Lattice study of properties of QCD in extreme conditions: temperature and density, rotation, magnetic field

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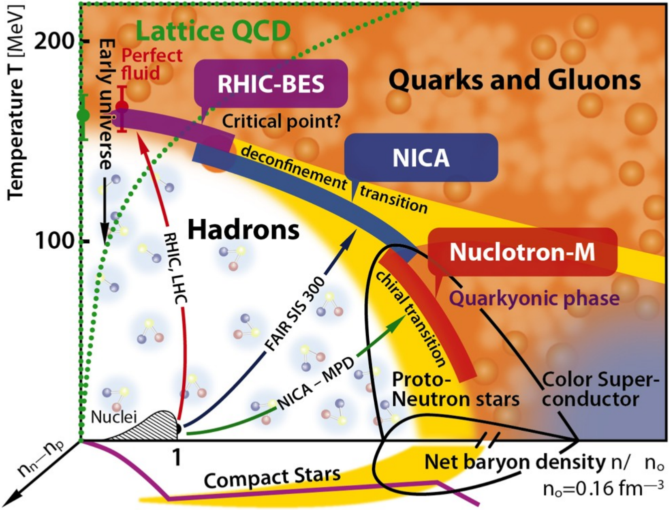
Outline

- Phase diagram in (μ, T, B) space
- Equation of state of dense QCD in external magnetic field
- Properties of rotating gluodynamics

QCD phase diagram

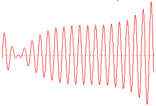


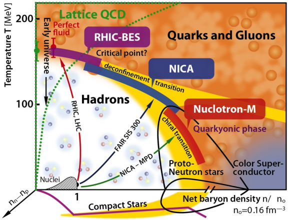
$T - \mu$ plane

Sign problem!

$Z = \int DA_\mu D\bar{\psi} D\psi e^{-S[\bar{\psi},\psi,A_\mu]}$

$\mu_B \Rightarrow S \notin \mathbb{R}$





Analytical continuation: $\mu_B \to i\mu_I$ - no sign problem!

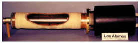
$$M(\mu_l) = A + B\mu_l^2 + \ldots \Rightarrow M(\mu_B) = A - B\mu_B^2 + \ldots$$

Magnetic fields in nature

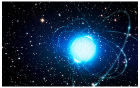
Souvenir magnet — 5×10^{-3} T

Max permanent magnet — 1.25 T

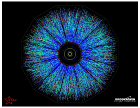
Magnetic field to levitate a frog — 16 T



Human produced pulsed magnetic field — 2.8×10^{3} T



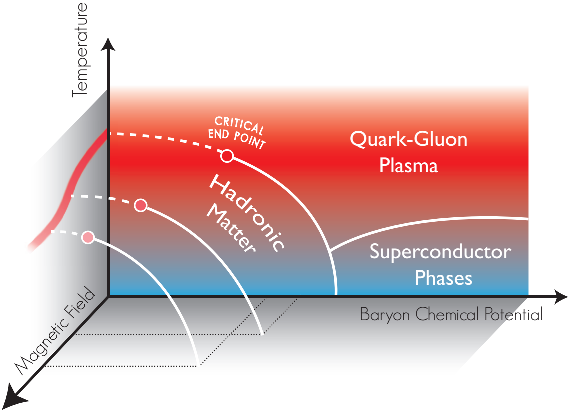
Magnetars — $10^{8} - 10^{11}$ T



Heavy ion collisions — $10^{14}\text{T} \sim m_\pi^2$

Wikipedia

QCD and Magnetic Field

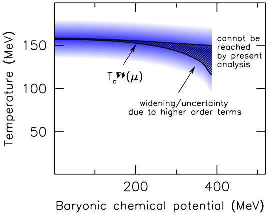


[P.Costa, M.Ferreira, C. Providência, 2018]

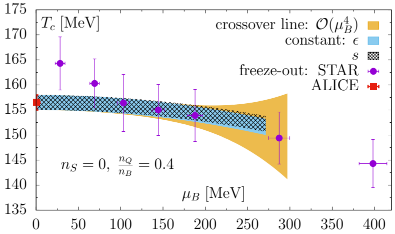
What can Lattice tell us about the phase diagram in μ, B, T?
CEP in magnetic fields?

$T - \mu$ plane

- Curvature of pseudocritical line
  $T_c(\mu_B) = T_c(0) - A_2\mu_B^2 + O(\mu_B^4)$

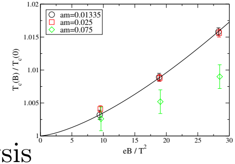


[R. Bellwied et al., 2015]



[HotQCD, 2018]

Lattice QCD in $T - B$ plane. (Inverse) Magnetic Catalysis



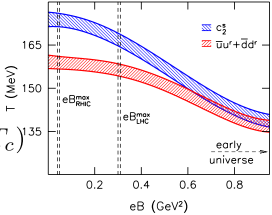
- Magnetic Catalysis

  [V. Gusynin, V. Miransky, I. Shovkovy, 1994]
- Lattice + heavy pions: Direct Magnetic catalysis

  [E.-M. Ilgenfritz et al., 2012] [M. D'Elia et al., 2010]
- Lattice + physical pions: Inverse Magnetic Catalysis

  [G. Bali et al., 2012] [F.Bruckmann et al., 2013]
- Recent studies with heavy pions

  [M. D'Elia et al., 2018] [G.Endrodi et al., 2019]
  - Inverse magnetic catalysis for T_c
  - Direct magnetic catalysis for $\bar{\psi}\psi$ (near T_c)



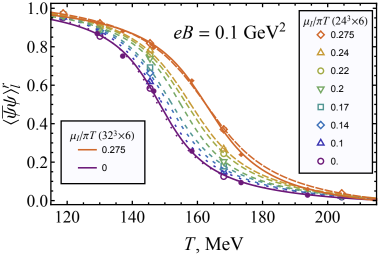
Not yet fully understood!

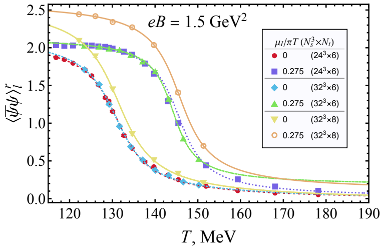
Details of lattice setup

- Our aim is to study QCD phase diagram in (μ, T, B) space
- Lattice size: $\underline{6 \times 24^3}$, 6×32^3, 8×32^3, 10×40^3
- Stout improved staggered $N_f = 2 + 1$ fermions
- Tree level Symanzik gauge action
- Physical masses of u, d, s quarks
- $q_u = 2/3e$, $q_d = q_s = -1/3e$
- $\mu_s = 0$, $\mu_u = \mu_d = \mu_l \to i\mu_l$
- $O(100)$ configurations per each T, μ_I, eB
- Multi-GPU code was created within RFBR 18-02-40126

the results are published in V.Braguta et al., Phys.Rev.D 100 (2019) 11, 114503

Observables: Chiral consensate





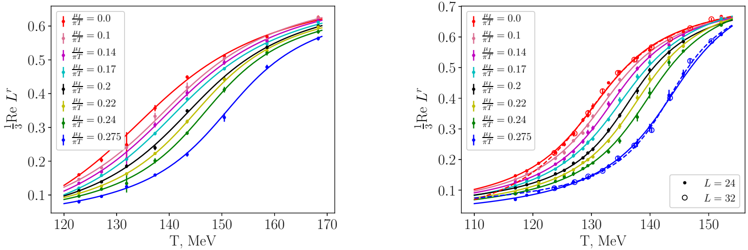
Inflection point:

$$O(T) = A + B \arctan\left(\frac{T - T_c}{\delta T_c}\right)$$

- eB grows, T_c decreases

Observables: Polyakov loop

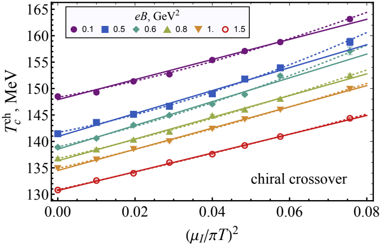
$eB = 0.8\mathrm{GeV}^2$ $\qquad$ $eB = 1.5\mathrm{GeV}^2$

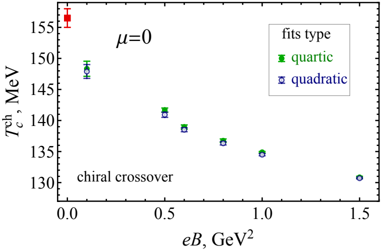


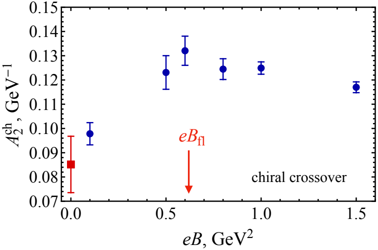
$$O(T) = A + B \arctan\left(\frac{T - T_c}{\delta T_c}\right)$$

- eB grows, T_c decreases

Chiral phase transition vs $\mu_I/\pi T$

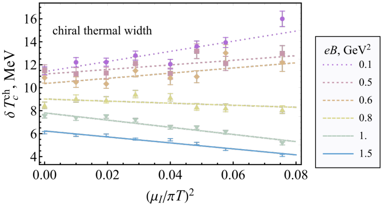






$$T_c(\mu_B, B) = T_c(0, B) - A_2(B)\mu_B^2 + O(\mu_B^4)$$

Width of the chiral phase transition



CEP(?) at $eB = 0$

$(T, \mu_B) \sim$

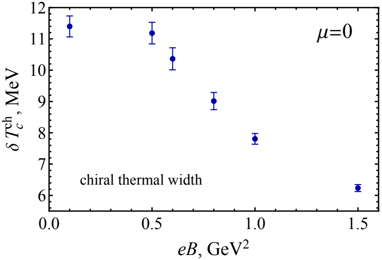
$(100(25), 800(140))$ MeV

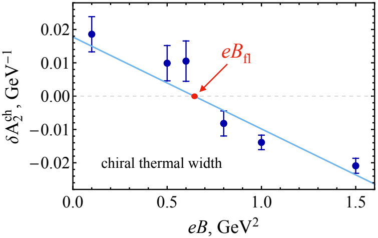
FRG: $(107, 635)$ MeV

[W. Fu, J. Pawlowski, F. Rennecke, 2019]

Holography: $(89, 724)$ MeV

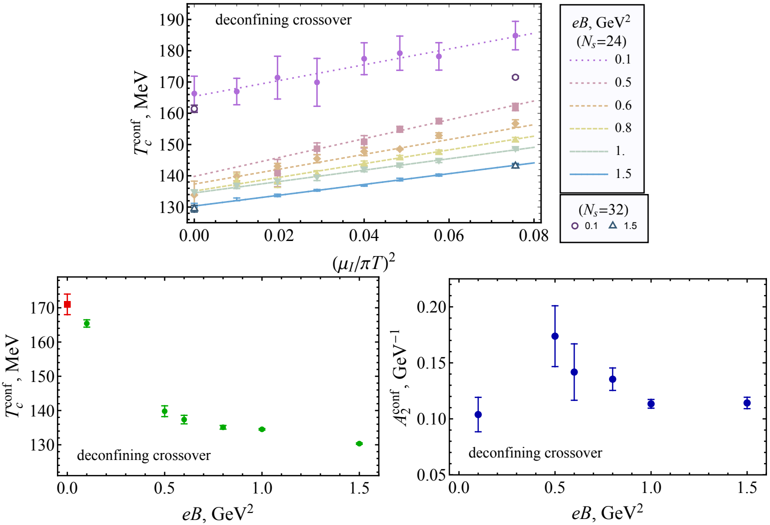
[R.Critelli et al., 2017]





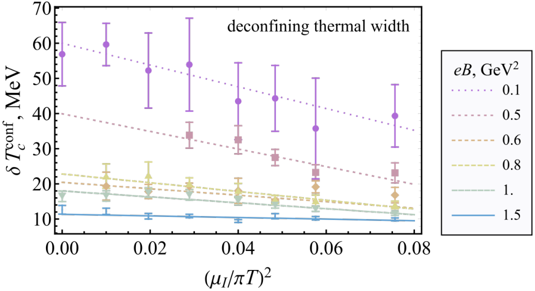
$$\delta T_c^{ch}(\mu_B, B) = \delta T_c^{ch}(0, B) - \delta A_2^{ch}(B)\mu_B^2 + O(\mu_B^4)$$

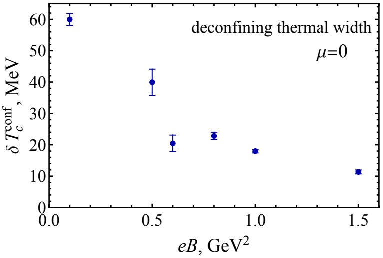
Confining crossover, critical temperature T_c

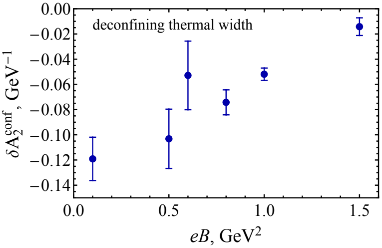


$$T_c(\mu_B, B) = T_c(0, B) - A_2(B)\mu_B^2 + O(\mu_B^4)$$

Confining crossover, width δT_c







$$\delta T_c^{\rm conf}(\mu_B, B) = \delta T_c^{\rm conf}(0, B) - \delta A_2^{\rm conf}(B)\mu_B^2 + O(\mu_B^4)$$

EoS of dense QCD in external magnetic field

- EoS is important for different applications
- EoS was studied in (μ, T) plane
  S. Borsanyi et al., Phys. Lett. B 730, 99 (2014) [arXiv:1309.5258 [hep-lat]]
  A. Bazavov et al., Phys.Rev.D 95 (2017) 5, 054504, [arXiv: 1701.04325 [hep-lat]]

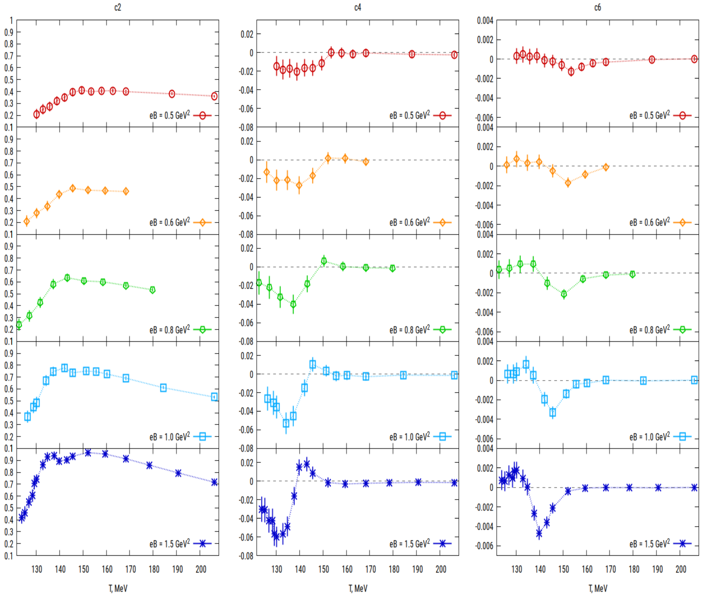
  J. N. Günther et al., Nucl. Phys. A 967, 720 (2017) [arXiv:1607.02493 [hep-lat]]
- EoS at $B \neq 0$ (but $\mu = 0$) was calculated

  G. S. Bali et al., JHEP 08, 177 (2014) [arXiv:1406.0269 [hep-lat]]
- Dimensional reduction in magnetic field $\Rightarrow$
  EoS strongly depends on magnetic field
- Our aim is to study EoS in (μ, T, B) space

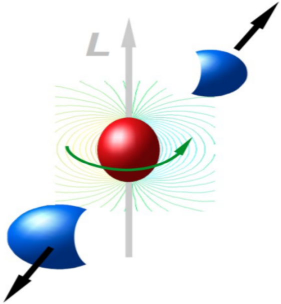
EoS of dense QCD in external magnetic field

- $p = -\frac{\Omega}{V} = \frac{T}{V} log Z$
- Cannot be measured directly
- Derivatives of $log Z$ can be measured: $n = \frac{\partial p}{\partial \mu}$
- $\frac{p}{T^4} = c_0(T) + c_2(T)\left(\frac{\mu}{T}\right)^2 + c_4(T)\left(\frac{\mu}{T}\right)^4 + c_6(T)\left(\frac{\mu}{T}\right)^6 + O(\mu^8)$
  $\frac{n}{\mu T^2} = 2c_2(T) + 4c_4(T)\left(\frac{\mu}{T}\right)^2 + 6c_6(T)\left(\frac{\mu}{T}\right)^4 + O(\mu^6)$
- $\mu_s = 0$, $\mu_u = \mu_d = \mu_l \to i\mu_l$
- Mostly we used 6×24^3 lattice
- Results are preliminary!

EoS of dense QCD in external magnetic field



Rotation of QGP in heavy ion collisions

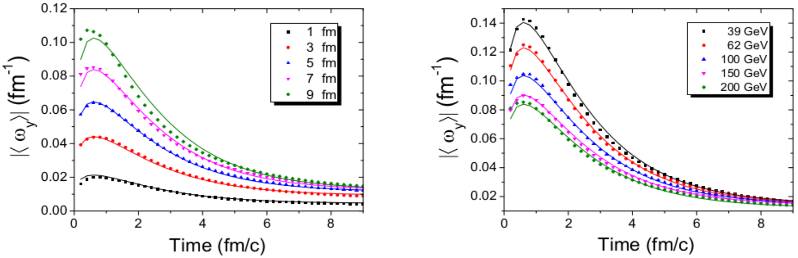


- QGP is created with non-zero angular momentum in non-central collisions

published in V.Braguta et al., JETP Letters 112(1):6-12, 2020

more details in the talk of A. Roenko, Friday, 23 October

Rotation of QGP in heavy ion collisions



Hydrodynamic simulations (arxiv:1602.06580)

- Au-Au: *left* $\sqrt{s} = 200$ GeV, *right* $b = 7$ fm,
- $\Omega \sim 20$ MeV ($v \sim c$ at distances 7 fm)
- Relativistic rotation of QGP

How relativistic rotation influences QCD?

Study of rotating QGP

- Rotating QGP at thermodynamic equilibrium
  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field
- **Boundary conditions are very important!**

Recent works

- Arata Yamamoto, Yuji Hirono, Phys.Rev.Lett. 111 (2013) 081601
- S. Ebihara, K. Fukushima, K. Mameda, Phys. Lett. B 764 (2017) 94–99
- M.N. Chernodub, Shinya Gongyo, Phys.Rev.D 95 (2017) 9, 096006
- M.N. Chernodub, Shinya Gongyo, JHEP 01 (2017) 136
- Hui Zhang, Defu Hou, Jinfeng Liao, e-Print: 1812.11787 [hep-ph]
- Yin Jiang, Jinfeng Liao, Phys.Rev.Lett. 117 (2016) 19, 192302

...

Common features

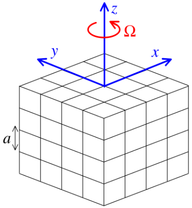
- The studies are carried out in NJL (chiral transition)
- Critical temperature of the chiral phase transition drops with angular velocity
- Explanation: polarization of the chiral condensate (Phys.Rev.Lett. 117 (2016) 19, 192302)
- **Confinement/deconfinement transition was not considered**

Details of the simulations

- Gluodynamics is studied at thermodynamic equilibrium in external gravitational field
- The metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Geometry of the system: $N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$



Details of the simulations

Boundary conditions

- **Periodic b.c.:**
  - $U_{x,\mu} = U_{x+N_i,\mu}$
  - Not appropriate for the field of velocities of rotating body
- **Dirichlet b.c.:**
  - $U_{x,\mu}\big|_{x\in\Gamma} = 1, \quad A_\mu\big|_{x\in\Gamma} = 0$
  - Violate Z_3 symmetry
  - Not appropriate for the field of velocities of rotating body
- **Neumann b.c.:**
  - $U_P\big|_{P\in\Gamma} = 1, \quad F_{\mu\nu}\big|_{x\in\Gamma} = 0$

Details of the simulations

Sign problem

$$S_G = \frac{1}{2g_{YM}^2}\int d^4x \mathrm{Tr}\big[(1-r^2\Omega^2)F^a_{xy}F^a_{xy} + (1-y^2\Omega^2)F^a_{xz}F^a_{xz} +$$

$$+(1-x^2\Omega^2)F^a_{yz}F^a_{yz} + +F^a_{x\tau}F^a_{x\tau} + F^a_{y\tau}F^a_{y\tau} + F^a_{z\tau}F^a_{z\tau} -$$

$$-2iy\Omega(F^a_{xy}F^a_{y\tau} + F^a_{xz}F^a_{z\tau}) + 2ix\Omega(F^a_{yx}F^a_{x\tau} + F^a_{yz}F^a_{z\tau}) - 2xy\Omega^2 F_{xz}F_{zy}\big]$$

- The Euclidean action has imaginary part (**sign problem**)
- Simulations are carried out at imaginary angular velocities $\Omega \to i\Omega_I$
- The results are analytically continued to real angular velocities
- This approach works up to sufficiently large Ω ($\Omega < 50$ MeV)

Details of the simulations

The critical temperature

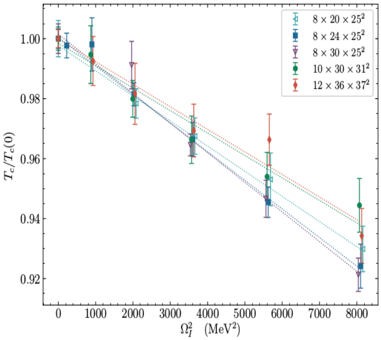
- Polyakov line

$$L = \left\langle \mathrm{Tr} \mathcal{T} \exp \left[ig \int_{[0,\beta]} A_4 \, dx^4 \right] \right\rangle$$

- Susceptibility of the Polyakov line

$$\chi = N_s^2 N_z \left(\langle |L|^2 \rangle - \langle |L| \rangle^2 \right)$$

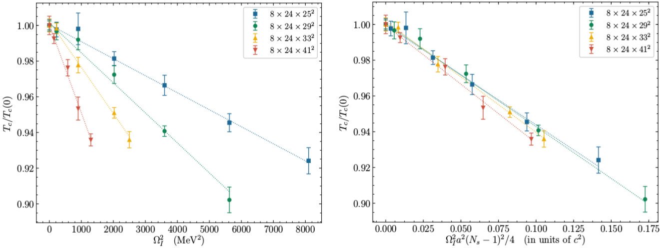
Results of the calculation



- The results can be well described by the formula ($C_2 > 0$)

$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2\Omega_I^2 \Rightarrow \frac{T_c(\Omega)}{T_c(0)} = 1 + C_2\Omega^2$$

- **The critical temperature rises with angular velocity**
- The results weakly depend in lattice spacing and the volume in z-direction



- The results can be well described by the formula

$$\frac{T_c(\Omega)}{T_c(0)} = 1 - B_2 v_I^2, \quad v_I = \Omega_I (N_s - 1) a/2, \quad C_2 = B_2 (N_s - 1)^2 a^2/4$$

- **Periodic b.c.:** $B_2 \sim 1.3$
- **Dirichlet b.c.:** $B_2 \sim 0.3$
- **Neumann b.c.:** $B_2 \sim 0.5$

Conclusion

- QCD phase diagram in (μ, T, B) space
  - Critical temperatures drops with magnetic field and baryon density
  - First observation of the inverse magnetic catalsis in dense matter
  - Our estimation for the CEP is $(T, \mu) \sim (100(25), 800(140))$ MeV
- Equation of state of dense QCD in external magnetic field
  - QCD equation of state strongly depends on magnetic field
- The influence of relativistic rotation to confinement/deconfinement transition
  - Critical temperature of the confinement/deconfinement transition rises with Ω
  - Critical temperature of the chiral transition drops with Ω
  - One needs to include dynamical quarks to see who wins

THANK YOU!