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Computer Modeling of a Compact Isochronous Cyclotron

V. L. Smirnov

Joint Institute for Nuclear Research, Joliot-Curie 6, Dubna, Moscow oblast, 141980 Russia
e-mail: vsmirnov@jinr.ru

Abstract—The computer modeling methods of a compact isochronous cyclotron are described. The main stages of analysis of accelerator facilities systems are considered. The described methods are based on theoretical fundamentals of cyclotron physics and mention highlights of creation of the physical project of a compact cyclotron. The main attention is paid to the analysis of the beam dynamics, formation of a magnetic field, stability of the movement, and a realistic assessment of intensity of the generated bunch of particles. In the article, the stages of development of the accelerator computer model, analytical ways of assessment of the accelerator parameters, and the basic technique of the numerical analysis of dynamics of the particles are described.

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1. INTRODUCTION

It is impossible to imagine contemporary science without wide application of mathematical modeling. The essence on this methodology involves replacement of the initial object with its "image"—a mathematical model—and further study of the model using computational-logic algorithms implementable on computers [1]. This method of perception, construction, and design combines many advantages of both theory and experiment. Working with a model instead of the object itself allows investigating its properties in all kinds of situations without serious consequences, relatively fast, and without significant costs (advantages of theory). At the same time, computational experiments with models of objects allow deep and detailed study of the objects to a sufficient level using the power of contemporary computational methods and technical facilities of information science not available for purely theoretical methods (advantages of experiment).

Posing the question of mathematical modeling of some object generates a precise plan of actions. It can be conventionally split into three stages: model–algorithm–program. At the first stage, an "equivalent" of the object mathematically reflecting its most important properties is constructed, which represents laws which it obeys and connections peculiar to its components.

The second stage is the selection (or development) of an algorithm for implementing the model on a computer. The model is represented in a form convenient for applying numerical methods; then a sequence of computational and logic operations is defined for the purpose of finding the desired values with a given accuracy. Computational algorithms must not distort the main properties of the model and, consequently, of the initial object, and they must be effective and adaptable to peculiarities of the problems being solved and the computers used.

At the third stage, programs which "translate" the model and algorithm into machine language are created. They also must be efficient and adaptable. Programs can be called an "electronic" equivalent of the object under consideration which is appropriate for direct testing on a "test facility"—a computer.

Having created the triad "model–algorithm–program," a researcher acquires a universal, flexible, and an inexpensive tool which first of all is debugged and tested in "trial" computational experiments. After verification of the adequacy (sufficient suitability) of the triad to the initial object, the model is subjected to different detailed experiments which provide all required qualitative and quantitative properties and characteristics of the objects. The modeling process is accompanied by enhancement and refinement of all parts of the triad as needed.

Accelerator physics as a science dealing with complicated and expensive facilities cannot do without computer modeling, which today has firmly taken its place in the science at all stages of design and modernization of accelerator facilities. Numerical modeling is a reliable, effective, and often the only way to analyze and optimize different accelerator systems.

The last decades are characterized by active creation, development, and application of accelerators, mostly of compact cyclotrons [2]. Under the classification "compact," one should understand the general definition of cyclotrons with a dee. The main fields of application of such cyclotrons include research on nuclear physics, generation and study of radioactive ions, applications in medicine and for detecting explosive substances, and use of cyclotrons as injectors into other accelerators. Compactness, simplicity of maintenance, and relatively low cost were factors that led to the

overall demand for cyclotrons. There are centers all over the world engaging in development, modernization, and operation of cyclotrons and research on them.

Independently of the target of application of a cyclotron, whether nuclear physics or applied research, an accelerator facility represents a complicated system. Creation of a compact up-to-date cyclotron may cost tens of millions of dollars, and mistakes at the design stage are unacceptable.

Without a doubt, the most important part is to create computer models of installations approximating real objects as close as possible and to model a cyclotron as a whole in a complex with systems of beam injection and beam extraction. During computation of beam dynamics, it is necessary to take realistic three-dimensional fields of structural elements of the installation and the influence of the particle self-field into account. In order to provide a high-quality beam with sufficient intensity, it is necessary to include computation of centering of accelerated orbits, computation of frequencies of betatron oscillations, and consideration of losses of particles on structural elements of the facility in analysis of particle dynamics. There are commercial and authorial software packages intended for modeling different systems of a compact cyclotron: injection lines, central zones, and acceleration and extraction zones. These programs are based on the basics of physical motion of charged particles in electromagnetic fields and have different approaches to describing such motion, starting from the matrix approach and finishing with numerical integration of equations of motion. Electromagnetic fields set analytically, as well as three-dimensional field maps computed in third-party programs, can be used for operation. All existing and designed accelerator facilities have "virtual" models with the help of which they can be modeled with a definite level of realism. Programs developed for modeling beam dynamics [3, 4, 5, 6] can either be written in universal programming languages or be based on known software packages such as MATLAB, Mathematica, and Borland. Though most of existing programs are created for specific accelerators, nevertheless their basic modeling methods have a common character. This work describes modeling methods applicable to a compact cyclotron [7–9] both at accelerator design stage and during modernization of an already existing facility.

2. CYCLOTRON COMPUTER MODEL

In order to have the possibility of a detailed analysis of particles in a cyclotron, it is necessary to create a realistic computer model of an accelerator facility which would describe its main characteristics. Actually, such a model must digitally replicate a real existing accelerator and include the following elements:

—electromagnetic fields of structural elements of the facility;

—parameters of the acceleration regime;

—model of the particle beam as a physical object with the corresponding characteristics;

—software which simulates the motion of accelerated particles in the facility;

—software for modeling real systems of beam diagnostics existing for the accelerator;

—geometric structure of the facility to account for particle losses.

As a rule, an accelerator facility contains the following beam formation and control elements in addition to the main cyclotron magnet and system of accelerating dees:

—electrostatic and magnetic quadrupoles applied for focusing the beam into injection lines and during extraction;

—electrostatic deflectors (including spherical) for rotating and focusing the beam during its injection into the cyclotron;

—radio frequency bunchers located in the injection lines and grouping ions of the beam in the longitudinal direction to obtain a higher density of particles in the operating range of wave phases of high-frequency voltage on dees (further, RF phases);

—solenoids used for controlling the beam cross section in the injection line as well as for the beam matching in the central part of the cyclotron;

—spiral electrostatic inflector required for transferring the beam from the plane of the axial injection line into the median plane of the cyclotron;

—phase slits located in the central zone of the accelerator in the domain of first revolutions and serving as a barrier for ions that are "in the tail" of particle distribution versus the accelerating RF field phases as a result of energy deficit;

—current correction coils needed for formation of an isochronous magnetic field;

—harmonic coils which as a rule are located near the center and extreme radius required for introduction of additional harmonics into the main magnetic field necessary for the beam extraction and correction of lower harmonics of magnetic field errors;

—elements for correcting a negative gradient of the main magnetic field in the particle extraction region (further, gradient correctors);

—electrostatic deflectors and magnetic channels composing the system of ion extraction from the cyclotron.

All structural elements existing in accelerator facility must be included in the composition of its computer model. Each element is represented in a three-dimensional (in some cases, in two-dimensional) field map obtained by means of calculation or from measurements [10]. In the case of fields having different kinds of symmetry, it is necessary to import only the periodic part of the field map for saving machine mem-

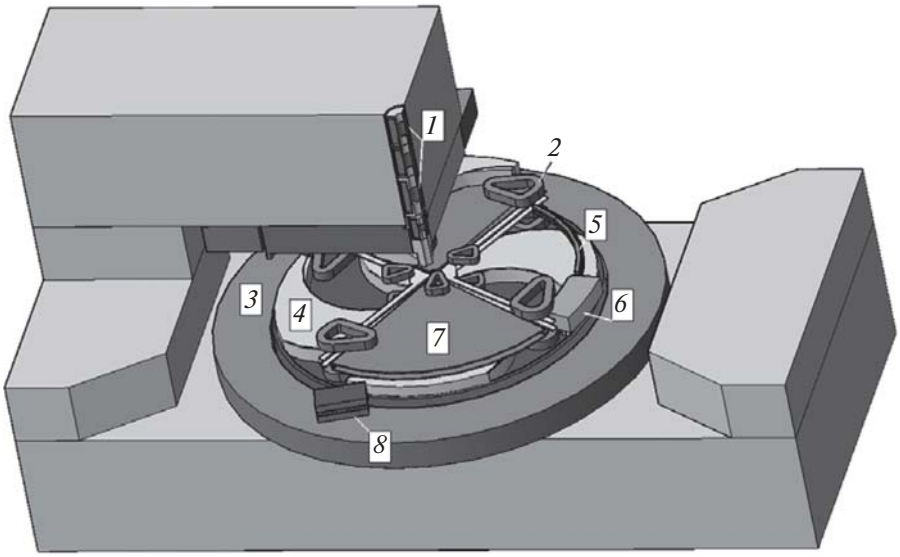

Fig. 1. Computer model of a compact cyclotron: (*1*) solenoids, (*2*) harmonic coils, (*3*) main winding, (*4*) sector forming magnetic field, (*5*) electrostatic deflector, (*6*) magnetic channel, (*7*) accelerating dees, (*8*) corrector of magnetic field decline.

ory. For example, in the case of a solenoid, it is sufficient to have a magnetic field map in coordinates r, z, where the axis of the solenoid coincides with z axis and r is the coordinate along the radius of the solenoid. If only the distribution of the main magnet field is available in the median plane, then a three-dimensional field map of the magnet can be reconstructed using the corresponding procedures [11].

In order to take particle losses on the surfaces of a structural element into account, it is necessary to introduce a description of its geometry into the computational program either by analytic surfaces or by exporting the corresponding geometry from a program used for 3D graphics.

Figure 1 shows an example of a computer model of a compact cyclotron.

An important feature of the model is the possibility of adjusting the parameters of the acceleration regime: current in coils, voltage amplitudes on electrical elements, etc. There also must be the possibility of adjusting the positions of structural elements if such a feature is included in the facility design.

The next element of a computer model is the object being modeled, the beam of ions, which is replaced with a distribution of macroparticles describing its physical characteristics: emittance, velocity, charge, intensity, etc. Naturally, the availability of software packages with the corresponding mathematical support capable of modeling beam dynamics in superposition of electromagnetic fields with allowance for such peculiarities as beam space charge is important. The possibility of modeling diagnostic systems such as phase sensors and measurement probes of beam current allows one to calibrate the created model and make sure that it corresponds to the actual facility. Such a procedure must be performed in order to have confidence in the data obtained by means of computation.

In this way, in the presence of elements described above, a "copy" of the actual accelerator facility is developed on a computer with the differences that the parameters of the acceleration regime are controlled by clicking a mouse button instead of using the cyclotron control desk. At the same time, it is essential that computations be performed in an idealized system which is free from inaccurate constructions, noises, and other uncertainties. However, the created maximally realistic computer model is a reliable tool for designing an accelerator.

Software which is an inseparable part of the computer model must be user-friendly and flexible with optimal consumption of PC resources and visualize the results obtained since it is responsible for simulation of physical processes [3].

3. COMPUTATION OF ELECTROMAGNETIC FIELDS OF STRUCTURAL ELEMENTS

In order to achieve maximal realism of the performed computations of particle dynamics, they must be carried out using the three-dimensional distribution of electric and magnetic fields of structural elements of the accelerator facility. If available, measured field maps can also be used. However, the most probable and widespread case involves computation of electromagnetic fields using one of numerous professional programs, such as Opera-Tosca [12], Mermaid [13], KOMPOT [14], and Poisson-Superfish [15], which allow computing electromagnetic fields in a static approximation. In computation of the radio-fre-

quency field of accelerating dees, it is necessary to consider the fact that the voltage amplitude in the accelerating gap may have a definite dependence on the radius, the consideration of which is required in some cases in modeling acceleration and extraction zones. Radio-frequency acceleration systems can be modeled using one of the specialized programs such as CST Studio Suite [16].

The first stage of the work in computing electromagnetic fields using the listed programs is developing a three-dimensional model of the system. This problem is solved by means of a detailed description of the geometric shape of the elements of the construction. The next stage is selection of grid size and distance of domain boundaries for setting the corresponding boundary conditions and splitting the main domain into subdomains where the most common changes in geometry of the problem will be introduced and the like operations. At splitting of the computational domain, it is necessary to reduce the grid size in places with a complicated configuration and small dimensions of the geometric structure. Thus, in computation of the magnetic field of the main magnet, it is necessary to pay special attention to the domain of the central ring shim. It is appropriate to create two separate models for the central zone containing tips and for the acceleration zone for a model of accelerating dees. In first case, the finest size of the finite element grid is set in the location of the first accelerating gaps. For computation of the central zone containing an internal source of ions, one more domain is distinguished between the source slit and extraction electrode. One should consider that the external surface of the plasma which fills the source of ions is located under the source potential; therefore, it is necessary to introduce "imaginary" filling of its internal part with a given source potential into the program for computing the electric field when modeling a source [17]. It is appropriate to increase the amount of elements in the domain of accelerating gaps and decrease their amount inside of the dee in the model for computing the electric field in the acceleration zone. The use of linear and quadratic elements for computing the field for magnetostatic problems provides different results. Using quadratic elements leads to a smoother field; however, the time spent for one computation increases significantly. Nowadays, computations of electromagnetic fields on computers are so precise that often there is no need for constructing full-scale models. At the same time, results of analysis of beam dynamics performed in computed fields can be considered as reliable with a high accuracy.

4. INTEGRATION OF EQUATIONS OF MOTION

Since an accelerator facility includes an axial injection line and transportation line of the extracted beam in addition to the cyclotron itself, it is appropriate to integrate equations of motion in the Cartesian coordinate system for universality of the computations. The motion of a particle with charge e in an electric field with strength $\vec{E}$ and applied magnetic field with induction $\vec{B}$ is described by the Newton–Lorentz equation [18]

$$\frac{d}{dt}(m\vec{v}) = e(\vec{E} + [\vec{v} \times \vec{B}]), \quad (1)$$

where right part represents the Lorentz force. Here, m is the mass, e is the charge, and $\vec{v}$ is the velocity of the particle.

Differentiating Eq. (1) and considering that the particle is relativistic, we obtain the equations of motion:

$$\ddot{x} = \frac{e}{m}\sqrt{1-\beta^2}$$

$$\times\left[+\left(1-\frac{v_x^2}{c^2}\right)Fx - \frac{v_x v_y}{c^2}Fy - \frac{v_x v_z}{c^2}Fz\right],$$

$$\ddot{y} = \frac{e}{m}\sqrt{1-\beta^2}$$

$$\times\left[-\frac{v_x v_y}{c^2}Fx + \left(1-\frac{v_y^2}{c^2}\right)Fy - \frac{v_x v_z}{c^2}Fz\right],$$

$$\ddot{z} = \frac{e}{m}\sqrt{1-\beta^2}$$

$$\times\left[-\frac{v_x v_z}{c^2}Fx - \frac{v_y v_z}{c^2}Fy + \left(1-\frac{v_z^2}{c^2}\right)Fz\right].$$

where

$$Fx = v_y B_z - v_z B_y + Ex,$$

$$Fy = v_z B_x - v_x B_z + Ey, \quad Fz = v_x B_y - v_y B_x + Ez.$$

Here, v_x, v_y, v_z, E_x, E_y, E_z, and B_x, B_y, B_z are the components of particle velocity, strength of electric field, and induction of magnetic field in the Cartesian coordinate system.

These equations describing particle motion are valid in any electromagnetic field. They were obtained without any simplifying assumptions. The program of analysis of the dynamics of a charged particle can be based on these equations. The integration method can be the Runge–Kutta method of fourth order of accuracy. In the case of using model particles of one type (with identical ratio between charge and mass), time can be selected as an independent variable in the integration method. If, however, the beam consists of different types of ions, then for particle confinement in the beam transport it is necessary to select distance as

an independent variable for taking forces of space charge of the beam into account.

5. CREATION OF MODEL OF MACROPARTICLE BEAM

A single-particle approximation is insufficient for estimating the beam intensity obtained in an accelerator facility; therefore, a computation of macroparticles transport is required. Generation of a beam of model ions with characteristics closest to real ions requires a separate consideration. The use of an external ion source usually provides data on measurements of the transverse emittance of the injected beam and the energy of ions. In such case, it is sufficient to select a method of filling a six-dimensional phase space volume of the beam set by the magnitude of emittances on its projections. In addition to uniform filling, two more basic models of beam distribution are common: Gaussian distribution and Kapchinskii–Vladimirskii (KV) distribution. In first case, the beam volume is filled separately in each of two-dimensional planes—projections of six-dimensional phase space with density

$$I(x, x') = \frac{1}{2\pi\sigma_x\sigma_y}\exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2}+\frac{x'^2}{\sigma_{x'}^2}\right)\right],$$

where σ_x and $\sigma_{x'}$ are the standard deviations of the distribution in the x and x' directions.

The KV distribution has a constant dependence on density of particles inside of the phase ellipse and is set as follows:

$$I(x, x') = \begin{cases} \dfrac{1}{\pi\varepsilon}, & if\ \gamma x^2 + 2\alpha x x' + \beta x'^2 \le \varepsilon, \\ 0, & \text{otherwise}, \end{cases}$$

where α, β, γ, and ε are the Twiss parameters of the ellipse.

Analysis of densities implies that four standard root-mean-square deviations contain 100% of the beam distributed according to the KV law and 86% of particles for the case of a Gaussian distribution. At generation of a longitudinal distribution in the absence of any dependence of longitudinal beam density on time (DC beam), it is sufficient to create a beam with size of 360 degrees of RF and set the boundary conditions of zero forces of beam space charge in the longitudinal direction. In the presence of a dependence of flow of moving particles on time (CW beam), one may generate a beam three bunches long (3 × 360 degrees of RF) and use such a beam during transport and select the internal bunch for analysis in order to obtain a more accurate computation of selffield forces of the beam.

In creating the initial distribution of the beam being injected from the internal source, it is necessary to consider the dependence of the extracted intensity on the amplitude of the extracting voltage, which obeys the Child–Langmuir law [19]:

$$I = \text{const}\sqrt{\frac{q}{m}}\frac{\cos^{3/2}(\theta)}{d^2},$$

where q is the ions charge, m is the mass, and d is the effective distance between the source slit and the extracting electrode, which also takes into account the electric field that is distributed inside of the axial gap of the electrode.

Thus, the initial model distribution of the beam in the slit of the internal source can be shown in the following way (Fig. 2):

—The domains inside of the given ellipses on phase planes (R, Pr) and (Z, Pz) and in configuration space (R, Z) are filled with a uniform distribution.

—The dependence of the amount of model particles on the phase of extracting RF voltage is set in the longitudinal direction.

—Particles of the beam begin moving depending on their initial RF phase.

Qualitative characteristics in analysis of the beam distribution at different points of the accelerator can be computed using the procedure of reconstructing the phase ellipse by the known distribution of particles [20], with the help of which the beam emittance can be determined by the formula

$$\varepsilon = \sqrt{\langle x^2\rangle\cdot\langle x'^2\rangle - \langle x\cdot x'\rangle^2},$$

where x and x' are the coordinates of a particle in the phase plane (x, x'). The Twiss parameters of the beam are computed from the emittance:

$$\alpha = -\frac{\langle x\cdot x'\rangle}{\varepsilon}, \quad \beta = \frac{\langle x^2\rangle}{\varepsilon}, \quad \gamma = \frac{\langle x'^2\rangle}{\varepsilon}.$$

6. FORMATION OF CONTROL MAGNETIC FIELD AND PHASE STABILITY

It is appropriate to start designing a cyclotron from selecting the level of the magnetic field. As a rule, for contemporary accelerators, one should rely on minimization of the dimensions and weight of the accelerator. Therefore, selection of level and shape of the magnetic field is based on approaching the limits of technical capabilities or physical limits of stability of particle motion. Thus, in creation of "warm" machines (working at room temperature), the field is chosen to be as high as possible in view of achievable current density in the connecting lead. Superconducting accelerators are characterized by another type of restrictors. The most prominent restriction in working with high magnetic fields (>3–4 T) is connected with the difficulty of creating an accelerator with a radially growing magnetic field, which provides stable axial motion of particles. Such a restriction is connected with the fact that

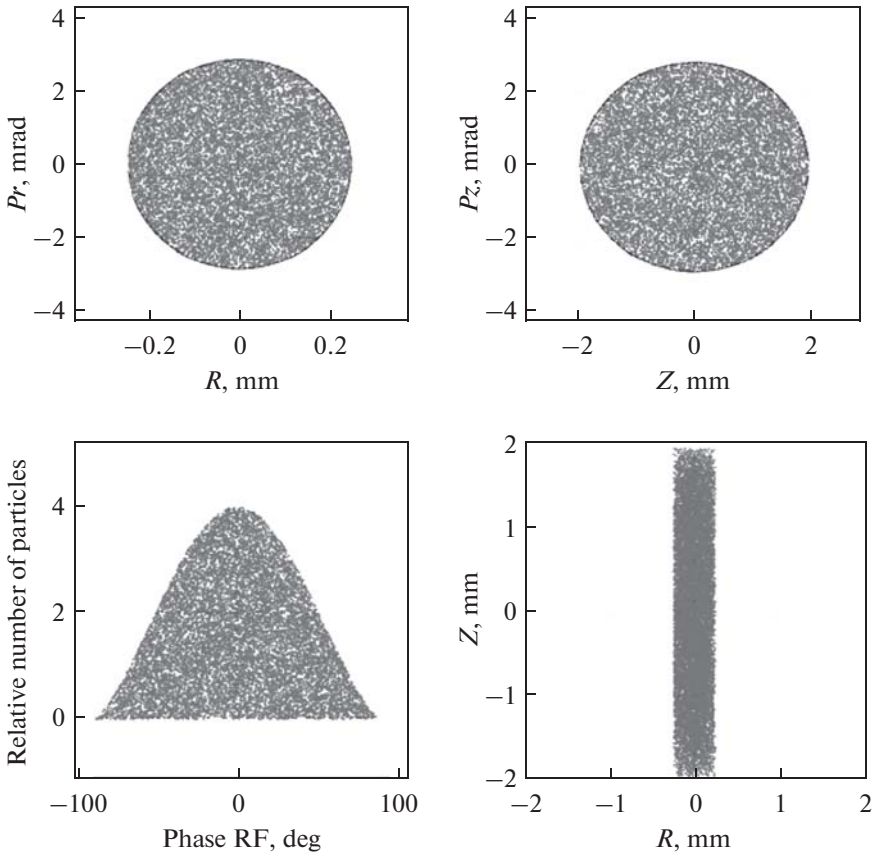

Fig. 2. Initial distribution of particles on slit of internal source.

the magnitude of magnetic flutter turns out to be insufficient for compensating the field sweep rate. Partially, the problem can be solved using spiral shims with a large spiral angle (greater than 60°) for creating spatial variation of the magnetic field. If one omits effects connected with nonlinearity of the magnetic field, compact isochronous cyclotrons with the field level in the center up to 4.5 T when using spiral shims up to 75° are realizable in practice [21]. As a rule, an Archimedes spiral is selected as the dependence of spiral angle on radius where the spiral angle grows from the center to an extreme radius. However, there are cases where smallness or absence of axial focusing on the initial radii becomes the critical factor. In such cases, it is appropriate to use a logarithmic dependence of spiral angle on radius.

The periodicity of the magnetic structure is selected on the basis of a rule in which the magnitude of magnetic flutter decreases with growth of the number of structure periods. So at low output energies of particles, the field is generally formed by means of three sector shims when using a strong magnetic field. Creation of a cyclotron for high energies faces restrictions related to crossing of dangerous resonances, primarily structural resonance $2Qr = N$, where N is the number of structure periods. At the same time, increasing the period number allows advancing into domain of high energies.

Using external axial beam injection is an additional restriction on the upper limit of the magnetic field magnitude in the center, which is related to the difficulty of injection of particles into the accelerator with the help of a spiral electrostatic inflector. In the case of the presence of an overly strong magnetic field, it seems impossible to perform the first revolution in the magnetic field without losing the beam completely. Exotic methods of injecting the beam using a spiral inflector [22] allow obtaining fields of 4–4.5 T.

At the initial stage of choosing the magnetic structure, it is appropriate to use the methodology based on analytical expressions and described in [23], where it was shown that variation of the amplitude of the magnetic field of a system of spiral shims is quite well described by an expression obtained from consideration of a field from an infinite system of straight shims:

$$H_{mN}(r) = \frac{8M}{m} \cdot \sin\left(mN\frac{\alpha}{2}\right)\left(e^{-m\frac{h_1}{\lambda_{eff}(r)}} - e^{-m\frac{h_2}{\lambda_{eff}(r)}}\right), \quad (2)$$

where $4\pi M$ is the magnetization of shims in the state of full saturation, N is the periodicity of the magnetic

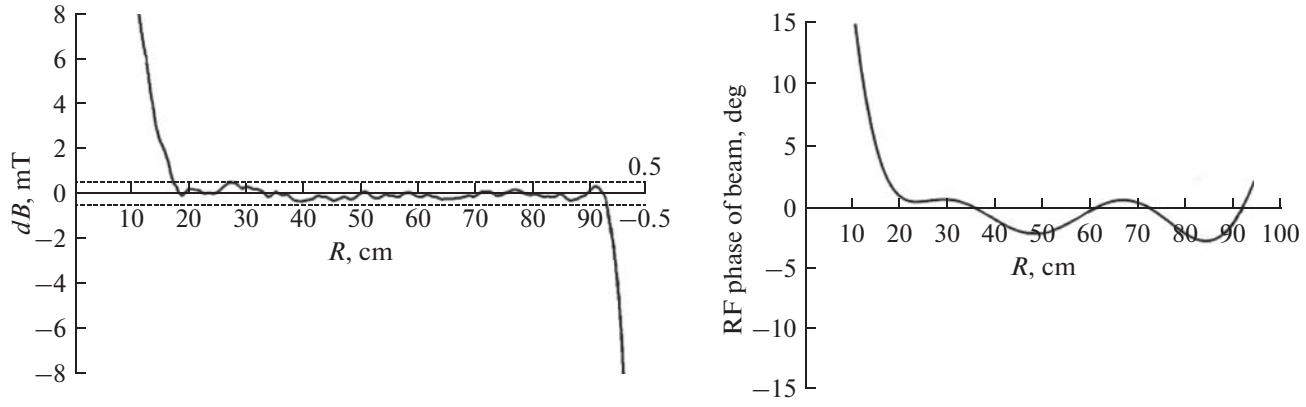

Fig. 3. Deviation of mean field from isochronous field (left) and phase motion of central part of beam in such field (right).

structure, h_1 and h_2 are the distances to the surfaces limiting shims in the axial direction, α is the azimuth dimension of a spiral shim (angle between lateral generator lines), $\lambda_{eff}(r) = \frac{\lambda(r)}{\sqrt{1+\left(\frac{N\lambda(r)}{r}\right)^2}}$, $\lambda(r) = \frac{r}{N\cdot\tan(\xi)}$, ξ is the angle between the tangent to a spiral and radius, and $m = 1, 2, 3, \ldots$.

Knowing the amplitude of variation of the magnetic field obtained in (2), one can determine the depth of magnetic field variation (flutter):

$$F(r) = \sum_m \frac{1}{2}\left(\frac{H_{mN}}{B(r)}\right)^2,$$

where $B(r)$ is the mean magnetic field.

The shape of the magnetic field is selected so as to provide stability of phase motion of particles. For cyclotrons designed for accelerating ions to energy higher than 16–20 MeV/nucleon, it is appropriate to form a mean magnetic field with deviation from an isochronous field no greater than $(5–10) \times 10^{-4}$ in order to provide a high-quality beam at extreme radius. The magnetic field is formed either by means of changing the azimuthal size of shims or by means of changing the axial gap between shims or poles. In cyclotrons with several acceleration regimes, correcting coils are included for formation of an isochronous field. Such coils can be located in magnet valleys, above sectors, and between sectors and pole tips in the form of rings of definite radius. Sometimes, coils are installed around sectors. Usually, correcting coils are not provided in cyclotron projects for acceleration of one type of ions to a fixed energy. Computation of particle dynamics requires changing the acceleration regime with tuning of the leading magnetic field. Setting of required magnetic fields is performed using currents of the main and correcting coils. Therefore, during construction of a computer model of a cyclotron, it is necessary to compute also impacts of correcting coils for different levels of the main magnetic field along with the field map of "iron" (computation of the magnetic field with only the main winding powered). Acquisition of the distributions of fields is followed by comparison of the computed and measured data if the latter exist. Incomplete correspondence of the computer model of a cyclotron to a real existing facility, insufficiency of data, and uncertainty in curves of magnetization of elements usually lead to discrepancy between computed and measured data. This difference can be minimized using so-called calibration coefficients to the field level created by coils, which usually constitute ~1–2% of the current magnitude.

After calibrating the field levels from coils and obtaining sufficient agreement between measured and computed field maps, one may use experimental current values for modeling a specific acceleration regime. In such case, the dynamics of particles is computed in a magnetic field corresponding to the field in experiment with fairly good accuracy.

The isochronism of the magnetic field can be demonstrated with the help of the dependence of the RF phase at passing of particles through acceleration gaps (or central line of accelerating dees) on the radius. Comparison of the computed phase obtained by the transport of particles in the considered magnetic field with values of phase probes which are usually installed in all cyclotrons shows correspondence of the created model to a real facility. In practice, it is impossible to achieve precise coincidence of real and isochronous fields. Therefore, deviation of phases from the given value takes place. For example, if the field for a typical "warm" compact proton cyclotron for energy of 30 MeV [8] is formed so that deviation from the isochronous field at all radii is not greater than 5 G, then this corresponds to deviation of the RF phase of the beam not greater than 5 degrees (Fig. 3).

The absence of flutter in the central part of the cyclotron is compensated by creation of a magnetic

field bump in this part, which leads to two main positive effects. Firstly, the mean magnetic field becomes decreasing with respect to radius, which results in stable axial motion of particles. Secondly, a magnetic field higher than the isochronous one leads to displacement of the beam phase in the negative direction. Thus, particles "pass" the first acceleration gaps at a positive RF phase, later being shifted to zero, which leads to additional focusing of particles in the axial direction at the first revolutions by the radio-frequency electric field. Depending on the configuration of the central domain of the cyclotron, the level of the magnetic field in the center rises to a magnitude from several tens of gauss to several hundred gauss.

In the case of designing an isochronous cyclotron for small energy, it is possible to form a mean magnetic field that is quasi-isochronous or even decreases with respect to radius. Since the output energy is small, the RF phase of the beam does not manage to go beyond the limits of the domain of acceleration. First of all, the maximum energy which can be obtained in such an accelerator depends on the rate of particle energy gain per revolution. So in the presence of one accelerating dee and voltage of 70 kV, protons in a decreasing field can be accelerated to the energy of 9 MeV [24]. At acceleration in strong magnetic fields with lack of variation of the magnetic field for axial focusing of particles, it is possible to create an increasing mean magnetic field with a sweep rate less than that of the isochronous field. The magnitude of growth of the field is chosen so as to provide axial focusing and to prevent drift of the beam RF phase beyond the limits of the domain of acceleration.

The RF phase during the acceleration process can be computed analytically using the following expression [27]:

$$\sin\phi(E) = \sin\phi_i + \frac{2\pi h}{qV}\int\left(\frac{\omega_i}{\omega} - 1\right)dE,$$

where ϕ_i is the starting phase, h is the harmonic of the accelerating field, q is the charge of particle, V is the amplitude of the accelerating voltage, and ω is the revolution frequency of a particle in the magnetic field.

The revolution frequency of particles can be computed from a linear approximation of the mean radius of equilibrium orbit [25]:

$$r = R - \frac{\varepsilon^2 R}{2(N^2 - 1 - n)(1 + n)} \times \left(2 - \frac{N^2}{2(N^2 - 1 - n)} + n + R\frac{\varepsilon'}{\varepsilon}\right),$$

where R is the energy radius of particle; $n = -\frac{dB}{dR}\cdot\frac{R}{B}$ is the sweep rate of the magnetic field; and ε is the depth of variation of the magnetic field, i.e., the ratio between the amplitude of the Nth harmonic of the magnetic field and the mean value of the field at a given radius.

The revolution period of of a particle in a closed orbit is determined by the expression

$$T = 2\pi r + \frac{R_a}{N}[\sin(N(2\pi - \tan(\alpha))) + \sin(N\tan(\alpha))],$$

$$R_a(r) = \frac{r\varepsilon}{N^2 - 1 - n}.$$

Here, α is the magnetic spirality, which differs from the geometric spirality determined by sector shims and equal to the angle between the radius vector and the tangent to the spiral. The unknown quantity can be computed from harmonic analysis of the magnetic field distribution.

7. ORBITAL STABILITY AND CENTERING

The frequencies of betatron oscillations can be computed using the following approximate analytical expressions [26]:

$$Q_z = \sqrt{n + F(1 + 2\cdot\tan^2\xi)\cdot\frac{N^2}{N^2 - 1}},$$

$$Q_r = \sqrt{1 - n + F(1 + \tan^2\xi)\cdot\frac{3N^2}{(N^2 - 1)(N^2 - 4)}},$$

where $F = \dfrac{\langle B(\theta)^2\rangle - \langle B(\theta)\rangle^2}{\langle B(\theta)\rangle^2}$.

Another way to estimate the frequencies is using an approach based on particle tracing, which can be implemented with the help of a well-known algorithm in the Cyclops program [27]. Use of the described methods provides similar results up to extreme radii, where some difference takes place. This is due to the fact that orbits of particles partially get into the domain of the edge magnetic field of the facility, which leads to change in the effective field gradient in orbits and correspondingly to change in dynamic characteristics of orbits.

Alternatively, the frequencies of betatron oscillations can be computed using a method which follows the classical definition as the number of oscillations around an equilibrium orbit per revolution performed by a particle in a magnetic field. According to this, it is sufficient to find an equilibrium orbit using any program for tracing particles. Then a small deviation from the starting coordinates of an equilibrium orbit with respect to radius (axial coordinate) and radial (axial) angle is set and an additional trajectory is computed by tracing of the particle. Then for computing, say, a radial betatron frequency, it is sufficient to subtract the

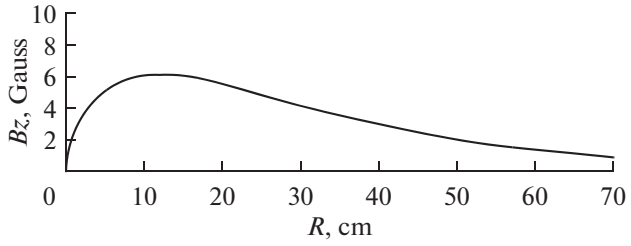

Fig. 4. Dependence of amplitude of first harmonic of magnetic field on radius.

coordinates of the equilibrium orbit from the coordinates of the orbit with initial deviation with respect to radius and compute the number of obtained oscillations. Frequency Qz is determined in the same way. In order to decrease the computational errors in determining the frequencies using the given method, it is desirable in computing the orbits to perform several dozen revolutions in the magnetic field, by the number of which one should divide the number of betatron oscillations. Such an approach is advantageous by the fact that it automatically takes nonlinearity of the magnetic field into account.

The final goal of computing the frequencies of betatron oscillations is analysis of the motion of the working point on the operation diagram. The magnetic system is designed in such a way that the working point would be located far from lines of dangerous resonances. The most dangerous resonances include $Qr = 1$ and $2Qz = 1$. The first one is often crossed at the beginning of acceleration in the domain of first revolutions. This resonance is excited by the first harmonic of the magnetic field and causes distortion of an accelerated orbit. Its analysis and formulation of requirements for maximum amplitude are performed by computing the beam dynamics in the presence of the first harmonic of the magnetic field. The first harmonic is set analytically and may have a dependence like the one shown in Fig. 4, which mainly reflects typical behavior of the first harmonic amplitude in the central zone, which is usually observable in measurements of the magnetic field. The amplitude of the harmonic maximum usually varies from 0 to 10–15 G, which corresponds to measurements of first harmonic level in a compact cyclotron, such as in [28]. The phase of the harmonic maximum is set to a constant along a preferred azimuth.

There are several methods to compute the amplitudes of radial oscillations during the acceleration process. Thus, the numerical value of amplitudes can be calculated as follows [29]. Upon intersection of the selected azimuth by a model particle during the acceleration process, the memory array is filled with coordinates of the intersection point and particle velocity. Then each such record becomes the starting data for computing the trajectory of the particle. At the same time, the accelerating voltage is neglected and the particle starting from the accelerated orbit performs revolutions in the magnetic field without increasing its energy in this case. The particle trajectory traces out an ellipse in space (R; Pr), whose semiaxis defines the value of the amplitude of radial oscillations of the particle. For cyclotrons without azimuthal variation of the magnetic field, it is more convenient to use a method based on computing the accelerated orbit of instantaneous circles to which the trajectory is tangent at each point during the acceleration process. For a centered orbit, centers of such circles will coincide with center of the coordinate system of the accelerator. For trajectories of particles having radial oscillations, the distance from the center of the cyclotron coordinate system to the center of the obtained circle is exactly the value of the oscillation amplitude.

Centering is an important parameter influencing the final dimensions of the beam. The centering value is directly related to the amplitudes of coherent radial oscillations of particles. There are numerous factors defining the level of centering of orbits in a cyclotron. The most important factor is construction of the central zone of the cyclotron, which defines the level of centering of an accelerated orbit. In the case of using an internal source of ions, the locations of the first accelerating gaps and the voltage on accelerating electrodes also influence the value of orbit centering besides the location of the source. The spiral inflector, which transfers the beam of particles from the vertical (in some cases horizontal) plane to the horizontal (vertical) plane, defines the parameters of trajectory of a particle being injected when using external injection of ions into the cyclotron. Precisely the construction of the inflector at the initial stage mostly influences the deviation of the particle trajectory from the centered orbit. The value of centering can be varied by the corresponding adjustment of the inflector geometry and voltage on its plates [30]. The value of the RF phase at which the particle passes the first accelerating gaps also directly influences the value of centering. The main determinative factor here is the geometry of the central part of the accelerating electrodes.

The optimal structure of the central domain can be determined using so-called “backward tracking” [31]. The method involves the determination of the ideal centered orbit at an extreme radius from which the particle starts. The voltage on the accelerating electrodes changes sign and the particle arrives at the central zone of the accelerator with decreasing energy. This is how the slowing down process is modeled for particle with a well-centered accelerated orbit. After that, having the trajectory of the particle inlet to the central domain, one can select such a geometry of the accelerating electrodes in the center and such parameters of the inflector (position of internal source) as to provide agreement of the injected particle trajectory with the centered one determined by the described procedure.

Variation of the voltages on the accelerating electrodes (dees) is an effective method to control the value of centering during acceleration. Introducing some asymmetry of voltage amplitudes on dees allows decreasing the amplitudes of radial oscillations of particles by many times.

Harmonic coils intended for increasing the output efficiency also can be used as a tool for improving centering. Coils located in the beam injection domain in the central part of the cyclotron are especially effective in this view. Introduction of harmonics of the magnetic field (usually first one) in the center of the accelerator impacts the value of coherent amplitudes of radial oscillations of particles during the acceleration process. The output efficiency can be increased by using such a setting of harmonic coils at which amplitudes of coherent radial oscillations increase. At the same time, overlapping areas of neighboring accelerated orbits appear, the RF phase is shifted, and an increase in the number of revolutions performed by particles to achieve the given nominal energy is observed. But the essence of the method lies in the fact that, in such a case, areas with high sparseness of orbits occur along with the regions of overlapping orbits. And the beam output coefficient can be increased by selection of the parameters of the acceleration regime so that an area with a large distance between neighboring orbits is found in the place of location of extraction system elements.

8. COMPUTATION OF SPACE CHARGE OF BEAM

After one sets the parameters of the acceleration regime on the basis of modeling of the injection, acceleration, and extraction of a central particle, the next stage is to analyze the motion of the particle beam. The single-particle approximation, which takes only external fields into account, is insufficient to compute the beam dynamics in modern accelerators. The effects of space charge of the accelerated beam must also be taken into account.

Overly large computational expenses for consideration of space charge forces by direct computation of forces acting on particles (particle-to-particle method) led to the appearance of a number of models used for numerical computations of the selffield of the beam. Each of such models has its disadvantages and computational accuracy is restricted by the requirement of an acceptable CPU time of computing the space charge effect. The most common numerical model is particle-in-cell [32]. This method uses the formalism of close-range interaction and field equations for the potential. As a result, the force is computed faster, but is usually less precise than in the particle-to-particle method. The field values determined in the whole space of the physical system are approximately represented as values at regularly located nodes of a grid which covers the domain occupied by particles. Differential operators such as the Laplacian operator ∇^2 are replaced by finite difference approximations on this grid. The potentials and forces at the location of the particle are computed by means of interpolation by an array of grid nodes. The grid densities are determined using the inverse procedure of distribution of particle characteristics (such as charge) to the nearest grid nodes in order to obtain grid values (such as charge density). The force is computed in three steps:

1. Distribution of charge on grid.

2. Solution of the Poisson equation on this grid.

3. Computation of forces by the grid potential and interpolation of forces at the point of location of particles.

Particle-in-cell methods are characterized by the fact that evolution of a system of particles at each time step is split into two stages. At one of the stages, firstly, the interaction of particles is computed at their fixed positions. Computation is performed on a stationary ("Euler") grid. Thus, this stage is called the Euler stage. The other, Lagrange, stage is devoted to integration on the next time step of the dynamic system whose right part is computed at the Euler stage.

Given the potentials on boundaries and distribution of charge density inside of the computational domain (space covered by the computational model), Poisson equation (1) completely defines the potential:

$$\nabla^2\varphi = -\rho/\varepsilon_0, \tag{3}$$

where φ is the potential and ρ is the charge density.

In order to represent Eq. (3) in a form convenient for numerical solution, it is necessary to apply finite differences. The derivative in the one-dimensional case is defined by the expression

$$\frac{d}{dx}f(x) = \lim_{h\to 0}\left\{\frac{f(x+h/2)-f(x-h/2)}{h}\right\}. \tag{4}$$

In the finite difference approximation, h does not approach zero, but takes some finite value which is chosen on the basis of sufficient precision and appropriate CPU time. The charge density, potential, and electric fields are represented by a set of values located with constant step h in the whole computational domain. The points where the values are written represent grid nodes.

As a result, the finite difference equations for the field and voltage have the form

$$\frac{\varphi_{p+1}-2\varphi+\varphi_{p-1}}{h^2} = -\frac{\rho_p}{\varepsilon_0}, \quad E_p = \frac{\varphi_{p-1}-\varphi_{p+1}}{2h}. \tag{5}$$

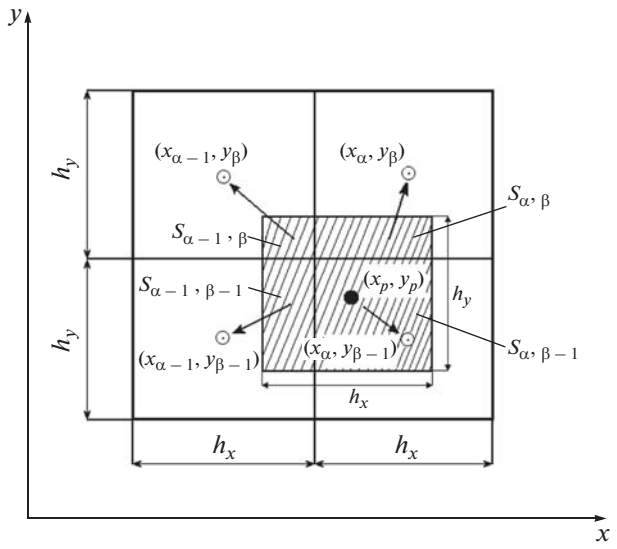

Fig. 5. Initial kernel of PIC model. Interpolation scheme of transferable factor to Euler grid.

Assume that the one-dimensional analog of charge density is defined as

$$\rho(x) = \sum_{j=1}^{N_p} q_j R(x - x_j), \tag{6}$$

where q is the charge of particle and summation is performed over all particles within computational domain.

A simple example of a one-dimensional kernel is

$$R(x) = \begin{cases} \dfrac{1}{2\Delta}, & |x| \le \Delta, \\ 0, & |x| > \Delta. \end{cases} \tag{7}$$

The kernel in the form (7) describes a particle with width of 2Δ with uniform distribution of density. In this case, the density obtained by Eq. (6) will be a piecewise constant function. At $\Delta = h/2$, this kernel corresponds to a one-dimensional PIC model.

The values of ρ_α at grid nodes are determined by the formula

$$\rho_\alpha = \sum_{j=1}^{N_p} q_j \bar{R}(x_\alpha - x_j), \tag{8}$$

where the grid kernel is

$$\bar{R}(x_\alpha - x_j) = \frac{1}{h} \int_{x_\alpha - \frac{h}{2}}^{x_\alpha + \frac{h}{2}} R(x - x_j) dx.$$

For a point particle with singular kernel $R(x - x_p) = \delta(x - x_p)$, where x_p is the position of the particle, we obtain

$$\bar{R}(x_\alpha - x_p) = \frac{1}{h} \int_{x_\alpha - \frac{h}{2}}^{x_\alpha + \frac{h}{2}} \delta(x - x_p)$$

$$= \frac{1}{h} \int_{x_\alpha - x_p - \frac{h}{2}}^{x_\alpha - x_p + \frac{h}{2}} \delta(y) dy = \begin{cases} \dfrac{1}{h}, & |x_\alpha - x_p| \le \dfrac{h}{2}, \\ 0, & |x_\alpha - x_p| > \dfrac{h}{2}. \end{cases} \tag{9}$$

This kernel represents a one-dimensional NGP model (nearest grid point model).

In formula (8), summation is performed over all particles. However, the contribution to the grid density at a node is made only by particles located in some neighborhood of the node. In the PIC model, the contribution to the given node is made by particles spaced less than h from it. That is, each point may impact the two nearest nodes.

In the two-dimensional case, the particle in the PIC model will take a rectangular shape with sides h_x and h_y parallel to the coordinate axes (x, y) (Fig. 5). Its Lebegue measure is area $s_p = h_x h_y$, where the factor carried by the particle is distributed uniformly with density of $\rho_p = q_p/s_p$.

The corresponding grid kernel similar to kernel (9) is written as

$$\bar{R}(x_\alpha - x_p, y_\beta - y_p)$$

$$= \begin{cases} \dfrac{1}{h_x h_y}\left(1 - \dfrac{|x_\alpha - x_p|}{h_x}\right)\left(1 - \dfrac{|y_\beta - y_p|}{h_y}\right), \\ |x_\alpha - x_p| \le h_x \quad |y_\beta - y_p| \le h_y \\ 0, \quad |x_\alpha - x_p| > h_x \quad |y_\beta - y_p| > h_y. \end{cases}$$

In the two-dimensional PIC model, the density of each particle is distributed using inverse bilinear interpolation between the four nearest grid nodes.

Solution of the Poisson equation for computing generalized fields at the Euler stage can be performed using the fast Fourier transform (FFT) [33]. The solution algorithm is as follows: first of all, the fast Fourier analysis (FFA) of the right part of the Poisson equation is performed; then, using an analytical expression, the transition from Fourier coefficients of the right part to coefficients of the left part is carried out; and finally, the fast Fourier synthesis (FFS) is performed for the left part. This solution method constrains the possible number of grid nodes, which must be equal to 2^n, $n \in N$.

Upon the necessity of modeling in domains with complicated geometry of boundaries or with large local gradients of generalized fields where loss of accu-

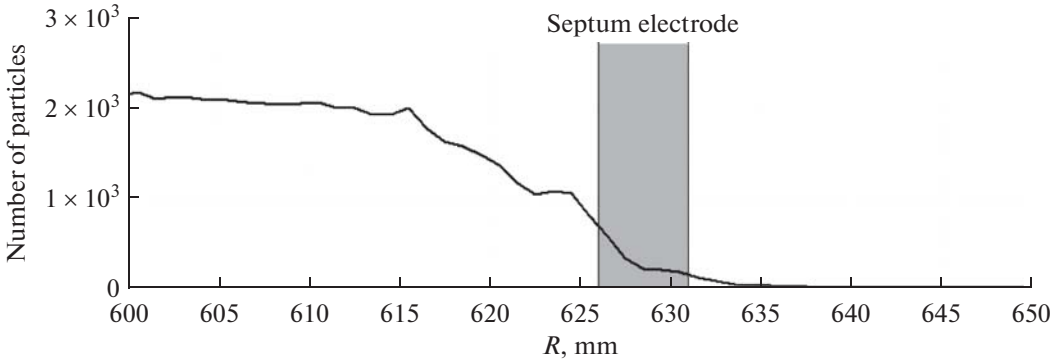

Fig. 6. Choice of position of electrostatic deflector septum on the basis of values of differential probe.

racy due to inflexible grid topology may lead to global distortions of the whole picture of the process being modeled, one should use structured nonuniform and unstructured grids designated by the common abbreviation BFC (boundary-fitted coordinates). Therefore, in recent decades, there has been development of different variations of the particle method for numerical modeling of processes in domains of complex shape based on using grids adequately describing the boundaries of the modeled domain. An approach based on nonstructured grids does not restrict the geometry of boundaries or connectivity of the computational domain. Generation of such grids is easily automated, which allows dynamically adapting them to local peculiarities of the solution. Over the last years, numerous publications [34, 35] have already been devoted to developing algorithms of the particle method on nonstructured grids.

The basic structural elements in the plane case are triangular elements obtained using some triangulation of the computational domain, and in the three-dimensional case, tetrahedrons. At the same time, grid nodes are represented by vertices of triangles or tetrahedrons. Each finite element is defined by a set of basic functions (shape functions) whose number is equal to the number of nodes (vertices) of the element.

Tests and computations show that, in compact cyclotrons even at a relatively low number of revolutions performed by particles during acceleration, the influence of space charge affects the characteristics of the extracted beam at currents of the injected beam of several hundred microamperes. Thus, modeling of beam dynamics taking forces of the particle self-field into account makes computations more realistic.

9. MODELING THE EXTRACTION SYSTEM

The accelerated beam can be directed to an internal target or to a stripping foil located in the vacuum chamber of the accelerator or extracted to an external target. In first case, when using liquid or gaseous targets, it is sufficient to create an additional separation between neighboring orbits in the domain of location of the target. Beam deflection to the target is defined such that losses of particles on the target wall restricting the body of the target from the side of the circulating beam are acceptable. In the case of using an external target, an extraction system usually consisting of electrostatic deflectors and magnetic channels is required. Additional gradient correctors and magnetic or electrostatic quadrupoles can be used for focusing the beam at its passage of the edge field of the magnet. There are several ways to organize a sufficient beam deflection to the inlet of the extraction system [36]. All of them use the method of creating coherent oscillations of particles near a finite radius. The first of them is increasing the radial amplitudes by means of adding the first harmonic of the magnetic field near the crossing of resonance $Qr = 1$. This causes creation of separation between the last internal and extracted trajectories of particles. If the first harmonic is added directly in the crossing domain and after that the particle still continues accelerating, moving in the edge magnetic field, then the frequency of Qr decreases significantly (usually to 0.6–0.8). As a result, the betatron phase between the two last revolutions differs considerably from 360 degrees and separation between orbits is proportional to $Qr - 1$. Naturally, at the same time, the particle must perform a small number of revolutions in the edge field. Such a method is called a precessional method. The regenerative method of orbit oscillation is based on excitation of resonance $2Qr = 2$. Such a resonance is controlled by the second harmonic of the field. The method is quite complicated as compared with those described above and is not widely applied in cyclotrons.

At the initial stage, it is appropriate to use an analytical method of setting the fields of elements of the extraction system. Since the particle trajectory in a small angular azimuthal size is close to an arc, it is possible to use the domain contained in a ring segment for setting the localization domain of the electromagnetic field (Fig. 6). The angular size of such a domain corresponds to the azimuthal size of the extraction element and the difference between the internal and external radii corresponds to the radial size of the element.

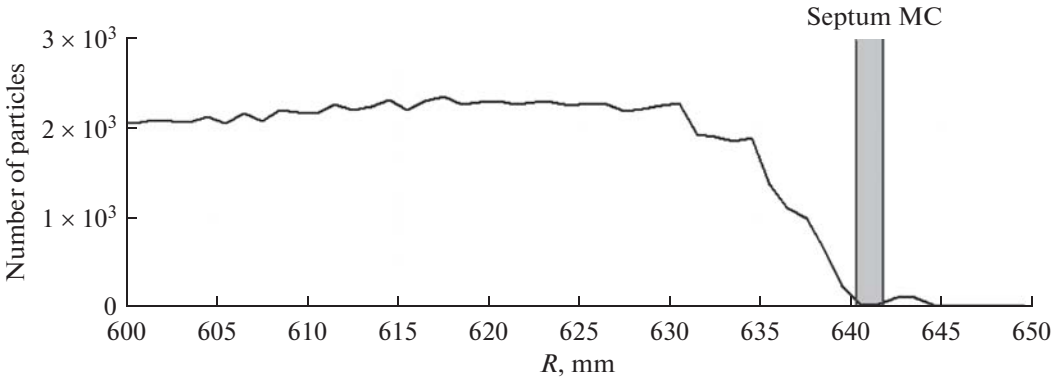

Fig. 7. Choice of position of magnetic channel septum on the basis of values of differential probe.

Such a method of setting the field domain is convenient for the fact that it can be determined by setting only the coordinates of three points which define the internal arc and the radial distance between arcs. The electromagnetic properties of the field can be described by setting only two magnitudes: the voltage on the central line and the gradient of the electric field depending on the radius in the case of electrostatic deflector; the magnetic field on the central line and the gradient of the field along radius in the case of the magnetic channel.

In order to perform a realistic computation, the program analyzing the particle dynamics must provide the possibility of using three-dimensional fields of elements of the extraction system. Choice of the geometric structure of an element is based on the required characteristics of electromagnetic fields obtained at stage of computing with the help of analytics. At the initial stage of selection of the structure of magnetic elements, one may use the methodology described above and based on description of the field using an infinite system of linear shims [23]. After that, the computation is performed using one of the programs for computing three-dimensional electromagnetic fields and the obtained field is exported to the program for computing the dynamics. The latter must include tools for controlling the location and orientation of the extraction element represented by the distribution of the three-dimensional field.

Choice of the location of extraction elements can be based on the coordinates of the trajectory of a central particle, which is determined in advance and around which the main mass of particles of the beam is grouped. Sometimes it is convenient to use data obtained in modeling a differential probe, which represents a detector of rectangular cross section with dimensions of several millimeters in a real accelerator facility. Such a probe moves along the radius with a step comparable with its dimensions and registers the intensity of the circulating beam. From the side of external radii, it can be restricted by a thick plate, where accelerated ions die. Modeling of such a probe shows the distribution of intensity of the circulating beam along radius on the selected azimuth. On the basis of these data, one can determine the installation position of the septum of the extraction element (see Fig. 6). At the same time, obviously, radii with the highest sparseness of orbits are characterized by lower values of probe readings. For example, on the azimuth of the input to the passive magnetic channel, located behind the electrostatic deflector, the deflected beam leaves a separate "track" on the curve of the dependence of intensity on radius, which makes it possible to determine the installation position of the septum of the channel (Fig. 7).

10. ESTIMATION OF LOSS OF PARTICLES ON SURFACES OF STRUCTURAL ELEMENTS

In order to determine qualitative and quantitative characteristics of the beam along with computation of forces of space charge of the beam, it is necessary to be able to estimate the losses of particles on the surfaces of structural elements contained in the accelerator facility. Estimation of the total efficiency of beam transmission through the cyclotron is the key issue for designing an accelerator. By transmission of the beam through a structural element, we mean the ratio between the beam intensity at the output and the intensity at the input. Multiplying local transmission through structural nodes of the cyclotron, one can obtain an estimate of the total transmission of the beam through the accelerator. Transmission of a structural node usually means the percentage ratio between the beam intensity at the output of the node and the intensity at its input. From the computational point of view, transmission efficiency is a value defined by the amount of losses of macroparticles taking part in modeling of the dynamics on the surfaces of structural elements.

In computations, a particle is considered as lost if its trajectory intersects the surface of a structural element. The computer model of the accelerator is prepared in such a way that it includes the description of

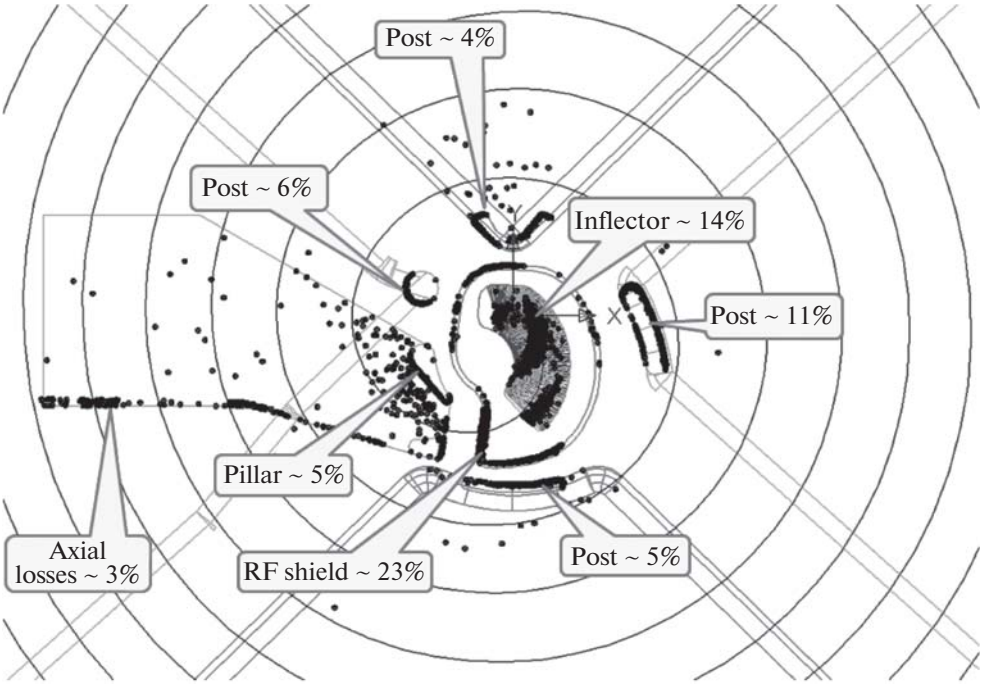

Fig. 8. Graphical representation of distribution of lost particles in central domain of compact cyclotron. Black spheres are lost particles.

all structural elements contained in the system. From the computational point of view, the most widespread and simplest method of defining the intersection of a structural element by a particle is replacement of the element by a set of surfaces described by analytical dependences such as circles, cylinders, sectors, and parallelepipeds. Setting of such surfaces makes it easy to determine whether the trajectory of a macroparticle intersects them. For example, such a description can be applied to solenoids, vacuum chamber of the cyclotron, electrostatic deflectors, etc.

In the case where element geometry has a complicated structure, it is quite difficult or completely impossible to represent it by a set of analytical surfaces. This problem can be solved as follows. It is necessary to represent the surface of a three-dimensional body as a set of triangles whose amount is chosen to be sufficient for the describing geometry of the element. Then intersection of the particle trajectory with some triangle of the surface is checked. At each iteration step of integration of the equations of motion, there are data on the particle coordinates at two sequential moments of time. The trajectory of the particle between these two moments represents a straight line set by the coordinates of two points in space. The problem is reduced to determining the intersection of the line and triangle set in space. Such a problem can be solved in multiple ways, among which one should select the least expensive from the computational point of view. The described method is convenient not only for the possibility of estimating the beam transmission coefficient. Under the assumption that lost macroparticles can be represented as spheres, then domains of beam losses on surfaces of structural elements can be graphically visualized. This requires constructing structural elements of the accelerator and spheres responsible for lost particles in one of the programs for 3D graphics (Fig. 8). This allows visually tracking and optimizing the "problem" zones of the accelerator.

The surface of a three-dimensional body can be easily represented by a set of triangles using commercial software for constructing three-dimensional bodies such as AutoCAD, Solid Edge, and SolidWorks. Structural elements are converted into the well-known format for representing surfaces of three-dimensional bodies *.3ds, which represents a set of triangles describing the surface. It is necessary to remember that the accuracy of estimating the amount of lost particles on the surface of a real element depends on the level of detail of the surface description (number of triangles). At creation of the file of the geometry in the *.3ds format, it is necessary to reach a compromise between the number of triangles for accurate description of the structural element and the time required for determining the possible intersection of the trajectory of a particle with at least one of them.

Information about the number of macroparticles lost during transport and acceleration defines the efficiency of beam transmission through the accelerator facility. In the case of representing structural elements in the form of a sufficient amount of surface triangles,

the transmission can be computed to a high degree of accuracy. Comparison of computed and experimental results of beam intensity at node points of the system allows obtaining information on the correspondence of the created computer model to a real existing accelerator.

11. ESTIMATION OF LOSSES OF PARTICLES IN RESIDUAL GAS

In accelerators which accelerate negatively charged ions such as H^- and D^-, a significant fraction of particle losses is due to charge exchange in the residual gas in the vacuum chamber of the accelerator. This is especially problematic for cyclotrons with an internal source owing to the difficulty of creating a vacuum of sufficient level in the vacuum chamber of such an installation. The amount of ions lost as a result of interaction with the gas depends on the atomic density of the gas, the cross section of ion–gas interaction, the pressure in the chamber, and the length of the path traversed by a particle. The output parameters in the analytical model of gas losses [37] are the variety of gas, the dependence of pressure on spatial coordinate in the vacuum chamber, and information about the trajectory of motion of an accelerated particle (coordinate and velocity). The amount of lost particles can be estimated by the formula

$$dN = \sigma n N v dt, \qquad (10)$$

where N is the initial amount of ions, dt is the time of motion of particle in the gas, n is the atomic density of the gas, v is the velocity of ion motion, and σ is the cross section of ion–gas interaction.

A more realistic estimate of losses requires knowledge of the gas content in the vacuum chamber and the dependence of pressure on spatial coordinate. In the case where such data are absent, the variety of gas can be represented by oxygen as the gas having the largest cross section of interaction with ions and providing an upper estimate, and nitrogen or hydrogen as gases obviously present in the cyclotron chamber. The cross sections of interactions of ions with the gas are known and the dependence of their magnitude on particle energy can be found in [38]. The atomic density of the gas is determined from the expression

$$P = nkT,$$

where P is the gas pressure, V is the gas volume, n is the atomic density, k is the Boltzmann constant, and T is the temperature. The density at the temperature of 300 K can be computed by the formula

$$n[\mathrm{m}^{-3}] = 3.3 \times 10^{22} \cdot P[\mathrm{Torr}].$$

The trajectory of an ion is computed using one of the programs for tracing particles. The gas pressure, which generally has a dependence on the spatial coordinate, can be determined for the computed particle coordinates. Known velocities of an ion make it possible to obtain its energy and consequently determine the cross section of interaction with the residual gas. The amount of losses of ions is determined by Eq. (10) stepwise along the particle trajectory, where the input parameter at each step is represented by the amount of particles at the previous step with further integration of all losses.

12. CONCLUSIONS

The article describes the basic techniques of modeling a compact isochronous cyclotron from setting of the system using tracing of a central particle to studying the motion of a beam of particles. Special attention was paid to the process of translation of theoretical algorithms to machine language. The described methods allow creating a computer model of an accelerator facility which can be used to prepare a design of a new accelerator or adjust parameters of an already existing facility. Using one of the programs for creating three-dimensional bodies of a mechanical model of an accelerator makes it possible to compute electromagnetic fields and estimate losses of particles on the surfaces of structural elements by exporting the geometric structure. In modeling, it is important to pay attention to the main features of the dynamics of particles which influence quantitative and qualitative characteristics of the beam. A correctly created computer model is a powerful and reliable tool for studying the physical processes occurring in an accelerator facility.

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