Quantum-field effects of acceleration and vorticity in QCD: polarization, geometry and statistics

grant 18-02-40056 mega

1903.09697, 2003.11119, 1911.04545

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The Conference "RFBR Grants for NICA",
20 to 23 October 2020, Online Event

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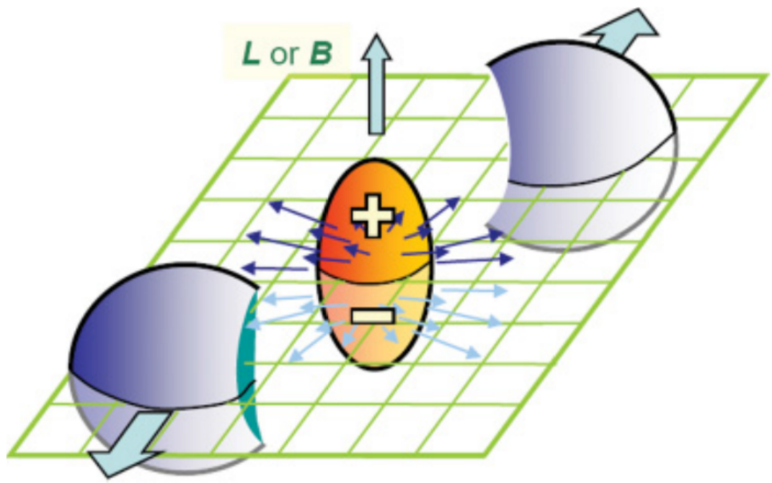
Introduction: motivation from heavy ion physics

In noncentral collisions of heavy ions on the particle colliders (NICA, RHIC), **huge magnetic fields** and a **huge angular momentum** arise. *Differential* rotation - different at different points: **vorticity** and vortices.

– Rotation 25 orders of magnitude *faster* than the rotation of the Earth:

the vorticity is about 10^{22} sec^{-1}



- **Acceleration** is of the **same order** of magnitude as vorticity (as the components of the same tensor): contributes to polarization.

[I. Karpenko and F. Becattini, Nucl. Phys., A982:519–522, 2019]

- *Another mechanism*: accelerations due to the tension of the hadron string (acceleration is much higher – of the order of Λ_{QCD})

$$e^+e^- \to \gamma^* \to q\bar{q} \to \text{hadrons}$$

[P. Castorina, D. Kharzeev, H. Satz, Eur. Phys. J., C52:187–201, 2007]

Polarisation as a sign of the vorticity

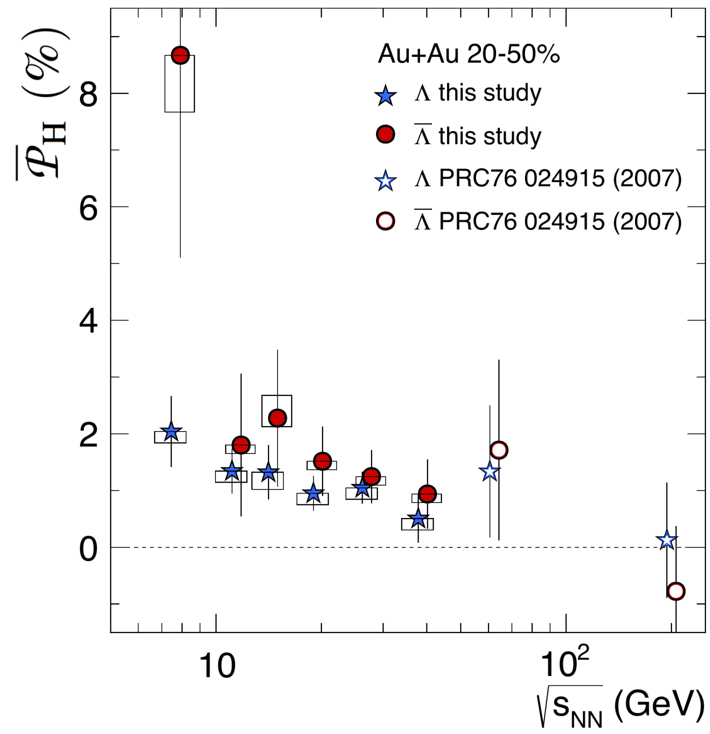
The **vorticity** leads to **polarization** of hadrons:

1. Through the chiral vortical effect (CVE) and chiral anomaly (**Dubna group**)
[M. Baznat, K. Gudima, A. Sorin, O. Teryaev, Phys.Rev.C 97 (2018) 4, 041902]

CVE: $Q_5^s \sim < \Pi_0^{\Lambda,lab} >$

2. Acceleration also leads to polarization
[F. Becattini, V. Chandra, L. Del Zanna, E. Grossi Annals Phys., 338:32–49, 2013]

$$\langle \Pi_\mu(p) \rangle \simeq \frac{1}{8} \epsilon_{\mu\rho\sigma\tau} \frac{p^\tau}{m} \frac{\int d\Sigma_\lambda \, p^\lambda \, n_F(1-n_F)\partial^\rho \beta^\sigma}{\int d\Sigma_\lambda \, p^\lambda n_F}$$



[Nature, 548:62–65, 2017]

Introduction: quantum anomalies in hydrodynamics

gauge part

Chiral anomaly

gravitational part

$$\nabla_\mu j_5^\mu = -\frac{Q^2 e^2}{16\pi^2\sqrt{-g}}\varepsilon^{\mu\mu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} + \frac{1}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\mu\rho\sigma}R_{\mu\nu\kappa\lambda}R_{\rho\sigma}{}^{\kappa\lambda}$$

Holography

- *[K. Landsteiner, E. Megias, F. Pena-Benitez. Phys. Rev. Lett. 107, 021601 (2011).]*
- *[M. Stone and J. Kim, Phys. Rev. D98, no. 2, 025012 (2018).]*
- *[S. P. Robinson, F. Wilczek Phys.Rev.Lett. 95 (2005) 011303 MIT-CTP-3561 gr-qc/0502074.]*

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Although the gravitational chiral anomaly **is not important** in **volume**, it is significant at the **boundary** or at the **horizon**.

Gauge chiral anomaly and CVE

$$j_5^\mu = n_5 u^\mu + \frac{\mu^2+\mu_5^2}{2\pi^2}\omega^\mu + \frac{\mu}{2\pi^2}B^\mu$$

[D. T. Son and P. Surowka, Phys. Rev. Lett. 103 (2009) 191601.]

[A. V. Sadofyev, V. I. Shevchenko and V. I. Zakharov, Phys. Rev. D 83 (2011) 105025.]

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Gravitational part

$$j_5^\lambda = \left(\frac{T^2}{6} + \frac{\mu^2+\mu_5^2}{2\pi^2}\right)\omega^\lambda$$

Introduction: problems

Vorticity	$\leftrightarrow$	**Magnetic field**
Acceleration	$\leftrightarrow$	**Electric field**

It is logical to consider *acceleration* effects in addition to vorticity.

Problems

1) Quantum effects in hydrodynamics in the case of **acceleration**?

2) Quantum anomalies and chiral vortical effect in the case of **higher spins** (spin 1, 3/2, etc.)? Relevant for spin dynamics of vector degrees of freedom (photons, gluons).

Emergent conical geometry in the Zubarev density operator

Zubarev density operator for a medium with a thermal vorticity tensor

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu} + \zeta\widehat{Q}\right]$$

Generators of Lorentz transformations

$$\widehat{J}^{\mu\nu} = \int_\Sigma \mathrm{d}\Sigma_\lambda \left(x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu}\right)$$

$$\varpi_{\mu\nu}\hat{J}^{\mu\nu} = -2\alpha^\rho \hat{K}_\rho - 2w^\rho \hat{J}_\rho$$

$\hat{K}^\mu$ – boost (associated with **acceleration**)

$\hat{J}^\mu$ – angular momentum (associated with **vorticity**)

Temperature and **acceleration** are **independent** parameters: different temperature ranges can be considered in an accelerated medium

in contrast of:

Unruh effect: if a Minkowski vacuum is created, then medium temperature **is related** to acceleration

$$T_U = \frac{|a|}{2\pi}$$

[M. Buzzegoli, E. Grossi, and F. Becattini, JHEP, 10: 091, 2017]

Effects of acceleration from Zubarev operator: results (*m=0*)

$$\langle \hat{T}^{\mu\nu} \rangle^0_{\text{fermi}} = \Big(\frac{7\pi^2 T^4}{60} + \frac{T^2|a|^2}{24} - \frac{17|a|^4}{960\pi^2}\Big) u^\mu u^\nu - \Big(\frac{7\pi^2 T^4}{180} + \frac{T^2|a|^2}{72} - \frac{17|a|^4}{2880\pi^2}\Big) \Delta^{\mu\nu} + \mathcal{O}(a^6)$$

$$\langle \hat{T}^{\mu\nu} \rangle^0_{\text{real}} = \Big(\frac{\pi^2 T^4}{30} + \frac{T^2|a|^2}{12} - \frac{11|a|^4}{480\pi^2}\Big) u^\mu u^\nu - \Big(\frac{\pi^2 T^4}{90} - \frac{T^2|a|^2}{18} + \frac{19|a|^4}{1440\pi^2}\Big) \Delta^{\mu\nu} + \Big(\frac{T^2}{12} - \frac{|a|^2}{48\pi^2}\Big) a^\mu a^\nu + \mathcal{O}(a^6)\,.$$

The energy-momentum tensor **vanishes** at the **Unruh temperature**

$$\langle \hat{T}^{\mu\nu} \rangle = 0 \qquad (T = T_U)$$

[F. Becattini, Phys. Rev. D 97, no. 8, 085013 (2018)]

Corresponds to the results of **non-perturbative** calculation:

- talk of A. Palermo
- *[F. Becattini, M. Buzzegoli, A. Palermo, e-Print: 2007.08249]*

Thus, a consequence of the **Unruh effect** is **justified**.

The duality of statistical and geometric approaches

The following expressions were obtained for the vacuum value of T_2^2 in spacetime with a cosmic string [V. P. Frolov and E. M. Serebryanyi, Phys. Rev. D 35, 3779 (1987)]

String:

$$\langle T_2^2 \rangle_{s=0} = \frac{\nu^4}{480\pi^2 r^4} + \frac{\nu^2}{48\pi^2 r^4} - \frac{11}{480\pi^2 r^4}$$

$$\langle T_2^2 \rangle_{s=1/2} = \frac{7\nu^4}{960\pi^2 r^4} + \frac{\nu^2}{96\pi^2 r^4} - \frac{17}{960\pi^2 r^4}$$

Passing to the Euclidean Rindler spacetime, we obtain

The coincidence will be for **massive** fields as well

Rindler:

$$\rho_{s=0} = \frac{\pi^2 T^4}{30} + \frac{T^2 |a|^2}{12} - \frac{11|a|^4}{480\pi^2},$$

$$\rho_{s=1/2} = \frac{7\pi^2 T^4}{60} + \frac{T^2 |a|^2}{24} - \frac{17|a|^4}{960\pi^2}$$

= Zubarev

The obtained expressions **correspond** to the energy calculated using the **Zubarev density operator** in inertial frame.

Unruh effect from statistics

The effects of **vorticity** are controlled by **anomalies**
[D. Kharzeev, K. Landsteiner, Andreas Schmitt, and Ho-Ung Yee. Strongly Interacting Matter in Magnetic Fields. Lect. Notes Phys., 871:pp.1–624, 2013]

I. **Acceleration** effects are controlled by the **Unruh effect**.

But the Unruh effect also follows from the **gravitational anomaly** [S. P. Robinson, F. Wilczek, Phys. Rev. Lett., 95:011303, 2005].

II. Therefore, the effects of **acceleration** are also controlled by **anomalies**.

Instability below Unruh temperature

Instability below Unruh temperature: acceleration as imaginary chemical potential

The density operator and the covariant Wigner function [F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, Annals Phys., 338:32-49, 2013] lead to the **integral representation** of energy density

$$\rho = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} = 2\int \frac{d^3 p}{(2\pi)^3}\left(\frac{|\mathbf{p}| + ia}{1 + e^{\frac{|\mathbf{p}|}{T} + \frac{ia}{2T}}} + \frac{|\mathbf{p}| - ia}{1 + e^{\frac{|\mathbf{p}|}{T} - \frac{ia}{2T}}}\right)$$

$$+4\int \frac{d^3 p}{(2\pi)^3} \frac{|\mathbf{p}|}{e^{\frac{2\pi|\mathbf{p}|}{a}} - 1} \qquad (T > T_U)$$

in red: *modifications* compared to the *Wigner function*

- In the first integral, the **acceleration** enters as an **imaginary chemical potential** $\pm\frac{ia}{2}$ [G.P., O. Teryaev, V. Zakharov, Phys. Rev. D 98, no. 7, 071901 (2018)].

Motivation:

- **Exact match** with the fundamental result from the density operator at $T > T_U$.
- True limit at $a \to 0$.
- Some terms can be obtained directly from the Wigner function.

Instability below Unruh temperature: jump of the derivative

Comparing the two temperature regions

$$\rho = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} \qquad T > T_U$$

$$\rho = \frac{127\pi^2 T^4}{60} - \frac{11 T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} - \pi T^3 a + \frac{T a^3}{4\pi} \qquad T < T_U$$

we see that the values in $T = T_U$ coincide, also the first derivatives turn out to be the same, however, a **jump** occurs in the **second derivative**:

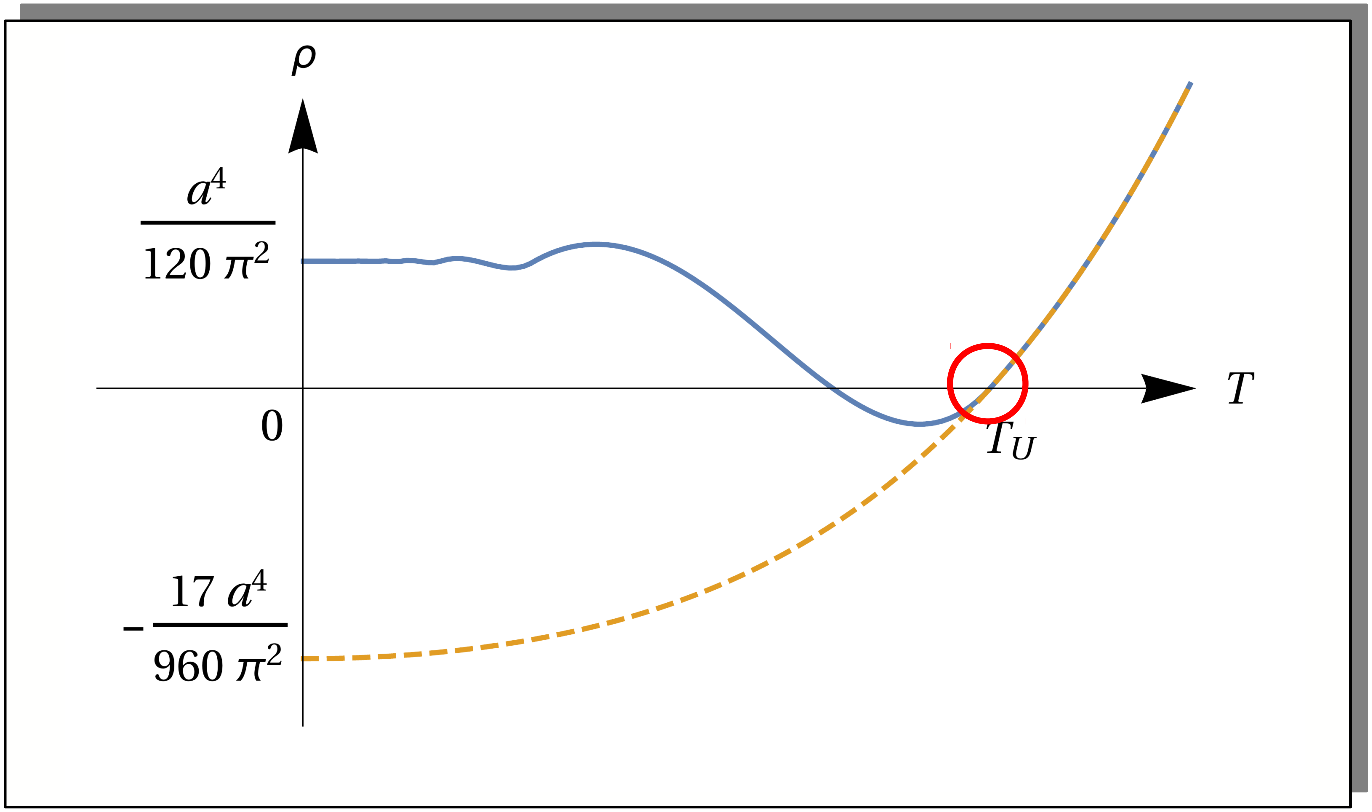
$$\rho_{T>T_U}(T \to T_U) = \rho_{T<T_U}(T \to T_U) = 0\,,$$

$$\frac{\partial}{\partial T}\rho_{T>T_U}(T \to T_U) = \frac{\partial}{\partial T}\rho_{T<T_U}(T \to T_U) = \frac{a^3}{10\pi}\,,$$

$$\frac{\partial^2}{\partial T^2}\rho_{T>T_U}(T \to T_U) \neq \frac{\partial^2}{\partial T^2}\rho_{T<T_U}(T \to T_U)\,,$$

[G. P., O. V. Teryaev, V. I. Zakharov, Phys. Rev., D100(12):125009, 2019]

Instability below Unruh temperature: jump of the derivative



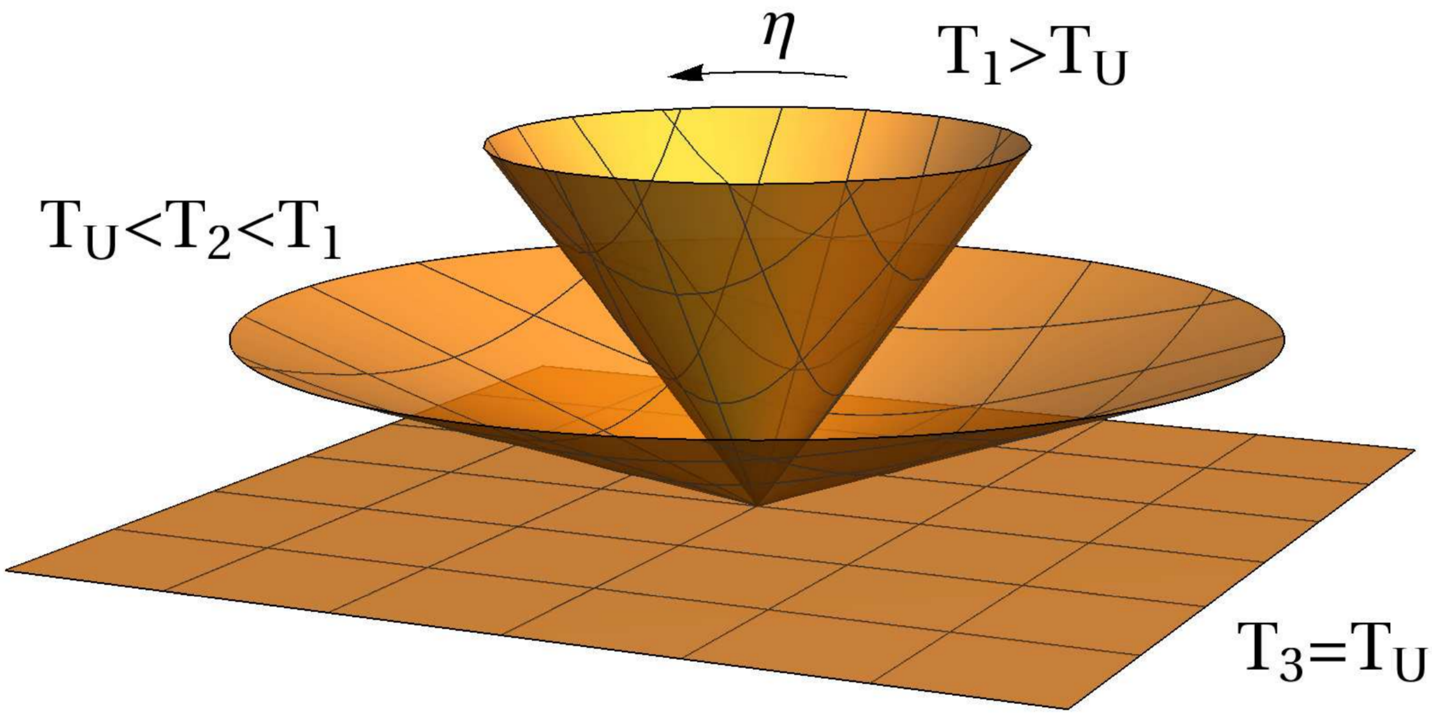
The solid blue line is the energy density as a function of temperature, corresponding to the integral representation.

The dashed orange line - the result of the fundamental perturbative calculation based on the density operator.

[G. P., O. V. Teryaev, V. I. Zakharov, Phys. Rev., D100(12):125009, 2019]

Instability at Unruh temperature: source in geometry

In the geometrical language at $T = T_U$ the **cone** transforms into a **plane**.
Statistical instability is accompanied by a qualitative **change** in **geometry**.



[G. P., O. V. Teryaev, V. I. Zakharov, Phys. Rev., D100(12):125009, 2019]

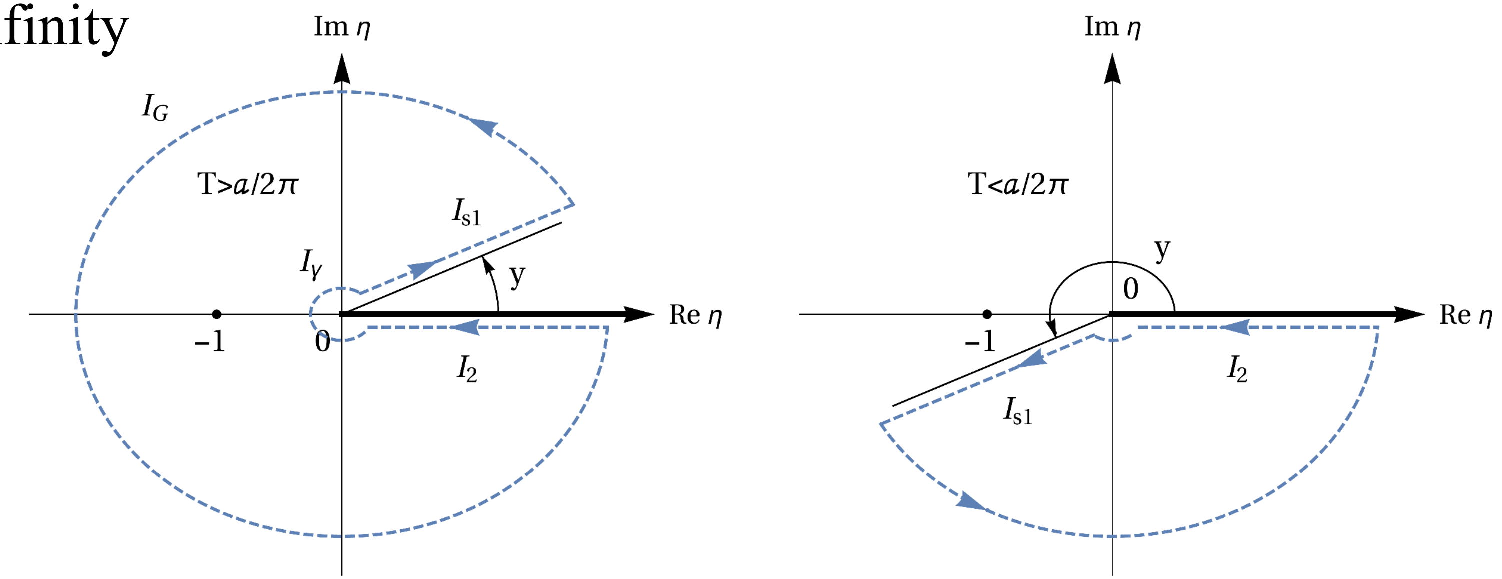
Instability at Unruh temperature

To study the transition through instability point, let's move on to the variable

$\eta = e^{\frac{|\mathbf{p}|}{T} + \frac{i|a|}{2T}}$, then the energy will be expressed through integrals of the form

$$I_{s1} = \int_{I_{s1}} \frac{\eta^D d\eta}{(\eta+1)^2} F(-iy)$$

Where $D = \dfrac{\partial}{\partial(-iy)}$ is the derivative operator. The integrand contains the **pole of the Fermi distribution** in the plane of the **complex momentum**. Integrals can be closed at infinity



Chiral vortical effect

CVE for spin 1: problem of the factor 2

- Chiral current for **spin 1** (*magnetic helicity*):

$$K^\mu = \frac{1}{\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma} A_\nu \partial_\rho A_\sigma$$

Other definitions of chirality for spin 1:

[M. N. Chernodub, et al. Phys.Rev. D98 (2018) no.6, 065016]

- **Chiral anomaly for spin 1:**

[A. I. Vainshtein, A. D. Dolgov, V. I. Zakharov, and I. B. Khriplovich, Sov. Phys. JETP 67 (1988) 1326, Zh. Eksp. Teor. Fiz. 94 (1988) 54]

$$< \nabla_\mu K^\mu > = \frac{1}{96\pi^2} R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma}$$

- According to ***[M. Stone and J. Kim, Phys. Rev. D98, no. 2, 025012 (2018).]*** the coefficient before the anomaly determines the coefficient T^2

$$\frac{(CVE)_{photons}}{(CVE)_{Weyl\ fermions}}\Big|_{black\ hole} = 4$$

$$\frac{(CVE)_{photons}}{(CVE)_{Weyl\ spinor}}\Big|_{Kubo\ relation} = 2$$

[A. Avkhadiev, et al. Phys.Rev. D96 (2017) no.4, 045015]

CVE for spin 1: sources of the problem of the factor 2 and possible solutions

- **Infrared** effects and **zero mode**?

Landau levels in *magneic* field:

$$E_n = \pm\sqrt{2H(n+1/2)+P_3^2+H\sigma_3}$$

In the gravitational case the gyromagnetic ratio is **two times smaller** and there is **no zero mode for spin ½.** *However it may exist for spin 1:*

$$E_{min} = 0\,, \quad spin\ 1\,,\ gravity$$

- **Regularization** method?

$$1/R \ll \Omega \ll T$$

one may consider *another* case:

$$\Omega \ll 1/R \ll T$$

[M. N. Chernodub, et al. Phys.Rev. D98 (2018) no.6, 065016]

- The **duality** $T \leftrightarrow \frac{a}{2\pi}$ of temperature and acceleration?

$$J_{CVE} \sim c_1 T^2\Omega + c_2 a^2 \Omega$$

may also give *cubic dependence* on spin S^3 (next slide)

Higher spins

Gravitational anomaly for **arbitrary** spin:

[M. J. Duff, Cambridge Univ. Press, 1982, preprint Ref.TH.3232-CERN]

[A.I. Vainshtein, A.D. Dolgov, V. I. Zakharov, and I.B. Khriplovich, Sov. Phys. JETP 67 (1988) 1326, Zh. Eksp. Teor. Fiz. 94 (1988) 54]

$$\nabla_\mu K^\mu_S = \frac{(-1)^{2S}(2S^3 - S)}{192\pi^2\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

- Used in supersymmetry
- Checked in other approaches for 3/2

Prediction for **vortical** chiral **current** (CVE) of **arbitrary spin**:

$$\vec{K}_S = \frac{(-1)^{2S}(2S^3 - S)}{3} T^2 \vec{\Omega} \ (gravitational \ anomaly)$$

The statistical approach **does not** reproduce the **cubic dependence** S^3

[X. G. Huang, et al. JHEP 03, 084 (2019)]

Conclusions

Conclusions

1. **Quantum** corrections with **acceleration** in the energy-momentum tensor (scalar and Dirac fields, massive and massless) are found.

These corrections:

- **controlled** by the **Unruh effect**
- match the predictions of field theory in **spacetime** with **conical singularity** (cosmic strings)

2. A **jump-like behavior** of the observables is observed around the Unruh temperature.

3. There is a *discrepancy* between the predictions of the **gravitational anomaly** and the **statistical** calculations in the case of the chiral vortical effect for **spin 1** (and higher spins).

Conclusions

The physics of *chiral phenomena* is looking for manifestations of the fundamental effects of **quantum field theory** and **general relativity** in **hydrodynamics** and opens up a unique opportunity to study **quantum anomalies** on the present experimental level.

Thank you for attention!