Phase diagram structure and kaon-to-pion ratios in the entanglement SU(3) PNJL model in Breit-Wigner and Beth-Uhlenbeck approaches

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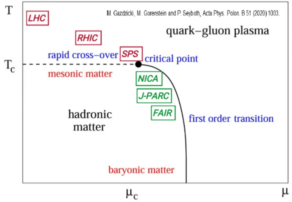
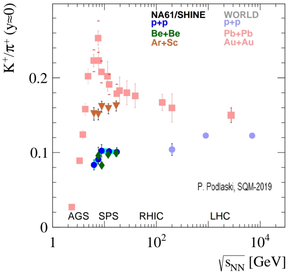
Content

Preface





- Onset of deconfinement (possible signal): characteristic enhanced production of pions $\Rightarrow$ suppression of the strangeness-to-pion ratio (a jump to plateau is a signal of deconfinement and QGP formation (SMES M. Gazdzicki, M.I. Gorenstein, Acta Phys. Pol. B30, 2705 (1999)).
- Low energies: first order transition (?)
  the quick increase at low energies is a result of the partial chiral symmetry restoration (A. Palmese, et al. PRC 94, 044912 (2016)- PHSD; K. Bugaev - statistical model; J. Nayak - microscopic model).
- The system-size dependence of the K/π ratio (was shown by PHSD group))
  was found by NA69/SHINE collaboration
- The fireball creation (?)

Motivation

- The most intriguing region of the QCD phase diagram is a subject of the nonperturbative study.
- We need a model that is capable to describe the matter properties at finite T and μ_{B} in the nonpeturbative region.
- The NJL model is a successful effective model, which describes the spontaneous chiral symmetry breaking, formation of the quark condensate and the chiral phase transition.
- Polyakov loop extention solves the problem of a lack of deconfinement.
- To describe the fluctuation we need to go beyond mean field approximation (that's why we use the Beth-Uhlenbeck approach).

SU(3) PNJL model

The Lagrangian (Ratti, Thaler, Weise PRD (2006), P. Costa et al. PRD79, 116003 (2009); A. Friesen et al. Phys.-Usp 59, 367 (2017)):

$$\begin{aligned}\mathcal{L} &= \bar{q}\,(\,i\,\gamma^{\mu}\,D_{\mu}\,-\,\hat{m}-\gamma_0\mu)\,q+\frac{1}{2}\,G_s\,\sum_{a=0}^{8}[\,(\,\bar{q}\,\lambda^{a}\,q\,)^2\;+\;(\,\bar{q}\,i\,\gamma_5\,\lambda^{a}\,q\,)^2\,]\\ &+\;K\,\{\det[\bar{q}\,(\,1\,+\,\gamma_5\,)\,q\,]+\det[\bar{q}\,(\,1\,-\,\gamma_5\,)\,q\,]\,\}-\mathcal{U}(\Phi,\bar{\Phi};T)\end{aligned}$$

$D_{\mu}=\partial^{\mu}-iA^{\mu}$, where A^{μ} is the gauge field with $A^{0}=-iA_4$ and $A^{\mu}(x)=G_sA^{\mu}_{a}\frac{\lambda_a}{2}$
The effective potential has to reproduce the Lattice calculation in the pure gauge sector:

$$\begin{aligned}\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} &= -\frac{b_2\left(T\right)}{2}\bar{\Phi}\Phi-\frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right)+\frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2,\\ b_2\left(T\right) &= a_0+a_1\left(\frac{T_0}{T}\right)+a_2\left(\frac{T_0}{T}\right)^2+a_3\left(\frac{T_0}{T}\right)^3\;.\end{aligned}$$

We can:

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$\mathrm{D}_{\mu} = \partial^{\mu} - \mathrm{i}\mathrm{A}^{\mu}$, where A^{μ} is the gauge field with $\mathrm{A}^0 = -\mathrm{i}\mathrm{A}_4$ and $\mathrm{A}^{\mu}(\mathrm{x}) = \mathrm{G_s}\mathrm{A}^{\mu}_{\mathrm{a}}\frac{\lambda_{\mathrm{a}}}{2}$
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We can:

- explain and describe spontaneous chiral symmetry breaking as $\mathrm{m_q} = \mathrm{m}_0 + 2\mathrm{G_s} < \bar{\mathrm{q}}\mathrm{q} >$;

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$$b_2\left(T\right)=a_0+a_1\left(\frac{T_0}{T}\right)+a_2\left(\frac{T_0}{T}\right)^2+a_3\left(\frac{T_0}{T}\right)^3.$$

We can:

- explain and describe spontaneous chiral symmetry breaking as $m_q=m_0+2G_s<\bar{q}q>$;
- simulate the confinement/deconfinement transition

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We can:

- explain and describe spontaneous chiral symmetry breaking as $m_q=m_0+2G_s<\bar{q}q>$;
- simulate the confinement/deconfinement transition
- build the phase diagram with crossover at low chemical potential and 1st order transition at high chemical potential ($m_0\neq 0$),

The mean-field approximation

The grand potential density:

$$\begin{aligned}\Omega &= \mathcal{U}(\Phi,\bar{\Phi};T) + G_s \sum_{i=u,d,s} \langle \bar{q}_i q_i \rangle^2 + 4K \langle \bar{q}_u q_u \rangle \langle \bar{q}_d q_d \rangle \langle \bar{q}_s q_s \rangle - 2N_c \sum_{i=u,d,s} \int \frac{d^3p}{(2\pi)^3} E_i - \\ &- 2T \sum_{i=u,d,s} \int \frac{d^3p}{(2\pi)^3} (N^+_\Phi(E_i) + N^-_\Phi(E_i))\end{aligned}$$

with the functions

$$\begin{aligned}N^+_\Phi(E_i) &= \mathrm{Tr}_c \left[\ln(1 + L^\dagger e^{-\beta(E_i - \mu_i)})\right] = \left[1 + 3\left(\Phi + \bar{\Phi} e^{-\beta E_i^+}\right) e^{-\beta E_i^+} + e^{-3\beta E_i^+}\right], \\ N^-_\Phi(E_i) &= \mathrm{Tr}_c \left[\ln(1 + L e^{-\beta(E_p + \mu_i)})\right] = \left[1 + 3\left(\bar{\Phi} + \Phi e^{-\beta E_p^-}\right) e^{-\beta E_p^-} + e^{-3\beta E_p^-}\right],\end{aligned}$$

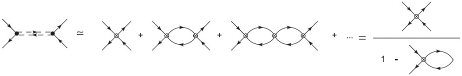
where $E_i^\pm = E_i \mp \mu_i$, $\beta = 1/T$, $E_i = \sqrt{{p_i}^2 + m_i^2}$ is the energy of quarks and $\langle \bar{q}_i q_i \rangle$ is the quark condensate.
The equations of motion

$$\frac{\partial \Omega}{\partial \sigma_f} = 0, \quad \frac{\partial \Omega}{\partial \Phi} = 0, \frac{\partial \Omega}{\partial \bar{\Phi}} = 0.$$

and gap equations:

$$m_i = m_{0i} + 4G < \bar{q}_i q_i > +2K < \bar{q}_j q_j >< \bar{q}_k q_k >$$

The mesons mass $\mu_B = 0$



The meson masses are defined by the Bethe-Salpeter equation at $P = 0$

$$1 - P_{ij}\Pi^P_{ij}(P_0 = M, P = 0) = 0 \ ,$$

with

$$P_\pi = G_s + K \langle \bar{q}_s q_s \rangle \ , \quad P_K = G_s + K \langle \bar{q}_u q_u \rangle$$

and the polarization operator:

$$\Pi^P_{ij}(P_0) = 4 \left((I^i_1 + I^j_1) - [P_0^2 - (m_i - m_j)^2] \ I^{ij}_2(P_0) \right) ,$$

where

$$I^i_1 = iN_c \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m_i^2}, \quad I^{ij}_2(P_0) = iN_c \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m_i^2)((p+P_0)^2 - m_j^2)}$$

When $T > T_{Mott}$ $(P_0 > m_i + m_j)$ the meson $\rightarrow$ the resonance state $\rightarrow$
$P_0 = M_M - 1/2i\Gamma_M$.

The mesons mass $\mu_B = 0$: result

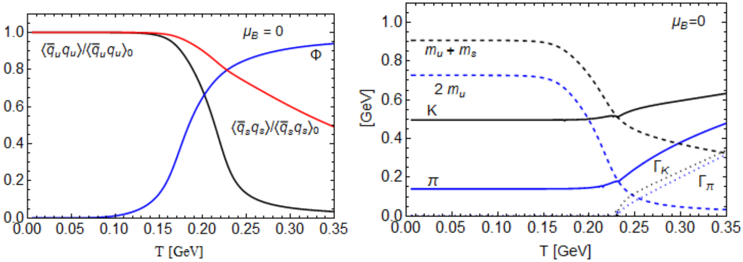
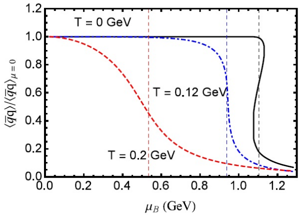
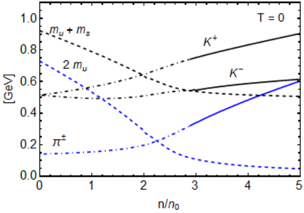


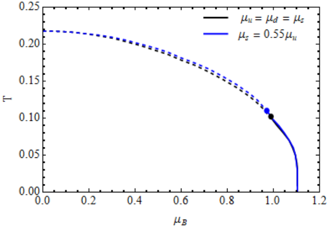
Figure 1: The mass spectra at zero mu$_B$

The model with finite μ_{B} and density

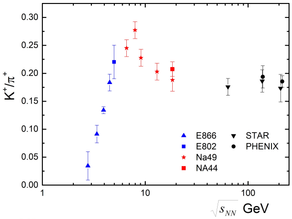
$$\mu_{\mathrm{u}} = \mu_{\mathrm{d}}; \mu_{\mathrm{s}} = 0.55\mu_{\mathrm{u}}, \quad \mu_{\mathrm{B}} = 3\mu_{\mathrm{u}}, \quad \rho_{\mathrm{B}} = \frac{\rho_{\mathrm{u}} + \rho_{\mathrm{d}} + \rho_{\mathrm{s}}}{3}$$

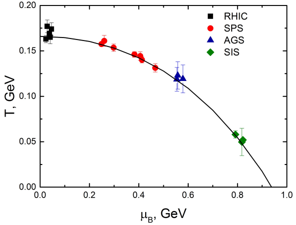






Can we apply the model to the 'horn' description?

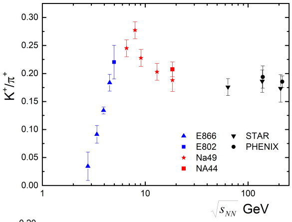


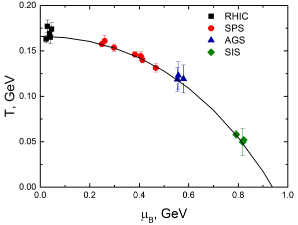


The model approach:

- all mesons were created during hadronization and we skip the rescattering, decays and so on..
- freeze-out line is coincide with the chiral phase transition line (it is an ansatz)
- Experiment: for each energy of collision we can find T^* and μ_B^* of the freeze-out
- Experiment: we can rescale the data as function of T^*/μ_B^*
- Theory: now we can calculate the kaon to pion ratio as a function T/μ_B where T and μ_b are chosen along the phase transition line.

Can we apply the model to the 'horn' description?





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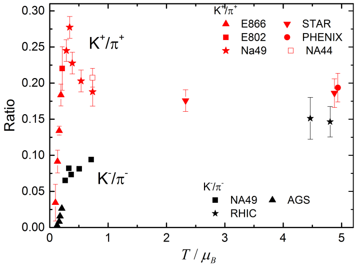
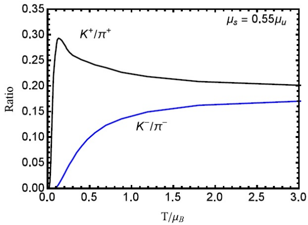
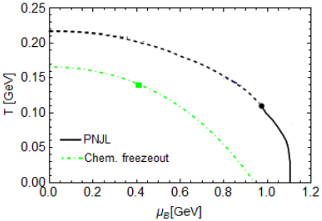


Kaon to pion ratio in PNJL model (Breit-Wigner approximation)





$$
\begin{aligned}
n_{K^\pm} &= \int_0^\infty p^2 dp \frac{1}{e^{(\sqrt{p^2+m_{K^\pm}} \mp \mu_{K^\pm})/T} - 1}, \\
n_{\pi^\pm} &= \int_0^\infty p^2 dp \frac{1}{e^{(\sqrt{p^2+m_{\pi^\pm}} \mp \mu_{\pi^\pm})/T} - 1}.
\end{aligned}
$$

with parameters:

$\mu_\pi = 0.135$ (M. Kataja, P.V. Ruuskanen PLB 243, 181 (1990))

$\mu_K = \mu_u - \mu_s$ (see for example A. Lavagno and D. Pigato, EPJ Web of Conferences 37, 09022 (2012)).

A. V. Friesen, Yu. L. Kalinovsky, V. D. Toneev PRC 99, 045201 (2019)

Phase diagram improvements and kaon to pion ratio

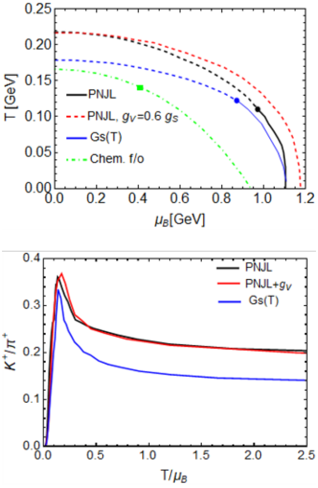


Figure 2: $\mu_s = 0.55\mu_u$

- introduction of a phenomenological dependence of $G_s(\Phi)$

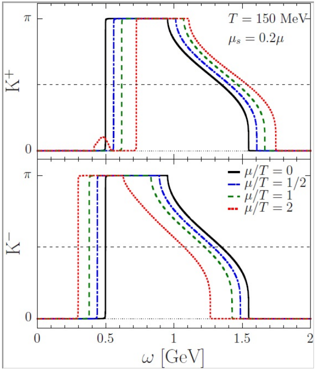
$$\tilde{G}_s(\Phi) = G_s[1 - \alpha_1 \Phi\bar{\Phi} - \alpha_2(\Phi^3 + \bar{\Phi}^3)]$$

  with $\alpha_1 = \alpha_2 = 0.2$. (Y. Sakai et al PRD 82, 076003 (2010), P. de Forcrand, O. Philipsen NPB 642, 290(2002), A. Friesen et al. IJMPA30, 1550089 (2015).)
- the effect of vector interaction:

$$\mathcal{L}_V = -\frac{1}{2} G_v \sum_{a=0}^{8} (\bar{q}\gamma_\mu \lambda^a q)^2,$$

$$\tilde{G}_v(\Phi) = G_v[1 - \alpha_1 \Phi\bar{\Phi} - \alpha_2(\Phi^3 + \bar{\Phi}^3)].$$

The medium effect to the kaon to pion ratio



- The meson spectra beyond the mean field approximation can be obtained from the "polar" representation

$$\mathcal{S}_{\rm ij}^{\rm M}(\omega, {\rm q}) = |\mathcal{S}_{\rm ij}^{\rm M}(\omega, {\rm q})| \, {\rm e}^{{\rm i}\delta_{\rm M}(\omega, {\rm q})}, \qquad (1)$$

with mesonic phase shift has the form

$$\delta_{\rm M}(\omega, \bar{\rm q}) = -\arctan\{\frac{{\rm Im}[\mathcal{S}_{\rm M}(\omega - {\rm i}\eta, \bar{\rm q})]^{-1}}{{\rm Re}[\mathcal{S}_{\rm M}(\omega + {\rm i}\eta, \bar{\rm q})]^{-1}}\}$$

- the bound state appears at the energy, where the phase shift jumps up by the value π (Levinson's theorem, Wergieluk:2012gd).

(see for discussion A. Dubinin, D. Blaschke, A. Radzhabov Phys. Rev. D 96, 094008 (2017))

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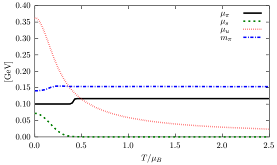
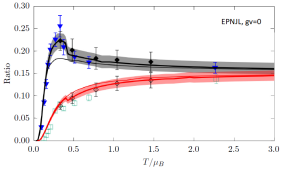
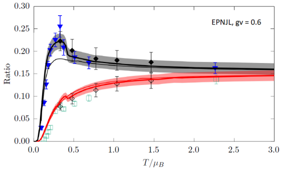






- The off-shell generalization of the number density of the bosonic species,

$$\mathrm{n_M(T)} = \frac{\mathrm{d_M}}{\mathrm{T}} \int \frac{\mathrm{dq\,q^2}}{2\pi^2} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \mathrm{g_M}(\omega)(1 + \mathrm{g_M}(\omega))\delta_\mathrm{M}(\omega$$

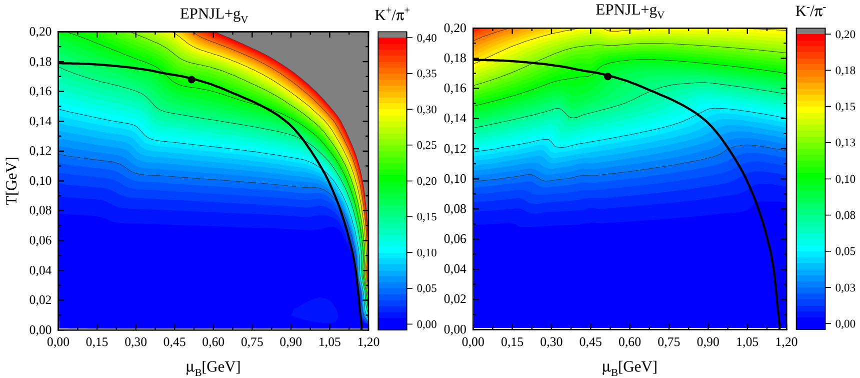
- the non-equilibrium medium dependent chemical potential for pion and strange quark (Wood-Saxon form, $\mathrm{x} = \mathrm{T}/\mu_\mathrm{B}$)

$$\begin{aligned} \mu_\pi(\mathrm{x}) &= \mu_\pi^{\mathrm{min}} + \frac{\mu_\pi^{\mathrm{max}} - \mu_\pi^{\mathrm{min}}}{1 + \exp(-(\mathrm{x} - \mathrm{x}_\pi^{\mathrm{th}})/\Delta \mathrm{x}_\pi))}, \\ \mu_\mathrm{s}(\mathrm{x}) &= \frac{\mu_\mathrm{s}^{\mathrm{max}}}{1 + \exp(-(\mathrm{x} - \mathrm{x}_\mathrm{s}^{\mathrm{th}})/\Delta \mathrm{x}_\mathrm{s}))}. \end{aligned}$$

parameters fitted to the experiment;

(see for discussion D. Blascke, G. Röpke, D.N. Voskresensky, Particles 3(2020) 29)

Meson fluctuation and K/π ratio



Results and discussion

- Splitting of kaons masses at high densities $\Rightarrow$ the difference in the behaviour of the K/π at low energies;
- The peak depends on properties of the matter (strangeness neutrality, or chemical baryon potential of strange quark);
- Both K/π ratios are almost unaffected to the change of the order of the chiral phase from first order to crossover (but more sensitive to the slope of the phase diagram);
- The sharpness of the "horn" is well explained by a Bose-enhanced pion production for $\sqrt{s_{\mathrm{NN}}} > 8$ GeV;
- Breit-Wigner approximation not good for yields in the vicinity of the Mott transition (i.e. freeze-out) because it neglects the continuum which Beth-Uhlenbeck approach takes into account.


Спасибо
Gracias
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Obrigado
Спасибо
Dank U
Grazie
Ευχαριστώ
Danke
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Thank You
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