Investigation of dense baryonic matter by lattice quantum chromodynamics methods

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OUTLINE

- Motivation
- QC_2D, lattice setup
- Confinement-deconfinement transition at zero temperature
- Gluon propagators
- Analytic continuation at T>T_c

- Decomposition of the static potential
- Lee-Yang zeros

Motivation

- There are no methods to simulate lattice QCD directly at nonzero μ_B (sign problem)

- Existing methods to obtain results at nonzero μ_B from lattice QCD:
  - Taylor expansion
  - Analytic continuation

Allow to obtain results up to $\frac{\mu_B}{T} < 2$ at most

Taylor expansion

HotQCD Collaboration, A. Bazavov et al. Phys.Lett.B 795 (2019) 15-21

HotQCD Collaboration, A. Bazavov et al. Phys.Rev.D 101 (2020) 7, 074502

HotQCD Collaboration, A. Bazavov et al. Phys.Rev.D 95, (2017) 5,054504

R. Bellwied et al. Phys.Rev.D 92, (2015) 11, 114505

Analytic continuation

BMW Collaboration C. Ratti et al. J.Phys.Conf.Ser. 1602 (2020) 1, 012011

BMW Collaboration S. Borsanyi et al. JHEP 10, 205 (2018)

C. Bonati et al.   Phys.Rev.D 98, no.5, 054510 (2018)

V. Vovchenko et al.   Phys.Lett.B 775, 71-78 (2017)

Motivation (cont.)

QCD-like theories without a sign problem may be used as laboratories for QCD at high density:

- Provide information on QCD in some regions of parameters
- Allow to check the range of applicability of available methods
- Allow to check predictive power of other approaches (DSE, FRG, effective actions,…) to nonperturbative QCD by comparison of their results for QCD-like theories with respective lattice results

Other lattice studies of QC_2D

Dedicated workshop YITP workshop 'Probing the physics of high-density and low-temperature matter with ab initio calculations in 2-color QCD' 3rd - 6th November, 2020, Online (zoom and Remo)

$N_f = 4$, staggered

- Kogut, Toublan and Sinclair, The phase diagram of four flavor SU(2) lattice gauge theory at nonzero chemical potential and temperature, Nucl. Phys. B 642 (2002) 181

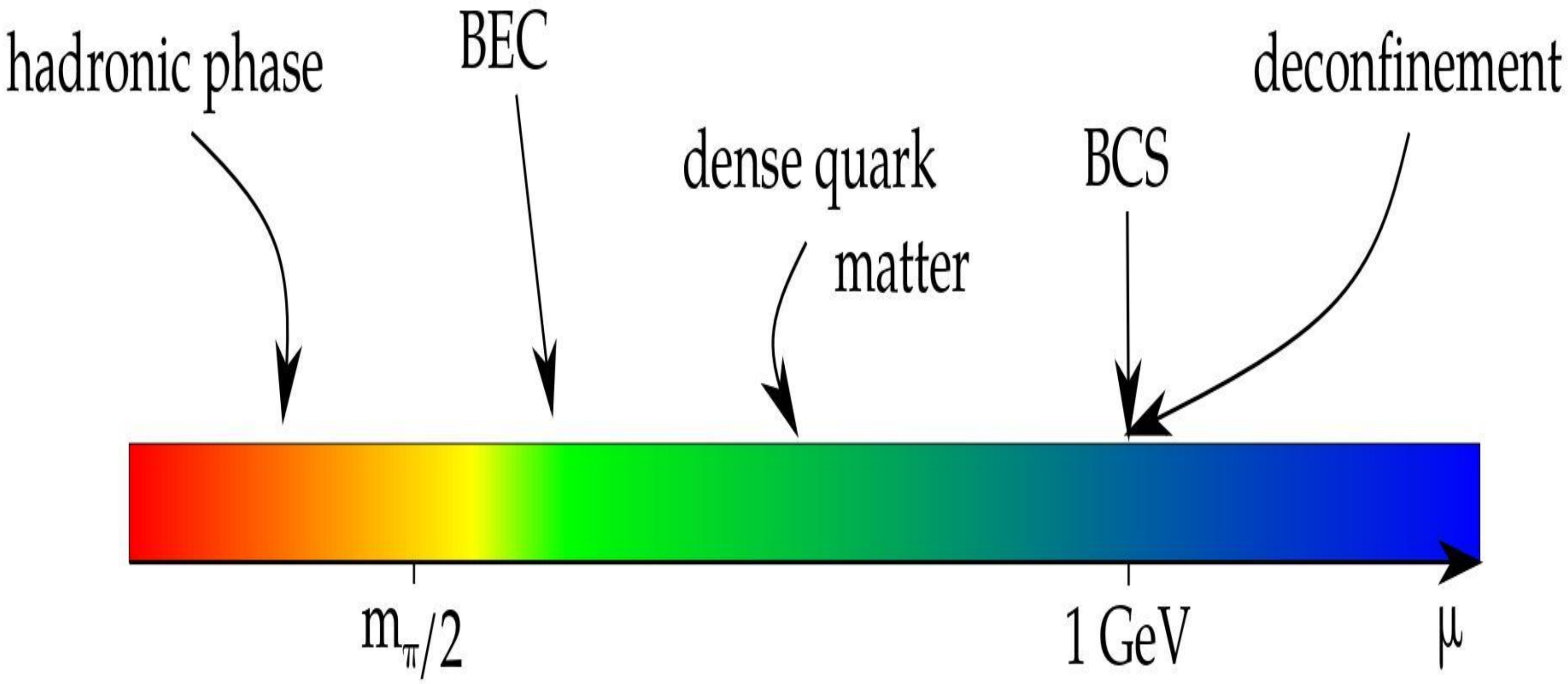
$N_f = 2$, staggered

- Braguta, Ilgenfritz, Kotov, Molochkov, Nikolaev, Study of the phase diagram of dense two-color QCD within lattice simulation, Phys. Rev. D 94 (2016)114510
- N. Yu. Astrakhantsev et al., Lattice study of static quarkantiquark interactions in dense quark matter, JHEP 05, 171 (2019), [arXiv:1808.06466].
- Wilhelm, Holicki, Smith, Wellegehausen and von Smekal, Continuum Goldstone spectrum of twocolor QCD at finite density with staggered quarks, Phys.Rev. D100, 114507 (2019), [arXiv:1910.04495].

$N_f = 2$, Wilson

- Cotter, Giudice, Hands and Skullerud, Towards the phase diagram of dense two-color matter, Phys. Rev. D 87 (2013) 034507
- Boz, Giudice, Hands, Skullerud, Dense two-color QCD towards continuum and chiral limits Phys.Rev.D 101 (2020) 7, 074506 • e-Print: 1912.10975 [hep-lat]
- K. Iida, E. Itou and T.-G. Lee, Two-colour QCD phases and the topology at low temperature and high density, JHEP 01, 181 (2020), [arXiv:1910.07872].

Phase Diagram of QC_2D at T=0



Simulation settings (old)

- SU(2) lattice QCD with $N_f = 2$ staggered Dirac operator
- Lattice size 32^4
- Lattice spacing $a = 0.044$ fm
- Pion mass $m_\pi = 740(40)$ MeV
- Range of μ values: $0 \le a\mu \le 0.5$

  or

  $0 \le \mu \lesssim 2000$ MeV

Lattice action

Lattice fermion action:

$$S_F = \sum_{x,y} \bar{\psi}_x M(\mu, m)_{x,y} \psi_y + \frac{\lambda}{2} \sum_x \left(\psi_x^T \tau_2 \psi_x + \bar{\psi}_x \tau_2 \bar{\psi}_x^T \right)$$

M is the staggered lattice Dirac operator,

λ- term is needed to make the di-quark condensate nonzero

Partition function:

$$Z = \int DU e^{-S_G} \cdot \left(\det(M^\dagger M + \lambda^2) \right)^{\frac{1}{4}}$$

Simulation settings (new)

- SU(2) lattice QCD with improved staggered Dirac operator
- Lattice size 40^4
- Lattice spacing 0.05 fm
- Pion mass m_π = 450 MeV
- Range of μ_q values $0 < a\mu_q$ < 0.5

  $0 < \mu_q$ < 1700 MeV
- Size in physical units $L \approx 2.0$ fm

Spectral representation of WL.

Confinement phase:

Ground state – hadron string up to distance $r = r_{sb}$,

then GS becomes 2 h-l mesons

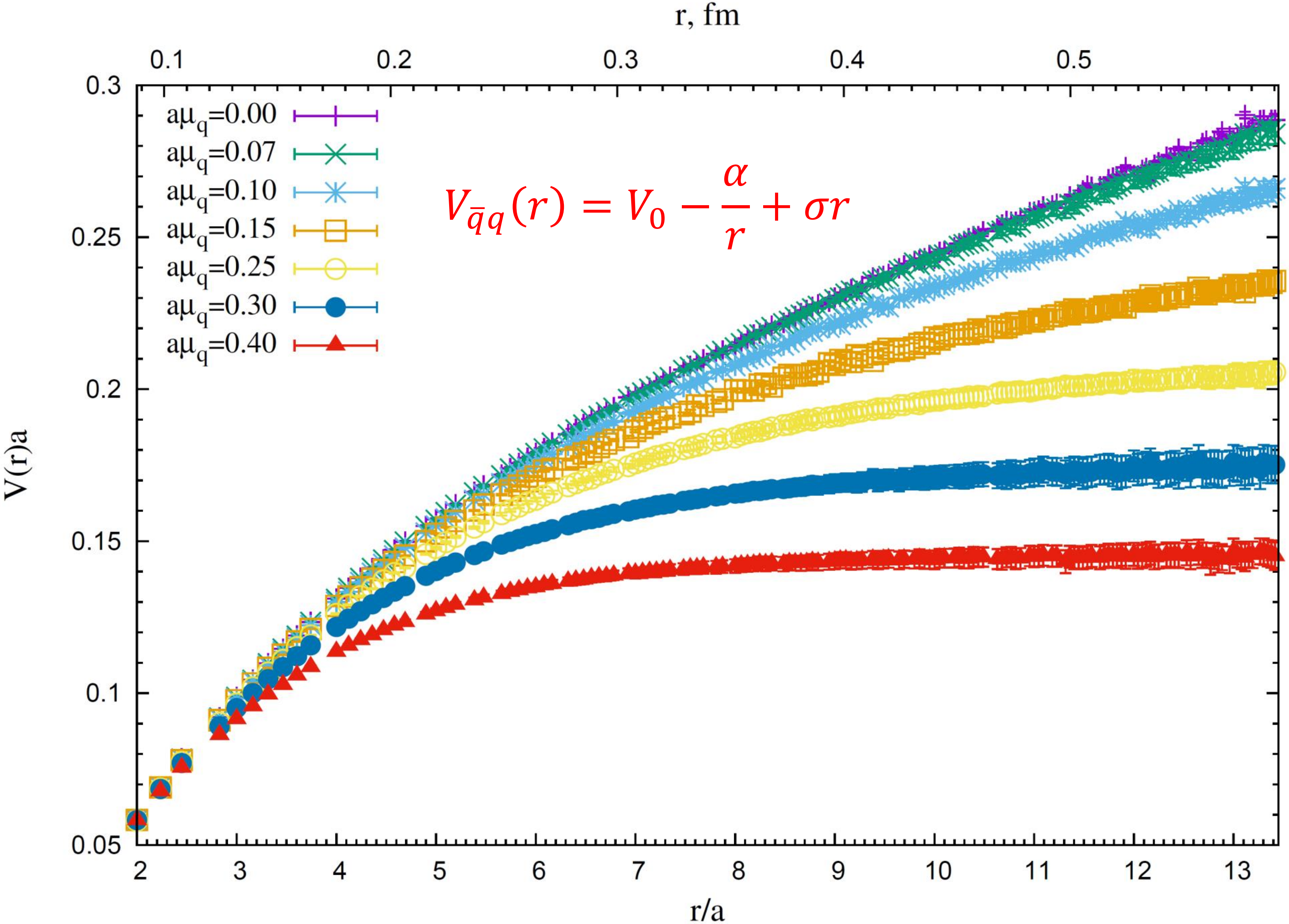
But WL has very small overlap with h-l meson state, C_{hl}<<1

For this reason we do not see string breaking, but clearly see

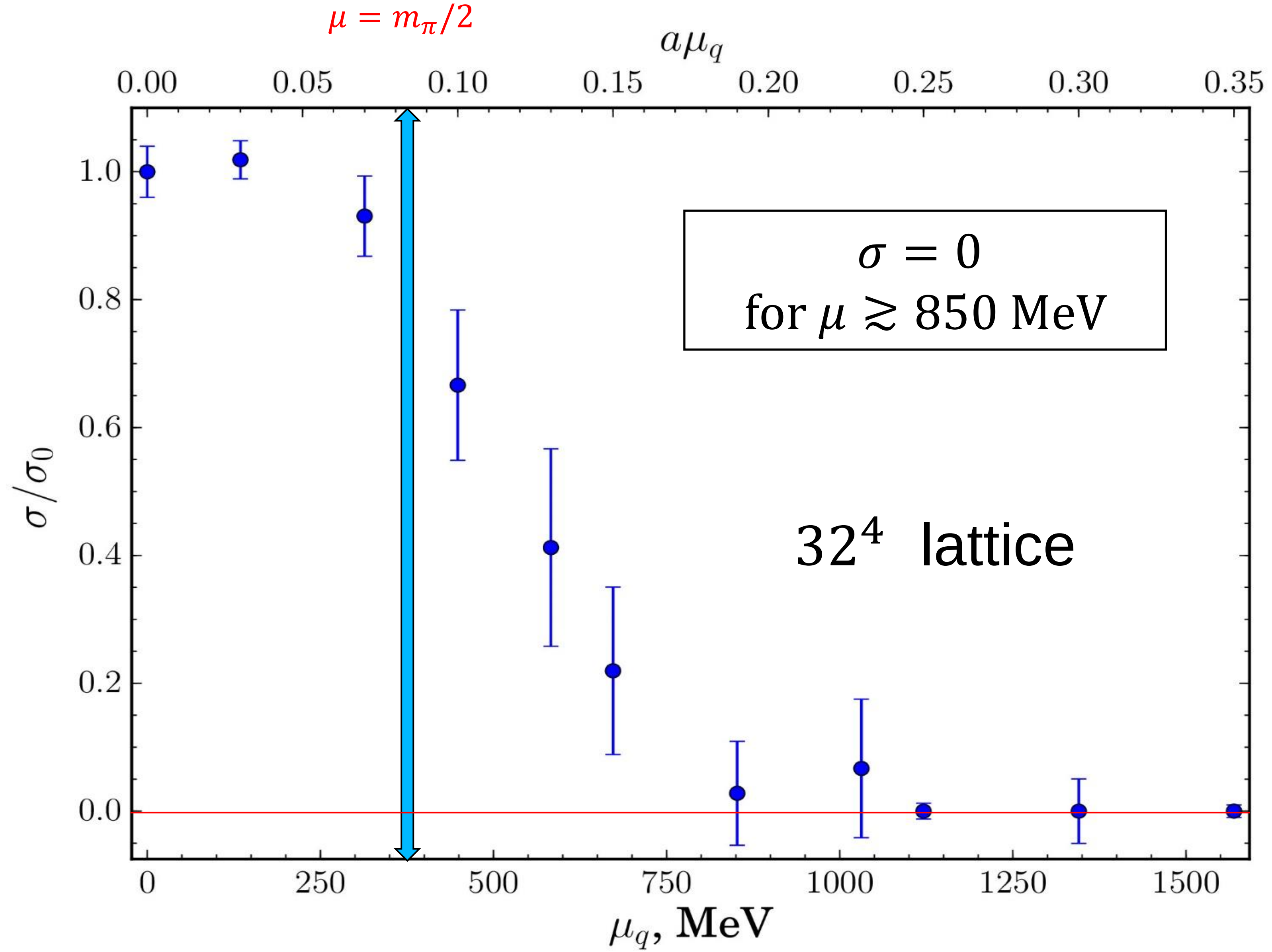
Hadron string state

Deconfinement phase:

Ground state – color interaction is screened, Debye screening



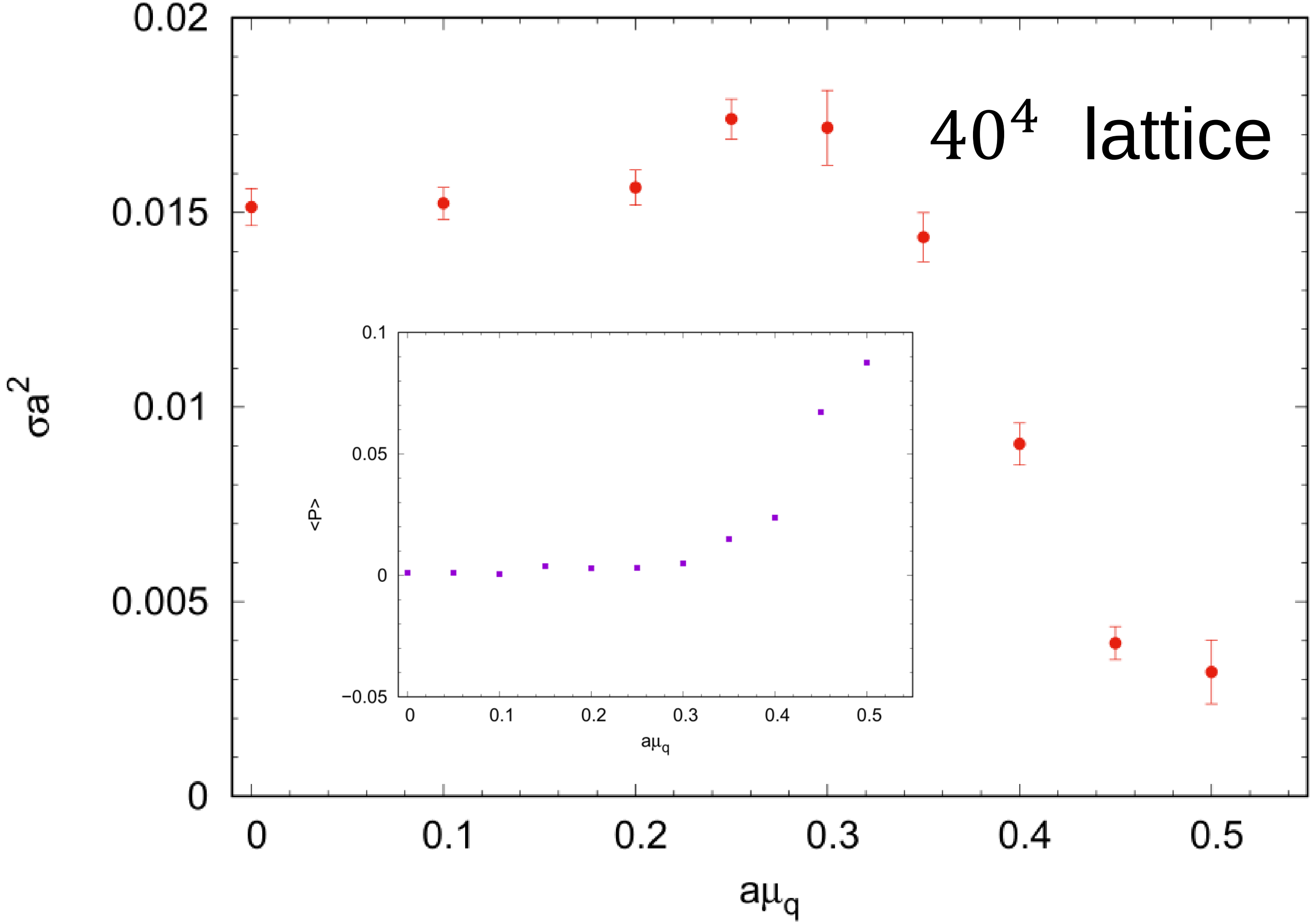
r, fm
0.1
0.2
0.3
0.4
0.5
0.3
0.25
0.2
0.15
0.1
0.05
V(r)a
2
3
4
5
6
7
8
9
10
11
12
13
r/a
$a\mu_q=0.00$
$a\mu_q=0.07$
$a\mu_q=0.10$
$a\mu_q=0.15$
$a\mu_q=0.25$
$a\mu_q=0.30$
$a\mu_q=0.40$
$V_{\bar{q}q}(r) = V_0 - \frac{\alpha}{r} + \sigma r$



JHEP 03 (2018) 161 • e-Print: 1711.01869 [hep-lat]

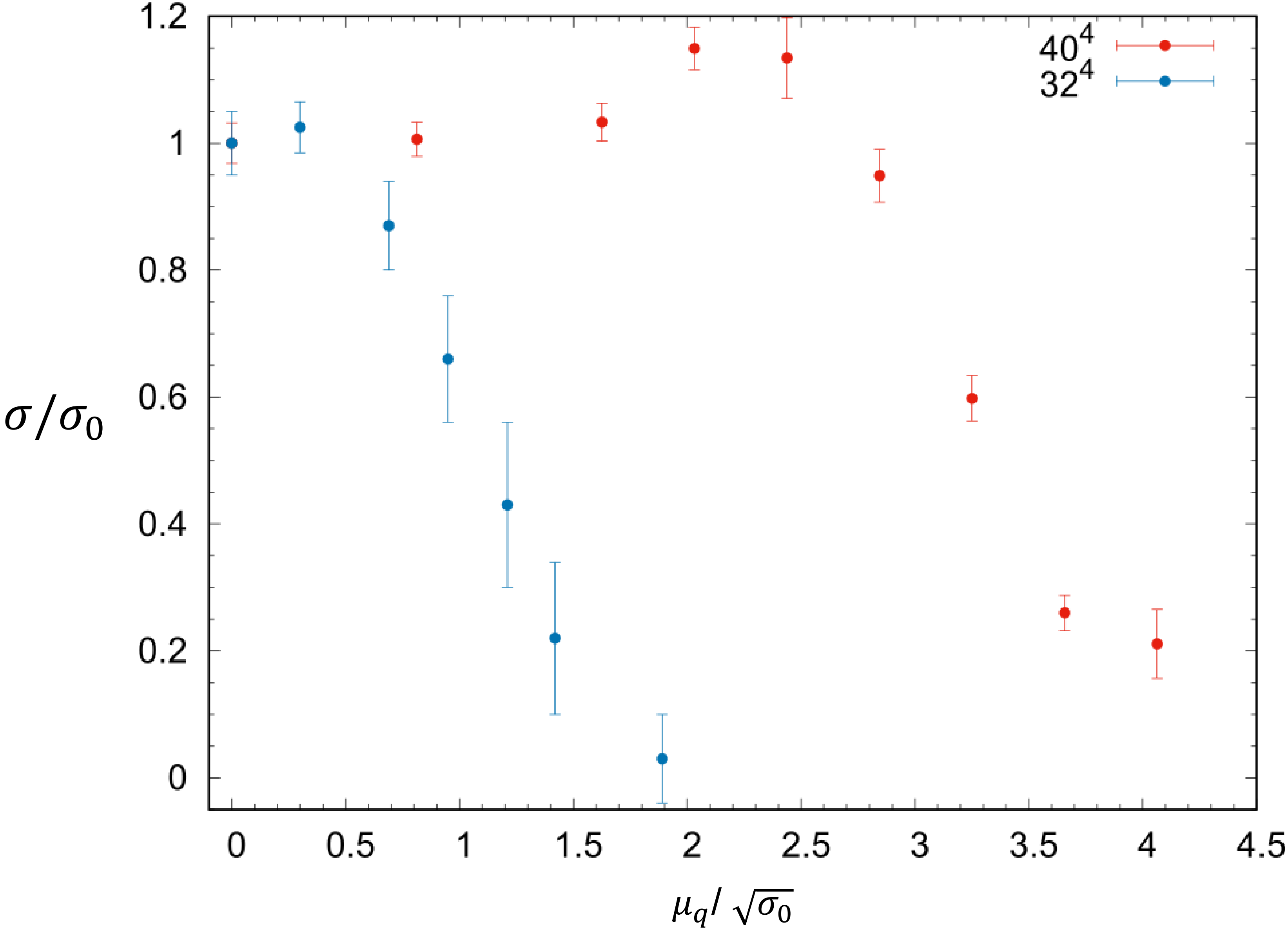
V. G. Bornyakov, V. V. Braguta, E.-M. Ilgenfritz, A. Yu. Kotov, A. V. Molochkov, and A. A. Nikolaev

The confinement-deconfinement transition thus happens in the range 850 MeV< μ<1100 MeV , (screening at μ > 1100 MeV)

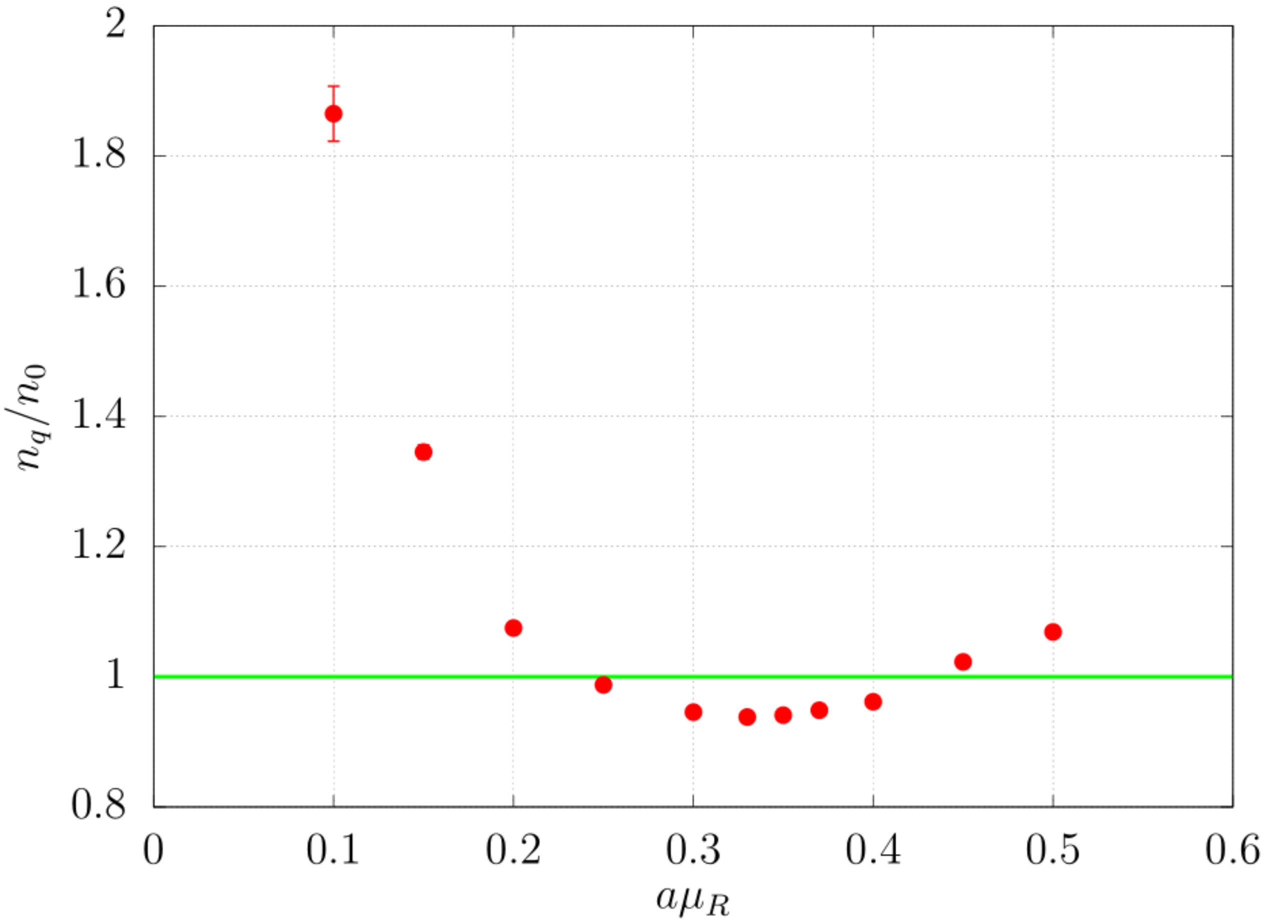


40^4 lattice
σa^2
aμq
0.02
0.015
0.01
0.005
0
0.1
0.2
0.3
0.4
0.5
<P>
0.05
−0.05

String tension for two lattices



Quark number density



For large enough μ_q data are close to the quark number density for free relativistic quarks $n_0 = 4/(3\pi^2\mu^3)$. This is similar to results on 32^4 *lattice*

Conclusions I

On the lattice with increased sized by ~15%

and with decreased pion mass by factor 1.7

we observed decreasing of σ to much larger μ_q : from ~800 MeV to ~1400 MeV

This change is to be explained

Our expectation: main effect comes from decreasing of m_π

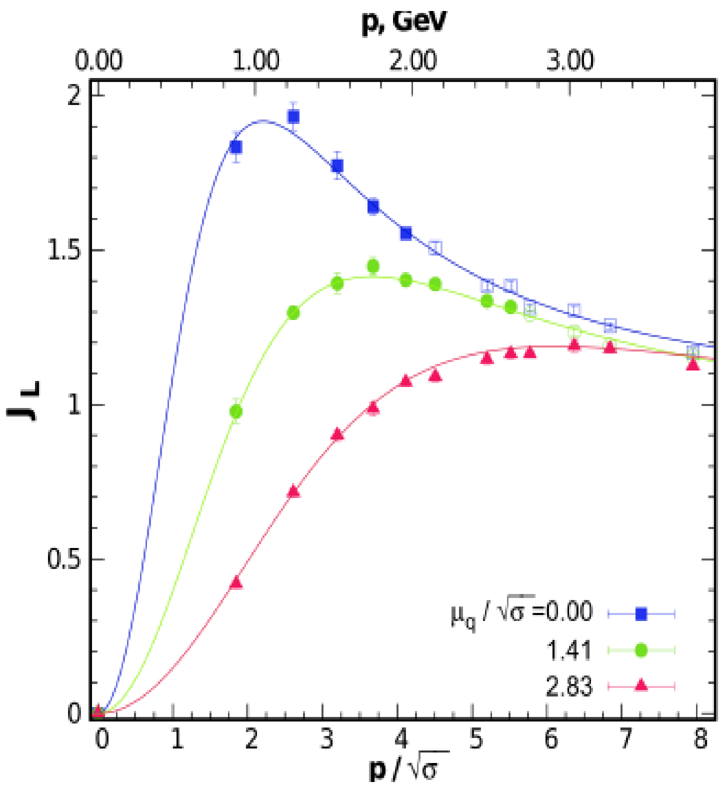
Gluon Propagators at T=0 (32^4 lattice)

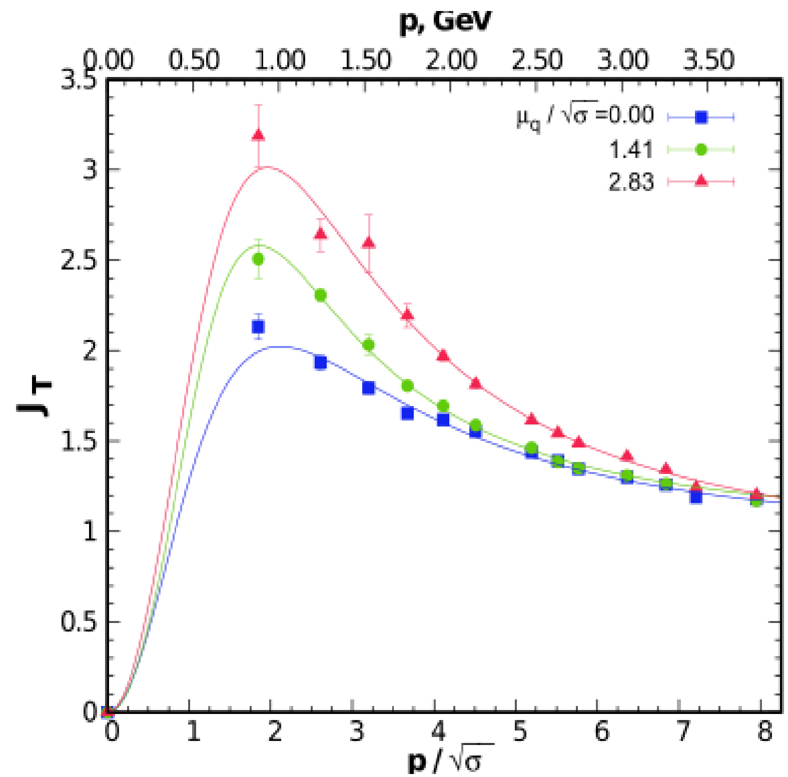
At nonzero μ_q the O(4) symmetry is broken and there are two tensor structures for the gluon propagator

$$D^{ab}_{\mu\nu}(p) = \delta^{ab}\left(P^{T}_{\mu\nu}\, D_T(p) + P^{L}_{\mu\nu}\, D_L(p) \right)$$

We consider the soft modes $p_4 = 0$ and use the notation $D_{L,T}(p) = D_{L,T}\left(0, \overrightarrow{|p|}\right)$

Gluon dressing functions

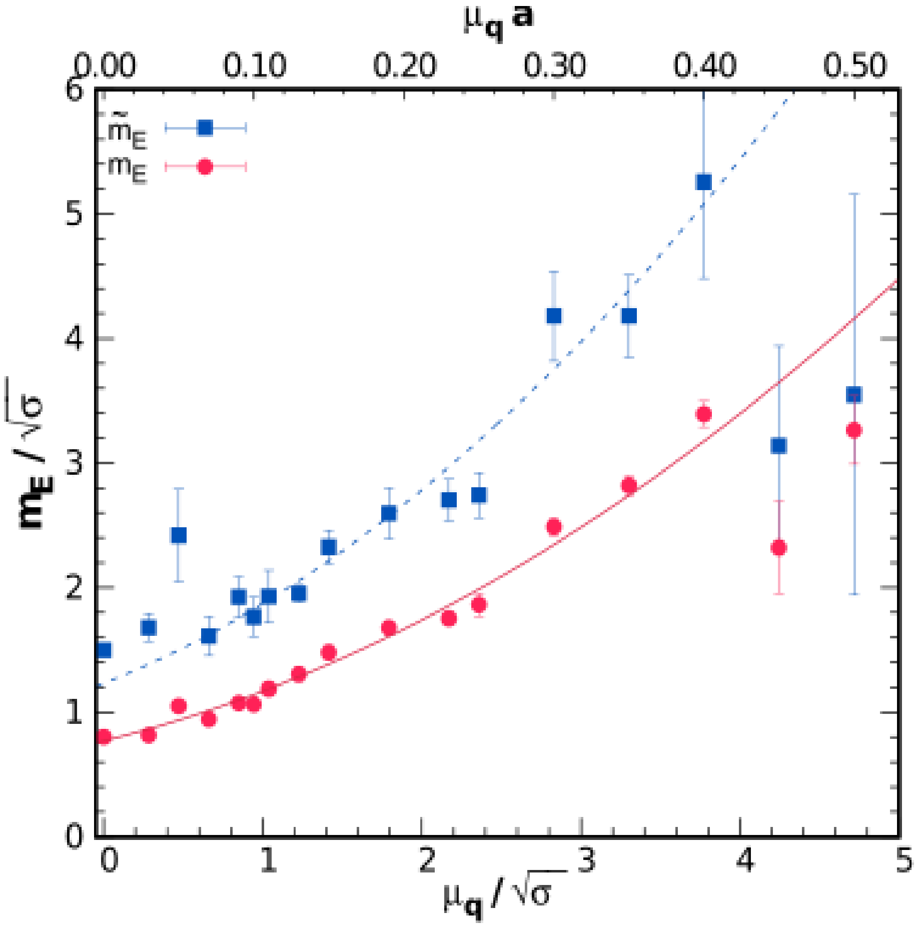




The maximum of J_L goes down and shifts to the right, thus approaching dressing function of a massive scalar particle

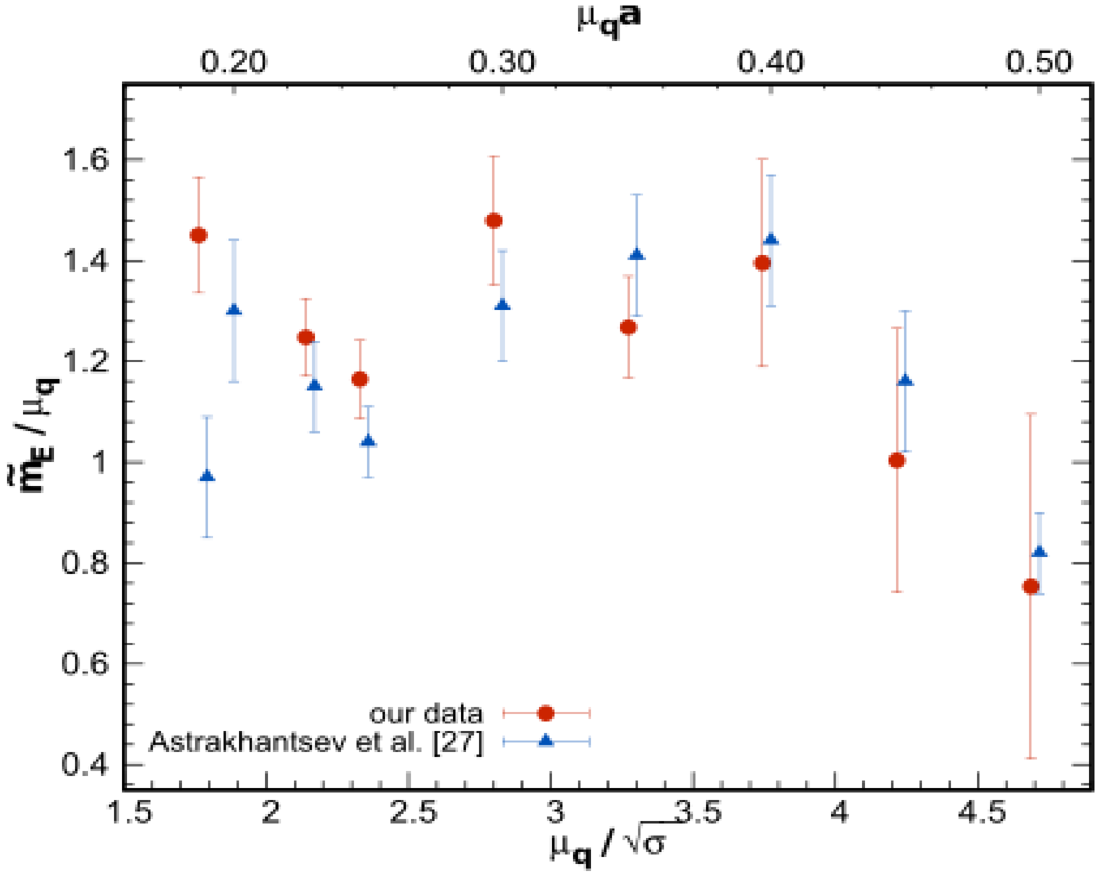
J_T shows instead infrared enhancement with increasing μ_q. This is in agreement with the disappearance of the magnetic field screening at extremely large quark chemical potential predicted in *D. T. Son, 1999*

Electric screening mass



$$m_{E,M}^2 = \frac{1}{D_{L,T}(0)} \qquad vs \qquad Z(\,\tilde{m}_{E,M}^2 + \,p^2) = \frac{1}{D_{L,T}(p)}$$

Comparison with screening mass m_D

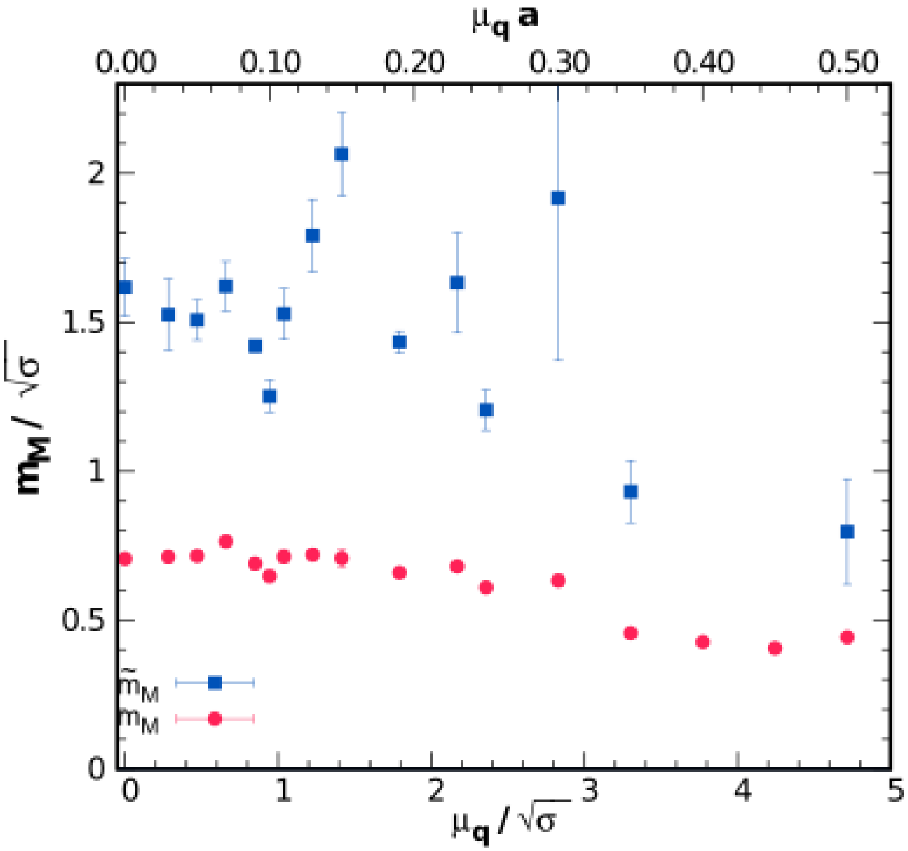


$\widetilde{m}_E$ and m_D vs. μ_q

m_D is computed from the singlet quark-antiquark potential at large distances using the Coulomb gauge.

- indication of gauge invariance of the electric screening mass
- ratio $\widetilde{m}_E / \mu_q$ is a slowly varying function of μ_q in a qualitative agreement with perturbation theory.

Magnetic screening mass



m_M is decreasing at large μ_q in the deconfinement phase

Conclusions II

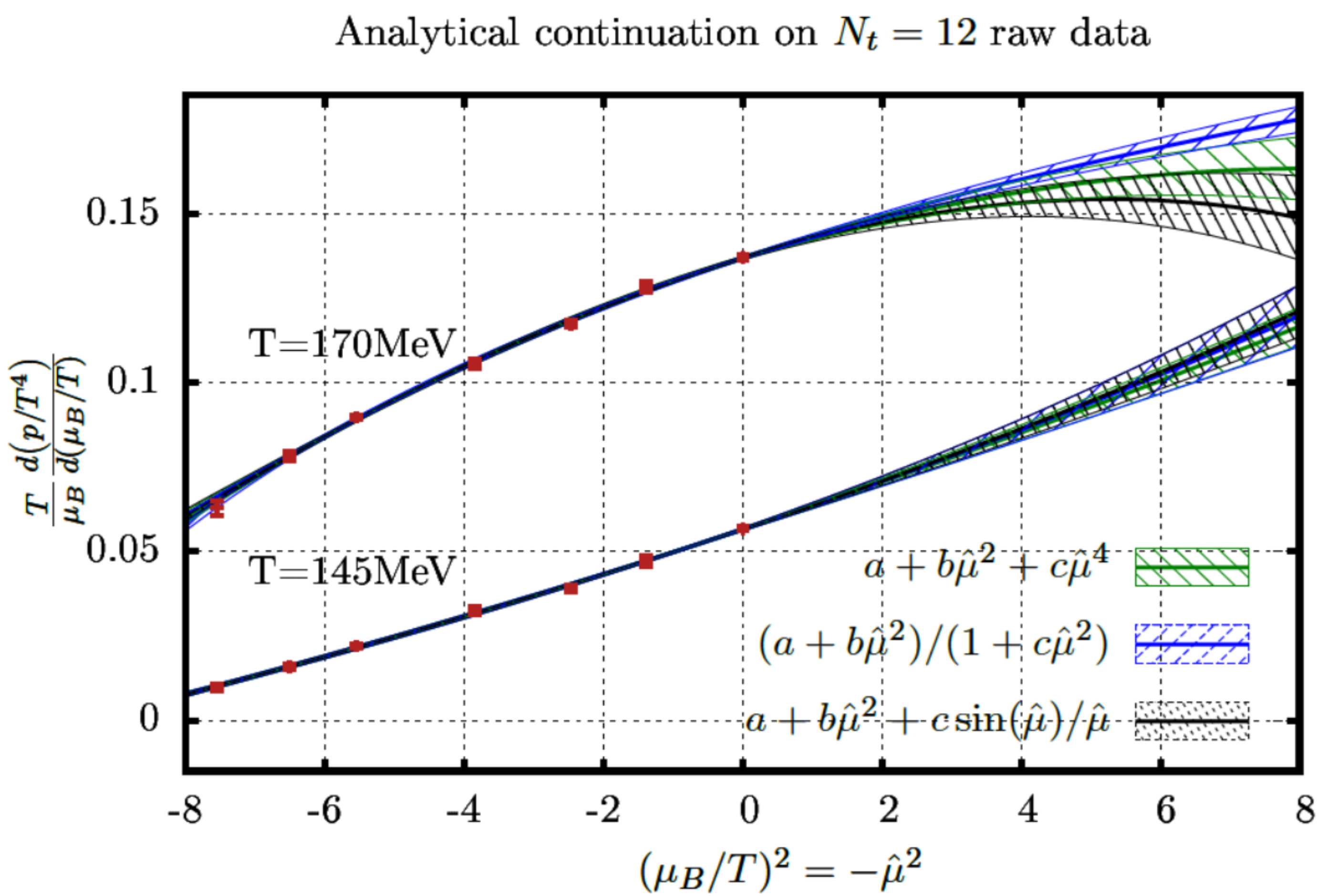
- dependence of D_L(p) on μ_q is analogous to its dependence on the temperature at T > Tc
- much weaker dependence of D_T(p) on μ_q with indication of the infrared enhancement at large μ_q
- $\widetilde{m}_E / \sqrt{\sigma}$= 1.50(4) at μ_q = 0 agrees with SU(2) and SU(3) gluodynamics
- $\widetilde{m}_E(\mu_q) \approx m_D(\mu_q)$ in the deconfinement
- decreasing of m_M at large μ_q

Analytic continuation at $T>T_c$

Goal: To find better way of doing analytic continuation for the quark number density

QC_2D allows to check analytic continuation results comparing with directly computed results

Example: Fodor et al. 2016, 2+1 QCD



Grand canonical partition function

$$Z_{GC}(\theta, T, V) = \sum_{n=-\infty}^{\infty} Z_C(n, T, V)\xi^n,$$

Quark number density

$$n_q/T^3 = N_c \frac{N_t^3}{N_s^3} \frac{2\sum_{n>0} nZ_n \sinh(n\theta)}{1 + 2\sum_{n>0} Z_n \cosh(n\theta)}$$

$$n_q/T^3 = N_c \frac{N_t^3}{N_s^3} \frac{2\sum_{n>0} nZ_n \sin(n\theta)}{1 + 2\sum_{n>0} Z_n \cos(n\theta)}$$

We consider two types of the fitting function:

$$n_q = \sum_{n=1}^{N} a_n \sin(2n\vartheta)$$ Fourier series

$$n_q = \mathcal{N} \frac{2\sum_{n=1}^{N} n Z_n \sin(2n\vartheta)}{1+2\sum_{n=1}^{N} Z_n \cos(2n\vartheta)}$$ canonical formula

Direct application is not fruitful

We employ models for coefficients a_n

cluster expansion model (CEM)

V. Vovchenko, J. Steinheimer, O. Philipsen, and H. Stoecker, Phys.Rev. D97 (2018) 114030

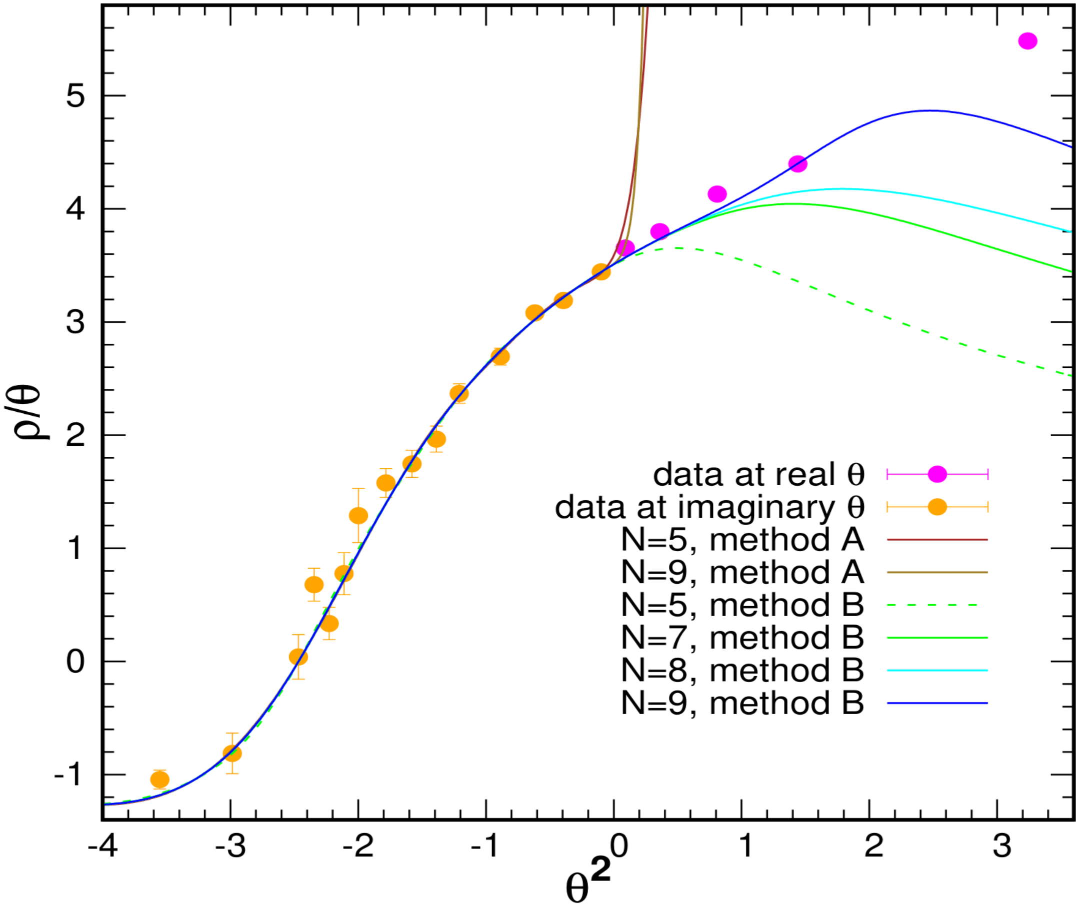
a_n = α_n^{SB} $\frac{[a_2]^{n-1}}{[a_1]^{n-2}}$ n=3,4…, α_n^{SB} known numbers related to SB limit

rational fraction model (RFM)

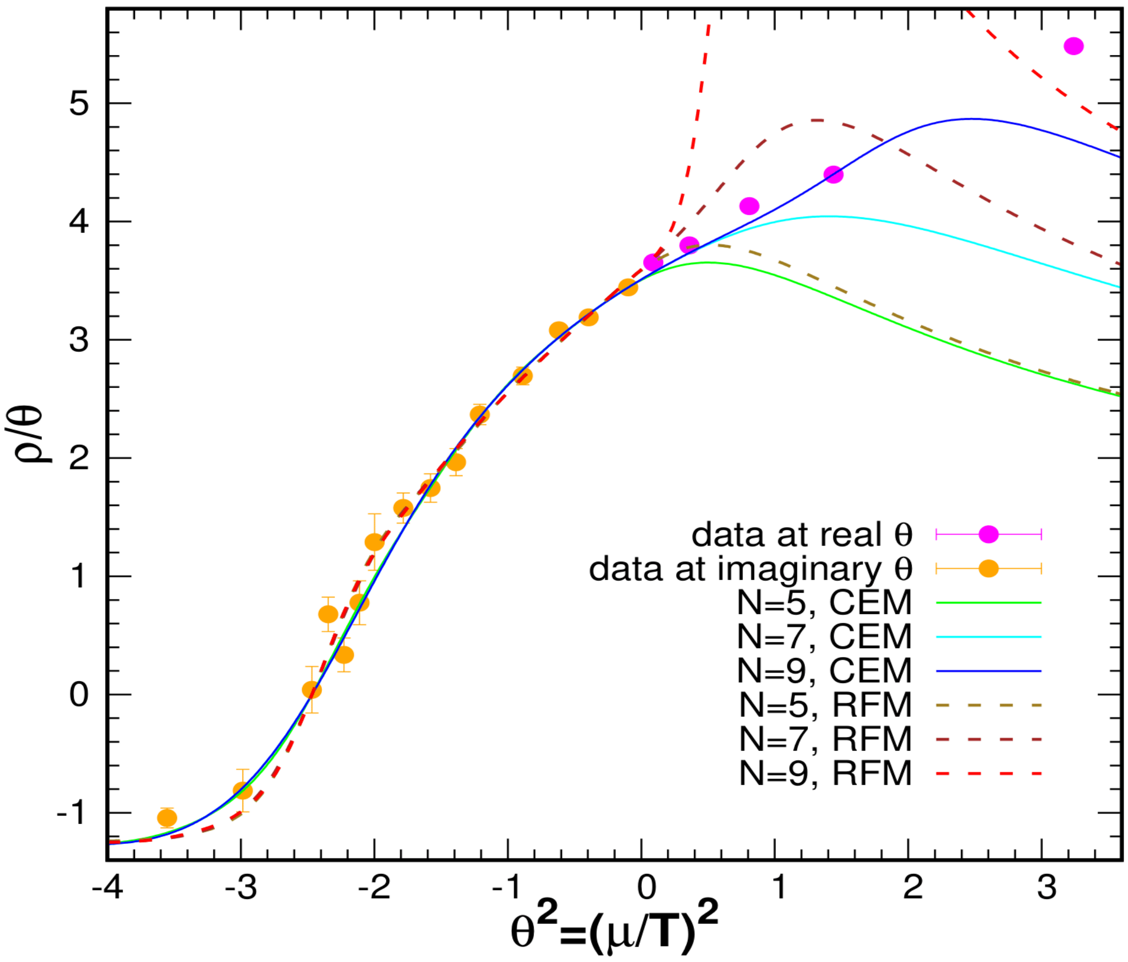
G. A. Almasi, B. Friman, K. Morita, P. M. Lo, and K. Redlich, Phys. Rev. D100 (2019) 016016

Both models have two free parameters and reproduce lattice resylts for a_n, n=1,2,34

Quark number density vs $(\mu_q\ /T)^2$



Comparison of CEM and RFM



Conclusions III

- using CEM for the Fourier coefficients we found:

1) the analytic continuation of the Fourier series has rather small range of agreement with our data at real μ_q: up to μ_q/T = 0.37

2) With canonical fitting function we found much wider range of agreement with numerical results at real μ_q: up to μ_q/T = 1.4

3) Z_n obtained from CEM take negative values at large n indicating that the model should be modified

- using RFM we did not find agreement with our results at real μ_q