Holographic Model for Light Quarks in Anisotropic Background

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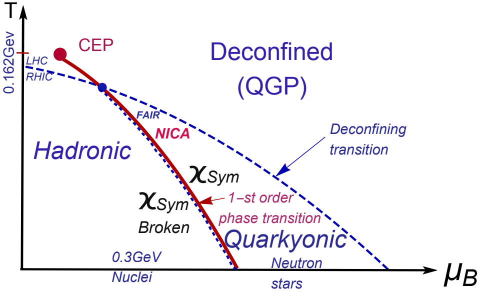
with I.Ya. Aref'eva and P. Slepov
arXiv:2009.05562 [hep-th]

Goal of Holographic QCD — describe QCD phase diagram

Requirements:

- reproduce the QCD results from perturbative theory at short distances,
- reproduce Lattice QCD results at large distances (~ 1 fm) and small density.

I.A. talk, Wednesday



Action and metric

$$\mathcal{S} = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g}\left[R - \frac{f_1(\phi)}{4}\,F_{(1)}^2 - \frac{f_2(\phi)}{4}\,F_{(2)}^2 - \frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

$$F_{\mu\nu}^{(1)} = \partial_\mu A_\nu - \partial_\nu A_\mu,\;\; A_\mu^{(1)} = A_t(z)\delta_\mu^0 \qquad F_{\mu\nu}^{(2)} = q\;dy^1\wedge dy^2,\;\; F_{23}^{(2)} = q$$

$$ds^2 = \frac{L^2}{z^2}\,\mathfrak{b}(z)\left[-\,g(z)dt^2 + dx^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_1^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_2^2 + \frac{dz^2}{g(z)}\right]$$

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I.A., K.R. JHEP **1805** 206 (2018)

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$\mathcal{A}(z)=-\,cz^2/4\rightarrow$ heavy quarks (b, t) Andreev, Zakharov (2006)

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$\mathcal{A}(z) = -\,cz^2/4 \to$ heavy quarks (b, t) Andreev, Zakharov (2006)

$\mathcal{A}(z) = -\,a\ln(bz^2+1) \to$ light quarks (d, u) Li, Yang, Yuan (2017)

Solution for anisotropic metric ansatz

$$\mathcal{A}(z) = -\,a\ln(bz^2+1) \qquad f_1 = e^{-cz^2-\mathcal{A}(z)}\; z^{-2+\frac{2}{\nu}}$$

Boundary conditions:

$$A_t(0)=\mu,\;\; A_t(z_h)=0, \qquad g(0)=1,\;\; g(z_h)=0, \qquad \phi(z_0)=0$$

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next talk by P.S.

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next talk by P.S.

$$A_t = \mu\ \frac{e^{cz^2}-e^{cz_h^2}}{1-e^{cz_h^2}}$$

$$g = 1 - \frac{\int_0^z \left(1+b\,\xi^2\right)^{3a} \xi^{1+\frac{2}{\nu}}\,d\xi}{\int_0^{z_h} \left(1+b\,\xi^2\right)^{3a} \xi^{1+\frac{2}{\nu}}\,d\xi} + \frac{2\mu^2 c}{L^2\left(1-e^{cz_h^2}\right)^2}\int_0^z e^{c\xi^2}\left(1+b\,\xi^2\right)^{3a}\xi^{1+\frac{2}{\nu}}\,d\xi\ \times$$
$$\times\left[1-\frac{\int_0^z \left(1+b\,\xi^2\right)^{3a} \xi^{1+\frac{2}{\nu}}\,d\xi}{\int_0^{z_h} \left(1+b\,\xi^2\right)^{3a} \xi^{1+\frac{2}{\nu}}\,d\xi}\ \frac{\int_0^{z_h} e^{c\xi^2}\left(1+b\,\xi^2\right)^{3a} \xi^{1+\frac{2}{\nu}}\,d\xi}{\int_0^{z} e^{c\xi^2}\left(1+b\,\xi^2\right)^{3a} \xi^{1+\frac{2}{\nu}}\,d\xi}\right].$$

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$$\phi = \int_{z_0}^{z} \frac{2\sqrt{\nu - 1 + (2(\nu-1) + 9a\nu^2)\, b\,\xi^2 + (\nu - 1 + 3a(1+2a)\nu^2)\, b^2\,\xi^4}}{(1+b\xi^2)\,\nu\,\xi}\, d\xi$$

$$\nu > 1: \; z_0 \to 0 \;\to\; \phi(z) \sim \int_0^z dz/z$$

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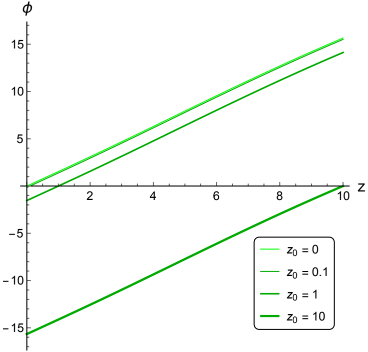
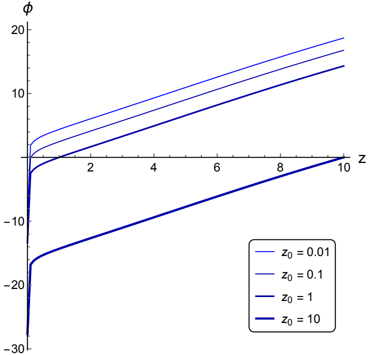
$\nu = 1$



$\nu = 4.5$



Black hole temperature $\longleftrightarrow$ QGP temperature (Maldacena)

$$T = \frac{|g'|}{4\pi}\Big|_{z=z_h} = \frac{1}{4\pi}\left| - \frac{\left(1+bz_h^2\right)^{3a} z_h^{1+\frac{2}{\nu}}}{\int_0^{z_h} \left(1+b\,\xi^2\right)^{3a} \xi^{1+\frac{2}{\nu}}\, d\xi} \left[1 - \frac{2\mu^2 c\, e^{2cz_h^2}}{L^2\left(1-e^{cz_h^2}\right)^2} \times \right.\right.$$

$$\left.\left. \times \left(1 - e^{-cz_h^2} \frac{\int_0^{z_h} e^{c\xi^2} \left(1+b\,\xi^2\right)^{3a} \xi^{1+\frac{2}{\nu}}\, d\xi}{\int_0^{z_h} \left(1+b\,\xi^2\right)^{3a} \xi^{1+\frac{2}{\nu}}\, d\xi}\right) \int_0^{z_h} \left(1+b\,\xi^2\right)^{3a} \xi^{1+\frac{2}{\nu}}\, d\xi \right]\right|$$

Thermodynamics

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$$\left.\left.\times\left(1 - e^{-cz_h^2}\,\frac{\int_0^{z_h} e^{c\xi^2}\left(1+b\,\xi^2\right)^{3a}\xi^{1+\frac{2}{\nu}}\,d\xi}{\int_0^{z_h}\left(1+b\,\xi^2\right)^{3a}\xi^{1+\frac{2}{\nu}}\,d\xi}\right)\int_0^{z_h}\left(1+b\,\xi^2\right)^{3a}\xi^{1+\frac{2}{\nu}}\,d\xi\right]\right|$$

Entropy $\longleftrightarrow$ muliplicity of process (Landau)

$$s = \left(\frac{L}{z_h}\right)^{1+\frac{2}{\nu}}\frac{\left(1+bz_h^2\right)^{-3a}}{4}$$

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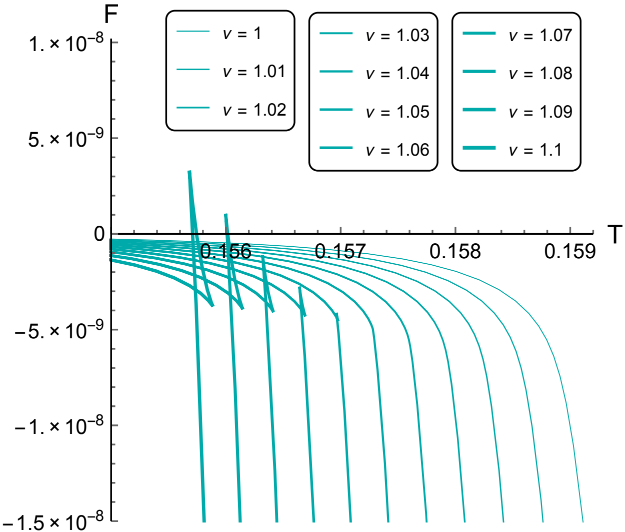
Entropy $\longleftrightarrow$ muliplicity of process (Landau)
Free energy behavior $\longrightarrow$ phase transitions

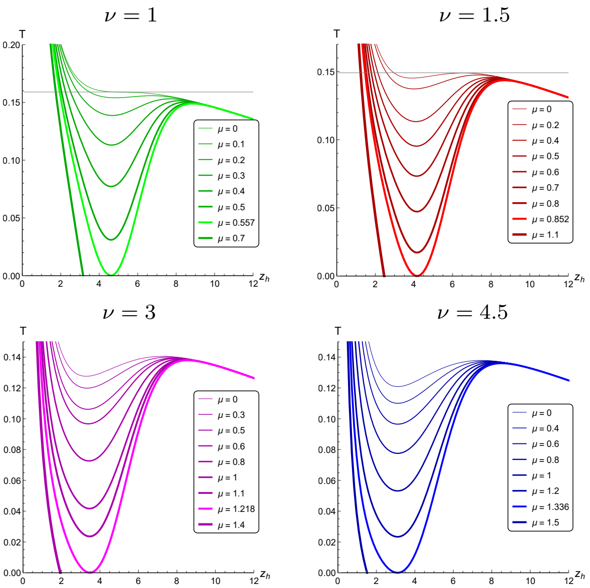
$$s=\left(\frac{L}{z_h}\right)^{1+\frac{2}{\nu}}\frac{\left(1+bz_h^2\right)^{-3a}}{4}\qquad\qquad F=\int_{z_h}^{z_{h_2}}s\,dT=\int_{z_h}^{z_{h_2}}s\,T'dz$$

Peculiar features of model $\longleftarrow$ in $T(z_h)$ *(I.A. talk, Wednesday)*

Free energy: $\mu = 0$



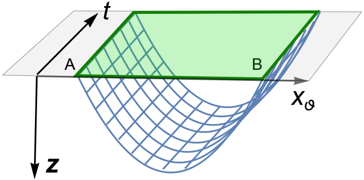
Temperature

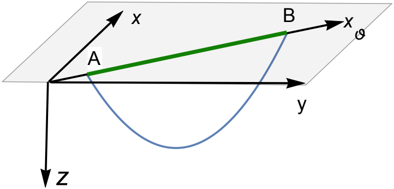


Temporal Wilson loops

Nambu-Goto action for strings

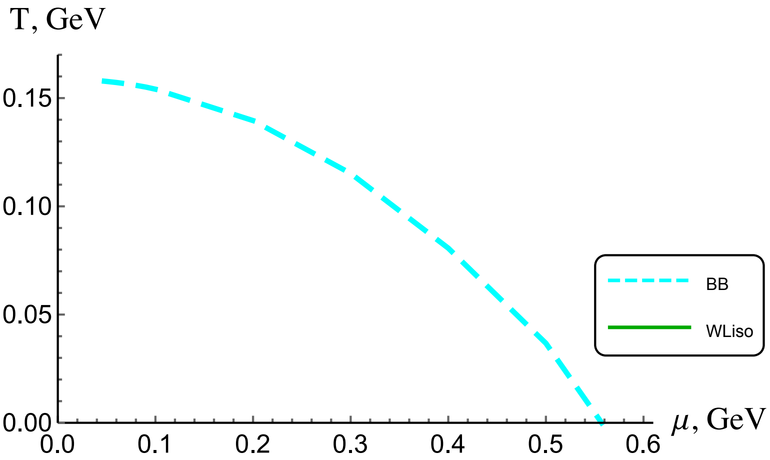
$$S \sim \sigma_{DW}\ \ell$$





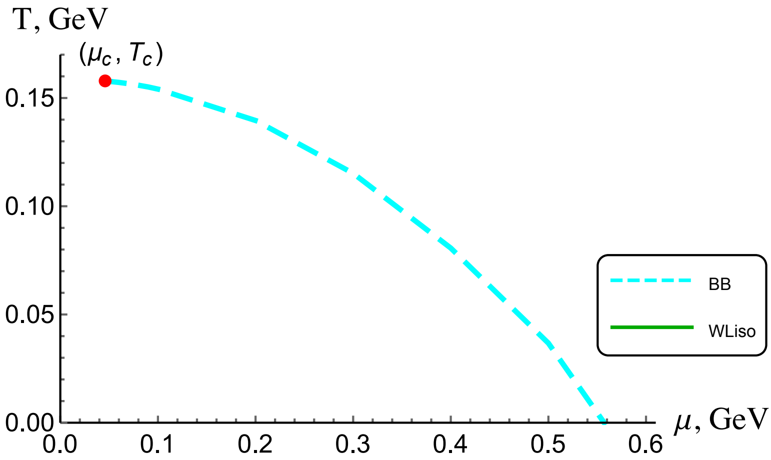
$$X^0 \equiv t,\ X^1 \equiv x = \xi\cos\theta,\ X^2 \equiv y_1 = \xi\sin\theta,\ X^3 \equiv y_2 = const,\ X^4 \equiv z = z(\xi)$$

Phase diagram: $\nu = 1$



Phase diagram: $\nu = 1$

CEP: $\mu_c = 0.04779,\ T_c = 0.1578$



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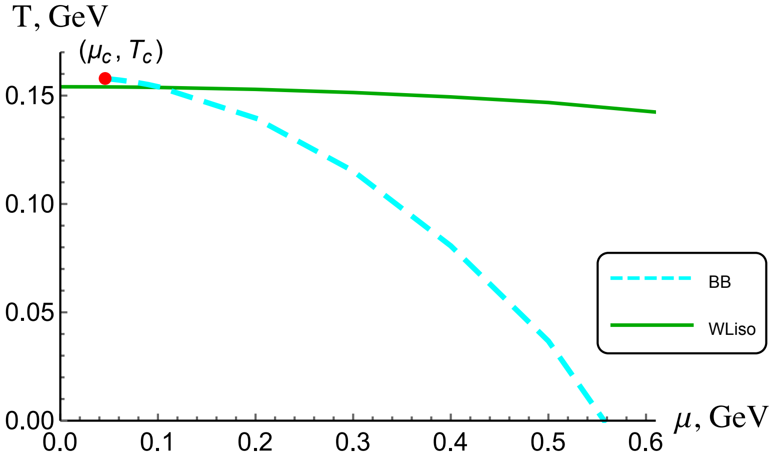

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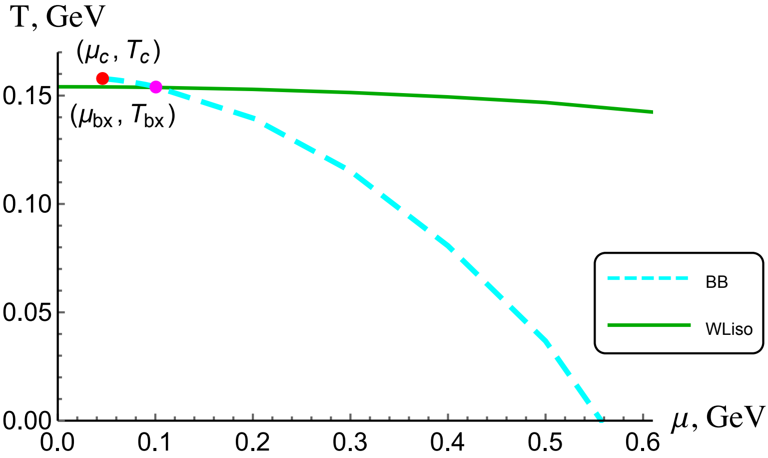

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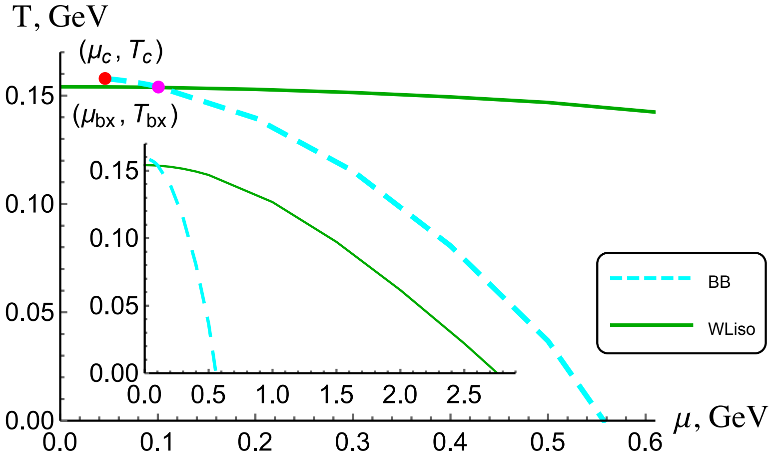
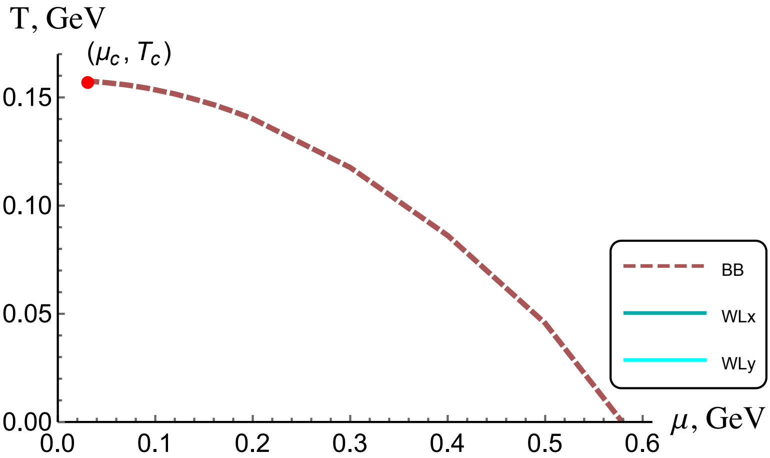


Phase diagram: $\nu = 1.03$

CEP: $\mu_c = 0.031,\ T_c = 0.1575$



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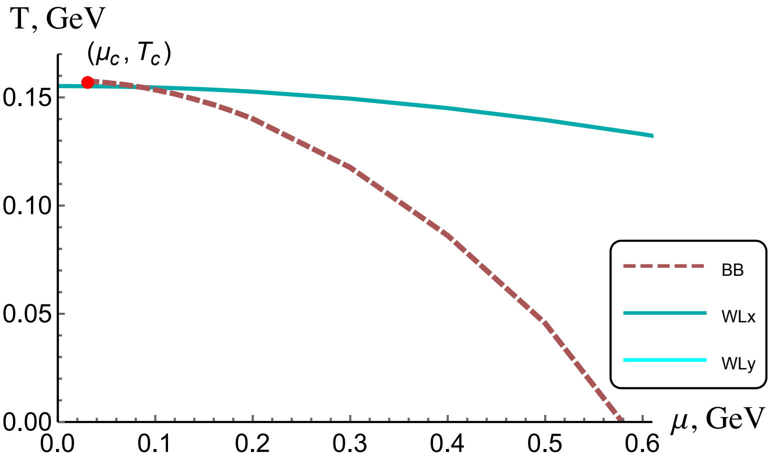

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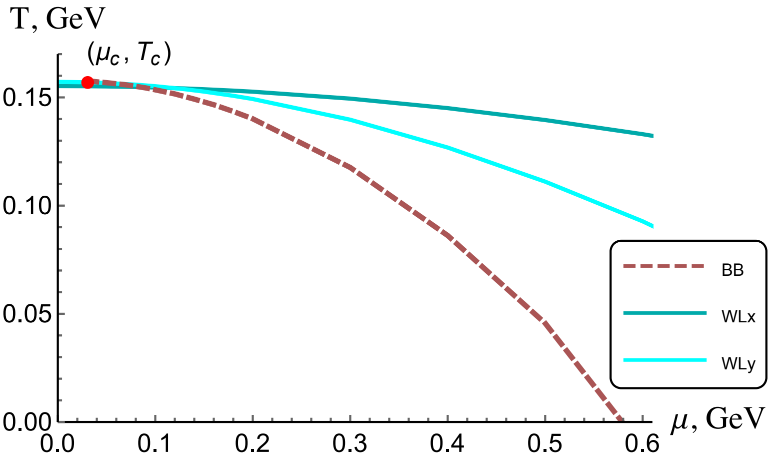

Phase diagram: $\nu = 1.03$

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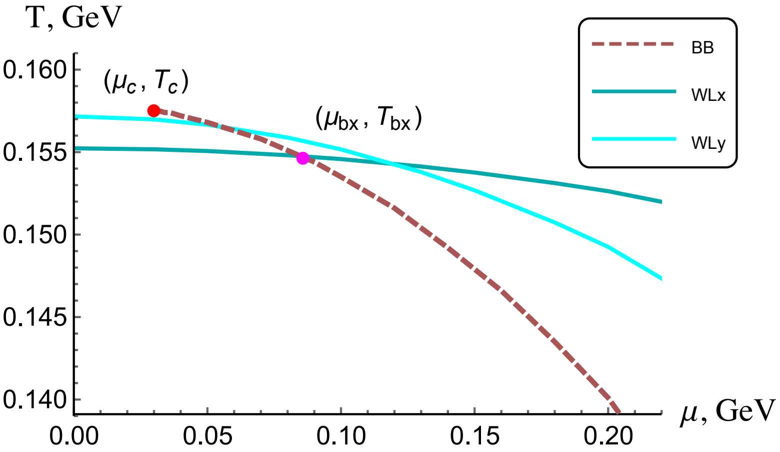

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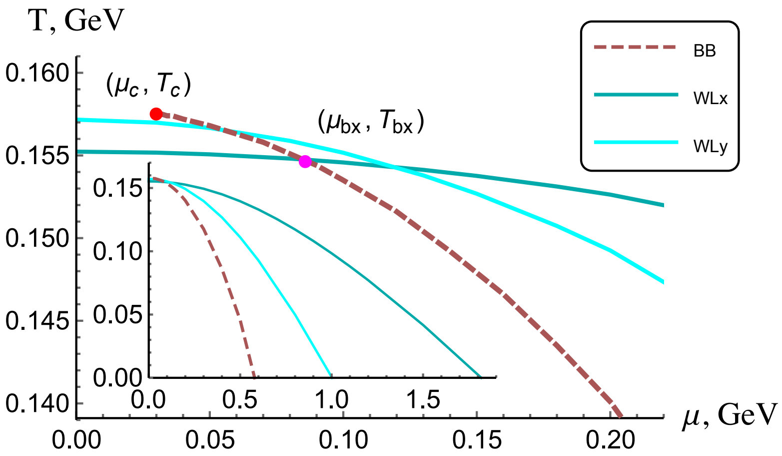
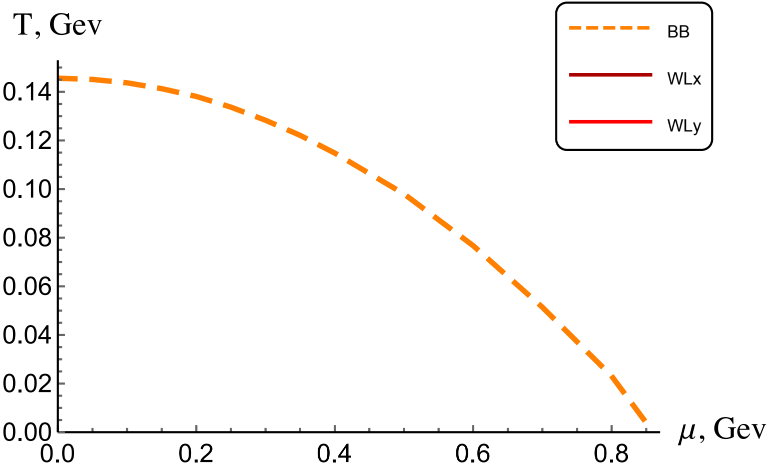


Phase diagram: $\nu = 1.5$

No CEP



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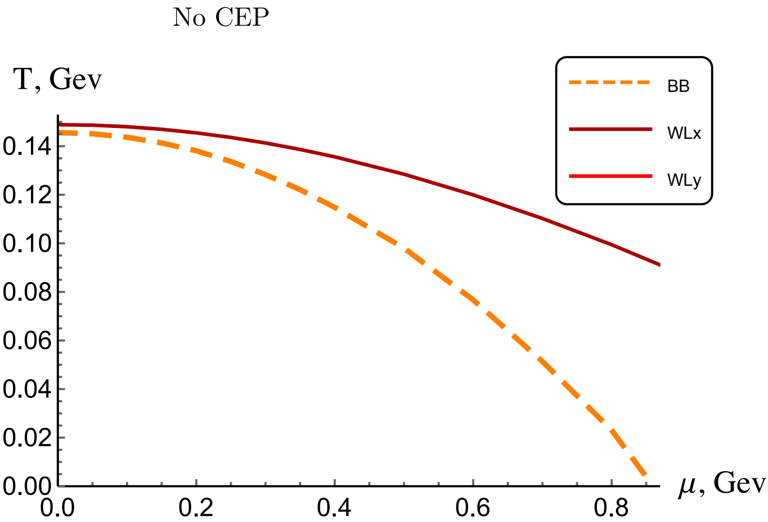

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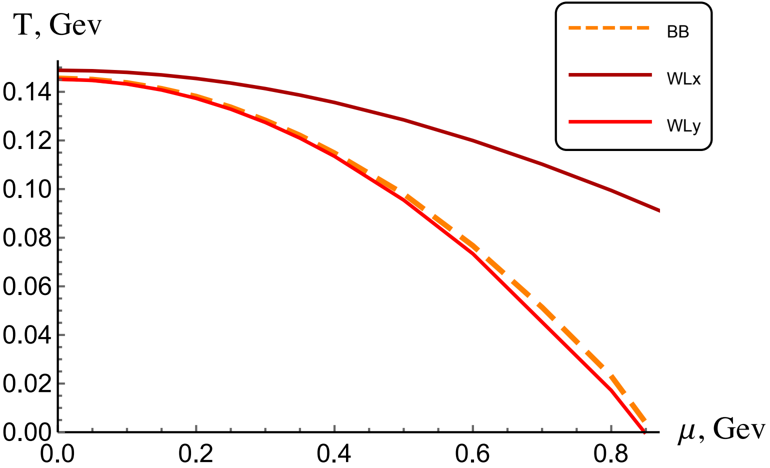
No CEP   No BY-crossection



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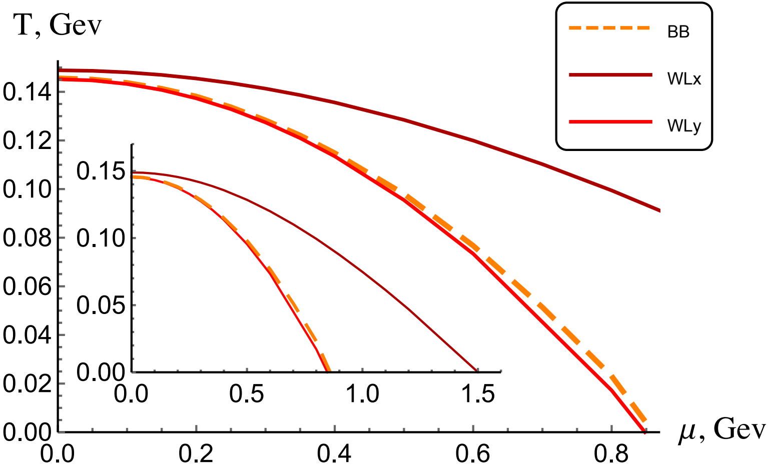
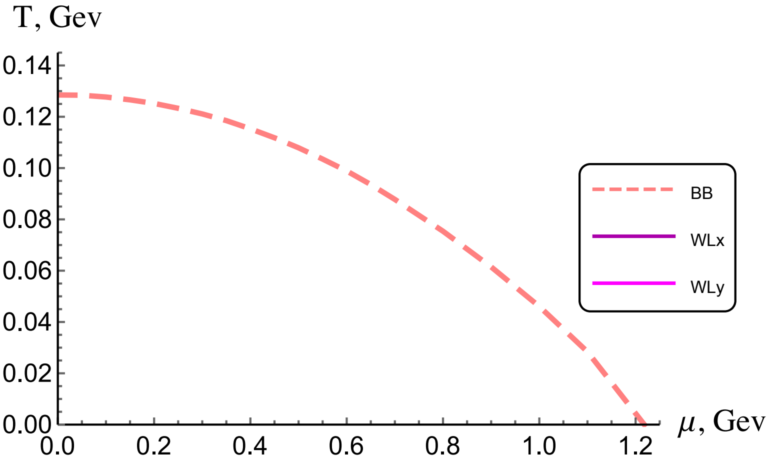


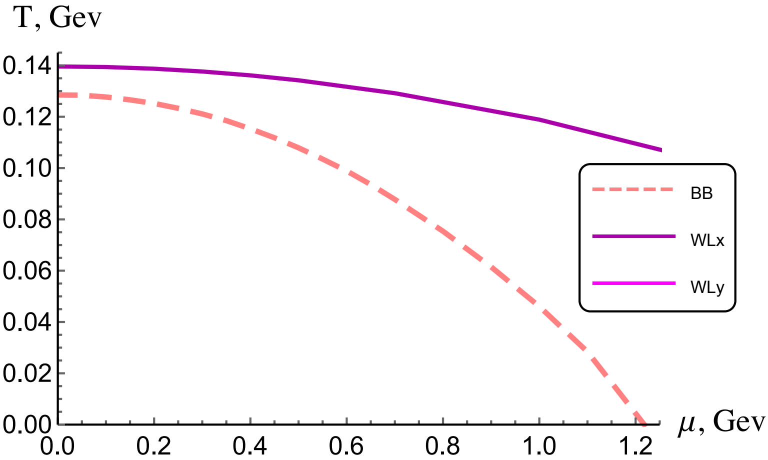
Phase diagram: $\nu = 3$

No CEP



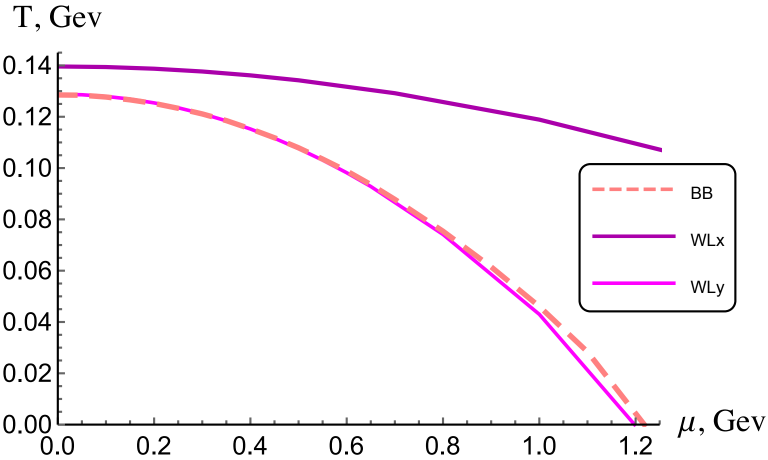
Phase diagram: $\nu = 3$

No CEP



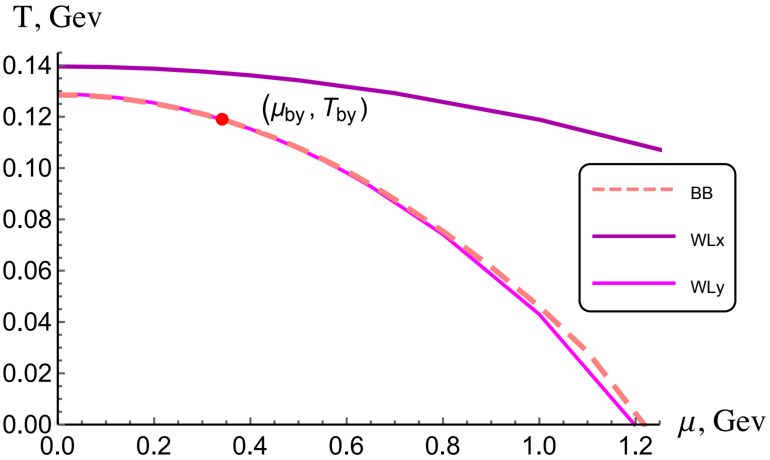
Phase diagram: $\nu = 3$

No CEP



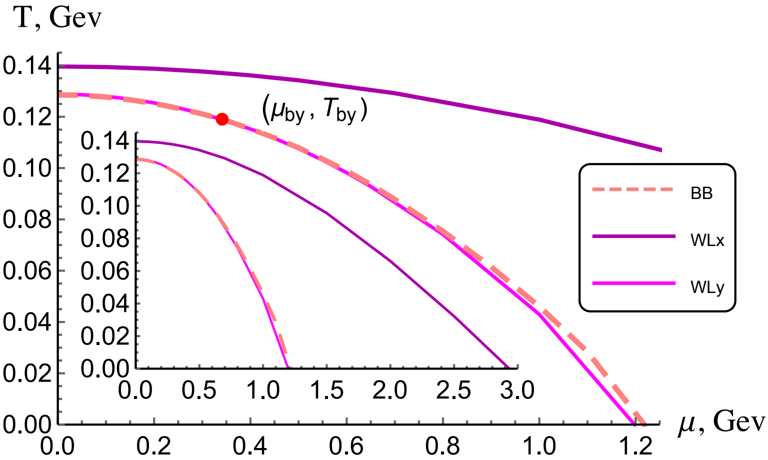
Phase diagram: $\nu = 3$

No CEP $\mu_{by} = 0.3244, \ T_{by} = 0.1198$



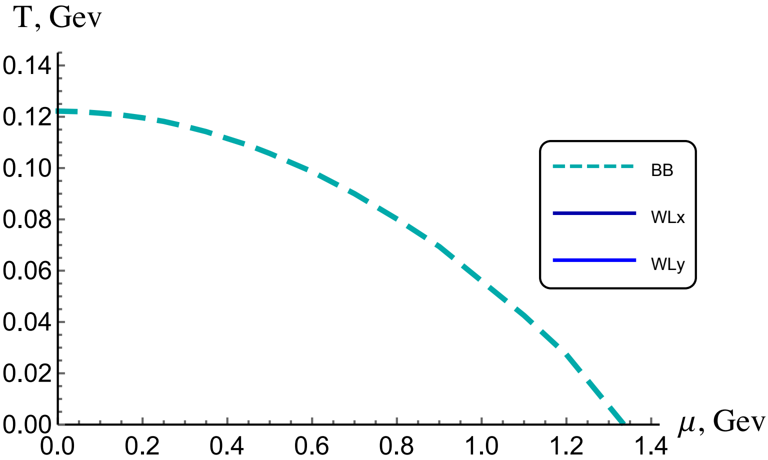
Phase diagram: $\nu = 3$

No CEP $\qquad \mu_{by} = 0.3244, \; T_{by} = 0.1198$



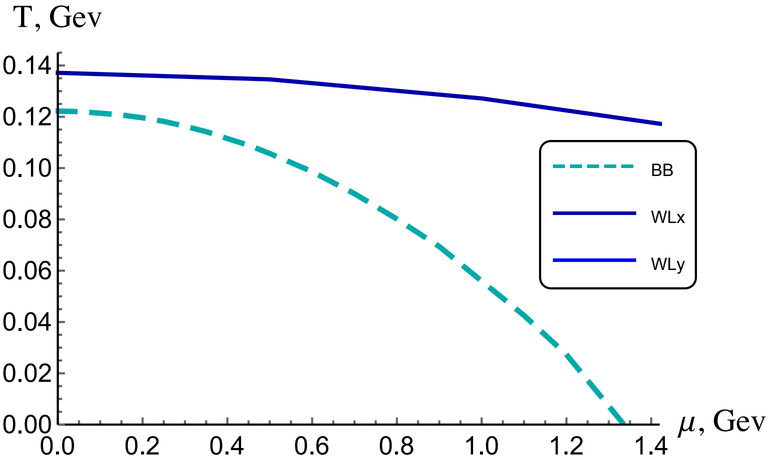
Phase diagram: $\nu = 4.5$

No CEP



Phase diagram: $\nu = 4.5$

No CEP



Phase diagram: $\nu = 4.5$

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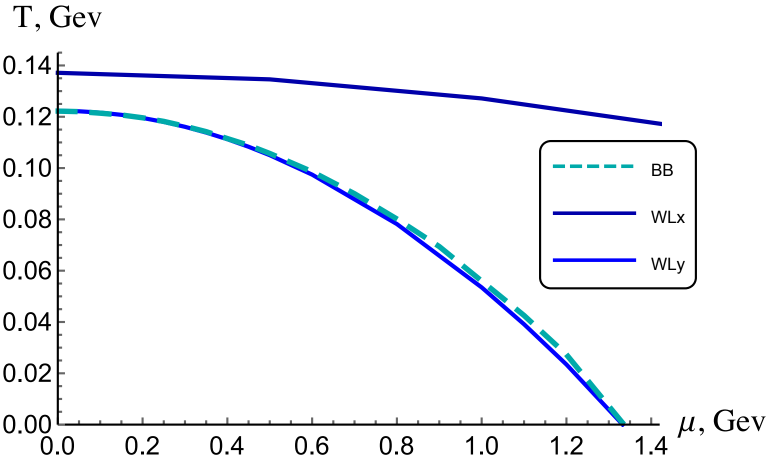
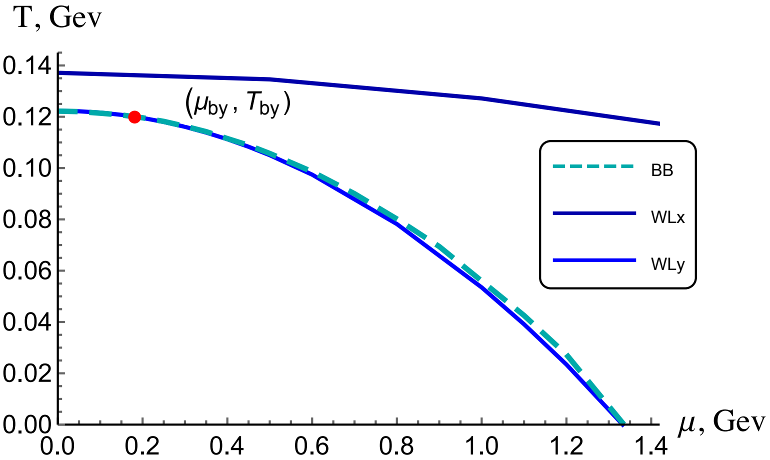


Phase diagram: $\nu = 4.5$

No CEP $\qquad \mu_{by} \approx 0.1748, \ T_{by} \approx 0.1201$



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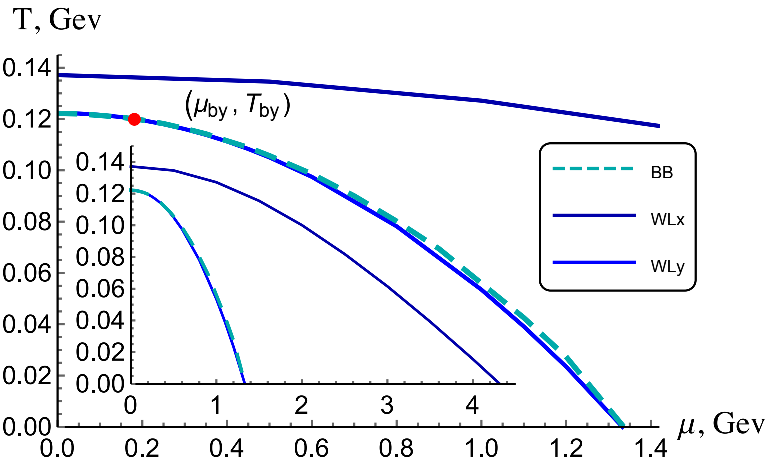

Conclusions

For light quarks

1. 1-st order (Hawking-Page-like) phase transition line
   - starts from critical point for $\nu < 1.05$ and from $\mu = 0$ for $\nu \geq 1.05$,
   - does not break at a relatively high temperature, but lasts till $T = 0$.
2. Longitudinal orientation of quarks pairs does not contribute to confinement/deconfinement phase transition.
3. Transfer of the main role in the phase transition smooth, without jumps (as it was in heavy quarks model).

Conclusions

For light quarks

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Conclusions

For light quarks

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Conclusions

For light quarks

1. 1-st order (Hawking-Page-like) phase transition line
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What's next?

Hybrid model: heavy&light quarks — all together now

Thank you for your attention

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