Stability of shock waves in anisotropic hydrodynamics

Kovalenko Aleksandr, Lebedev Physical Institute

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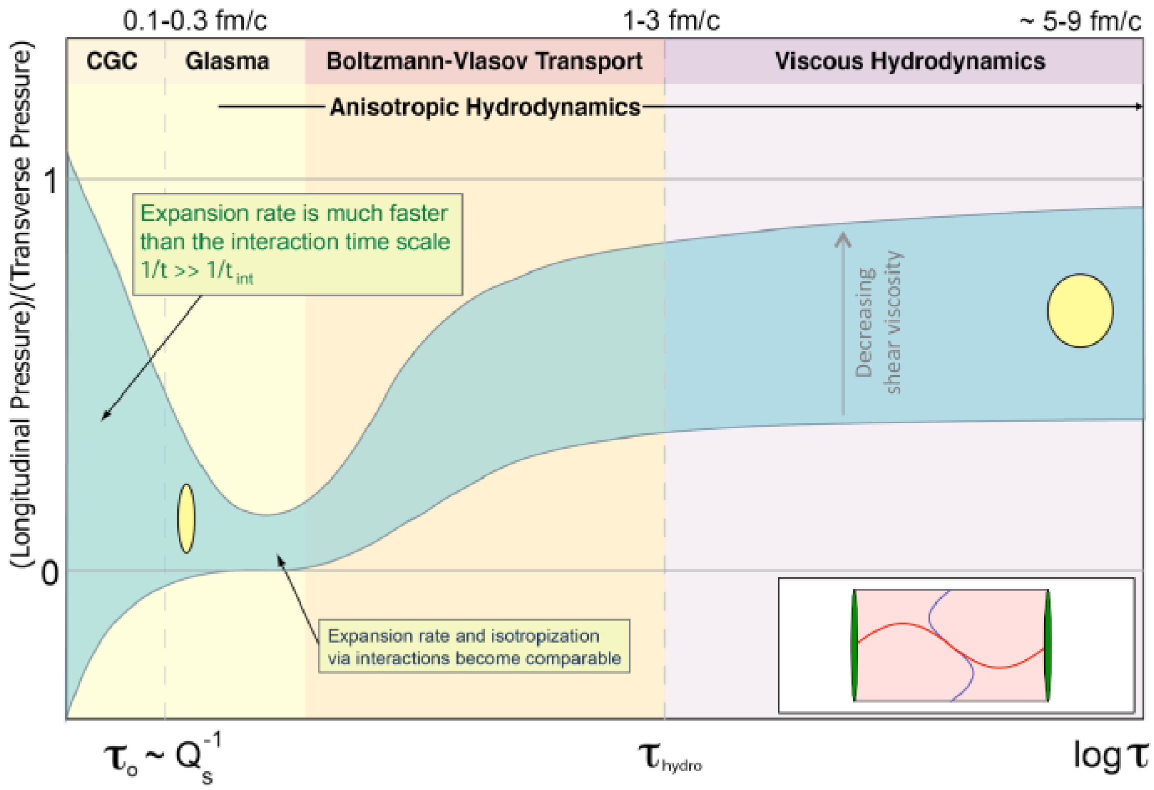
Anisotropy in heavy-ion collisions

$$P_{\parallel}/P_{\perp} \lesssim 0.3$$



Michael Strickland. «Anisotropic Hydrodynamics: Three lectures.» arXiv:1410.5786

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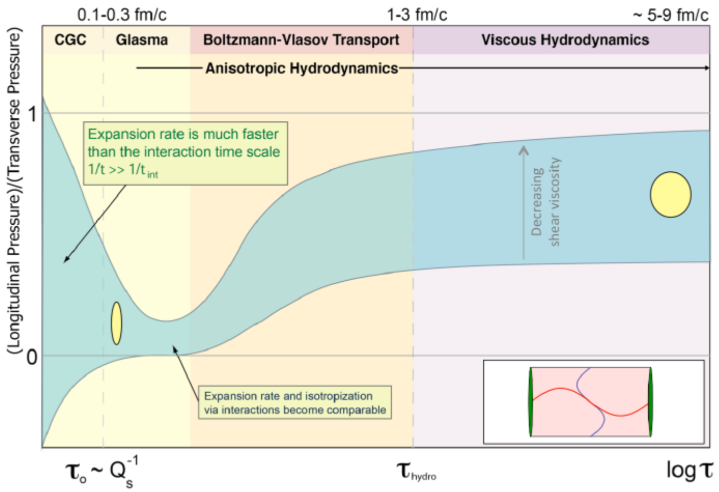

- anisotropy required large correction to the viscos hydrodynamics
- there are regions of the phase space where the one-particle distribution function is negative

Michael Strickland. «Anisotropic Hydrodynamics: Three lectures.» arXiv:1410.5786

Relativistic anisotropic hydrodynamics

$$f_{aniso}(x,p) = f_{iso}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\Lambda(x)}, \frac{\mu(x)}{\Lambda(x)}\right)$$

$$\varepsilon = 2P_\perp + P_\parallel$$

ultra-relativistic case

LRF: $p^\mu \Xi_{\mu\nu}(x) p^\nu \longrightarrow \mathbf{p}^2 + \xi(x) p_\parallel$

$$T^{\mu\nu} = (\varepsilon + P_\perp) U^\mu U^\nu - P_\perp g^{\mu\nu} - (P_\perp - P_\parallel) Z^\mu Z^\nu$$

where

$$U^\mu = (u_0 \cosh\vartheta, u_x, u_y, u_0 \sinh\vartheta)$$
$$X^\mu = (u_\perp \cosh\vartheta, \frac{u_0 u_x}{u_\perp}, \frac{u_0 u_y}{u_\perp}, u_\perp \sinh\vartheta)$$
$$Y^\mu = (0, -\frac{u_y}{u_\perp}, \frac{u_x}{u_\perp}, 0)$$
$$Z^\mu = (\sinh\vartheta, 0, 0, \cosh\vartheta)$$

$\longrightarrow$

$$U^\mu_{LRF} = (1,0,0,0)$$
$$X^\mu_{LRF} = (0,1,0,0)$$
$$Y^\mu_{LRF} = (0,0,1,0)$$
$$Z^\mu_{LRF} = (0,0,0,1)$$

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Relativistic anisotropic hydrodynamics

$$P_\perp(\xi) = R_\perp(\xi) P_{iso}$$

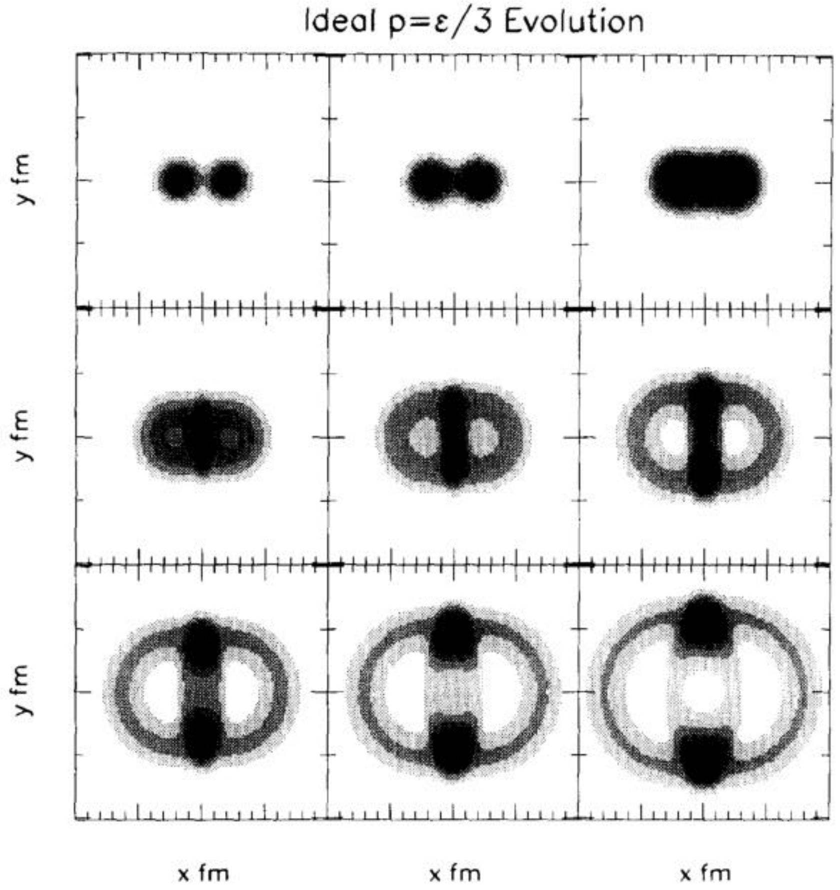
$$P_\parallel(\xi) = R_\parallel(\xi) P_{iso}$$

$$R_\perp(\xi) = \frac{3}{2\xi}\left(\frac{1+(\xi^2-1)R(\xi)}{1+\xi}\right)$$

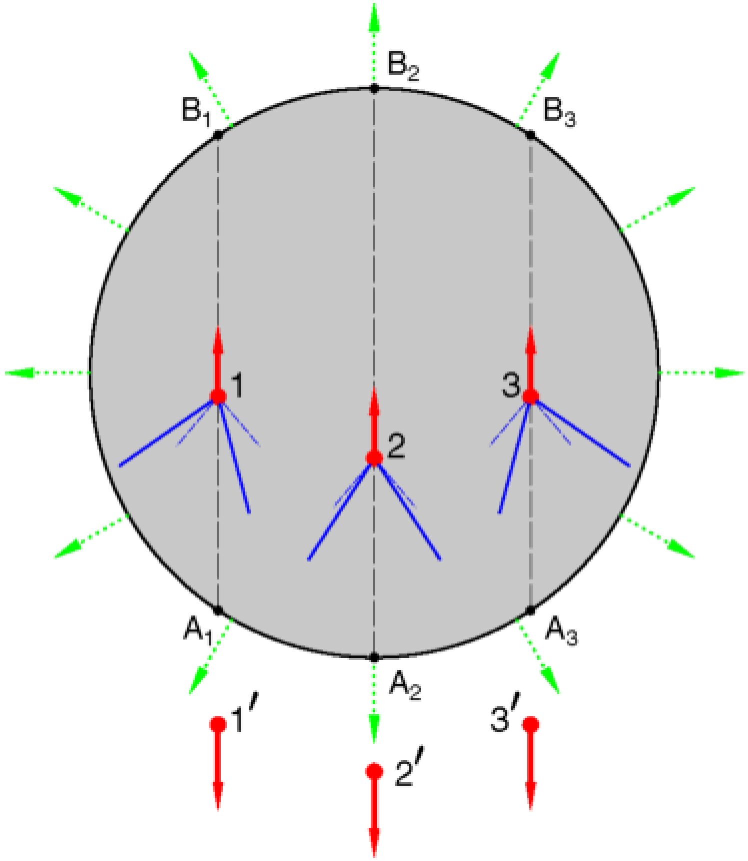
$$R_\parallel(\xi) = \frac{3}{\xi}\left(\frac{(\xi+1)R(\xi)-1}{1+\xi}\right)$$

$$R(\xi) = 2R_\perp(\xi) + R_\parallel(\xi) = \frac{1}{2}\left(\frac{1}{1+\xi} + \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right)$$

Shock waves in heavy-ion collisions

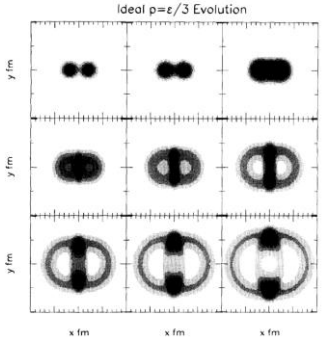


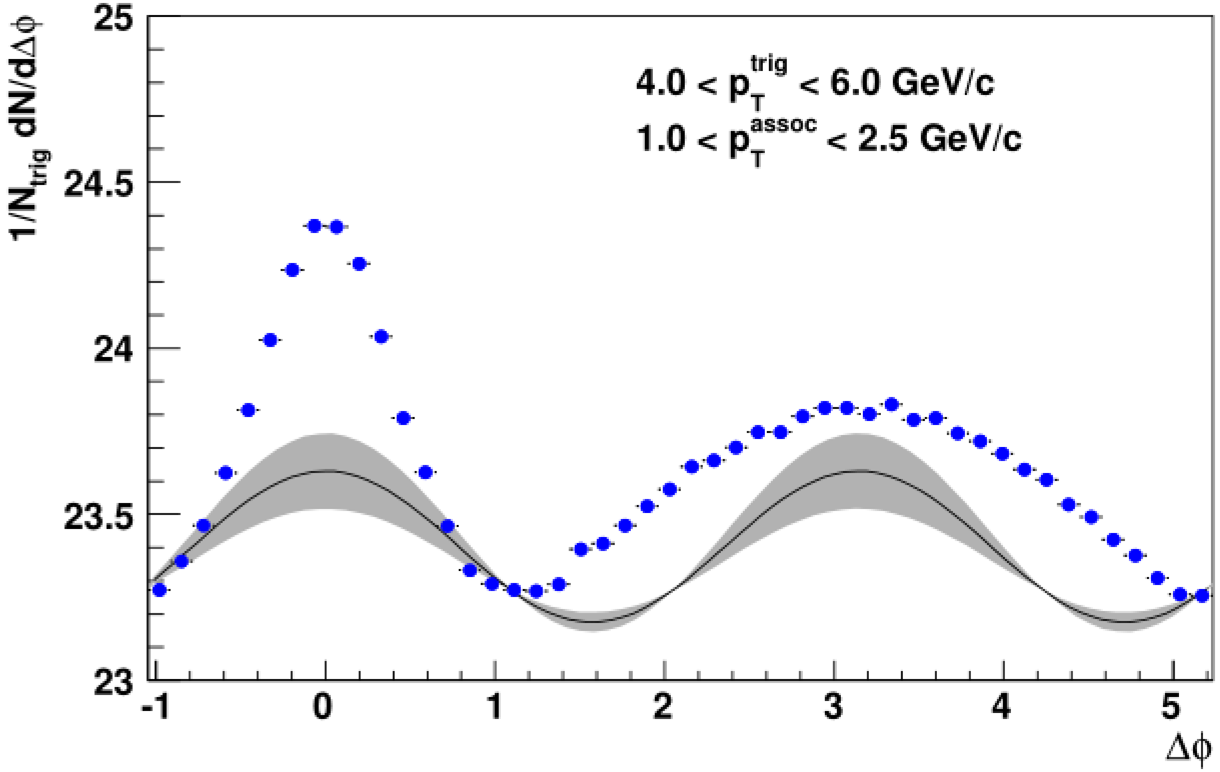
Miklos Gyulassy, Dirk H. Rischke, Bin Zhang
«Hot spots and turbulent initial conditions of quark-gluon plasmas in nuclear collisions»

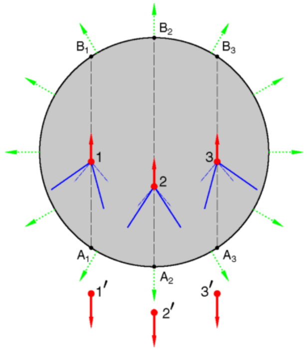


Satarov, L.M. and Stoecker, Horst and Mishustin, I.N.
«Mach shocks induced by partonic jets in expanding quark-gluon plasma»

Shock waves in heavy-ion collisions





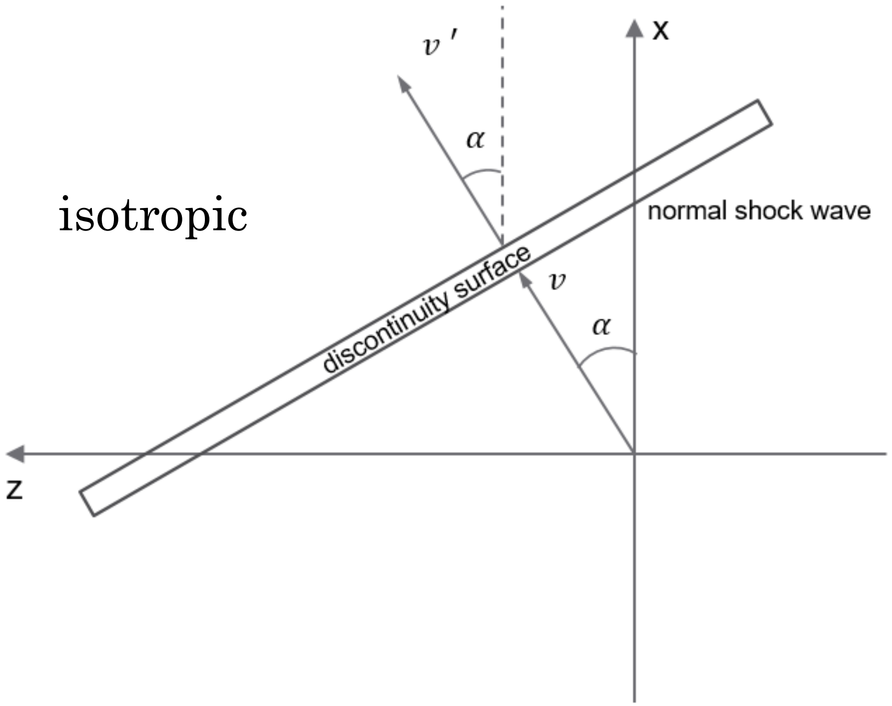


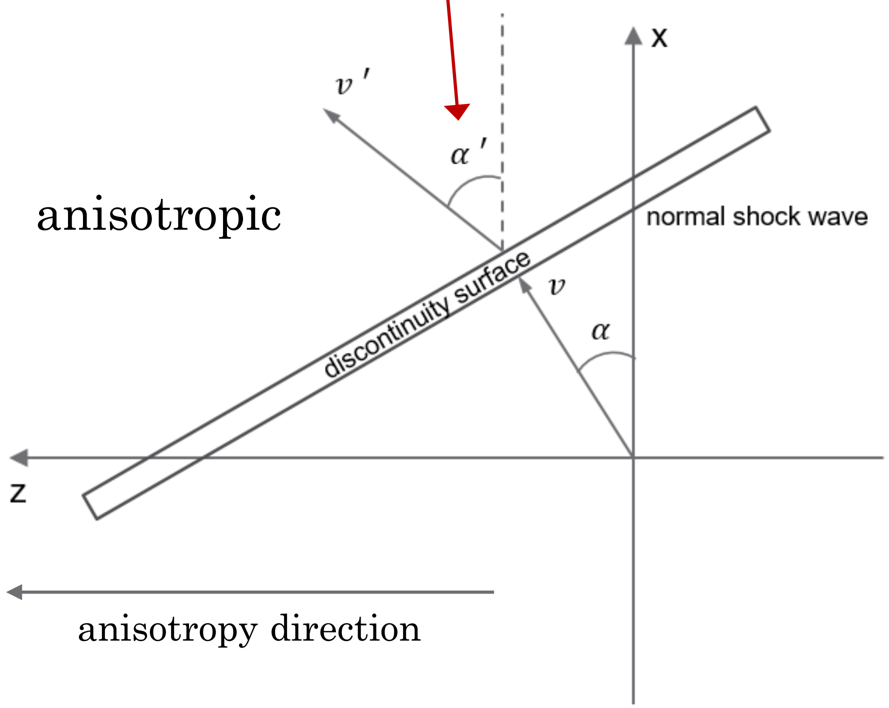
«Azimuthal di-hadron correlations in d+Au and
Au+Au collisions at √ s NN = 200 GeV from STAR»

Junction condition

$$T_{\mu\nu}N^{\mu} = T^{'}_{\mu\nu}N^{\mu}$$

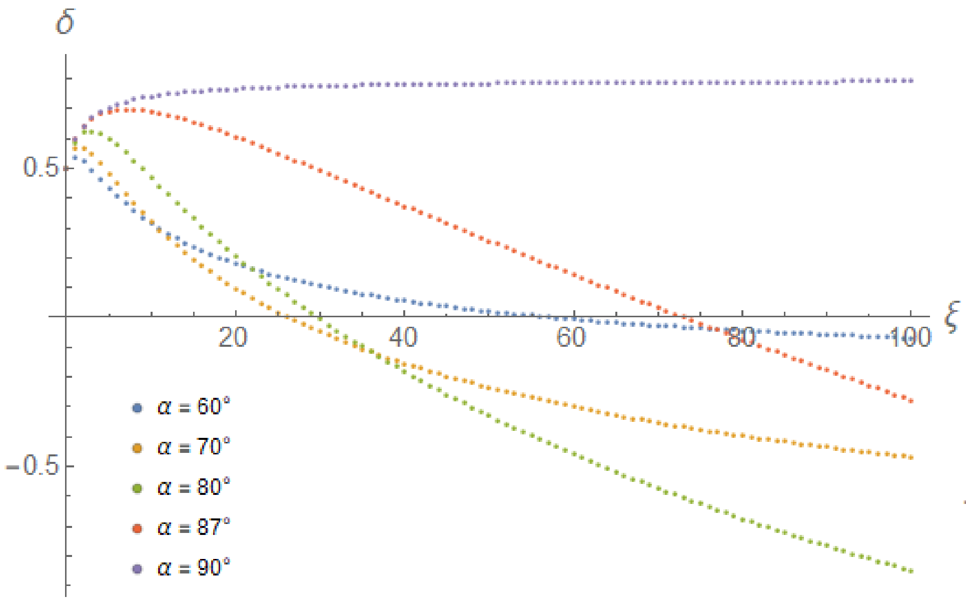
$N^{\mu} = (0, \cos\alpha, 0, \sin\alpha)$ - normal vector to the discontinuity surface

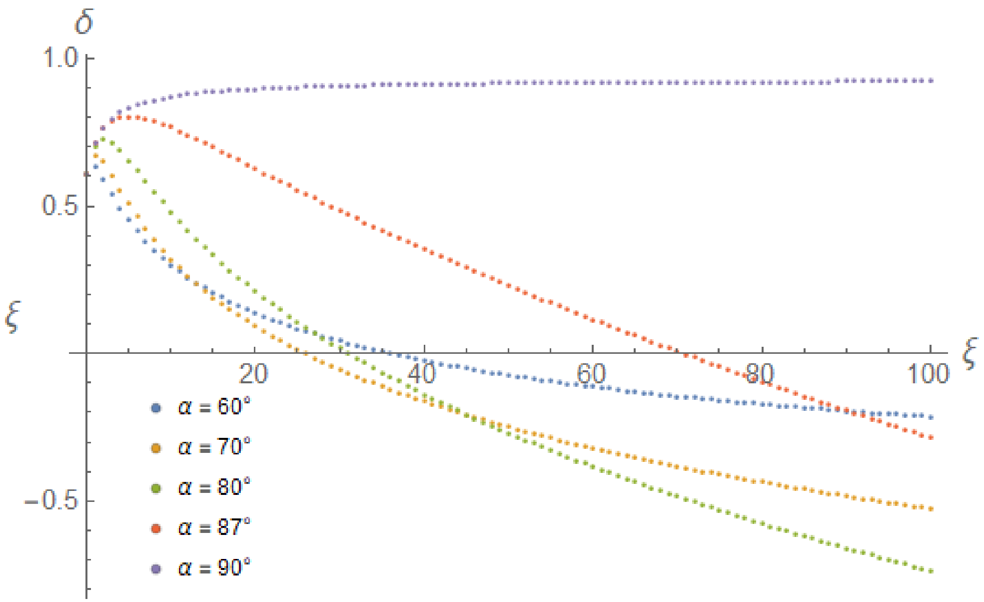




Junction condition

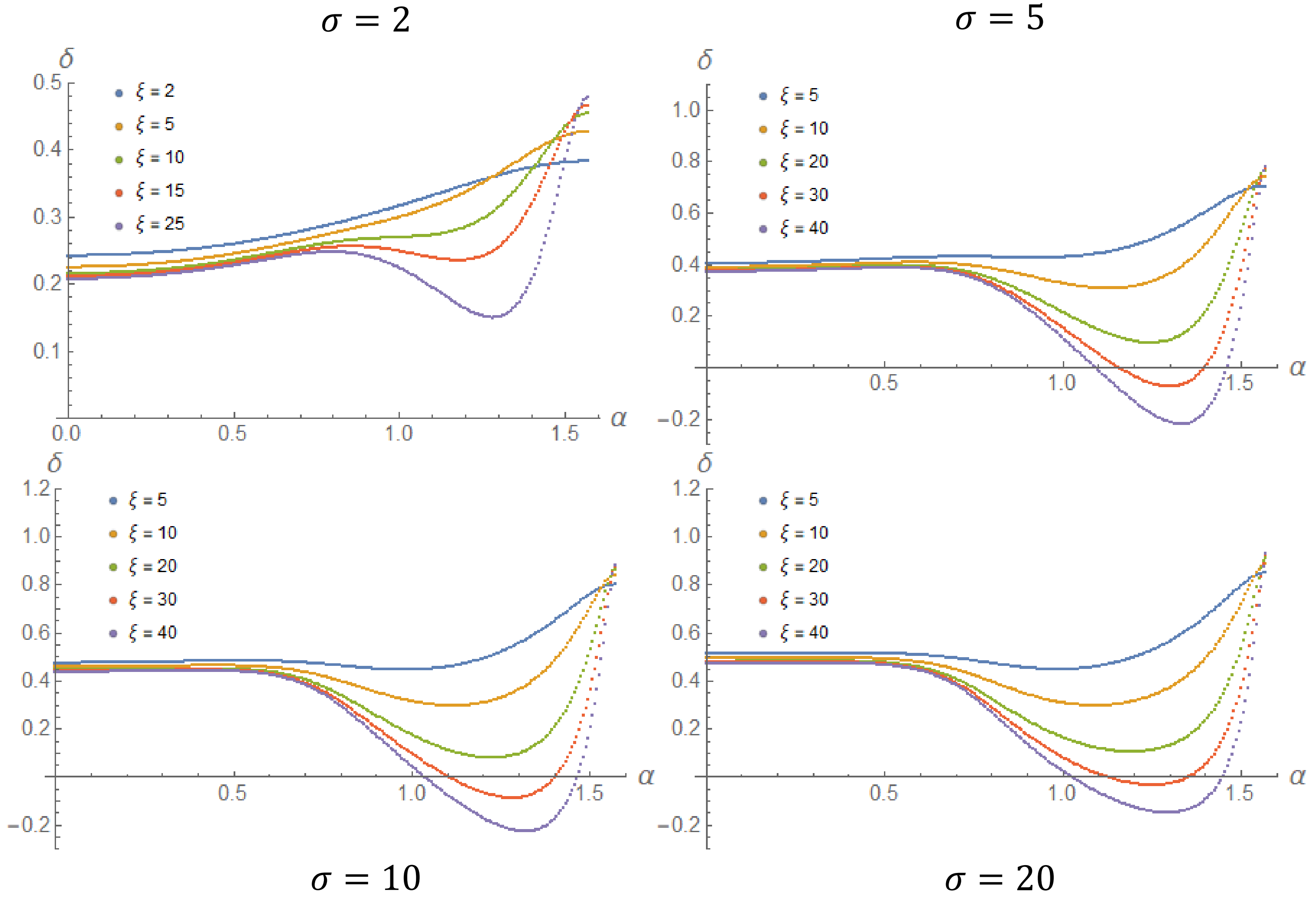
$$\delta = \frac{v - v'}{v} \qquad \sigma = P'_{iso}/P_{iso}$$





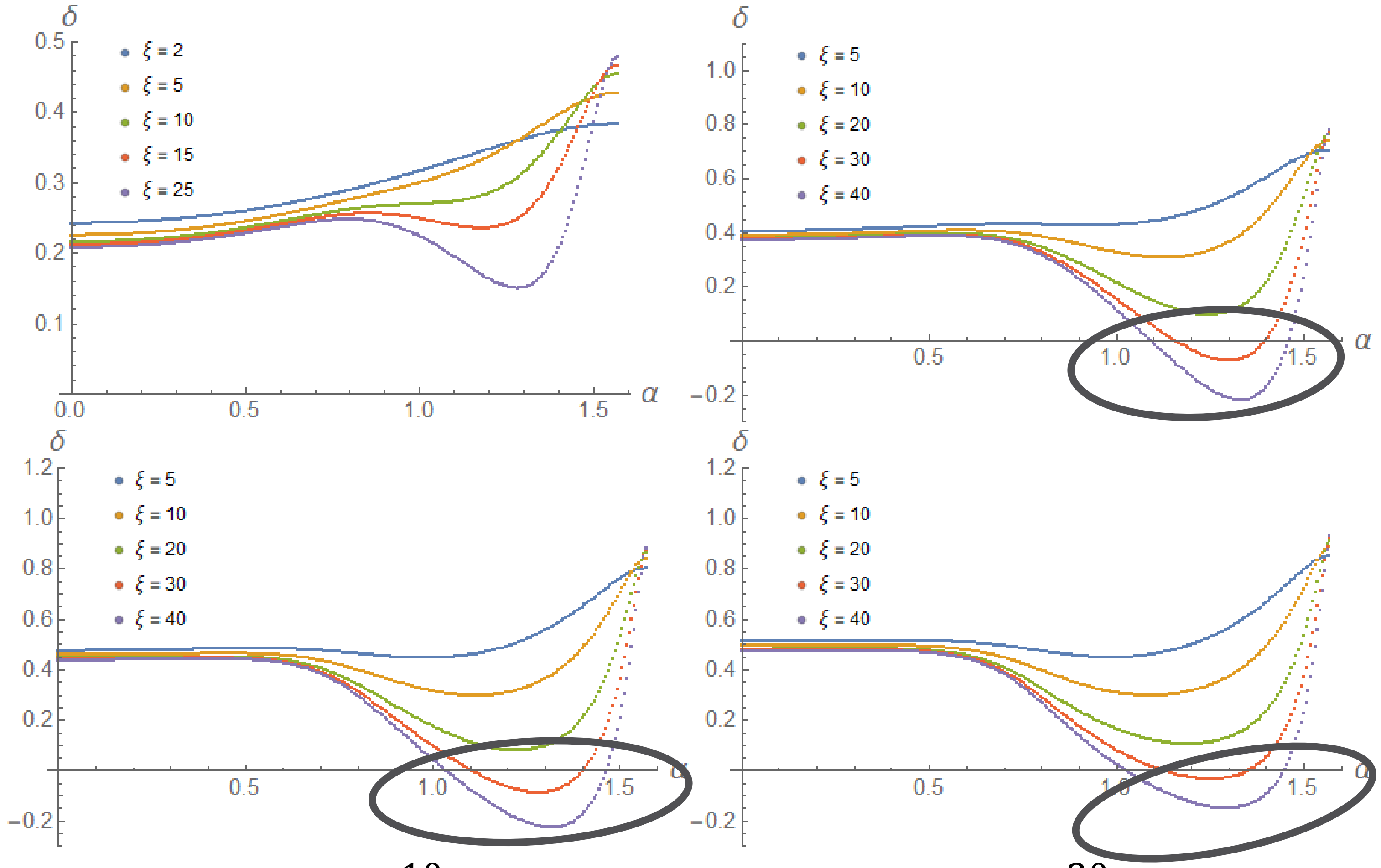
$\sigma = 5$ $\sigma = 15$

Junction condition



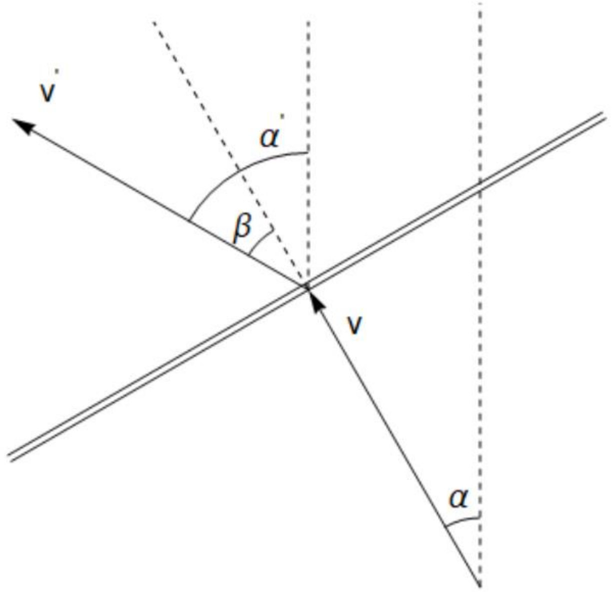
Junction condition

$\sigma = 2$ $\sigma = 5$

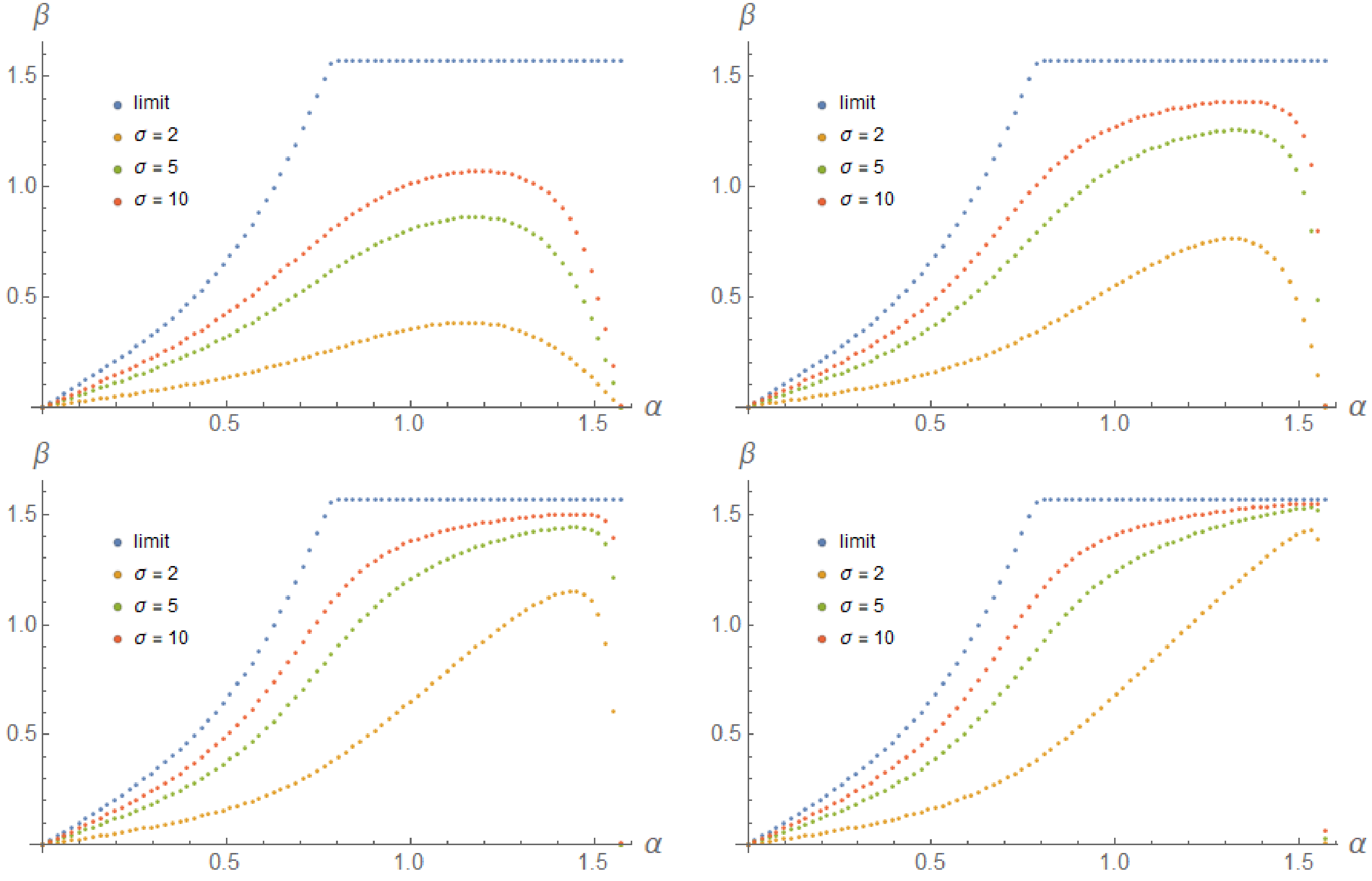


$\sigma = 10$ $\sigma = 20$

Junction condition

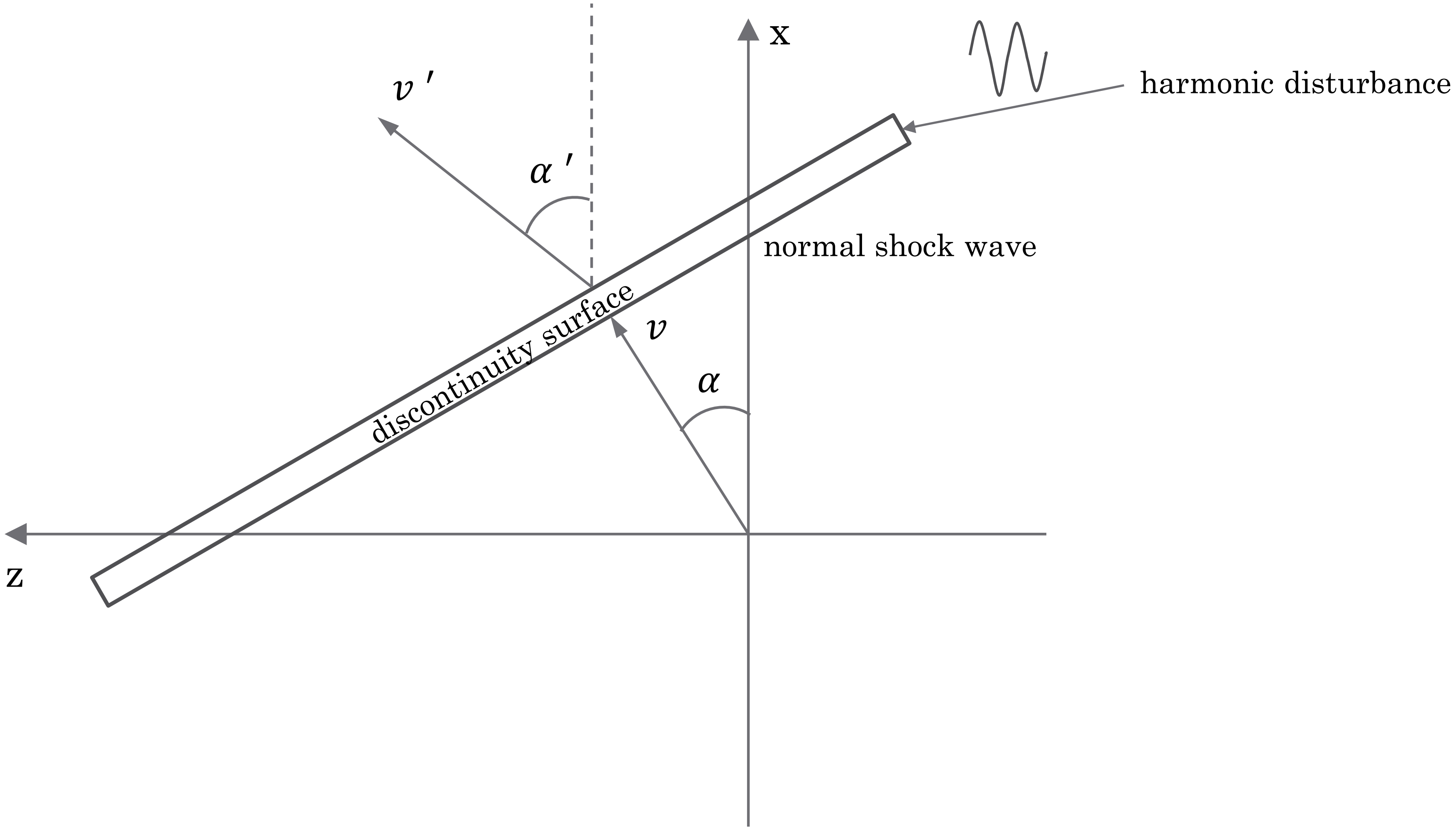


$\xi = 5$ $\xi = 20$

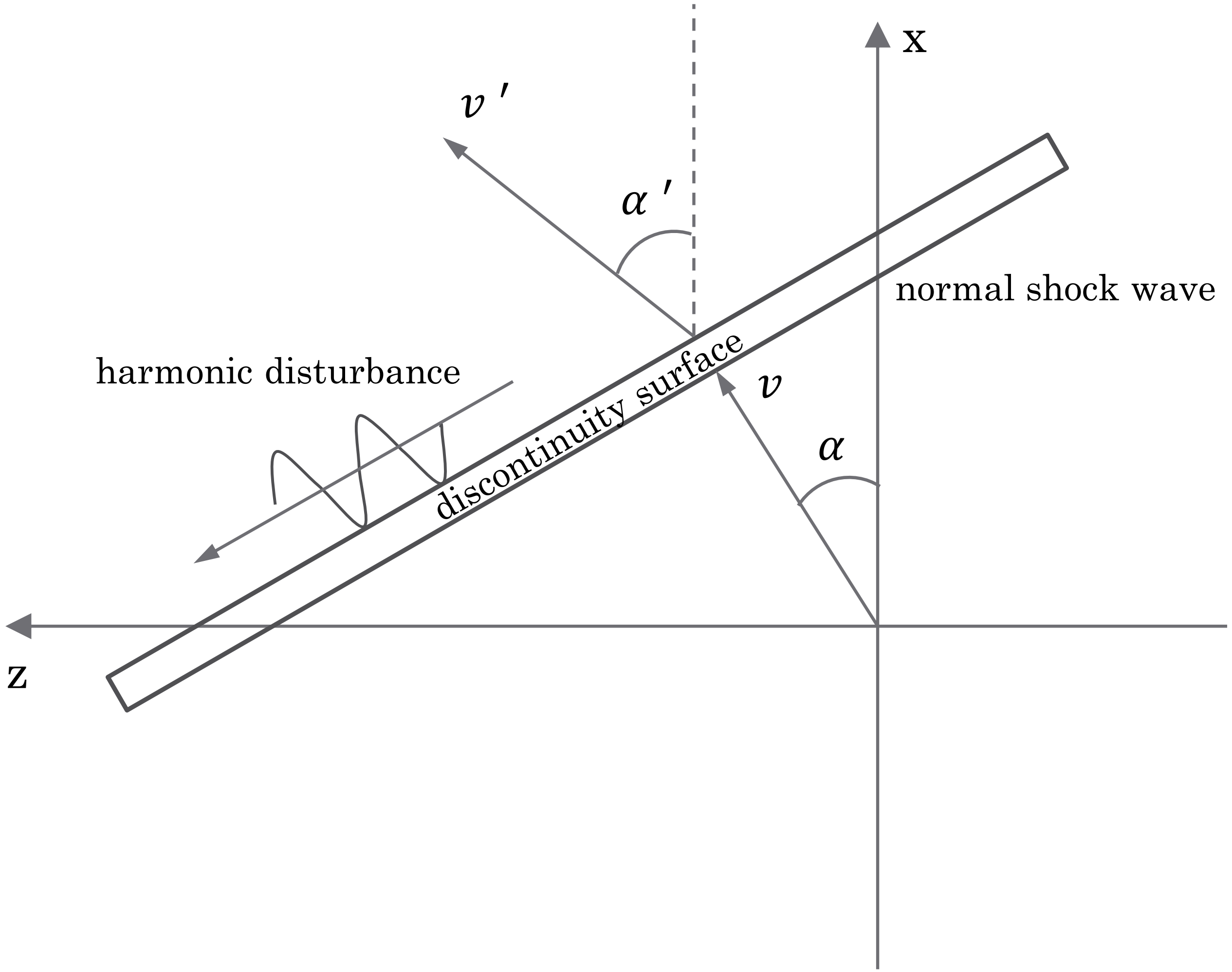


$\xi = 100$ $\xi = 1000$

Linear stability



Linear stability



Linear stability

Harmonic wave with small amplitude along the discontinuity surface

$$F(x,z) = z\sin\alpha + x\cos\alpha - \lambda e^{i\left[\omega t - k(z\cos\alpha - x\sin\alpha)\right]} = 0$$

Disturbance affect only on quantities behind the discontinuity surface A'

$$\delta A' \sim e^{i\left(\omega t - k_x x - k_z z\right)}$$

where $k_x = \tilde{k}\cos\alpha - k\sin\alpha$

$k_z = \tilde{k}\sin\alpha + k\cos\alpha$

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Stability conditions

$$\Im\,\omega \geqslant 0, \Im\,k_x \leqslant 0, \Im\,k_z \leqslant 0$$

Junction condition

$$T_{\mu\nu} N^{\mu} = T^{'}_{\mu\nu} N^{\mu}$$

N^{μ} - normal vector to the discontinuity surface

Using junction condition we can obtain variatio $\delta u^{'}_{x}, \delta u^{'}_{z}$

Linearized equation of motion

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Linearized equation of motion: $\partial_{\mu}\delta T^{\mu\nu} = ik_{\mu}\delta T^{\mu\nu} = 0$

$\cancel{\partial_{\mu}(nu^{\mu}) = 0}$

Linearized equation of motion

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Linearized equation of motion: $\partial_\mu \delta T^{\mu\nu} = ik_\mu \delta T^{\mu\nu} = 0$

+

numeric methods

$\cancel{\partial_\mu (n u^\mu) = 0}$

Results

1. Junction condition was studied in anisotropic case for normal shocks. The amplification and attenuation of shock wave was shown depending on the angle to the direction of the anisotropy. In contrast to the isotropic case, the angle of the direction of movement of flow behind shock wave does not coincide with the normal vector to the discontinuity surface, but increases according to the direction of anisotropy in space.
2. The linear stability of shock waves against small harmonic perturbations in anisotropic hydrodynamics was studied. Equations were obtained to estimate the stability of shock waves.

Thank you for your attention!