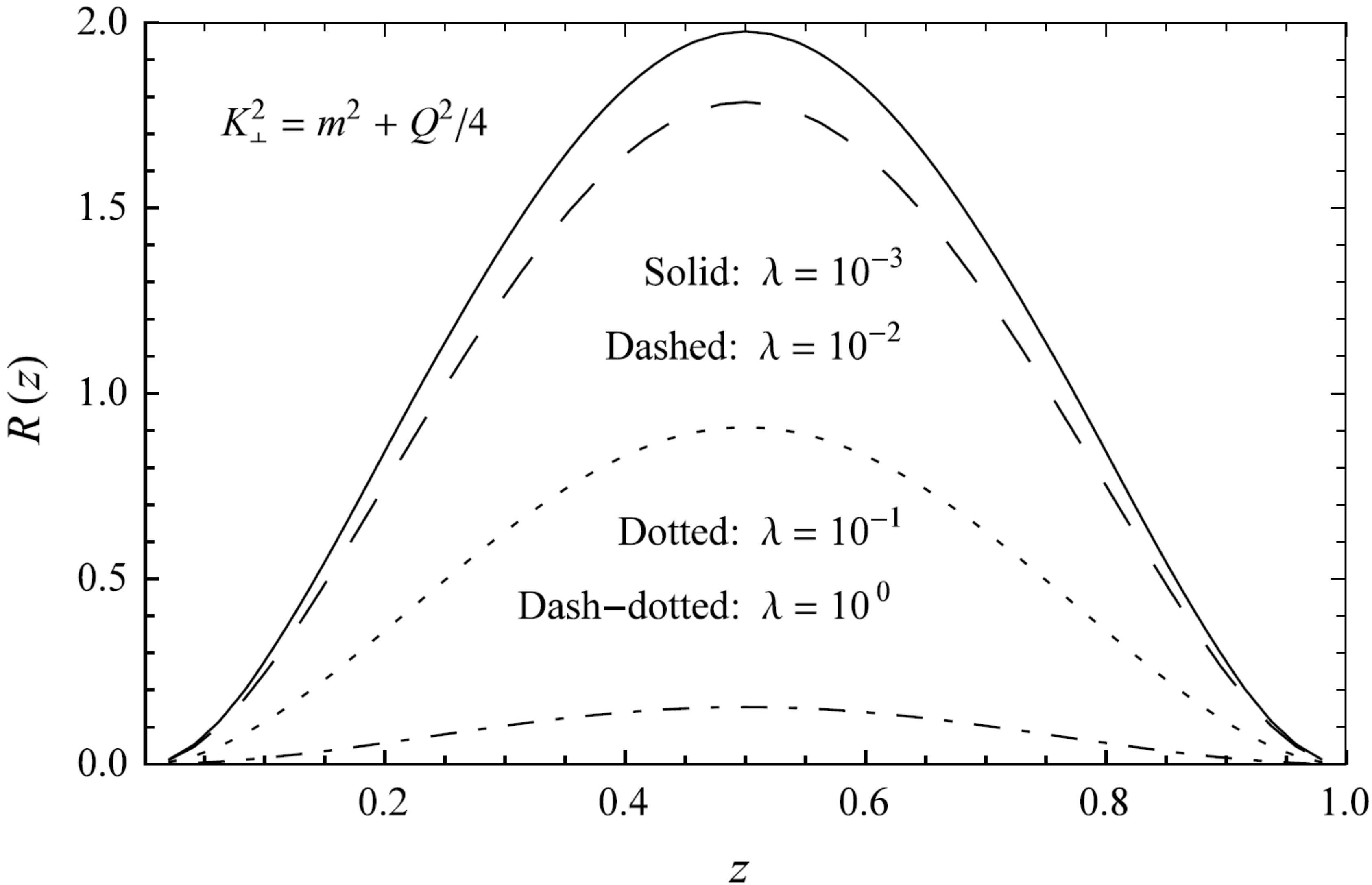
pQCD predictions for $R = F_L / F_T$ ratio (r = 0)

$$R\left(z, \vec{K}_\perp^2\right) = \frac{\mathrm{d}^3\sigma_L}{\mathrm{d}^3\sigma_T}\left(z, \vec{K}_\perp^2, r = 0\right) = \frac{\hat{B}_L}{\hat{B}_T}\left(z, \vec{K}_\perp^2\right)$$



pQCD predictions for $R = F_L / F_T$ ratio (r = 0)

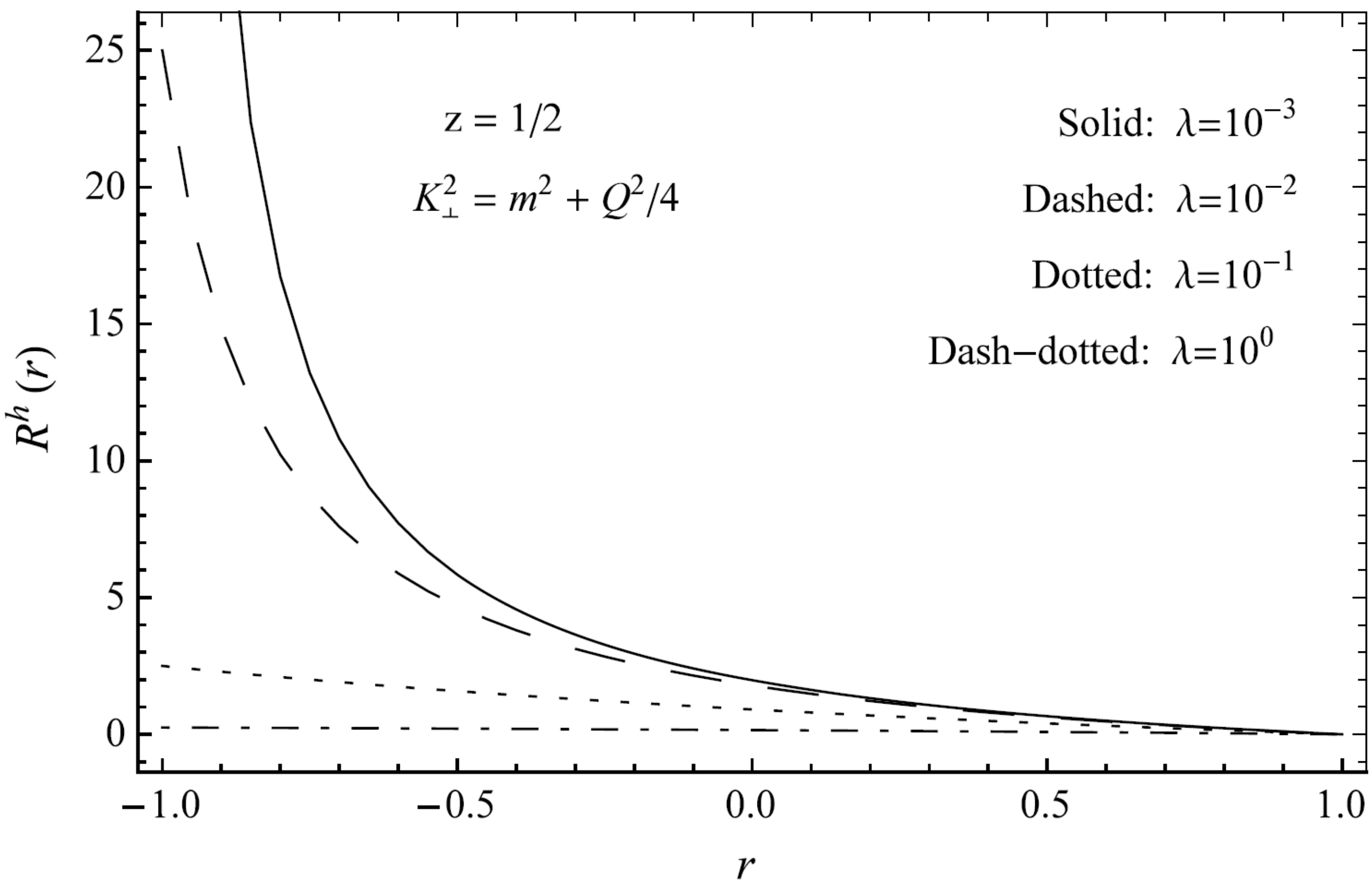
$$R\left(z = 1/2, \vec{K}_\perp^2 = m^2 + Q^2/4\right) = \frac{2}{1 + 12\lambda} \qquad \lambda = \frac{m^2}{Q^2}$$



pQCD predictions for $R = F_L / F_T$ ratio ($r \neq 0$)

$$R^h\left(z, \vec{K}_\perp^2, r\right) = \frac{\mathrm{d}^3\sigma_L}{\mathrm{d}^3\sigma_T}\left(z, \vec{K}_\perp^2, r\right) = \frac{\hat{B}_L}{\hat{B}_T}\frac{1 - 2r\hat{B}_L^h/\hat{B}_L}{1 - 2r\hat{B}_T^h/\hat{B}_T} \qquad r = \frac{\vec{q}_T^{\,2}}{2m_N^2}\frac{h_1^{\perp g}}{f_1^g}$$

$$R^h(r) \equiv R^h\left(z = 1/2, \vec{K}_\perp^2 = m^2 + Q^2/4, r\right) = \frac{2(1-r)}{1 + r + 4\lambda(3-r)}$$



$$\lambda = \frac{m^2}{Q^2}$$

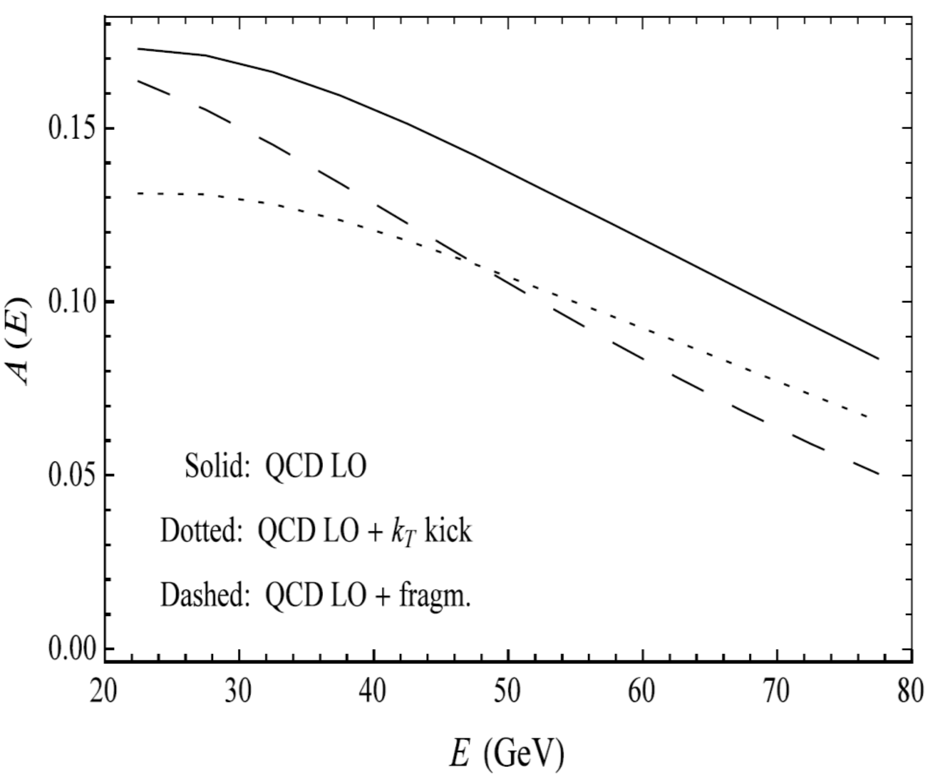
Conclusions:

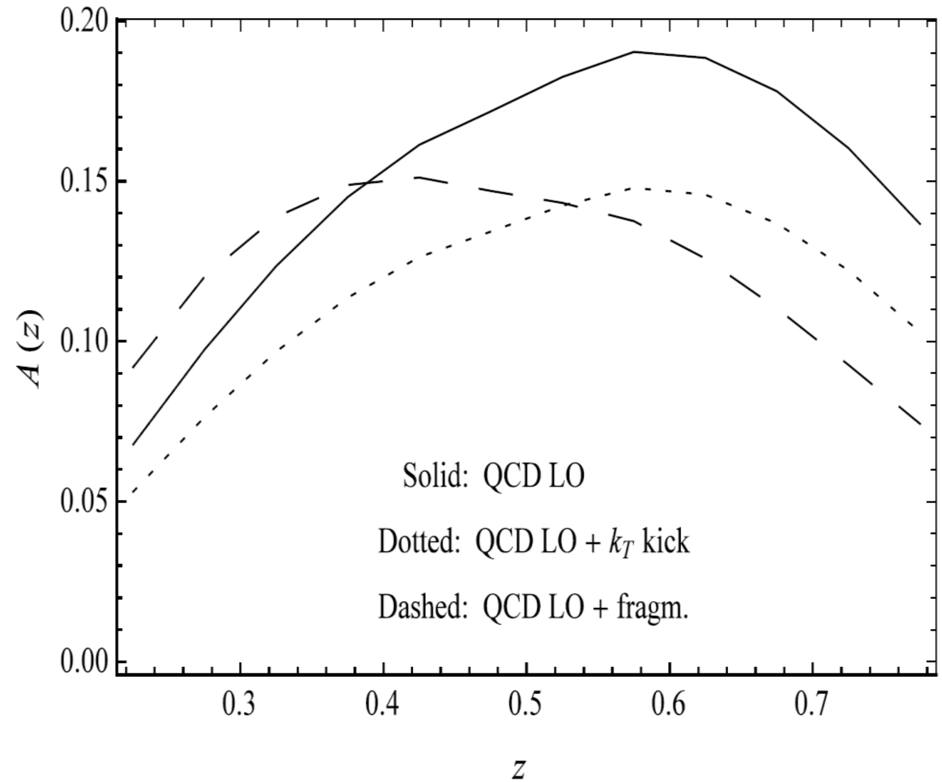
- When a linearly polarized gluon interacts with transverse virtual photon, the heavy-quark production plane is preferably orthogonal to the direction of the gluon polarization. For the longitudinal component, the momenta of emitted quarks and the gluon polarization lie in the same plane;
- The maximal values of the cos φ, cos2φ and $R = F_L / F_T$ quantities allowed by the photon–gluon fusion with unpolarized gluons are large: $(\sqrt{3}-1)/2$, $1/3$ and 2, respectively;
- These distributions are very sensitive to the linear polarization of gluons: their maximum values vary from 0 to 1 depending on $h_1^{\perp g}$, gluonic analogue of the Boer–Mulders function;
- We conclude that the cos φ, cos2φ and R distributions in heavy-quark pair leptoproduction could be good probes of the linear polarization of gluons inside unpolarized nucleon.


Thank You!

cos2ϕ asymmetry in charm electroproduction at COMPASS

cos2ϕ asymmetry in charm electroproduction can be measured at COMPASS : **Efremov, Ivanov, Teryaev, Phys.Lett. B 772 (2017), 283**





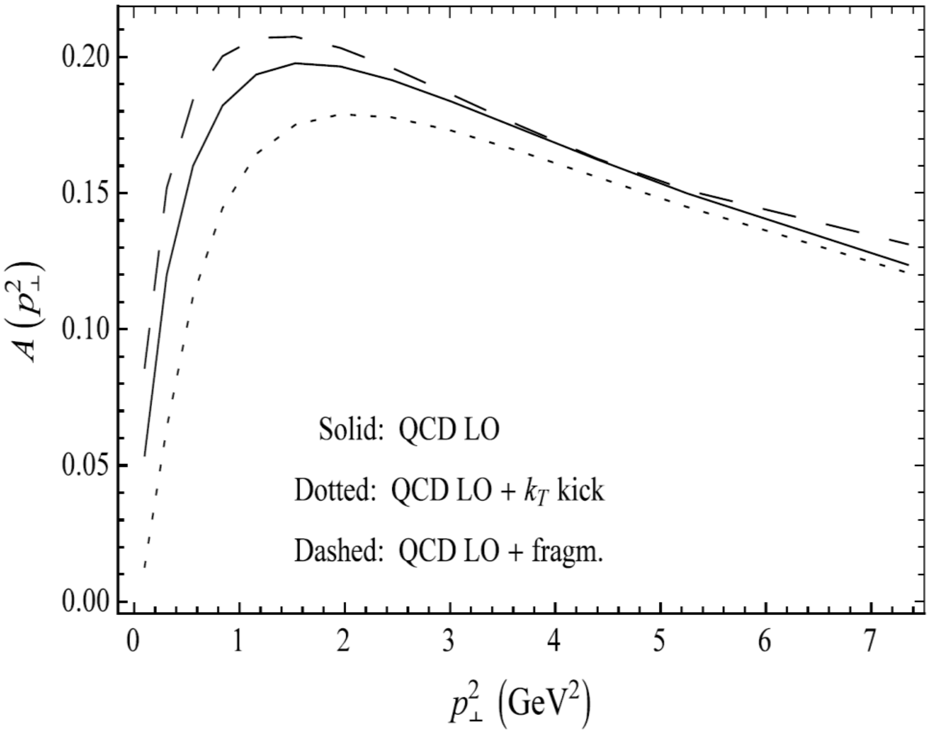
$$\nu = E_l - E_l'$$

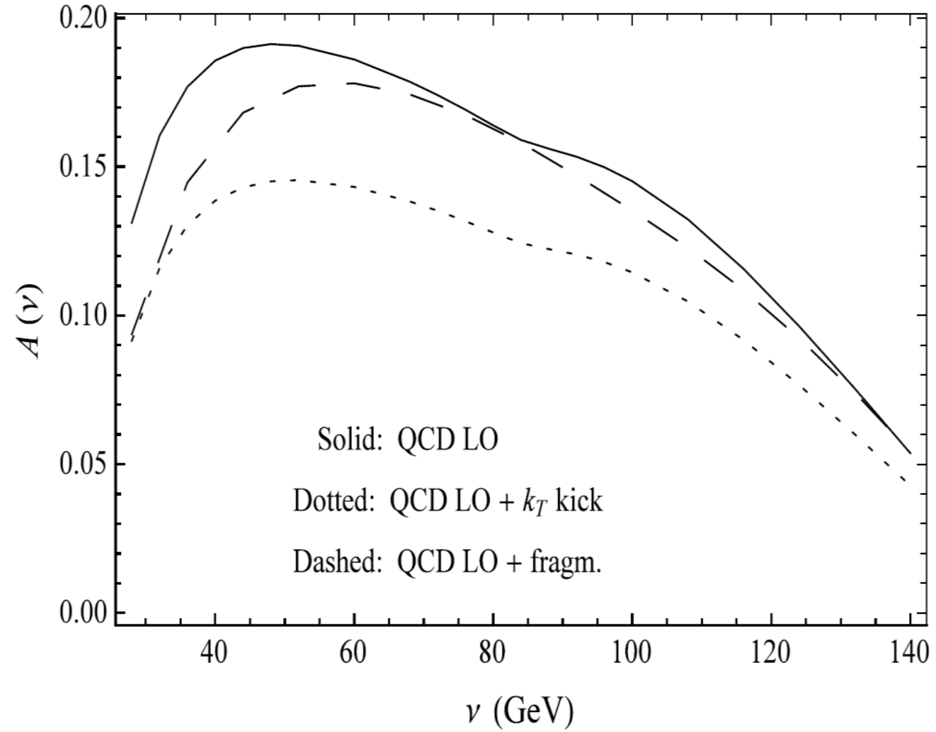
$$z = \frac{E}{\nu}$$

$\cos 2\phi$ asymmetry in charm electroproduction at COMPASS

COMPASS kinematics:

$$0.003 < Q^2 < 10 \text{ GeV}^2, \quad 3 \cdot 10^{-5} < x < 0.1, \quad 20 < E < 80 \text{ GeV}$$





$$\nu = E_l - E_l'$$

Azimuthal correlations in charm hadroproduction

To probe the TMD distributions in ***pp***- and ***AA***- collisions, the momenta of both heavy quark and anti-quark should be measured,

$$p_1(P_1) + p_2(P_2) \to Q(p_Q) + \bar{Q}(p_{\bar{Q}}) + X(p_X)$$

Corresponding cross section is:

$$\mathrm{d}\sigma \propto \sum_{a,b} \Phi_a(\zeta_a, k_{aT}) \otimes \Phi_b(\zeta_b, k_{bT}) \otimes \left| H_{ab\to Q\bar{Q}X}(k_a, k_b, p_Q, p_{\bar{Q}}) \right|^2$$

$$k_a^\mu \simeq \zeta_a P_1^\mu + k_{aT}^\mu, \qquad k_b^\mu \simeq \zeta_b P_2^\mu + k_{bT}^\mu$$

In this case, both quark and gluon densities do contribute at LO:

$$\Phi_g^{\mu\nu}(\zeta, k_T) \propto -g_T^{\mu\nu} f_1^g(\zeta, \vec{k}_T^2) + \left(g_T^{\mu\nu} - 2\frac{k_T^\mu k_T^\nu}{k_T^2} \right) \frac{\vec{k}_T^2}{2m_N^2} h_1^{\perp g}(\zeta, \vec{k}_T^2)$$

$$\Phi_q(\zeta, k_T) \propto f_1^q(\zeta, \vec{k}_T^2)\hat{P} + i h_1^{\perp q}(\zeta, \vec{k}_T^2) \frac{[\hat{k}_T, \hat{P}]}{2m_N}$$

The resulting cross section is: $\vec{K}_\perp = \frac{1}{2}\left(\vec{p}_{Q\perp} - \vec{p}_{\bar{Q}\perp}\right), \quad \vec{q}_T = \vec{p}_{Q\perp} + \vec{p}_{\bar{Q}\perp}$

$$\frac{\mathrm{d}^6\sigma}{\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}^2\vec{K}_\perp\mathrm{d}^2\vec{q}_T} = \mathcal{N}\left\{A + B\,\vec{q}_T^{\,2}\cos 2(\phi_\perp - \phi_T) + C\,\vec{q}_T^{\,4}\cos 4(\phi_\perp - \phi_T)\right\}$$

Pisano, Boer, Brodsky, Buffing, Mulders, JHEP 1310 (2013) 024

Schematically, the functions ***A***, ***B*** and ***C*** have the following structure:

$$\begin{aligned}
A &: \quad f_1^q \otimes f_1^{\bar{q}}, \qquad f_1^g \otimes f_1^g, \qquad h_1^{\perp g} \otimes h_1^{\perp g} \\
B &: \quad h_1^{\perp q} \otimes h_1^{\perp \bar{q}}, \qquad f_1^g \otimes h_1^{\perp g} \\
C &: \quad h_1^{\perp g} \otimes h_1^{\perp g}
\end{aligned}$$

The gluon fusion contribution to ***B*** is:

$$B^{gg} = \frac{N_c z(1-z)}{N_c^2 - 1}\left[z^2 - (1-z)^2 - 1/N_c^2\right]\left(1 - m^2/m_\perp^2\right) H^{gg}(x_1, x_2, \vec{q}_T^{\,2})$$

$$H^{gg} = \frac{1}{m^2\vec{q}_T^{\,4}}\int d^2\vec{k}_{1T}d^2\vec{k}_{2T}\delta^2(\vec{k}_{1T} + \vec{k}_{2T} - \vec{q}_T)\left\{\left[2(\vec{q}_T\cdot\vec{k}_{1T})^2 - \vec{q}_T^{\,2}\vec{k}_{1T}^{\,2}\right]\times\right.$$
$$\left. h_1^{\perp g}(x_1, \vec{k}_{1T}^{\,2}) f_1^g(x_2, \vec{k}_{2T}^{\,2}) + \left[2(\vec{q}_T\cdot\vec{k}_{2T})^2 - \vec{q}_T^{\,2}\vec{k}_{2T}^{\,2}\right] f_1^g(x_1, \vec{k}_{1T}^{\,2}) h_1^{\perp g}(x_2, \vec{k}_{2T}^{\,2})\right\}$$

Thank You!