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PHYSICS AND TECHNIQUE OF ACCELERATORS

Central Region Design in a Compact Cyclotron

V. L. Smirnov*
Joint Institute for Nuclear Research, Dubna, Moscow oblast, 141980 Russia
**e-mail: vsmirnov@jinr.ru*

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Abstract—Methods for designing and optimizing the central region of a compact cyclotron are described. Algorithms for the development of the geometric structure of the center, providing both an effective beam transmission and a high quality of the beam captured for the further acceleration, are given. Methods of computer modeling of the cyclotron central region are considered. The description of most basic elements of the beam control and shaping in modern cyclotrons and the methods for optimizing their parameters are given. Both cyclotrons with internal ion source and with external injection using spiral electrostatic inflector are considered.

Keywords: cyclotron, beam dynamics, computer modeling

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1. INTRODUCTION

The cyclotron central region is crucial from the viewpoint of formation of a beam of particles captured for further acceleration. Its design determines the transverse emittances and the longitudinal distribution of beam particles. Depending on the destination of accelerating installations, the requirements imposed on the beam quality are different. To satisfy these requirements, different elements of the beam formation, namely, phase slits, collimators, deflectors, and focusing elements, are arranged in the cyclotron center.

Optimizing the central region makes it possible to obtain accelerated beams of a higher quality, which is a determining factor in some cases, for example, in installations used in medicine. The design of the cyclotron central region largely determines the amplitudes of radial betatron oscillations of particles. Providing a good beam focusing owing to the efficient design of the central region is the most important problem at the stage of its elaboration. The resulting beam intensity directly depends on the efficiency of its transmission through the cyclotron center.

The design of the cyclotron central region is an integral part of the process of modeling of the cyclotron as a whole. Up to now, numerous compact cyclotrons are designed for different applications and set into operation. The acquired experience makes it possible to single out common methods for the design and optimization of the shape, position and parameters of the first few accelerating gaps, internal ion source, spiral inflector, and elements of beam formation at the first few turns. Most of these methods are described in this work.

2. INITIAL CONDITIONS

To provide maximally accurate calculations of the beam dynamics in the cyclotron central region, it is necessary to correctly specify the initial data. The searching for the charged particle trajectory is based on the integration of the equations of particle motion in the external electromagnetic field. To carry out ion tracing, it is necessary to know the distribution of the electric and magnetic fields of the setup and set the initial parameters of the beam. There is a diversity of programs to calculate the electromagnetic fields of structural elements, for example, Tosca/Opera [1], ANSYS [2], CST [3], as well as the programs to analyze the particle dynamic in cyclotrons [4].

At the initial stage of calculations, we can use the homogeneous magnetic field distribution. Such an approach is justified only for the region of a few first turns of particles in the cyclotron, where a priory there is no considerable variation of the magnetic field. At the next stage of calculations, to provide a more correct analysis of particle dynamics, in particular, to study their axial motion, it is necessary to specify the spatial magnetic field distribution. To calculate it, it is sufficient to correctly specify the boundary conditions in the calculation model and create a calculation mesh of a high quality in the finite element method. In this case, a possible insignificant inaccuracy in specifying the magnetic structure would lead only to small deviations of the field level and shape, which is uncritical for the central region design and does not influence the general pattern of particle motion.

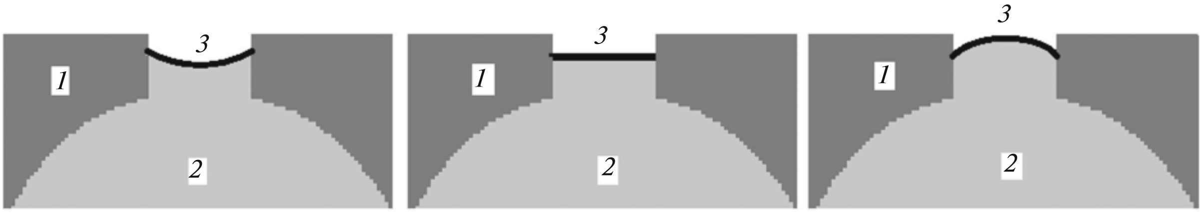


Fig. 1. Possible shapes of the plasma boundaries in the source slit: (*1*) source, (*2*) plasma inside the source, and (*3*) plasma boundary.

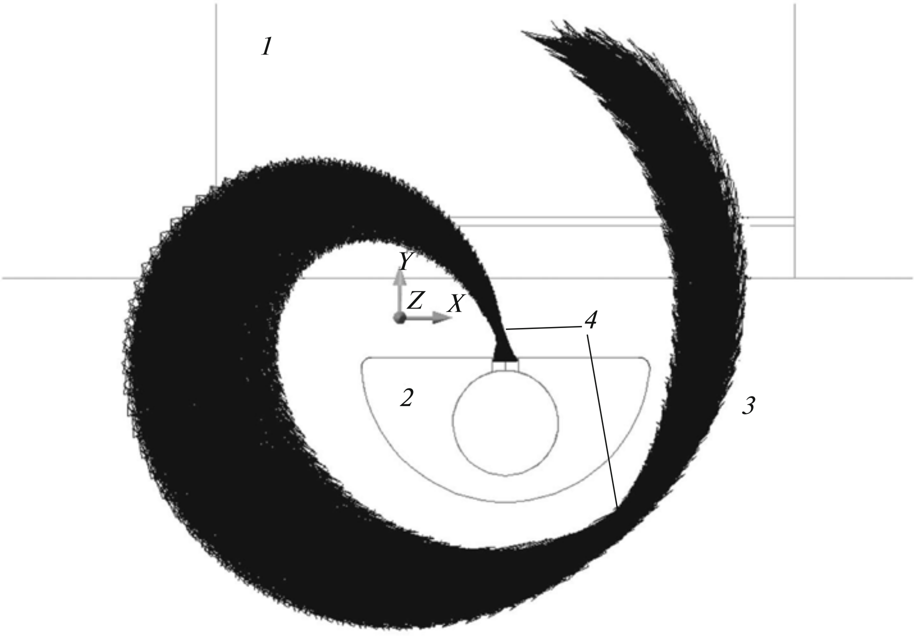


Fig. 2. Trajectories of beam particles at the first turn: (*1*) puller, (*2*) internal ion source, (*3*) particle trajectories, and (*4*) position of the beam focuses.

On the other hand, it is very important to elaborate a realistic model for calculations of the electric field of the accelerating system, since the particle beam parameters are mainly determined by the electric field in the central region. In the case of using the internal ion source, it is necessary to keep in mind that, inside the ion source, there is a boundary of plasma which is formed in a particular way in the source gap. It is necessary to pay attention to this fact when developing the calculation model of the electric field which includes the ion source. The beam is initially formed when passing through the accelerating gap, when the energy of particles is low and any change in the electric field leads to a considerable change in the ion trajectories. The plasma boundary in the source gap may be of three main types: concave, linear, and convex (Fig. 1). It is impossible to speak with certainty about a preference of one of them over the others. However, it is evident that, in each of these cases, the electric-field distribution in the gap will be different and the beam particles will be focused differently by the accelerating field lines.

The calculation and experimental data [5] show that the most probable electric-field distribution near the source gap is such that the beam particles are focused immediately after passing the ion source slit (Fig. 2). At that, the second focus position is situated approximately at a distance of a half revolution from the first focus.

Calculating the field, we can obtain such an effect by introducing an “imaginary” electrode located in the internal region of the source. Its diameter can be smaller than the inner diameter of the source. We can change the potential of the electrode surface to get the required effect, which is estimated either by analyzing the shape of equipotential lines or the results of particle tracing. Sometimes this problem may be solved by filling the internal region of the source with some material for which the source potential is given at the outer surface (Fig. 3). The initial particle-beam posi-

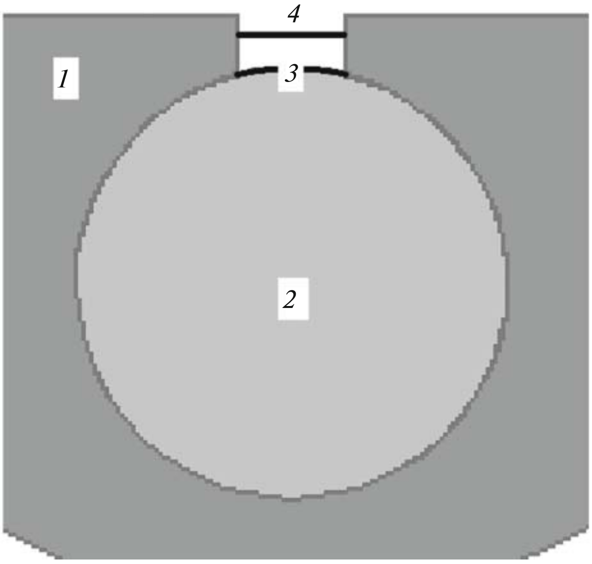


Fig. 3. Initial conditions for calculating the accelerating electric field in the internal source region: (*1*) source, (*2*) internal source region filled with plasma, (*3*) boundary of the imaginary electrode at which the source potential is given, (*4*) point from which the particles start moving in the calculation.

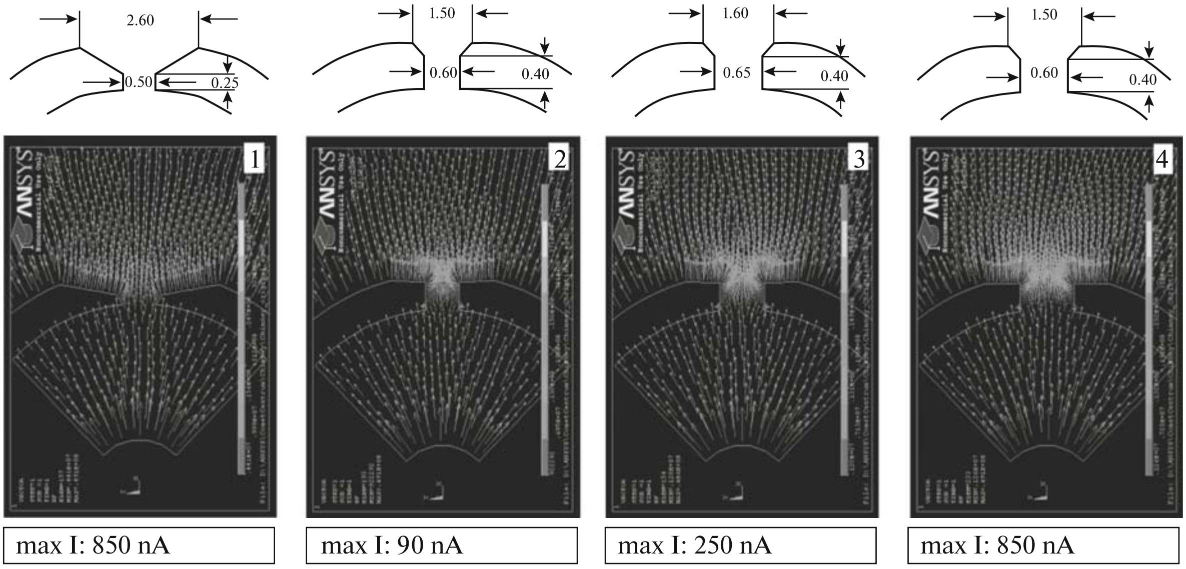
tion (the point at which the particles start moving) is given at some distance from the imaginary electrode surface.

The most often used shape of the source slit is that whose axial size is several times greater than that radial. From the viewpoint of electric field, not only are the source slit sizes important, but also the shape of the bevel edges. They are optimized by changing their inclination, which is usually 40°–60°. When analyzing the calculation model, it is necessary to pay attention to the field line distribution near the gap edges. In most software packages developed to calculate a field at the boundary of an element with an acute angle, for example, at the slit edge, the calculated field would have spikes conditioned by calculation errors. Therefore, it is necessary either to smooth the element edges or generate the initial particle distribution at some distance from such a region. The radial and axial sizes of the source slit and the value of its edges' bevel have a direct effect on the value of the current of the beam extracted from the source, which is confirmed by numerous calculations [6] and results of measurements (Fig. 4).

When using the external ion source with an axial injection line and a spiral electrostatic inflector, it is necessary to provide a sufficiently fine finite-element mesh for the region of the inflector fringe field. The ions passing through the inflector have a sufficiently high energy, so the problem of some uncertainties in the electric field distribution is not acute.

To calculate the beam dynamics, it is necessary to have information about the beam parameters either at a internal source slit or at the entrance to a spiral inflector. When generating the initial particle distribution in the beam injected from the internal source, it is necessary to take into account the dependence of the extracted beam intensity on the extraction voltage amplitude, which is described by the Child–Langmuir law [7]:

$$I = \mathrm{const}\sqrt{\frac{q}{m}}\frac{U^{3/2}}{d^2}, \quad (1)$$



max I: 850 nA

max I: 90 nA

max I: 250 nA

max I: 850 nA

Fig. 4. Results of calculations based on the analysis of the shape of the internal ion source [6], from which it follows that the best intensity of the extracted beam is provided for variants 1 and 4.

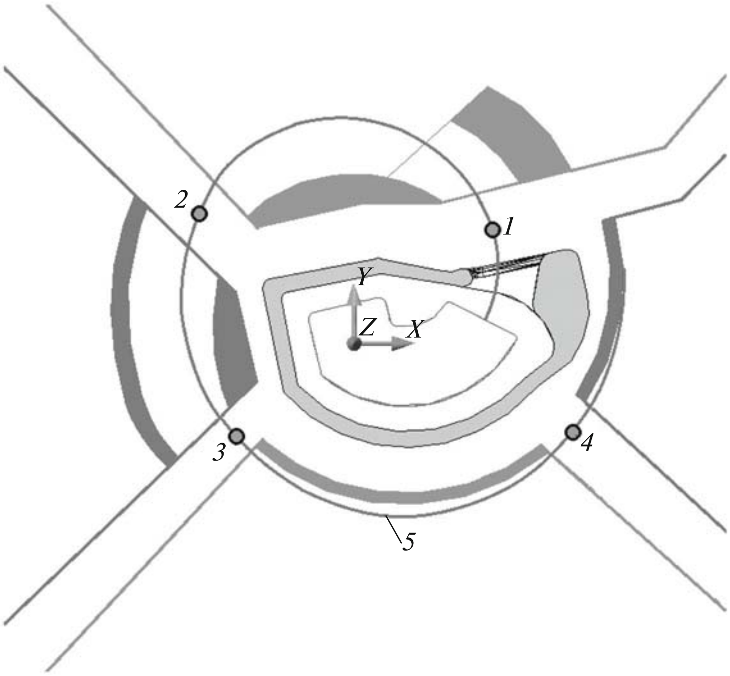


Fig. 5. Configuration of the central region in which an ion collides with the structure's wall because of the insufficient energy gain: (*1*, *2*, *3*, *4*) points at which the ion crosses the first, second, third, and fourth accelerating gap; (*5*) ion trajectory.

where q is the ion charge; m is its mass; U is accelerating voltage; and d is the effective distance between the source gap and the extracting electrode, whose estimation is made considering the fact that the electric field inside the electrode axial gap is nonzero.

Therefore, the model initial beam distribution at the internal source gap may be, for example, as follows:

(i) the regions inside the given ellipses on the phase planes (R, Pr) and (Z, Pz) and in the configuration space (R, Z) are filled according to the chosen way;

(ii) in the longitudinal direction, the dependence of the number of macro-particles on the phase of extracting radio-frequency (RF) voltage is described by (1);

(iii) beam particles start moving at the time moments determined by their initial RF phase.

Usually, for the external ion source, there are measured transverse emittances and energy of the injected beam particles. Hence, it is sufficient to choose the way to fill the 6-dimensional beam volume whose projections onto the corresponding coordinate planes are known. In addition to the uniform filling over the phase volume of the beam, two other distributions are also widely used, namely, the Gaussian and Kapchinsky–Vladimirsky (KV) distributions. In the first case, the filling of the beam volume is carried out separately in each of the two-dimensional planes—projections of the 6-dimensional space with the density:

$$I(x, x') = \frac{1}{2\pi\sigma_x\sigma_y}\exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{x'^2}{\sigma_{x'}^2}\right)\right], \qquad (2)$$

where σ_x and $\sigma_{x'}$ are the standard distribution deviations in the x and x' directions.

The KV distribution has a constant dependence of the particle number density inside the phase ellipse and is defined as follows:

$$I(x, x') = \begin{cases} \dfrac{1}{\pi\varepsilon}, & \text{if } \gamma x^2 + 2\alpha x x' + \beta x'^2 \le \varepsilon, \\ 0, & \text{otherwise.} \end{cases} \qquad (3)$$

where $\alpha, \beta, \gamma, \varepsilon$ are the Twiss parameters of the ellipse.

3. DESIGN OF TIPS OF ACCELERATING DEES

The priority task in designing the cyclotron center is to provide an efficient energy gain during the first few turns. There are several published works devoted to solving this problem [8–10]. We can single out a common algorithm for choosing the geometrical structure of the central region, which can be useful at the first stage of modeling. This algorithm is reduced to choosing the position of the accelerating gaps of the dees along the trajectory of the particle by analyzing its phase motion. As an example, let us consider a cyclotron designed to operate in such a regime when the RF phase incursion between two consequent passages through the accelerating gaps is 180°. Let us assume that the initial structure is chosen in such a way that the ion collides with a vertical post during the first turn because of an insufficient energy gain (Fig. 5). The problem is reduced to a change in the accelerating sys-

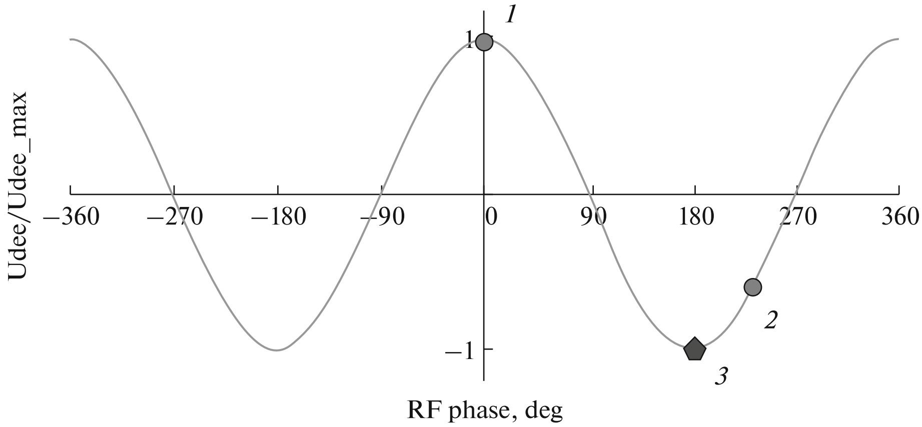


Fig. 6. Dependence of the accelerating voltage on the RF phase: (*1*) moment at which the particle crosses the first accelerating gap, (*2*) the particle crosses the second gap, (*3*) the required point at which the particle crosses the second accelerating gap.

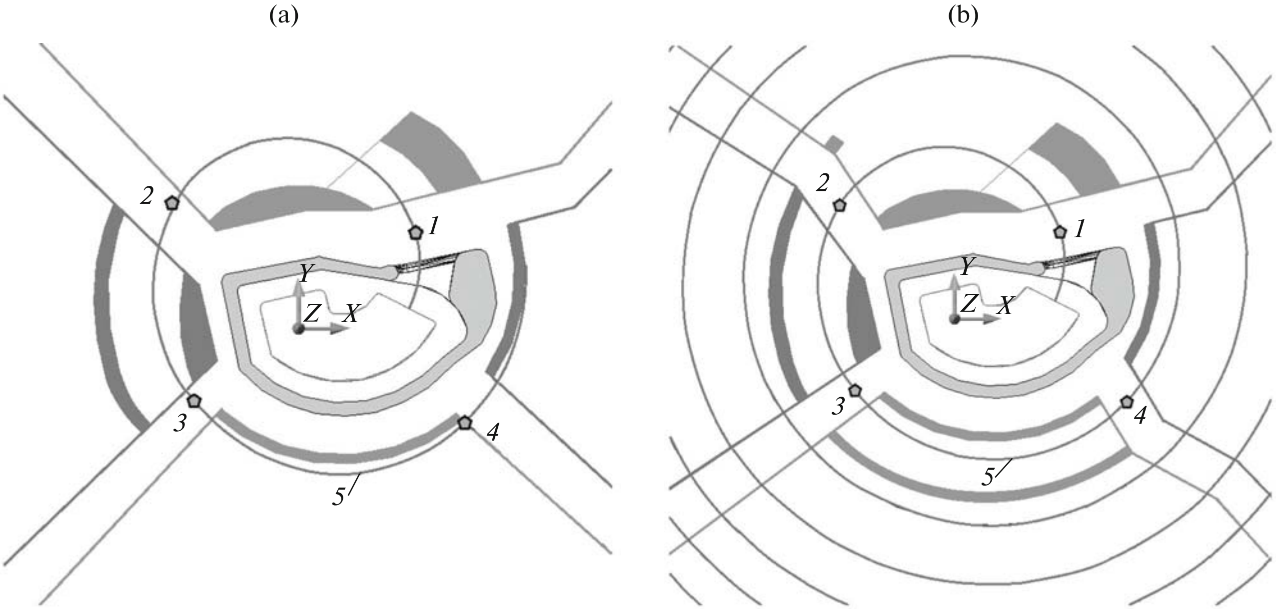


Fig. 7. Optimal particle positions against the background of the initial geometric structure (a) and modified configuration of the accelerating system (b): (*1*, *2*, *3*, *4*) optimal positions of the ion that crosses the first, second, third, and fourth accelerating gaps; (*5*) ion trajectory.

tem structure in such a way that the particle crosses the accelerating gaps at the moments of time corresponding to the maximum voltage at the dees. With this purpose, it is necessary to analyze the ion phase motion. In this case, the particle arrives to the second gap with an RF phase that corresponds to the accelerating voltage drop (Fig. 6).

The ion positions corresponding to the desired RF phase are calculated in a similar way for each accelerating gap (Fig. 7a). The geometric structure is modified in such a way that the centers of accelerating gaps coincide with the optimal particle positions calculated earlier (Fig. 7b). It should be considered that the change in the structure of one gap results in the change in the ion trajectory; therefore, the position of the next gap calculated earlier becomes nonoptimal. Therefore, it is necessary to carry out an iterative procedure that includes a recalculation of the electric-field distribution and the dynamics of particles. It is advisable to arrange accelerating gaps in such a way that they are perpendicular to the particle trajectory. It is also efficient to introduce into the model additional posts situated at the gap boundaries and forming an electric field in them.

4. PROVIDING THE BEAM CENTERING

Beam centering is the most important characteristic of the acceleration process in a cyclotron. The values of amplitudes of radial betatron oscillations of particles at the final radius directly influences the efficiency of beam extraction from the accelerator vacuum chamber. The oscillations can be corrected by modifying the magnetic field, for example, by introducing the first harmonics at the first radii. However, the determining contribution to the value of beam centering is made by the configuration of the cyclotron central region. The change in the position and shape of the accelerating gaps leads to the change in the particle trajectory and, thus, in the value of its centering. In most cases, it is sufficient to optimize the center structure, analyzing only the behavior of the central particle of the beam. Minimizing the amplitudes of free oscillations of the central ion results in both the reduction of the radial betatron oscillations of all beam particles and some reduction in incoherent oscillations of ions in the beam.

Solving the beam centering problem is of current importance for both cyclotrons with an internal source and for those using an axial injection line and a spiral inflector. There are two most commonly used methods to estimate the beam centering. This can be the analysis based on a calculation of the centers of instantaneous circles to which the particle trajectory is tangent [11]. The centers of such circles are located at some distance from the accelerator geometric center, depending on the magnetic field shape. The deviations of the centers of the circles contain the contribution to the orbit motion conditioned by the spatial variation of the magnetic field. The values of radial amplitudes can be estimated either visually or by calculating the average value of the circle-center coordinates. This method is intended to calculate the deviations of the centers of instantaneous orbits from the cyclotron geometry center rather than the amplitude of particle oscillations with respect to the relatively closed equilibrium orbit. In most cases, these values are equivalent. However, there are such structures where they are different, for example, the magnetic field in which the first harmonic is such that it is possible to find a closed equilibrium ion orbit. In such a field, the centers of instantaneous circles, to which the trajectory of the particle rotating exactly along the equilibrium orbit is tangent, will be displaced with respect to the cyclotron geometric center. However, it is evident that the particle does not perform radial oscillations relative to the equilibrium orbit. Therefore, it is necessary to use an additional algorithm that takes into account the displacement of the closed equilibrium orbit.

One widely used method to calculate the amplitudes of radial betatron oscillations of particles is based on the analysis of the behavior of the particle in the space (r; pr) when it is crossing some fixed azimuth. The ion that starts moving with some radius or angle deviation from the values corresponding to the initial values of the closed equilibrium orbit, after having made a sufficient number of turns, circumscribes an ellipse in the space (r; pr) (Eigen ellipse [12]). After the reduction of the obtained ellipse to the canonical form, its r-semiaxis determines the amplitude of particle radial oscillations. In the case of an oblique ellipse, if its Twiss parameters (β, γ) and emittance (ε) are known, the required semiaxis can be calculated as follows [13]:

$$A = \sqrt{\frac{\varepsilon}{2}}\left(\sqrt{\frac{\beta+\gamma}{2}+1}-\sqrt{\frac{\beta+\gamma}{2}-1}\right).$$

The algorithm for calculations of the oscillation amplitudes over the entire trajectory of the accelerated particle can be as follows. When calculating the acceleration of the ion crossing a chosen azimuth, the coordinates of the cross point and the direction of the particle velocity vector are registered into the memory array. Then, each of these data sets becomes initial for the new calculation of the particle trajectory. In this case, the accelerating voltage is not considered and the particle that starts moving from the accelerated orbit performs rotations in the magnetic field with no changes in its energy. Having made one rotation, the particle would cross the initial azimuth with coordinates and angles different from their initial values. After a certain number of rotations, the particle circumscribes the required self-consistent ellipse. In this case, there is no necessity to calculate the equilibrium accelerated orbit, since the described algorithm for the calculation of radial amplitudes does not use explicitly its coordinates.

When designing the cyclotron central region, it is reasonable to alternate the variation of its geometric structure with a calculation of amplitudes of the particle's radial betatron oscillations. The system configuration at the first rotation is determining. The variation in the azimuthal positions of accelerating gaps leads to the change in the accelerating voltage phase at which the particle crosses them. Therefore, the energy gain and the particle trajectory change when the particle passes through the accelerating gap. The angle between the normal to the accelerating gap and the line tangent to the trajectory are also of importance, as is the accelerating gap length, since the time-of-flight factor of the particle is small. The region of the first accelerating gap is the most important. The particle trajectory in this region can be efficiently corrected by varying the puller shape (Fig. 8).

To reduce the time spent at the initial stage, it is possible to use a visual similarity of the ion trajectory to the ideally centered orbit as a criterion of improving the beam centering. The ideally centered orbit can be obtained by using so-called "backward tracking" [14]. The gist of this method consists in the following. The closed equilibrium orbit is calculated near the final radius [15], where this orbit is most close to the equi-

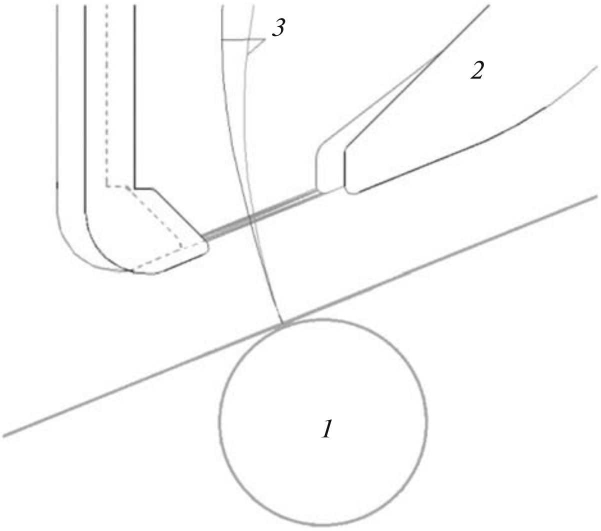


Fig. 8. Change in the particle trajectory by varying the puller position and the accelerating gap length: (*1*) ion source, (*2*) puller, and (*3*) particle trajectory.

librium accelerated orbit. The particle starts moving from the coordinates of the found orbit. To be more specific, it is necessary to use as the initial position the azimuth at which the particle RF phase is known, for example, the central line of the accelerating dee. The magnetic field direction and the voltage at the accelerating electrodes are such that, with decreasing energy, the particle arrives to the central region of the accelerator. Therefore, the process of deceleration of the particle with a well-centered orbit is modeled. The researcher obtains the required trajectory up to the first turns in the cyclotron central region, which serves as a visual guiding line when optimizing its structure. There are studies devoted to the automatic fitting of the central region parameters to obtain a well-centered beam [16]. The input parameters in these algorithms are the accelerated ion characteristics and the given magnetic field distribution.

The amplitudes of radial particle oscillations somehow depend on the relationship between the ion energy and the injection radius. The radius of the particle orbit at the first turn must be as close as possible to its energy radius, which can be calculated according to the formula

$$R = \frac{Mv}{QB}, \tag{4}$$

where M, Q, and v are the ion mass, charge and velocity, respectively; B is the value of the magnetic field.

In the internal source cyclotrons, the beam centering can be improved by varying either the radial position of the source or the value of the accelerating voltage amplitude at the dees.

5. MINIMIZATION OF PARTICLE LOSSES

Providing the maximally possible coefficient of beam transmission through the central region is one of the main problems of cyclotron design. There are different factors that influence the particle motion which should be considered. The radial beam losses are mainly determined by insufficient energy gain during the passage through the accelerating gaps. The axial behavior of particles depends on the focusing by both the electric and magnetic fields. As a rule, construction of the central region requires the existence of different posts that connect the upper and lower parts of the system and intersect the cyclotron median plane. Solving the problem of optimizing the position and shape of these posts is intended to reduce the losses of the useful part of the beam (the ions that will be captured for acceleration) and determine the localization of the regions where the ions that gained insufficient energy collide with the system elements. It is necessary to keep in mind that the modification of the geometric structure in the region of the first few accelerating gaps leads to a change in the electric field distribution, which requires its recalculation after having made the changes in the system structure. This effect is especially noticeable when optimizing the puller shape that specifies the initial parameters of particle trajectories of the first turns [12]. Usually, the main part of particles is lost at the puller walls. The change in the radial and axial gaps of the puller leads to changes in both the energy gain and the direction of particle motion. The axial aperture of the puller also influences the axial focusing of ions. Both the augmentation and reduction of the gap may be efficient; its value must be fitted individually for each concrete central region.

The axial motion of particles in the region of the first few turns is strongly influenced by the accelerating electric field. It can both focus the particles crossing the accelerating gap when the voltage at the accelerating dees is dropping and defocus them when the voltage is rising. Therefore, it is necessary to design the central region in such a way that the beam would cross the central region the maximum number of times when the accelerating voltage is dropping [17].

When using the external ion injection and a spiral inflector, the amplitudes of axial particle oscillations depend on the degree of optimization of the inflector structure, which must provide the optimal entry of the central particle to the median plane. The ion trajectory must intersect the median plane at the minimum angle. If this criterion is satisfied, the central particle trajectory coincides with the theoretically calculated central line of the inflector. The presence of the edge field at the inflector input and output increases its effective length. The ion trajectory deviates from the inflector central line, which results in an increase in the amplitude of axial particle oscillations in the acceleration process. This problem can be solved by short-

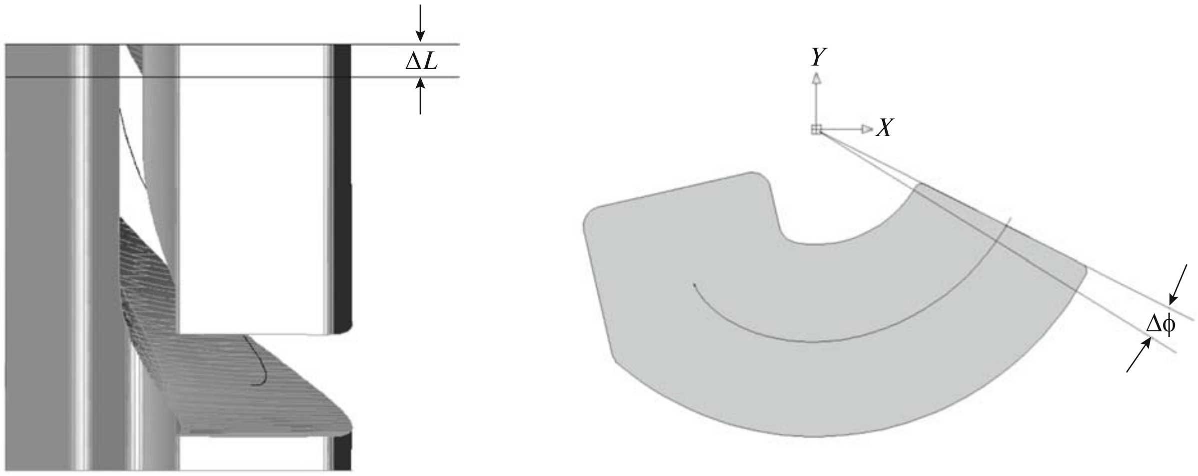


Fig. 9. Direction of cuts of the electrodes of the spiral electrostatic inflector at the input (a) and output (b) to match its effective length with the value calculated analytically.

ening the inflector electrodes. Usually, the value of the cut is a few millimeters at the input and a few degrees in the inflector coordinate system at the output (Fig. 9). The choice of the value of electrode cut is based on the ion tracing through the three-dimensional field of inflector. This is an iterative process of calculating the field and particle dynamics. It is important to carry out this procedure in the presence of the electric field of accelerating dees, since their field cardinally influences the particle motion at the inflector output. In addition, the geometric structure of the elements surrounding the inflector and changing the electric field distribution in the region of particle motion also influences the sought-after value of the cut. Therefore, the inflector optimization must be carried out with the presence of all main parts surrounding it. The final correction of the axial particle oscillation can be performed with the help of a small axial displacement of the inflector.

The beam going out of the inflector is divergent in the axial direction, which is sometimes the reason for a great percentage of axial particle losses. This problem may be partly solved by installing additional elements of beam focusing in the region between the inflector and the puller, for example, a magnetic quadrupole lens [18]. In this case, it is necessary to envisage a free space in the region of the first accelerating gap to arrange such an element there. It is also known that a spiral inflector with curved electrodes (Fig. 10) leads to a considerable reduction of the axial size of the beam. It is proved experimentally that the use of such an inflector may lead to an increase in transmission through the cyclotron central region [19]. In this case, the radial emittance of the beam increases; however, it is not principal for a number of accelerators.

The effect conditioned by using such an inflector is explained by the fact that the beam particles passing in the inflector gap undergo the impact of the electric field whose equipotential lines are directed in such a way that the beam at the output has an additional axial focusing. A similar effect, although to a less degree, can be obtained by displacing one of the electrodes of a classic spiral inflector to the axial direction so as to somewhat increase the inflector gap at the output. A considerable reduction of the beam axial defocusing at the inflector output may be obtained by modifying it in such a way that the width of the internal inflector electrode is smaller than that of the external electrode (Fig. 11). Here, we imply by the internal electrode the one to which direction the particle beam is deflected by the electric field. Varying the ratio of the internal inflector electrode width to that of the external elec-

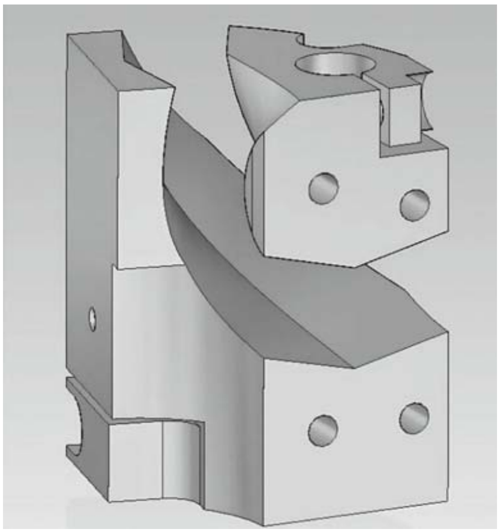


Fig. 10. Spiral inflector with curved electrodes to provide an additional axial focusing of the beam [18].

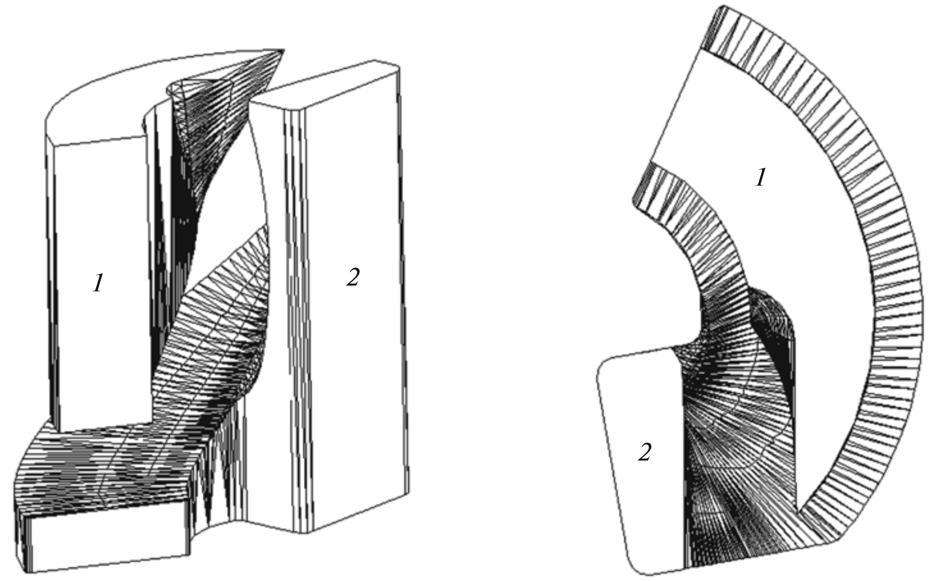


Fig. 11. Spiral inflector with different electrode widths: (*1*) internal electrode; (*2*) external electrode.

trode, we can get a required effect of the electric field on the beam characteristics. The value of the desired ratio is taken from the results of calculations of the beam dynamics; it proves to be about 1.7–2.

The losses of radial particles in cyclotrons using a spiral inflector depend largely on the energy of beam injection. An increase in the initial ion energy results in an increase in its orbit radius. However, in this case, the radial sizes of the spiral inflector also grow and, as a consequence, the sizes of the infrastructure which the ion must bypass at the first turn grow as well. When the injection energy decreases, the situation is opposite. Calculations can prove that the optimal case from the point of view of minimizing the beam radial losses is such a regime of operation at which the value of injection energy is smaller than the amplitude of accelerating voltage at dees. In this case, the sizes of the center structure are minimal and the particle, passing through the first few accelerating gaps, significantly increases the radius of its orbit. Therefore, under the condition of a bounded accelerating voltage, it is advisable to decrease the injection energy.

6. BEAM FORMATION SYSTEM

The destination of a great number of cyclotrons establishes particular requirements for the beam quality after passing the central region. Usually, the beam selection system at the first few turns consists of a set of phase slits, intensity modulating devices, and different collimators. The number of phase slits formed by several axial posts may vary from one to three–four. They are destined to separate a beam with a required radial size that is directly related to its phase parameter. The system of phase slits must provide the possibility of selection the phase range from a few degrees of RF to the maximum possible size defined by the acceptance of the central region of accelerator. The choice of the shape and positions of elements determines their functionality and operability. We can propose some simple recommendations which can be useful when designing the phase slits:

(i) a phase slit is the most functional if it is arranged at the point where the radial size of the beam is maximum;

(ii) the closer the slit is arranged to the center, the more effective it is and the lower the radiation losses;

(iii) if there are several slits, it is reasonable to arrange them at different turns and with an azimuthal difference of a half-period of the system; for example, in the hill and in the valley;

(iv) the elements must be arranged far from accelerating gaps; for example, along the central lines of the space between dees (Fig. 12).

The effect of using the phase slit can be estimated by analyzing the phase acceptance of the central region (the range of the initial RF phases of particles captured for further acceleration). Let us assume that we investigate a cyclotron with an internal ion source. In this case, the center phase acceptance usually consists of particles whose initial RF phases do not go beyond the range (–90; 0)°, if we are speaking about a case where the maximum extracting voltage corresponds to a phase of 0° (Fig. 13). Then, having arranged the slits and varying their size (the radial distance between the vertical posts), we can vary the

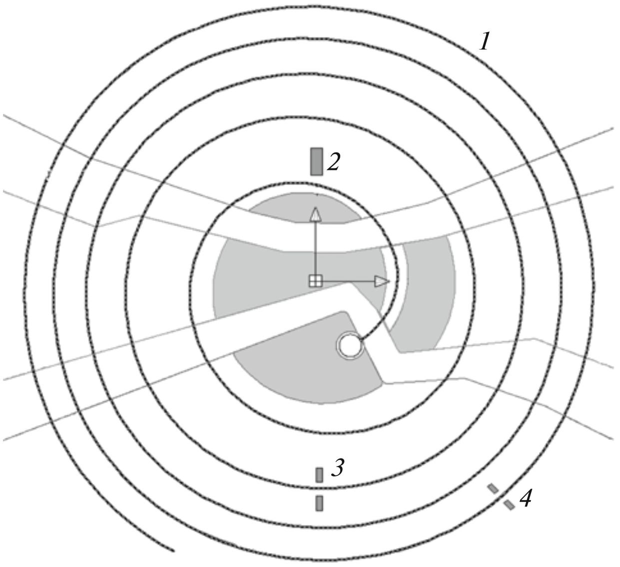


Fig. 12. Possible position of phase slits in the cyclotron central region: (*1*) central ion trajectory; (*2*, *3*, *4*) first, second, and third phase slit.

phase acceptance of the central region and separate the required phase size of the beam.

A more meticulous optimization of the position and sizes of phase slits can be carried out using the following procedure. Two initial particle distributions are generated. The first beam contains ions with the start RF phases from the phase range which must be separated by the phase slits. The second beam contains particles that have all RF phases except those which are contained in the first beam. With the help of a program analyzing the particle dynamics, the consequent tracing of both beams through the central region, which still does not contain phase slits, is carried out. The particle distribution is analyzed at the intersection of the possible azimuth of the first slit position. The optimality criterion for the radius at which the slit at the chosen azimuth is situated is the case when the overlap between the particles which must be separated and the remaining part of the beam is minimal (Fig. 14).

A similar procedure is applied to all other slits, provided that the beam passes through the structure containing the previous phase slits and the particles colliding with their walls come out of the further motion.

7. KILPATRICK CRITERION

Since the particle trajectory at the first turns is determined by the geometric structure of the central region, it is important at the initial stage of its design to estimate the maximum possible sizes of accelerating gaps. The consequent technical workup of the conceptual design and the prototype measurements will reveal the problems from the viewpoint of discharges if the gap sizes were chosen incorrectly. In this case, the design of the central region will be modified. For many years researchers were guided by a semiempirical formula called the Kilpatrick criterion [20, 21]:

$$f(MHz) = 1.64 \times E\left(MV/m\right)^2 \times e^{-8.5/E(MV/m)}. \quad (5)$$

Formula (5) gives the dependence of the frequency of accelerating system operation on the electric-field strength in the gap. However, with the development of the surface finish technique, operating vacuum characteristics, and other aspects influencing the dis-

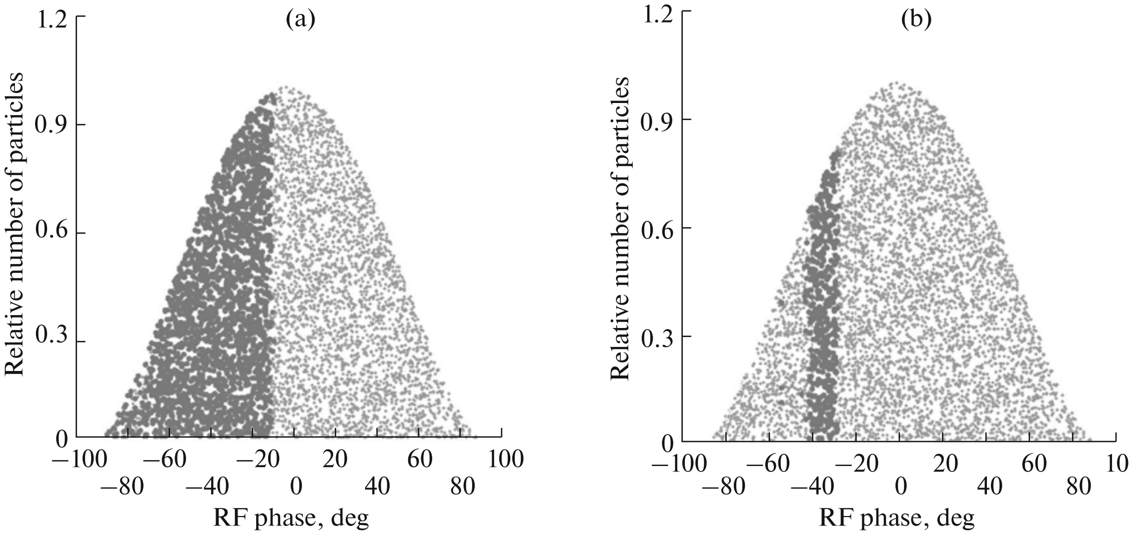


Fig. 13. Phase acceptance of the central region without phase slits (a) and in the case when the set of phase slits separates the 10°-bunch (b). Small red points indicate the ions extracted from the source. Large blue circles indicate the particles that will pass through the central region and be captured for further acceleration.

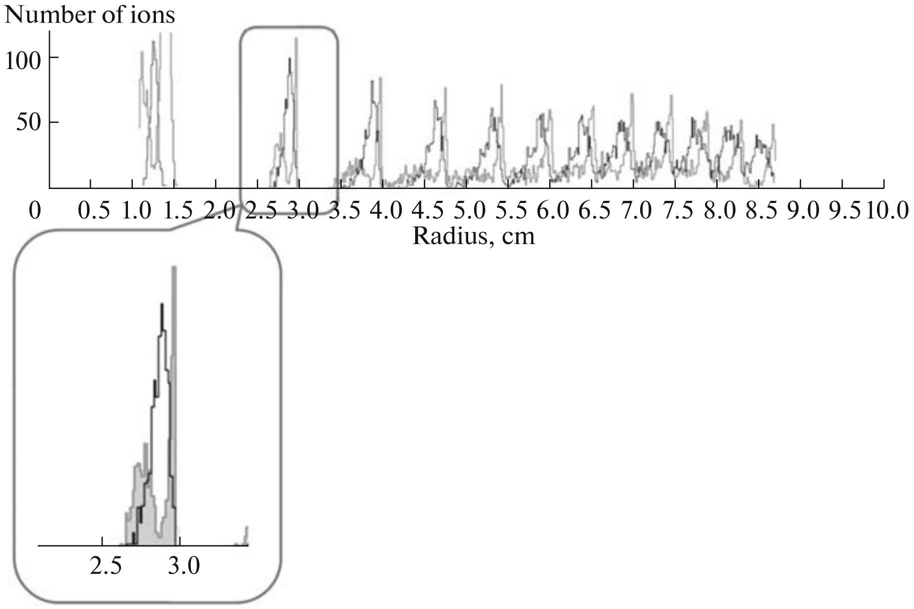


Fig. 14. Particle distribution in the azimuthal plane of the phase slit. The optimal range of radii to arrange the phase slit. The part of ions we need to separate is shown in black; the distribution of particles we need to cut off is shown in red.

charge value, the operating regimes in which the values of the electric-field strength exceed that obtained from (5) are achieved nearly at all modern accelerating installations. The electric strength of the vacuum gap is constantly being studied, mainly experimentally. The values obtained at different installations have quite a large spread [21] and depend on concrete experimental conditions. There are examples where the electric-field strengths reached the values exceeding the Kilpatrick criterion by tens of times. Nevertheless, the requirement of reliability of cyclotron operation leads to the situation where one can observe a common boundary of the maximum electric field strength in accelerating gaps in the cyclotron central region which exceeds the Kilpatrick criterion 1.3–1.4 times.

8. CONCLUSIONS

A broad experience in design and optimization of the central regions of compact cyclotrons allows us to single out some basic methods and algorithms which are used when designing the central region. In this paper we have described the methods for the design of a structure that satisfies the following principal criteria: increasing the beam transmission coefficient, ensuring a good beam centering, and obtaining a required quality of the accelerated beam. The results of this study may be useful at the initial stage of design and optimization of the central region of compact cyclotrons.

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