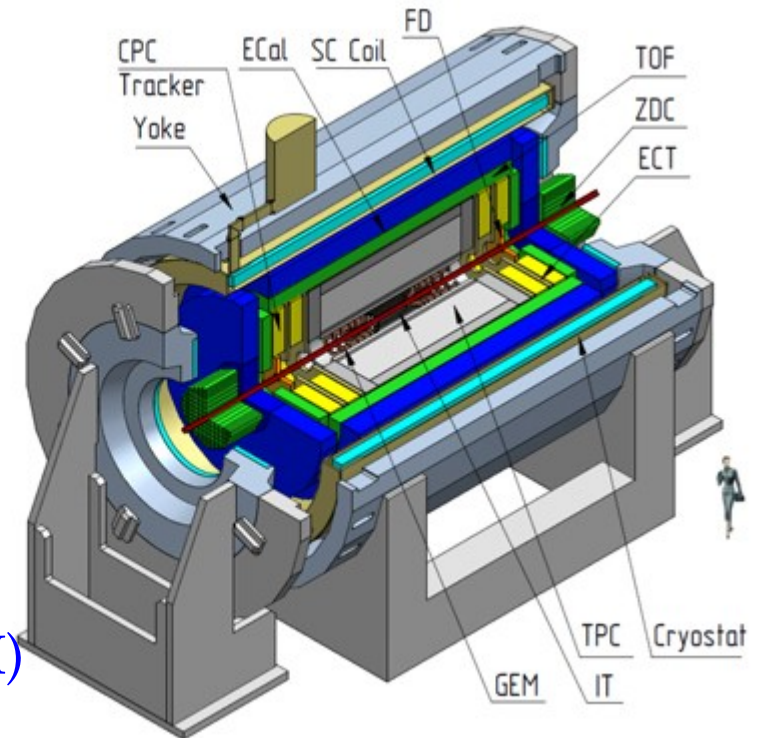# Study of strongly interacting matter properties at the energies of the NICA collider using the methods of femtoscopy and factorial moments

within the RFBR Mega Grant # 18-02-40044

People:
- <u>Ludmila Malinina (SINP MSU, JINR),</u>
- Konstantin Mikhaylov (ITEP & JINR), convener
- Pavel Batyuk (JINR),
- Grigory Nigmatkulov (NRNU MEPhI),
- Olga Kodolova (SINP MSU),
- Igor Lokhtin (SINP MSU),
- Gleb Romanenko (student, MSU),
- Marya Cheremnova (student, MSU)
– Yevheniia Khyzniak (PhD student, NRNU MEPhI)
– Anna Romanova (student, MSU)

Outline

- Femtoscopy & Factorial moments group activities

- Femtoscopy & Motivation

- Comparison with STAR BES pions for vHLLE and UrQMD

- Comparison of Pions and Kaons for vHLEE and UrQMD

- Tests with reconstructed data

- Conclusion

Femtoscopy & correlations activities within RFBR mega grant
"Study of strongly interacting matter properties at the energies of the NICA collider using the methods of femtoscopy and factorial moments"

Aim of the project:

Study of collective effects and dynamics of quark-hadron phase transitions via femtoscopic correlations of hadrons and factorial moments of particle multiplicity at NICA energies

- Factorial moments of the multiplicity distribution (See talk of Olga Kodolova)

- Software development (See talk of Grigory Nigmatkulov)

Activites in 2019-2020

- Simulation of Au+Au collisions with UrQMD and vHLLE models for different collision energies

- Development of the data analysis methods and software within MPD software environment

- Femtoscopic analysis for pions and kaons and factorial moments analysis for models with different Equation of State (EoS): first-order phase transition (1PT), crossover (XPT), no phase transition

- Investigation of the detector effects (track-merging and track-splitting in TPC) on femtoscopic measurements and on factorial moments analysis

Our software developments:

Package for Femtoscopy analyses:

✔ Inherited from STAR (StHbtMaker) and ALICE (AliFemto)
✔ Keeps the same hierarchy as in ALICE (PckgName/, PckgNameUser/, macros/)
✔ Works with ROOT 5 and 6
✔ Lighter than ancestors:
    ✔ Most of STAR-developed classes replaced with ROOT ones
    ✔ Better compression, smaller sizes
✔ Implemented running options (INDEPENDENT on experiment-dependent software):
    ✔ Standalone mode – compile with g++ (clang) and run on your "laptop"
    ✔ Maker; Tasks will be also implemented

Factorial moments:

Factorial moments analysis code inherited from Mirabel experiment is written

Data formats (DST):

✔ General-purpose data format for Monte Carlo generators - McDst
  (https://github.com/nigmatkulov/McDst)
    ✔ Similar to UniGen (developed at GSI)
    ✔ Lighter, faster, easy expandable, works with ROOT 5 and 6, g++ (clang)
    ✔ Possibility to add converters from other generators: Terminator, EPOS, AMPT, etc...
✔ Group has positive experience on the data format developments:
    ✔ (St)PicoDst format in STAR (standard data format for physics analysis)

Mini DST format:

Output data format derived from STAR has been incorporated to MpdRoot.

VHLLE interface software:

Allows to perform simulations with vHLLE+UrQMD model by simple and understandable way
(vHLLE_package/README.md)

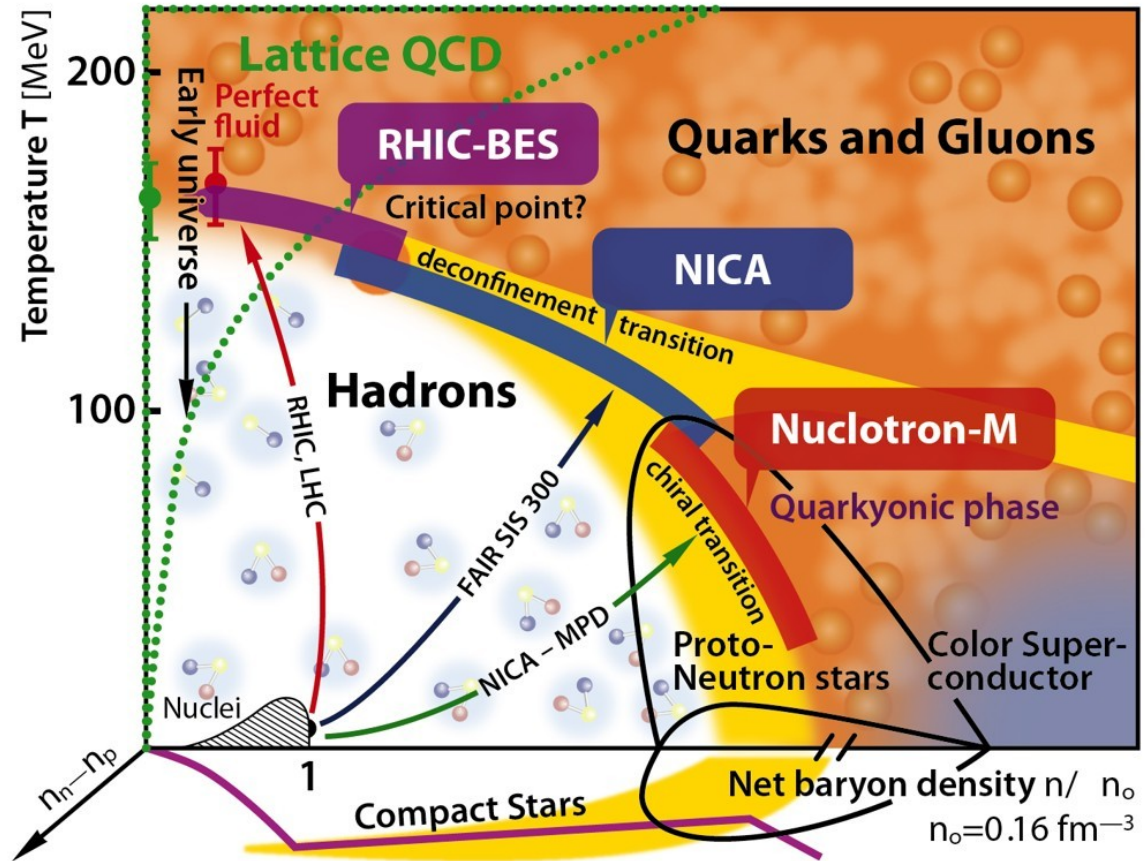# Motivation: Phase diagram QCD

- Crossover transition to QGP occurs at RHIC & LHC

- 1st order phase transition to QGP occurs at lower energies (?)

1 fm/c 5 fm/c 10 fm/c 10-15 fm/c

pre-equilibrium QGP hadronization freeze-out
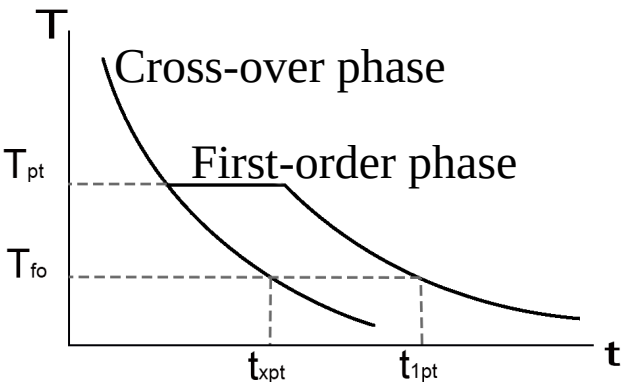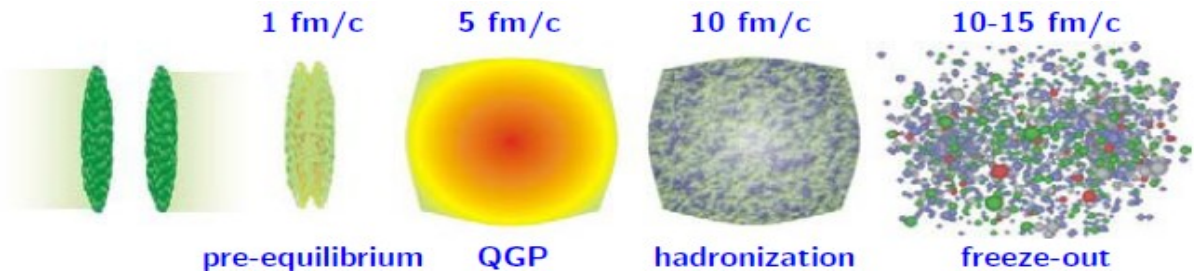
- Beam Energy Scan (BES) program: search for location of transition point from Crossover (XPT) to 1st order (1PT) phase transitions

- BES RHIC ($\sqrt{s}$=3-60 GeV)
- NA61@SPS (E_{lab}=10-158 GeV);
- projects: CBM@FAIR (GSI) и **MPD@NICA (JINR)**

Femtoscopy with vHLLE

Iu. Karpenko, P. Huovinen, H.Petersen, M. Bleicher, Phys.Rev. C.91, 2015, 064901

Pre-thermal phase → **Hydrodynamic phase** → **Hadronic cascade**

UrQMD

vHLLE
(3+1)-D viscous hydrodynamics

UrQMD

The transition to hydrodynamics occurs at a hyper-surface of constant longitudinal proper time τ_0
Minimal value of the starting time τ_0 is taken to be equal to the average time for the two colliding nuclei to completely pass through each other:

Parameters τ_0, $R_\perp$, R_η and η/s adjusted using basic observables in the RHIC BES-I region.

Model tuned by matching with existing experimental data from SPS and BES-I RHIC

EoS to be used in the model

- Chiral EoS — crossover transition
  J. Steinheimer et al., J. Phys. G 38, 035001 (2011)

- Hadron Gas + Bag Model 1st-order phase transition
  P. F. Kolb et al., Phys.Rev. C 62, 054909 (2000)

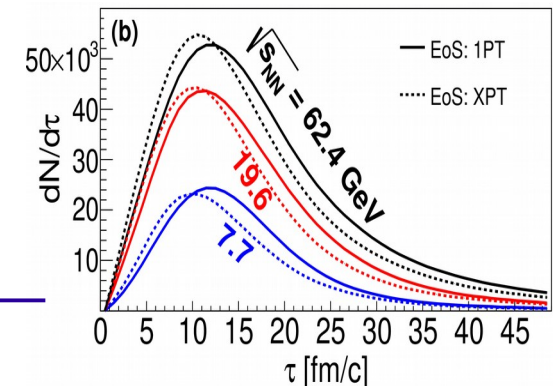Hydrodynamic phase lasts longer with 1PT, especially at lower energies but cascade smears this difference.

Fluid to particle transition at energy density = 0.5 GeV/fm^3

Pion emission time after hydrodynamic phase

after cascade

Can we probe this difference with femtoscopy ?

Femtoscopy

Correlation femtoscopy :

Measurement of space-time characteristics **R**, **cτ** of particle production using particle correlations due to the effects of quantum statistics (QS) and final state interactions (FSI)
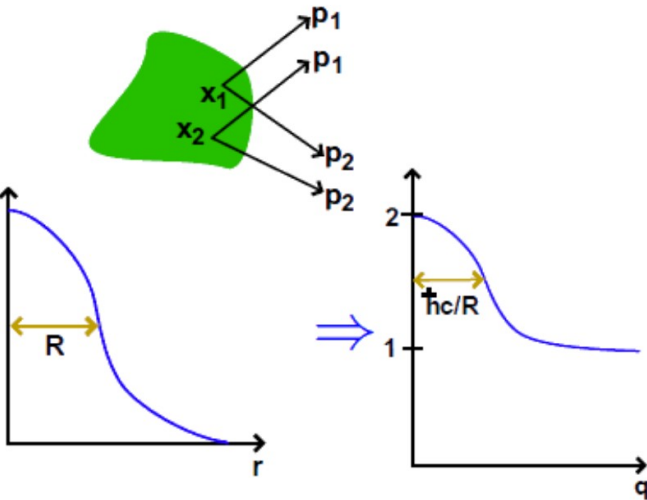
Two-particle correlation function:

theory:
$$C(q)=\frac{N_2(p_1,p_2)}{N_1(p_1)\cdot N_2(p_1)},\, C(\infty)=1$$

experiment:
$$C(q)=\frac{S(q)}{B(q)},\, q=p_1-p_2$$

$S(q)$ – distribution of pair momentum difference from same event
$B(q)$ – reference distribution built by mixing different events

Parametrizations used:

1D CF:
$$C(q_{inv})=1+\lambda\, e^{-R^2 q_{inv}^2}$$

R – Gaussian radius in PRF,
λ – correlation strength parameter

3D CF:
$$C(q_{out},q_{side},q_{long})=1+\lambda\, e^{-R_{out}^2 q_{out}^2 - R_{side}^2 q_{side}^2 - R_{long}^2 q_{long}^2}$$

R and q are in Longitudinally Co-Moving Frame (**LCMS**)
long ∥ beam; out ∥ transverse pair velocity v_T; side normal to out,long
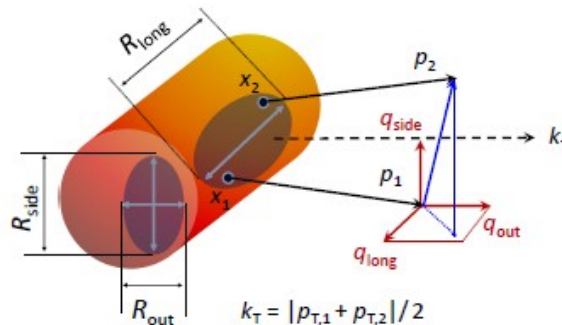
3D analysis:

R_{side} sensitive to geometrical transverse size.

R_{long} sensitive to time of freeze-out.

R_{out} / R_{side} sensitive to emission duration.

Motivation: Correlation femtoscopy.

- RHIC Beam Energy Scan program (BES-I):
  $\sqrt{s_{NN}}$ = 7.7, 11.5, 19.6, 27, 39 GeV
  - The search for the onset of a first-order phase transition in Au + Au collisions
  - Measured pion and kaon femtoscopic parameters: m_T-dependence of radii,
    flow-induced $x - p$ correlations

- NICA energy range: $\sqrt{s_{NN}}$ = 4 – 11 GeV
  - first collider measurements below 7.7 GeV
    - including K and heavier

- There is a lack of systematical high accuracy and high statistics studies of QGP phase transition for baryons enriched matter in the world.

- Study of this region will allow the better understanding of transition from quark degree of freedom (high energies) to nucleon degree of freedom (low energies)
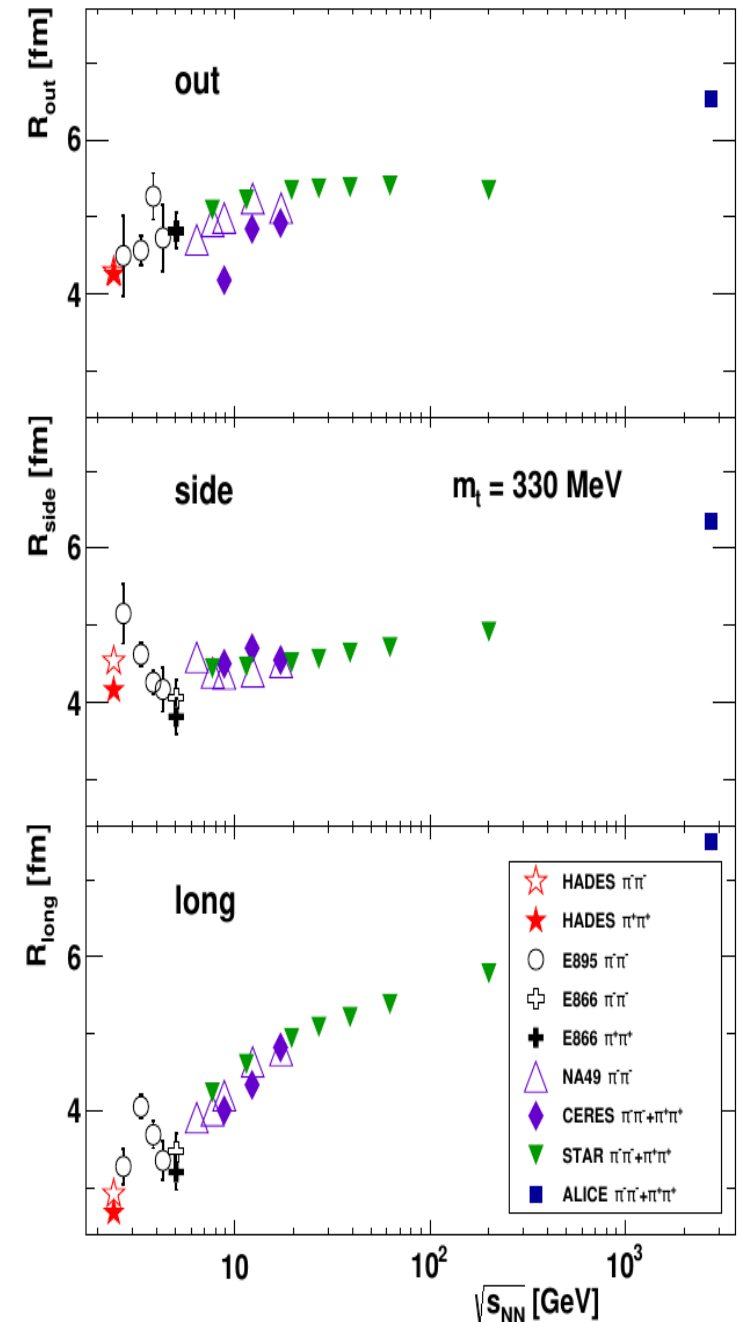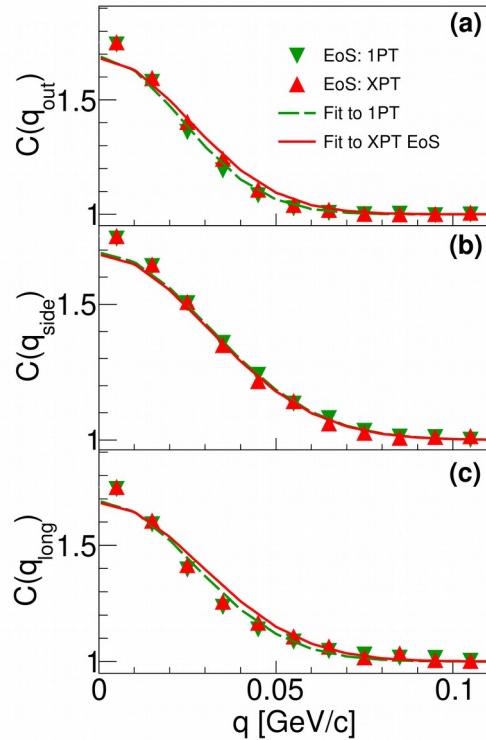
3D Pion radii with vHLLE model

Phys.Rev. C96 (2017) no.2, 024911

Projections of 3D Model CF
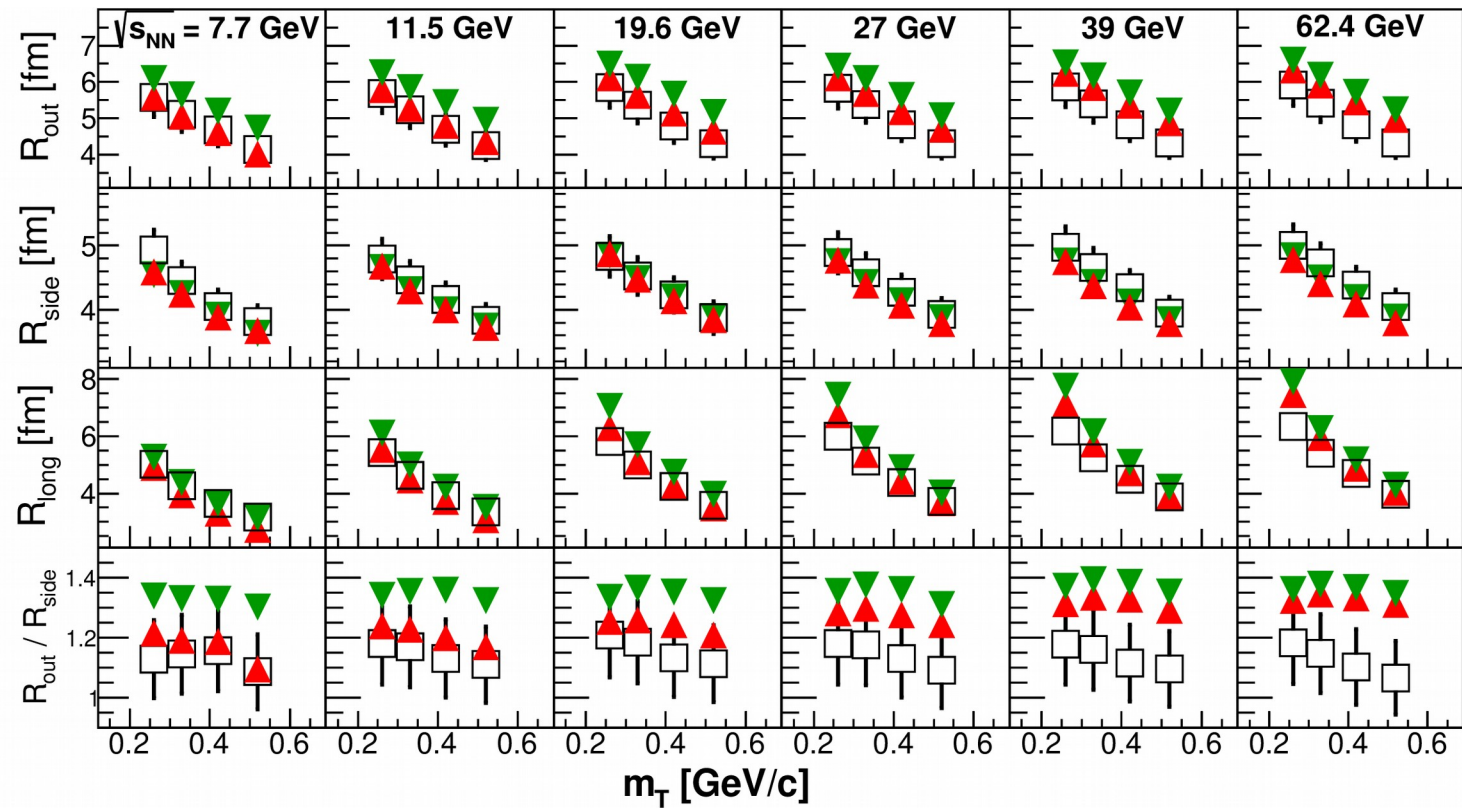

- Femtoscopic radii are sensitive to the type of the phase transition
- Crossover EoS describes better R(mT) dependencies
- **$R_{out,long}$ (1PT) > $R_{out,long}$ (XPT) by value of ∼1-2 fm.**
- R_{out} /R_{side} (XPT) agrees with STAR data points at 7.7 and 11.5 GeV, but then increases with increasing collision energy while ratio in data is independent with collision energy; R_{out} /R_{side} for 1PT systematically overestimate data and is independent on collision energy.

3D Pion Correlation Functions with vHLLE model

Analysis

1PT -green dots; XPT – red dots

- MDP femto package
  (see Grigory Nigmatkulov's talk)

- We performed analysis of
  pion and kaon correlations
  for vHLLE and UrQMD for
  different centrality ranges

- Au+Au, $\sqrt{s_{NN}}$ = 11.5 GeV
  central events vHLLE
- Standard 3D Gaussian fit used
- Projections of 3D pion correlation
  functions on out-side-long

- XPT CF projections on long direction
  are wider than 1PT ones

- CF become wider with increasing k_T
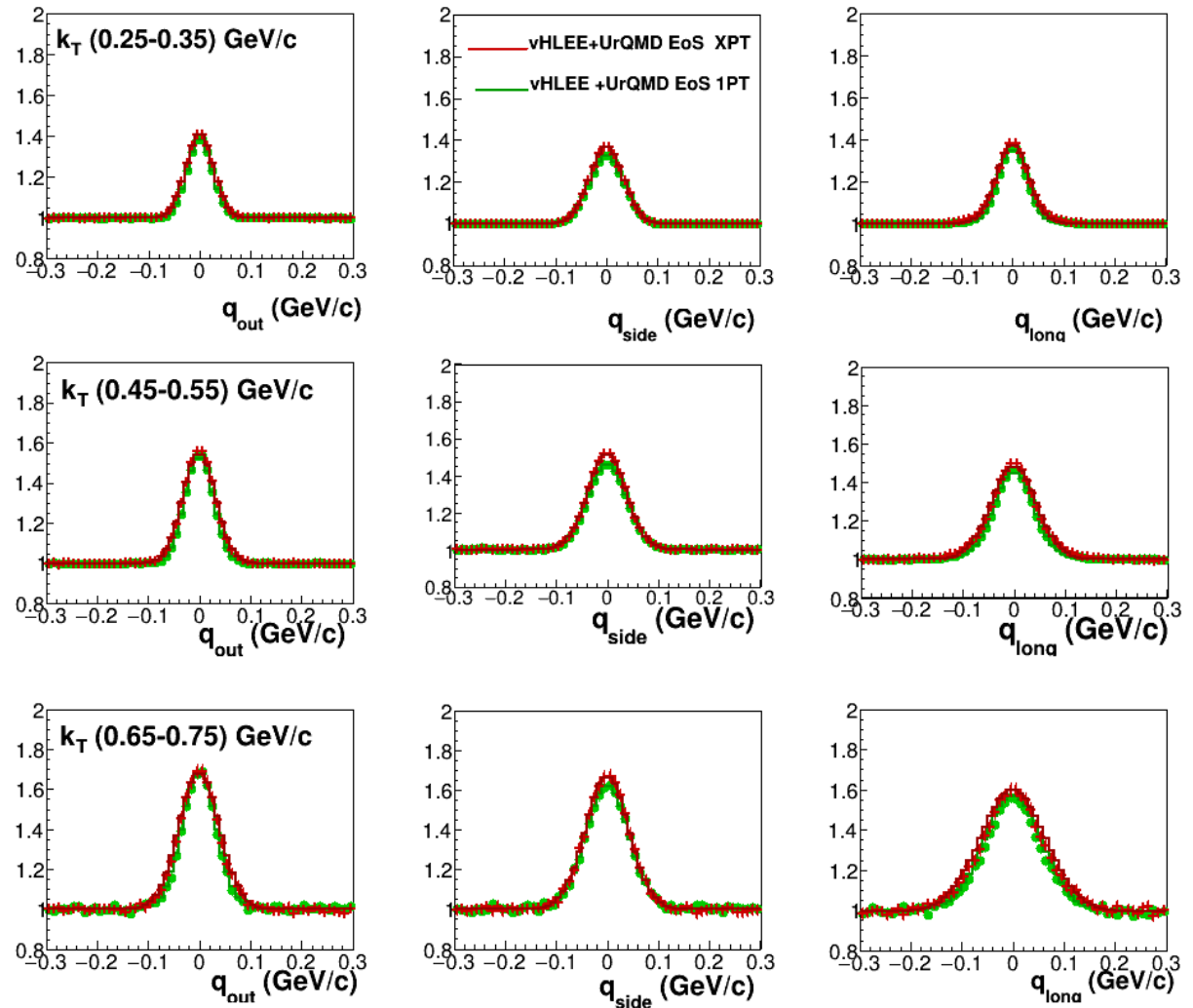
Pion radii with vHLLE and UrQMD models
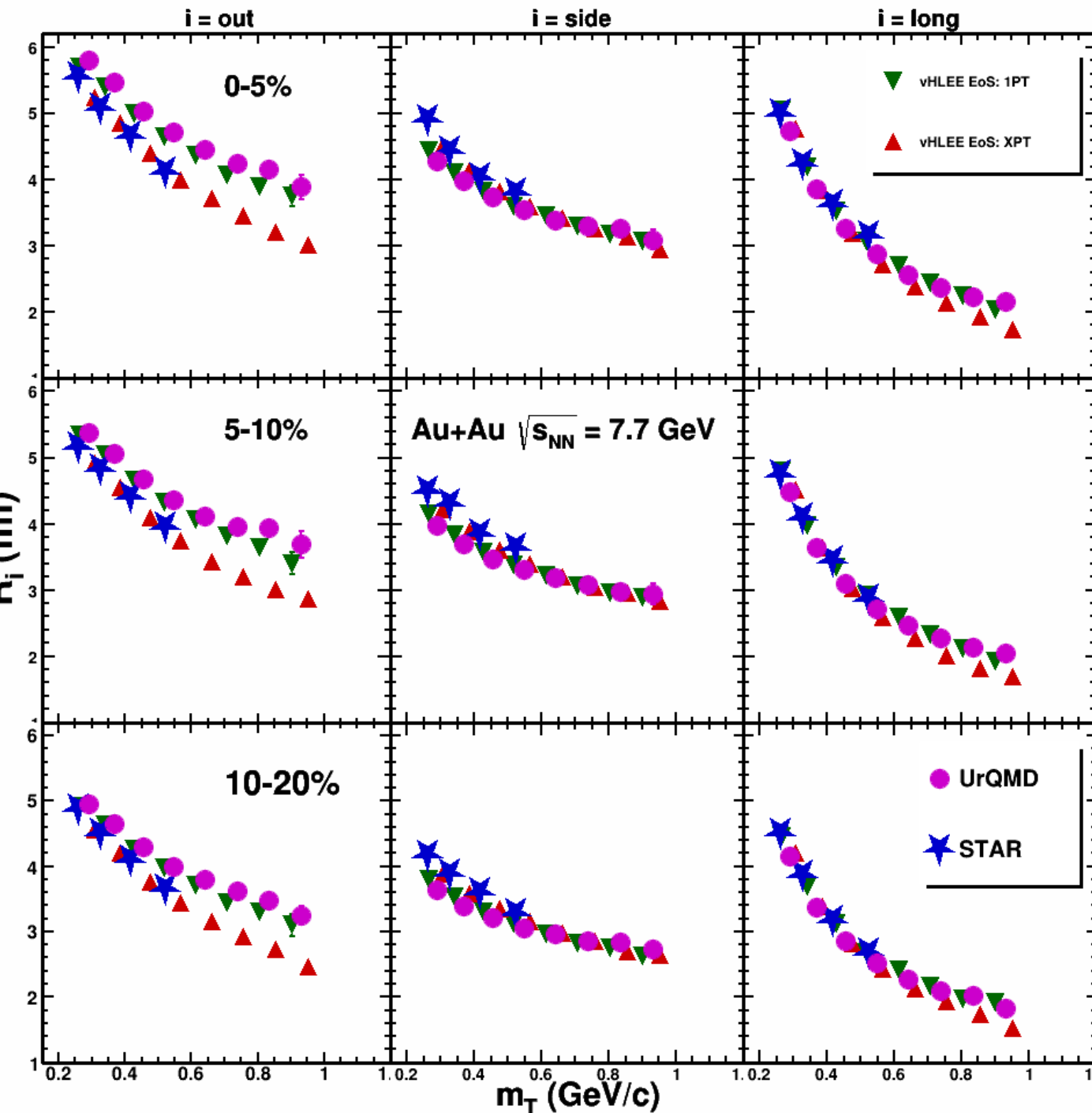
- Au+Au, $\sqrt{s_{NN}}$ = 7.7 GeV

- It is important to study k_T (m_T) dependence in larger interval than STAR

- Radii decrease with $m_T \rightarrow$ radial flow

- Increase size with increasing centrality $\rightarrow$ simple geometric picture of the collisions.

- Crossover EoS describes better $R_{out}(m_T)$

- $R_{out,long}$ (1PT) > $R_{out,long}$ (XPT)

- UrQMD is closed to 1PT

- The similar trends is observed for AuAu 11.5 GeV

Pion and kaon radii with vHLLE model
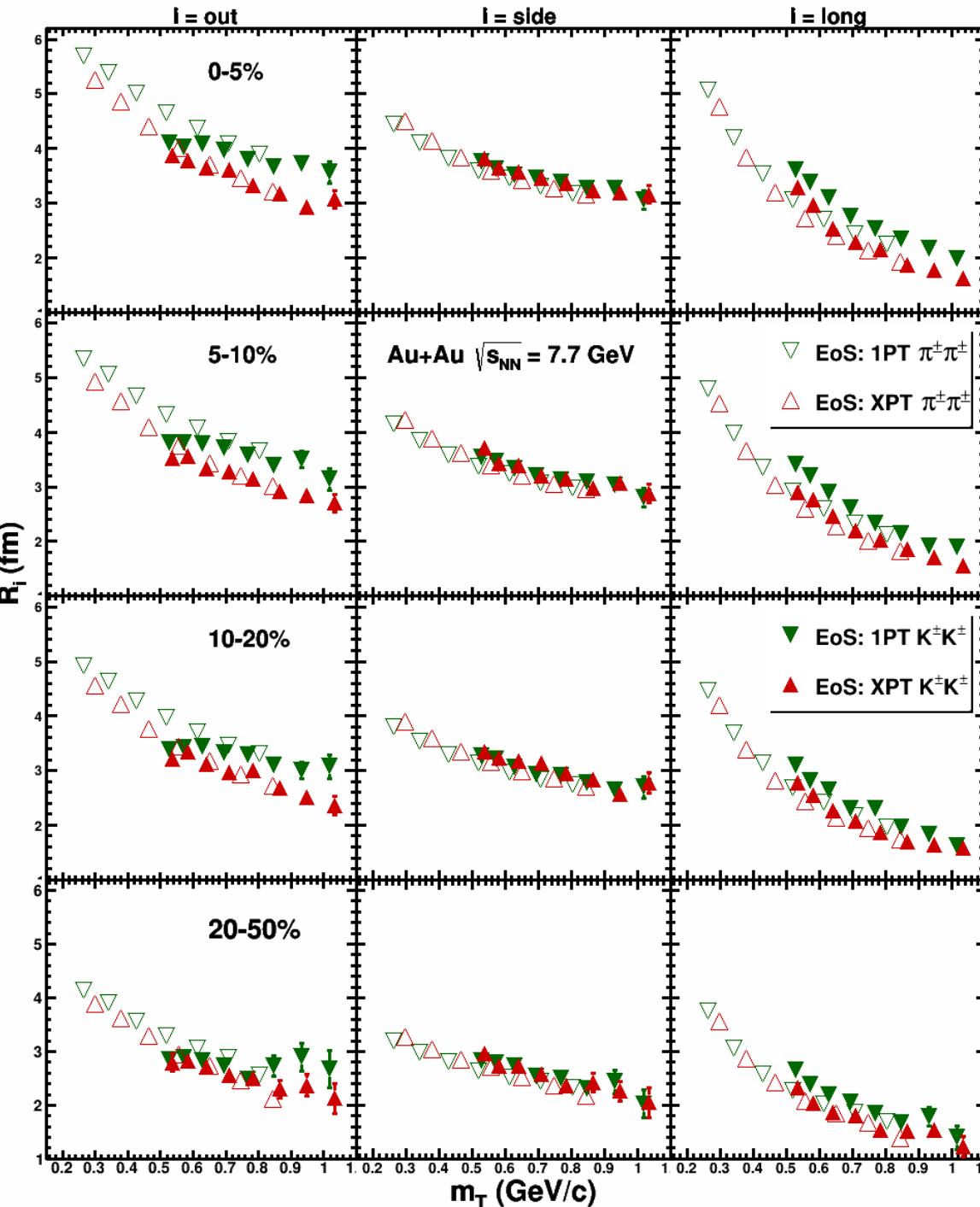
- Au+Au, $\sqrt{s_{NN}}$ = 7.7 GeV

- Approximate m_T scaling for R_{side}

- Similarly to pions : kaon radii decrease with m_T → radial flow ;

- **for 1PT EoS almost flat dependence R_{out} (m_T) is observed → weaker flow**

- $R_{out,long}$ **(1PT) > $R_{out,long}$ (XPT)**

- R_{long} kaon radii for XPT > R_{long} pion similarly to experiment (LHC & RHIC)

- Very different predictions of vHLLE model for different EoS → importance to study heavier than pions particles → kaons

- The similar trend is observed for AuAu 11.5 GeV

Pion and kaon radii with UrQMD model
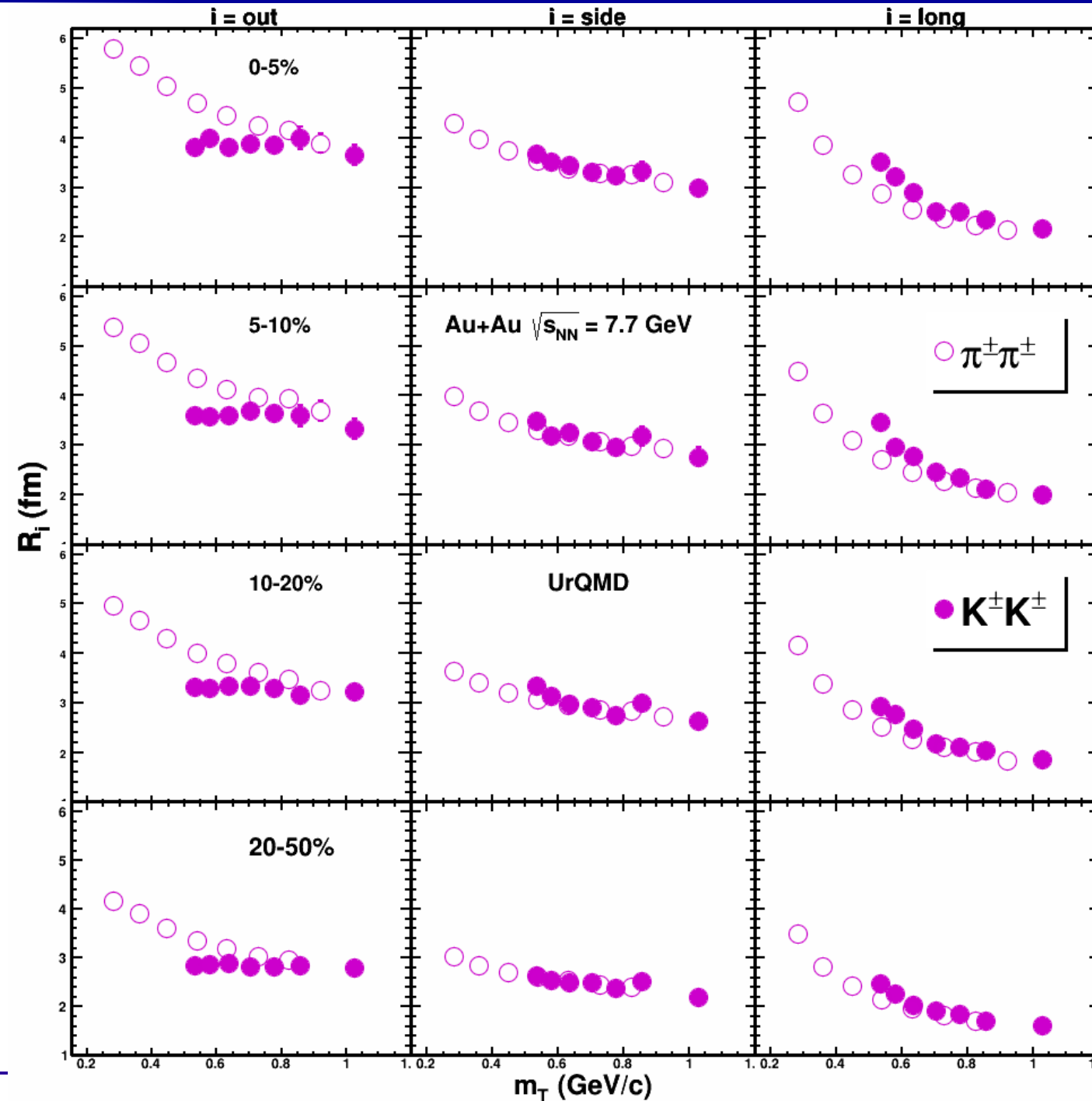
- Au+Au, $\sqrt{s_{NN}}$ = 7.7 GeV

- **kaon radii demonstrate almost flat behavior similarly to vHLEE with the 1PT EoS → weak flow**

- R_{long} kaon radii are larger than pion ones similarly to experiment (LHC & RHIC)

- The similar trend is observed for AuAu 11.5 GeV

Energy dependence of pion radii
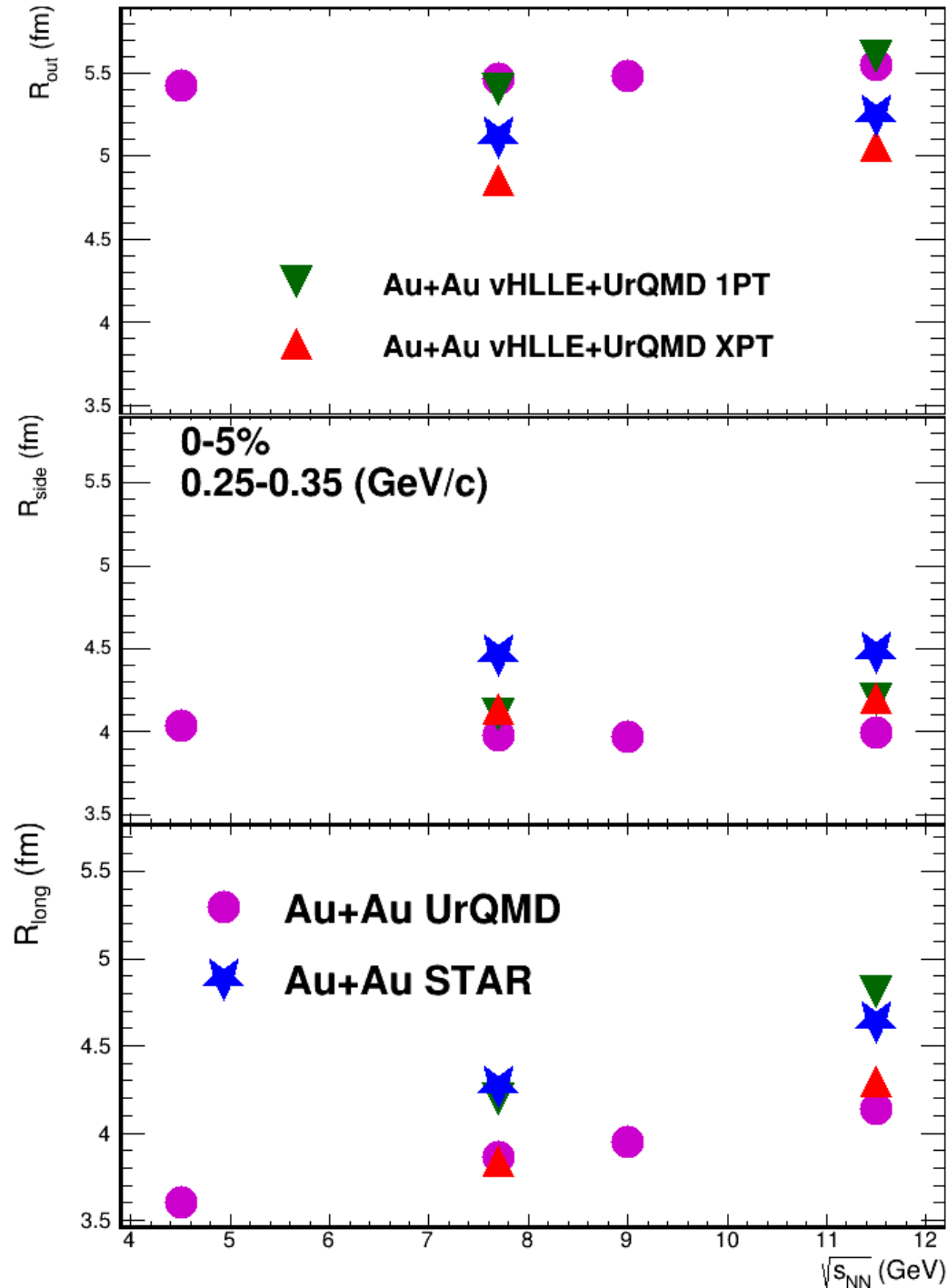
- NICA energy range: $\sqrt{s_{NN}}$ = 4 – 11 GeV

- We performed femtoscopic analysis of pions in this energy range using vHLEE and UrQMD models

- There is a lack of systematical high accuracy and high statistics studies of QGP phase transition for this region in the world !

# Analyis of reconstructed in MPD data

Comparison of AuAu and BiBi (UrQMD)
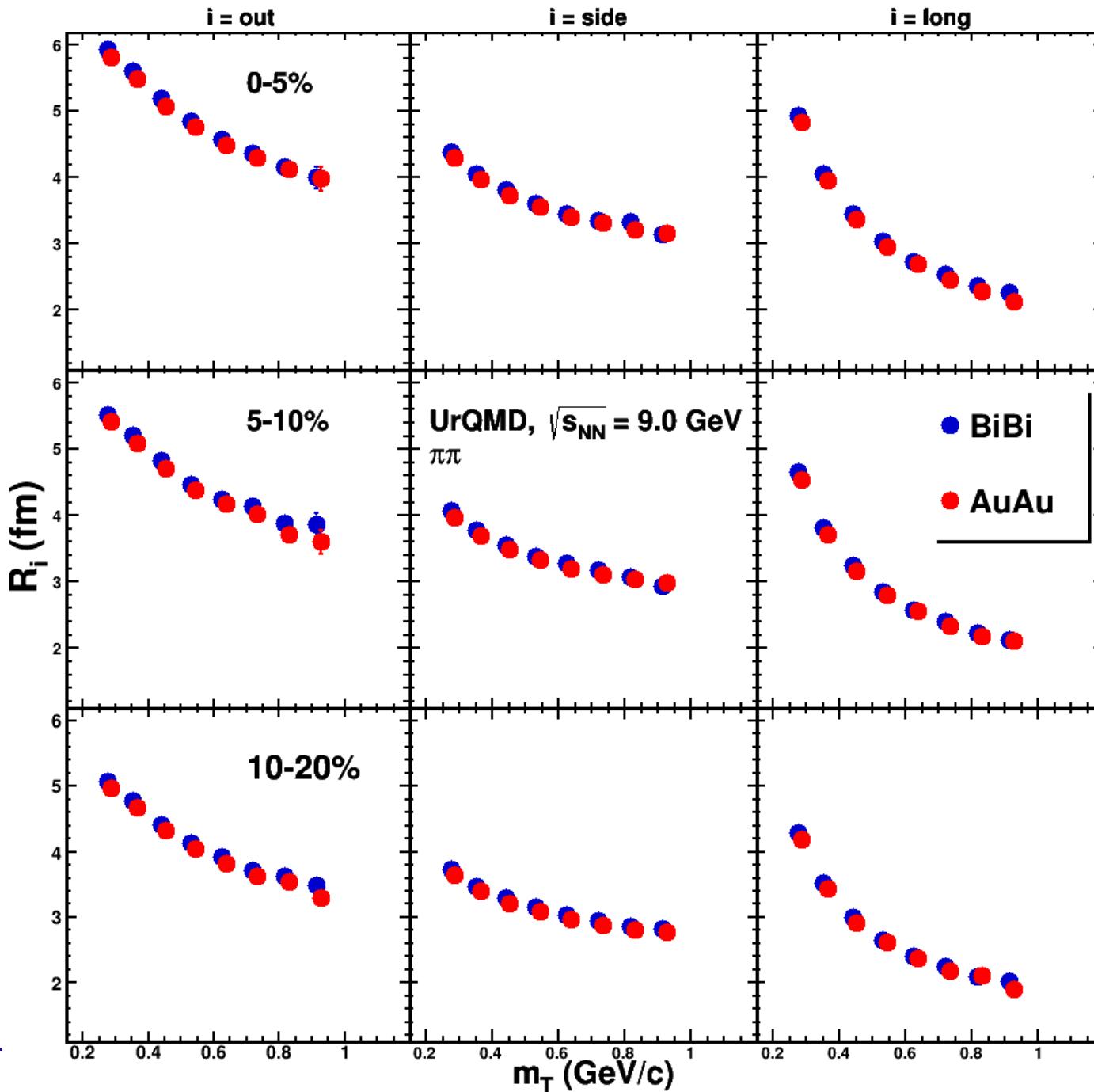
- Au+Au, and Bi+Bi at $\sqrt{s_{NN}}$ = 9 GeV

- Pion femtoscopic radii of Bi+Bi are larger than Au+Au ones by ~2-6%
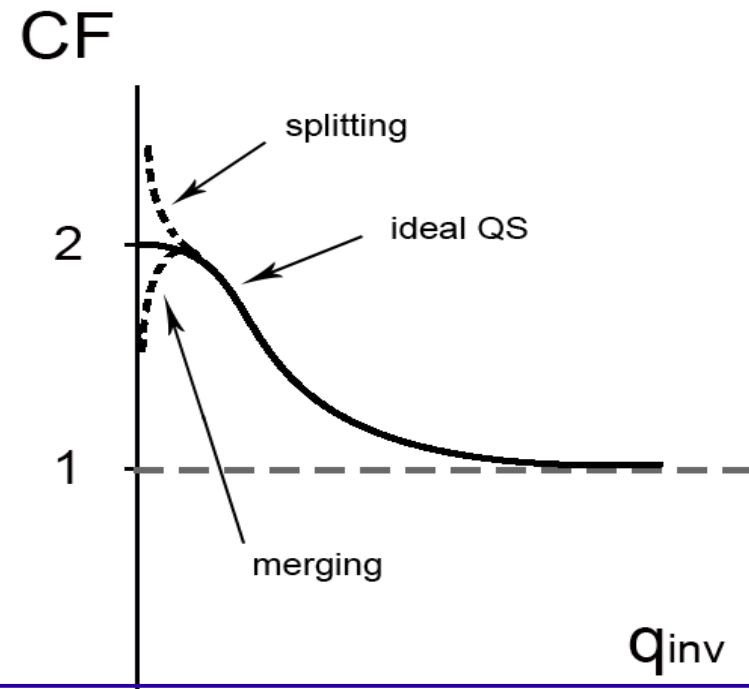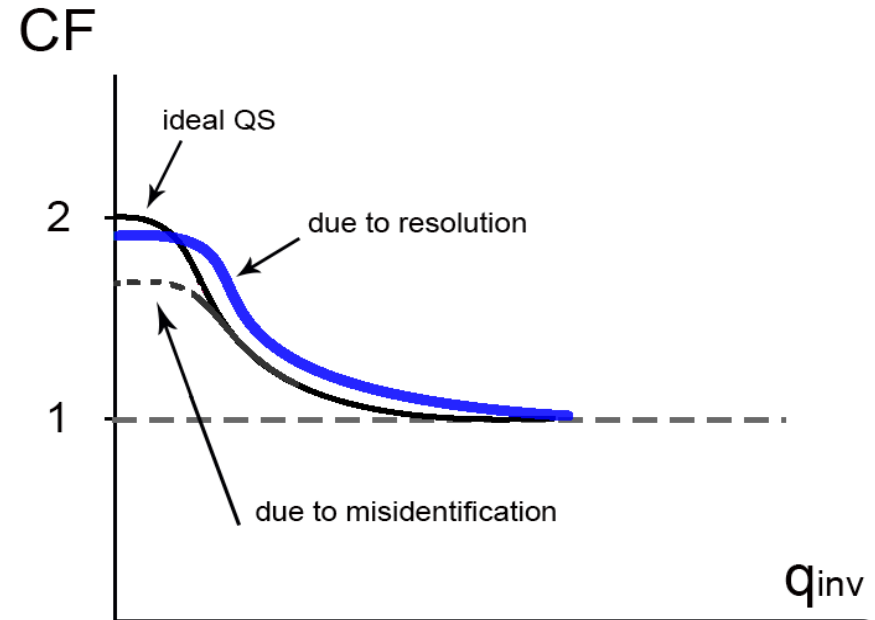
Analysis of reconstructed data using MPD software

- Dataset (reconstructed in MPD tracks) production:
  /eos/nica/mpd/sim/data/MiniDst/dst-BiBi-09GeV-mp07-20-pwg3-250ev/BiBi/09.0GeV-0-14fm/UrQMD/'

- 10 mln
  UrQMD Minimal Bias events
  BiBi 9 GeV
  - Mini Dst format

- 5 kT bins (0.15-0.65) GeV/c, with step 100 MeV

- Kinematic conditions for pions
  p_T (0.15-1.45) GeV/c
  |eta|<1.0

- Nhits TPC > 15
  DCA < 3 cm
  |VertexZ| < 75
  PID : Nsigma for pion selections in TPC & TOF = 2
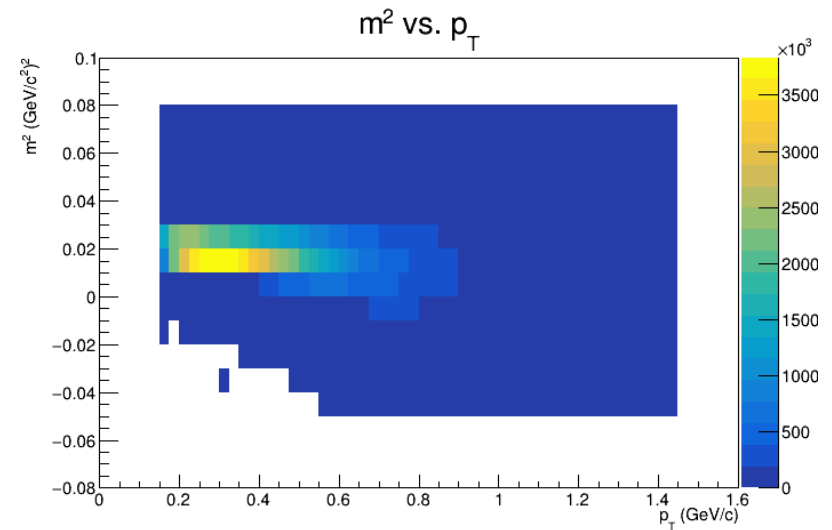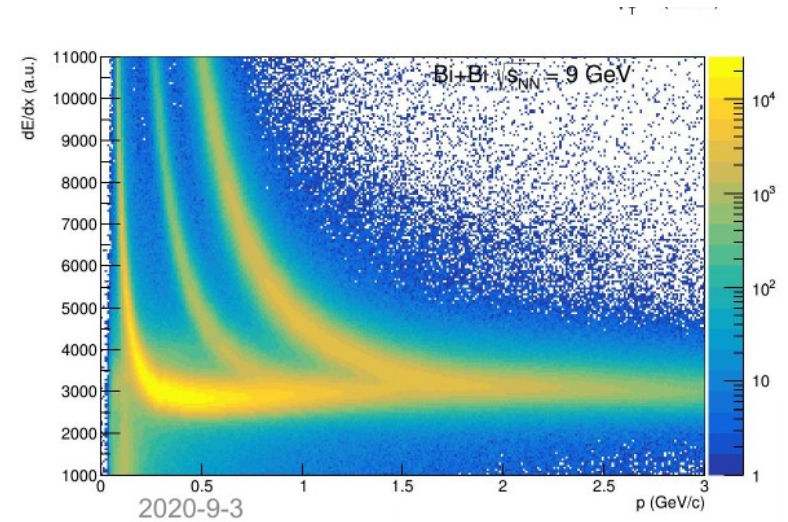
Detector reconstruction influence on CF

Detector reconstruction influences the shape of CF:

- Single track effects:
    - the momentum resolution effects smear CF, making it wider and extracted radii smaller
    - CFs should be corrected by resolution

    - the particle misidentification influences only λ-parameter of CF, radii do not change.
    - CF should be corrected by pair purity. Pair purity is obtained from particle purity

- Two track effects:
    - track splitting (one track is reconstructed as two)
    - track merging (two tracks are reconstructed as one)
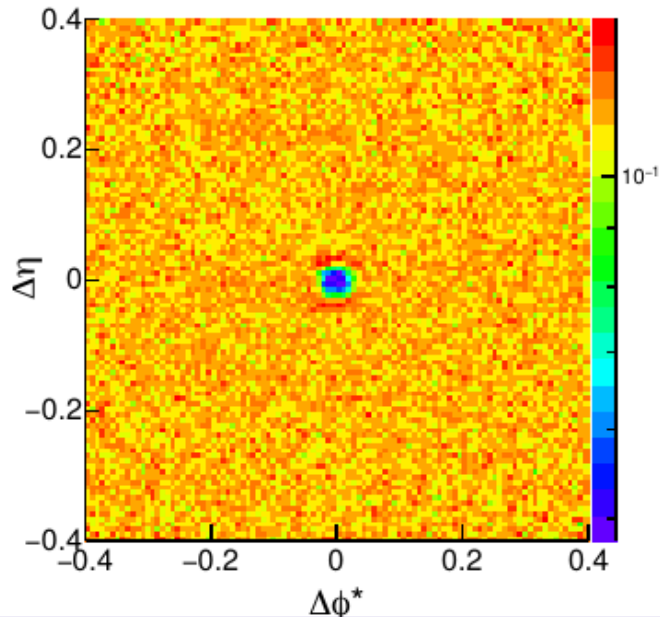   These effects are studied and the special pair cuts are used in the analysis.

In MPD FEMTO package are implemented different methods for study the two tracks effects which are widely used by STAR and ALICE collaborations.
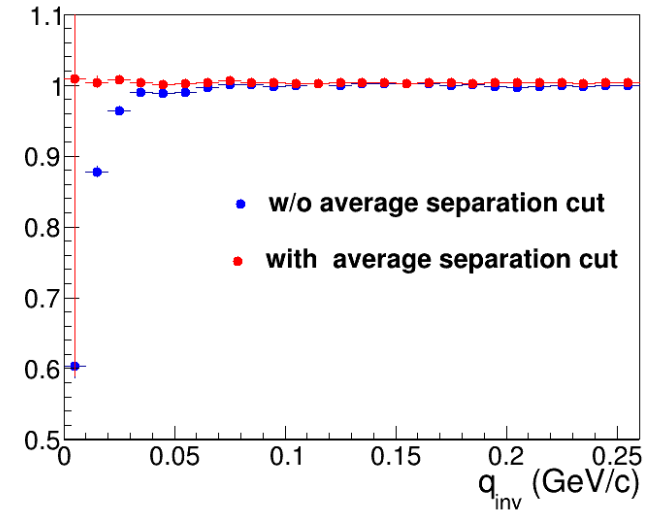

ππ correlation function w/o FSI
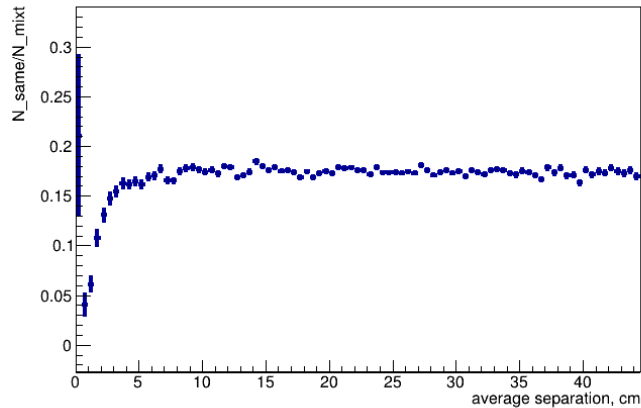
- The cut on Average Separation between two tracks in TPC was studied and is find to be > 7 cm

- The cut on cylindrial distance between tracks in TPC was tested and is find to be: Δη<0.04 and Δφ*<0.02

- Pion femtoscopic CF can be correctly reconstructed if two-tracks cuts are applied

Δη vs Δφ*

ππ correlation function (QS, r_0=3 fm)

χ^2 / ndf	56.75 / 47
r_0	2.985 ± 0.011
λ	0.9703 ± 0.0068
N	1.001 ± 0.000
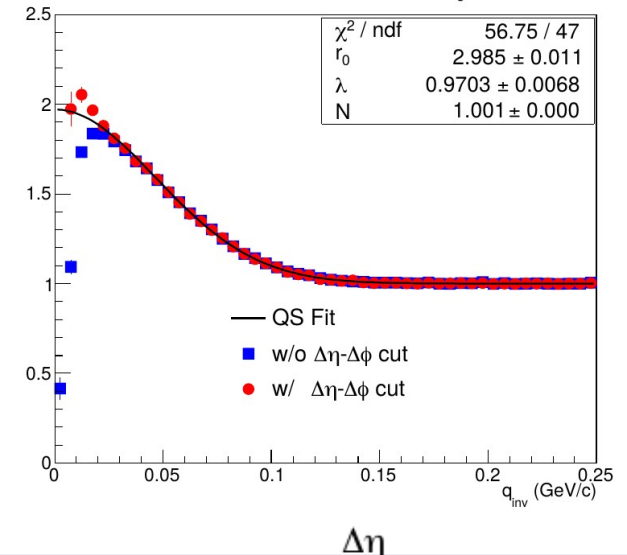
Conclusions & plans

- Study of collective effects and dynamics of quark-hadron phase transitions via femtoscopic correlations of hadrons and factorial moments of particle multiplicity at NICA energies was performed

- First results look promising and this study is planned to be continued.

- Development of the data analysis methods and software integrated in the Multi-Purpose Detector (MPD) software environment was performed and will be continued

- Our studies were presented in the MPD Physics Seminars on and in internatinal conferences WPCF2019, QFTHEP 2019, ICPPA-2020

Activities within RFBR grant 18-02-40044

- Three Master and 1 PhD student in Femto group

- PWG3 Meetings: 8 events(2019) and 4events(2020) →   https://indico.jinr.ru/category/346/

- MPD Physics Seminars:
L.Malinina. «Correlation femtoscopy at NICA» 21-11-2019
G.Nigmatkulov. «Looking at Data Stored in MpdDst» 21-11-2019
K. Mikhaylov «The first tests of MC data obtained using vHLLE model» 19-09-2019

- Conferences:
P. Batyuk. «Femtoscopy with identified particles for NICA/MPD». XIV WPCF, Dubna, 2019
K. Mikhaylov. «Correlation femtoscopy at NICA energies». XXIV QFTHEP, Sochi, 2019
P. Batyuk. "Correlation femtoscopy and factorial moments at theNICA energies". NICA-days 2019, Warsaw, 2019

- Publications:
K.Mikhaylov, P.Batyuk, O.Kodolova, L.Malinina, G.Nigmatkulov and G.Romanenko,
«Correlation femtoscopy at NICA energies», EPJ Web Conf. Volume 222, 2019, 02004
P. N. Batyuk, L. V. Malinina, K. R. Mikhaylov, and G. A. Nigmatkulov,
«Femtoscopy with Identified Charged Particles for the NICA Energy Range», Physics of Particles and Nuclei, 2020, Vol. 51, No. 3, pp. 252–257

Additional slides

Correlation Femtoscopy: Final State Interaction

pp

CF

Coulomb only

$k* = |\mathbf{q}|/2$
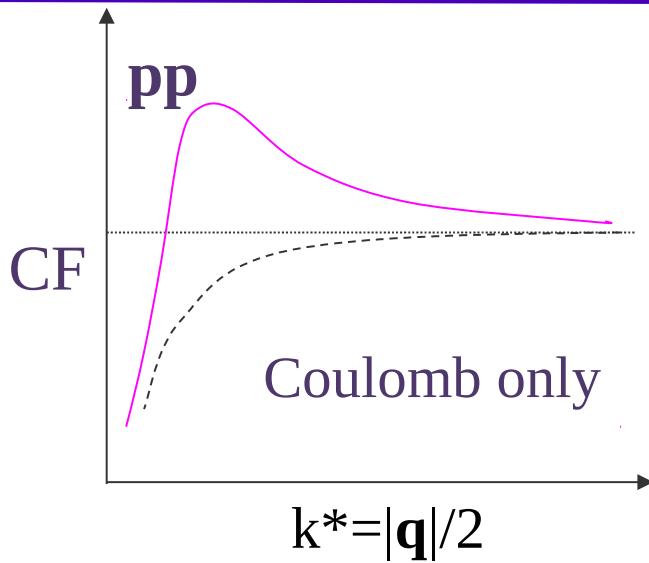
**FSI is sensitive to source size and scattering amplitude.
It complicates CF analysis but makes possible:**

- **Femtoscopy with nonidentical particles:** $\pi K, \pi p, \pi \Xi \dots$

- **Study of the "exotic" scatterings:** $\pi\pi, \pi K, KK, \pi\Lambda$

- **Study of the relative space-time asymmetries
of particles emission** $\pi K, pK, \pi\Xi$ ..
  (Lednicky, Lyuboshitz et al. PLB 373 (1996) 30)

Motivation: Correlation femtoscopy.

- It was predicted that for 1st order phase transition $R_{out}/R_{side} > 1$ & large R_{long} due to emission stalling during phase transition

(S. Pratt, Phys. Rev. D 33 (1986) 1314.G. Bertsch, M. Gong, M. Tohyama, Phys. Rev. C 37 (1988) 1896) D. H. Rischke and M. Gyulassy, Nucl. Phys. A608, 479 (1996)
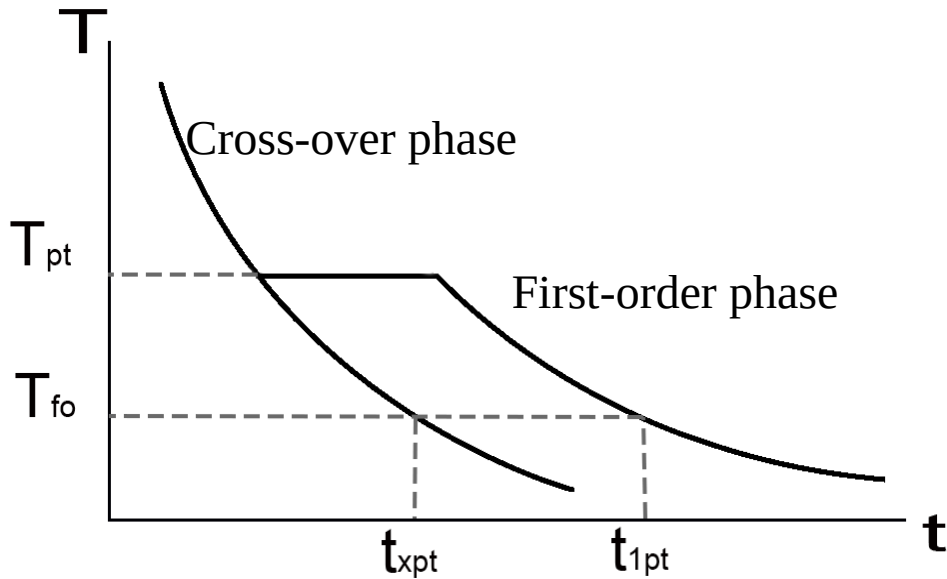

- RHIC Beam Energy Scan program (BES-I): √sNN = 7.7, 11.5, 19.6, 27, 39 GeV pion and kaon femtoscopic radii were measured
- New fix target results with sqrt(sNN)=4.5 GeV

Flow and interferometry results from Au+Au collisions at sqrt(sNN) = 4.5 GeV   STAR, 2007.14005
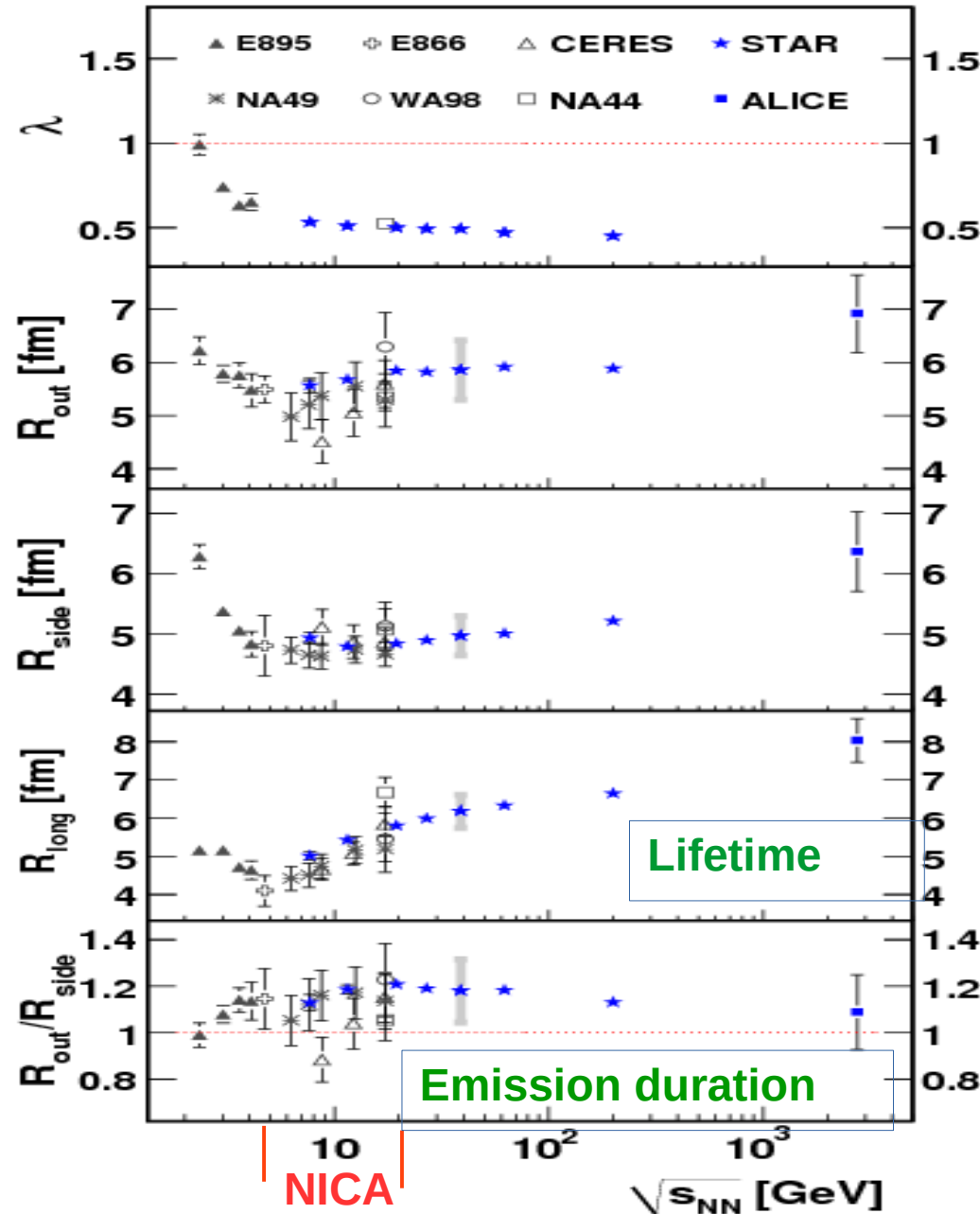
Femtoscopy with expanding source → m_T-dependence

- **x-p** correlations → interference dominated by particles from nearby emitters.
- Interference probes only parts of the source at close momenta – **homogeneity regions.**
- Longitudinal and transverse expansion of the source -> significant reduction of the radii with increasing pair velocity, consequently with k_T (or $m_T=(m^2+k_T^2)^{1/2}$)

Slow particle   *Fast particle*
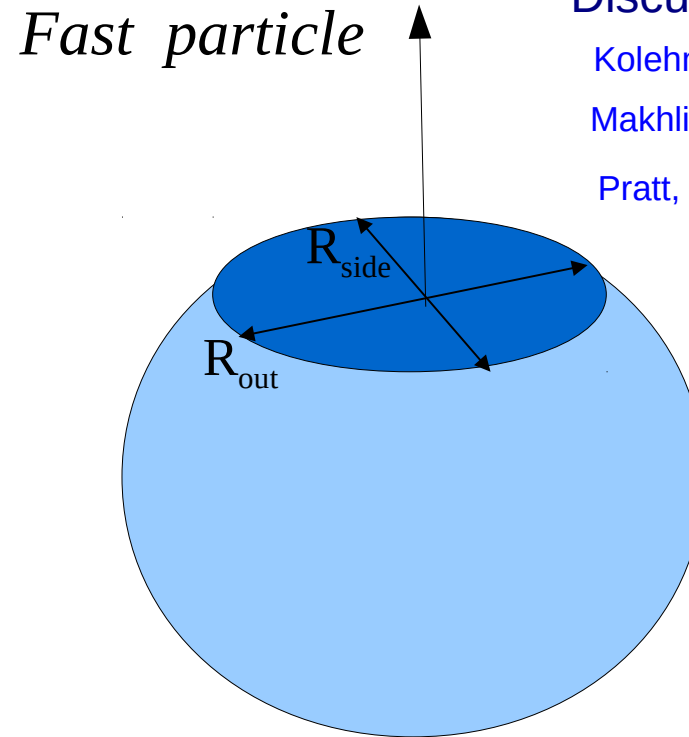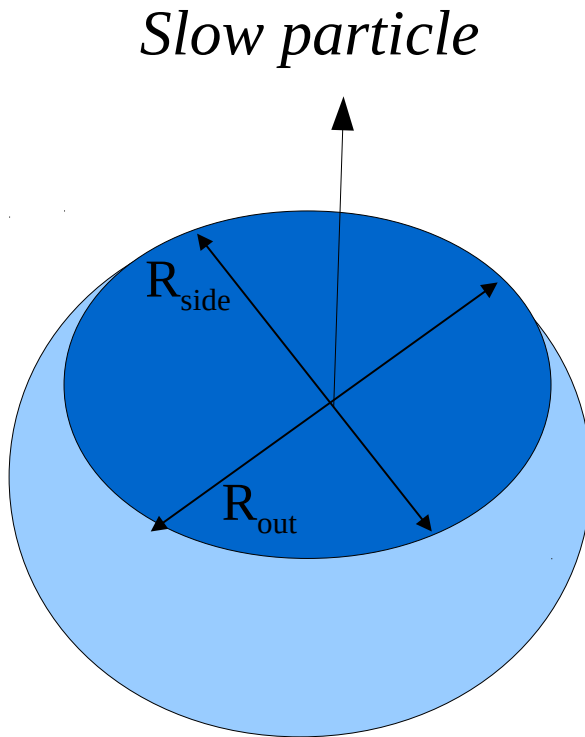
Discussed in e.g.:

Kolehmainen, Gyulassy'86

Makhlin-Sinyukov'87

Pratt, Csörgö, Zimanyi'90

$$R_{side}\sim R/(1+m_T\beta_T^2/T)^{\frac{1}{2}}$$

$$R_{long} = \tau\,(T/m_T)^{1/2}$$

$$R^2_{out}\sim R^2_{side}+1/2(T/m_T)^2\beta_T^2\tau^2$$
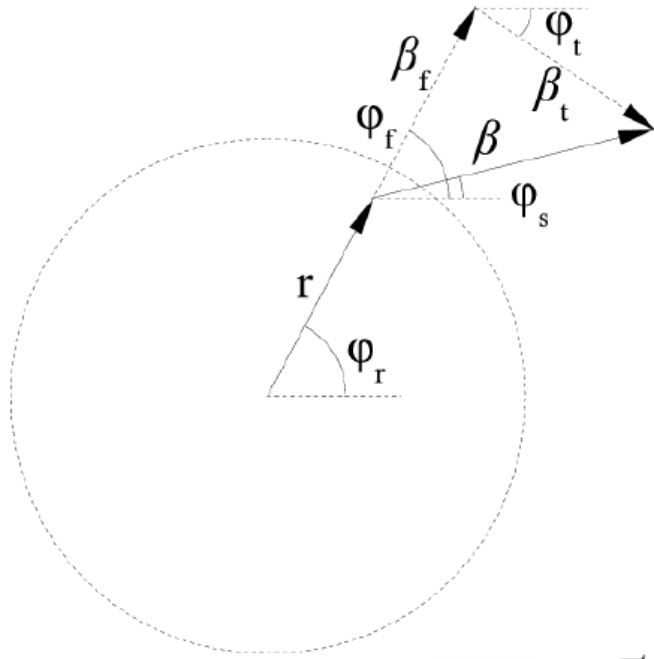
β_T collective transverse flow

assuming a longitudinal boost invariant expansion
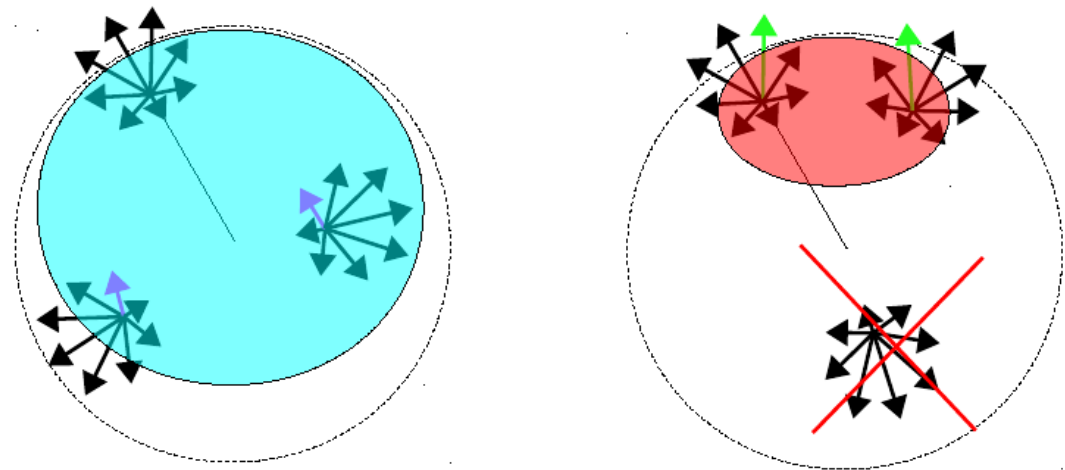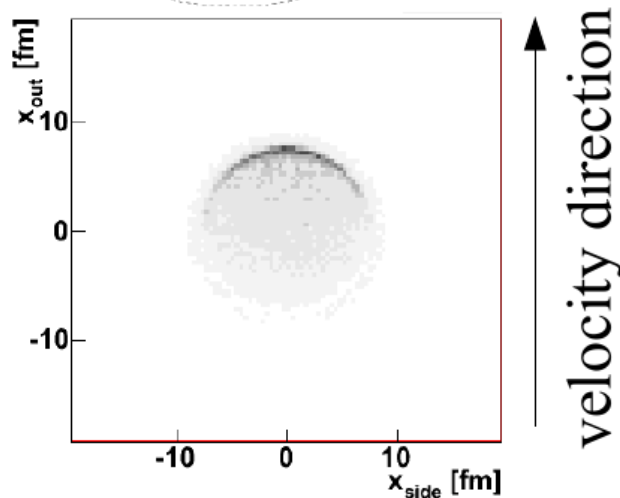
Femtoscopy with expanding source

Interference probes only parts of the source at close momenta – **homogeneity regions.**

- A particle emitted from a medium will have a collective velocity β_f and a thermal (random) one β_t

- As observed p_T grows, the region from where pairs with small relative momentum can be emitted gets smaller and shifted to the outside of the source

Motivation

expanding source: x:p correlations

Slow π

Fast π

π

K

p
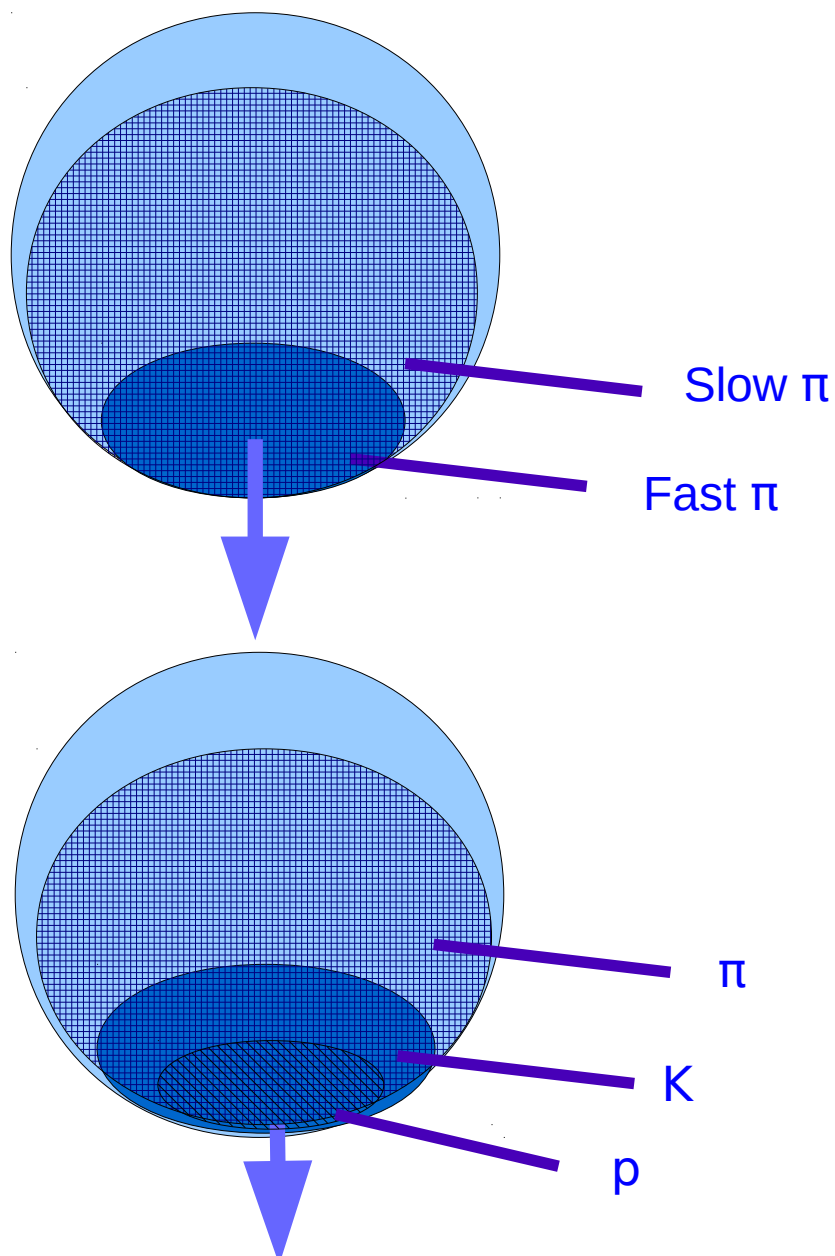
● Femtoscopy

  - Measure the spatial & temporal characteristics
of the particle emitting regions
  - Study collective dynamics, radial flow
  - Put constraints on system evolution models,
e.g. timescales & scattering parameters

● Femtoscopy of heavier particles -
complement to ππ

  - Strong constraints for hydrodynamic models
  predictions: they should work for heavier
  mesons and baryons.
  - Check for m_T dependence -> determine
  freeze-out conditions
  - Possibility to distinguish between different
  model scenario
  - ALICE collaboration extracted scattering
  parameters of many **"exotic" pairs:** pK, K+K-,
  pΛ, pΞ, pΣ, pΩ. →
   study pairs participating less in hadron rescattering
  phase and so more sensitive to EoS

L.Malinina,   RFBR grants for NICA Oct. 2020, JINR Dubna   **27**

Hybrid (hydro+hadron gas) vHLLE+UrQMD model

Pre-thermal phase $\rightarrow$ hydrodynamic phase $\rightarrow$ hadronic cascade

UrQMD

vHLLE

UrQMD

(3+1)-D viscous hydrodynamics

Iu. Karpenko, P. Huovinen, H.Petersen, M. Bleicher, Phys.Rev. C 91, 064901 (2015), arXiv:1502.01978,1509.3751 , talk QM2015

vHLLE code: free and open source, https://github.com/yukarpenko/vhlle, Comput. Phys. Commun. 185 (2014), 3016

The transition to hydrodynamical description occurs at a hyper-surface of constant longitudinal proper time τ_0
The minimal value of the starting time $\tau 0$ is taken to be equal to the average time for the two colliding nuclei to completely pass through each other:

$$\tau_0 = 2R/\sqrt{(\sqrt{s_{NN}}/2m_N)^2 - 1},$$

At $\tau = \tau 0$ energy, momentum and baryon/electric charges of hadrons are distributed to fluid cells ijk around each hadron's position according to Gaussian
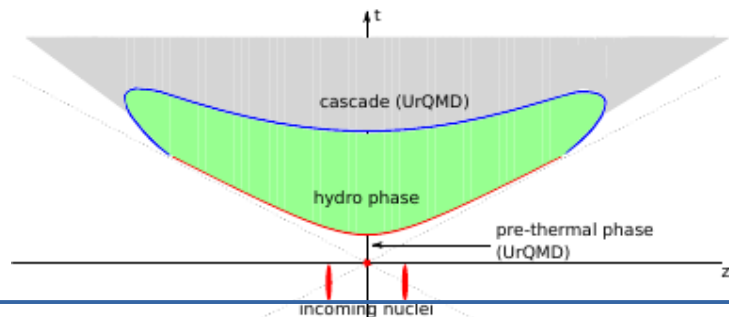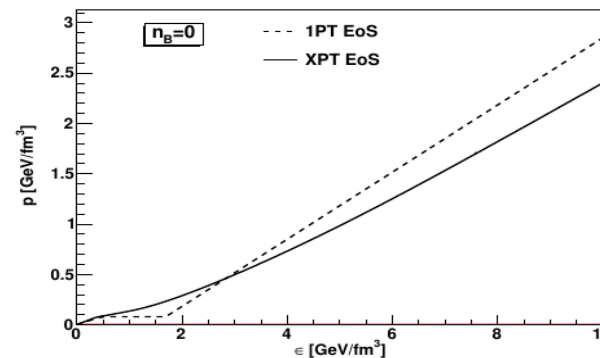

VHLLE (3+1)-D viscous hydrodynamics

HadronGas + Bag Model → 1st order PT (1PT)   P.F. Kolb, et al, PR C 62, 054909 (2000)
Chiral EoS → crossover PT (XPT)
   J. Steinheimer, et al, J. Phys. G 38, 035001 (2011)

Thermodynamic pressure as a function of energy density, evaluated at zero baryon density from the equations of state used in the hydrodynamic stage XPT & 1PT

Fluid to particle transition, or particlization, is set to happen at a hypersurface of constant (hydrodynamic) energy density sw = 0.5 GeV/fm3 ,

The particlization hypersurface is reconstructed with the CORNELIUS subroutine.

At this hypersurface, individual hadrons are sampled using the Cooper-Frye formula including shear viscous corrections to the distribution functions. The hadronic rescatterings and decays are treated with the UrQMD cascade.

Femtoscopy with vHLLE

Iu. Karpenko, P. Huovinen, H.Petersen, M. Bleicher, Phys.Rev. C.91, 2015, 064901

Pre-thermal phase	→	Hydrodynamic phase	→	Hadronic cascade

UrQMD

vHLLE
(3+1)-D viscous hydrodynamics

UrQMD

The transition to hydrodynamics occurs at a hyper-surface of constant longitudinal proper time τ_0
Minimal value of the starting time $\tau 0$ is taken to be equal to the average time for the two colliding nuclei to completely pass through each other:

Parameters τ_0 , $R_\perp$, R_η and η/s adjusted using basic observables in the RHIC BES-I region.

Model tuned by matching with existing experimental data from SPS and BES-I RHIC

EoS to be used in the model

• Chiral EoS — crossover transition
  J. Steinheimer et al., J. Phys. G 38, 035001 (2011)

• Hadron Gas + Bag Model 1st-order phase transition
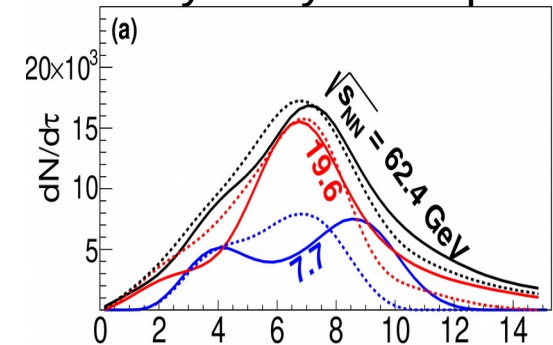 P. F. Kolb et al., Phys.Rev. C 62, 054909 (2000)

Hydrodynamic phase lasts longer with 1PT, especially at lower energies but cascade smears this difference.
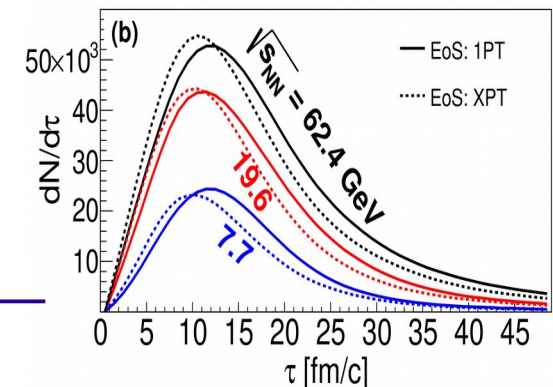
Can we probe this difference with femtoscopy ?

Fluid to particle transition at energy density = 0.5 GeV/fm^3
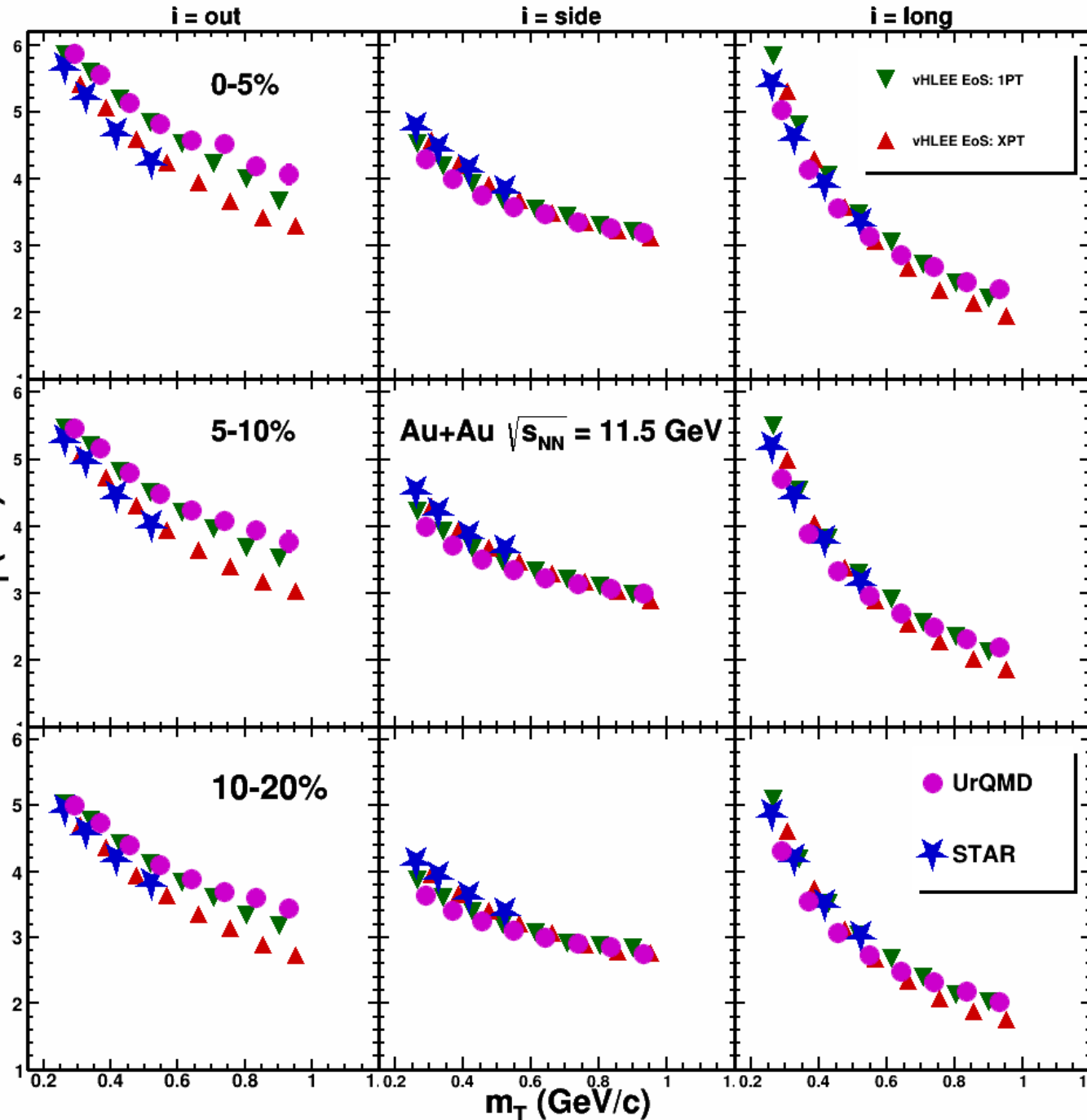
**Pion emission time**
after hydrodynamic phase


after cascade

Pion radii with vHLLE and UrQMD models

- Au+Au, $\sqrt{s_{NN}}$ = 11.5 GeV

- It is important to study k_T (m_T) dependence in larger interval than STAR

- Radii decrease with m_T → radial flow

- Increase size with increasing centrality → simple geometric picture of the collisions.

- Crossover EoS describes better $R_{out}(m_T)$

- **$R_{out,long}$ (1PT) > $R_{out,long}$ (XPT)**

- **URQMD is closed to 1PT**

- The similar trends is observed for AuAu 11.5 GeV

Comparison of pion and kaon radii for UrQMD
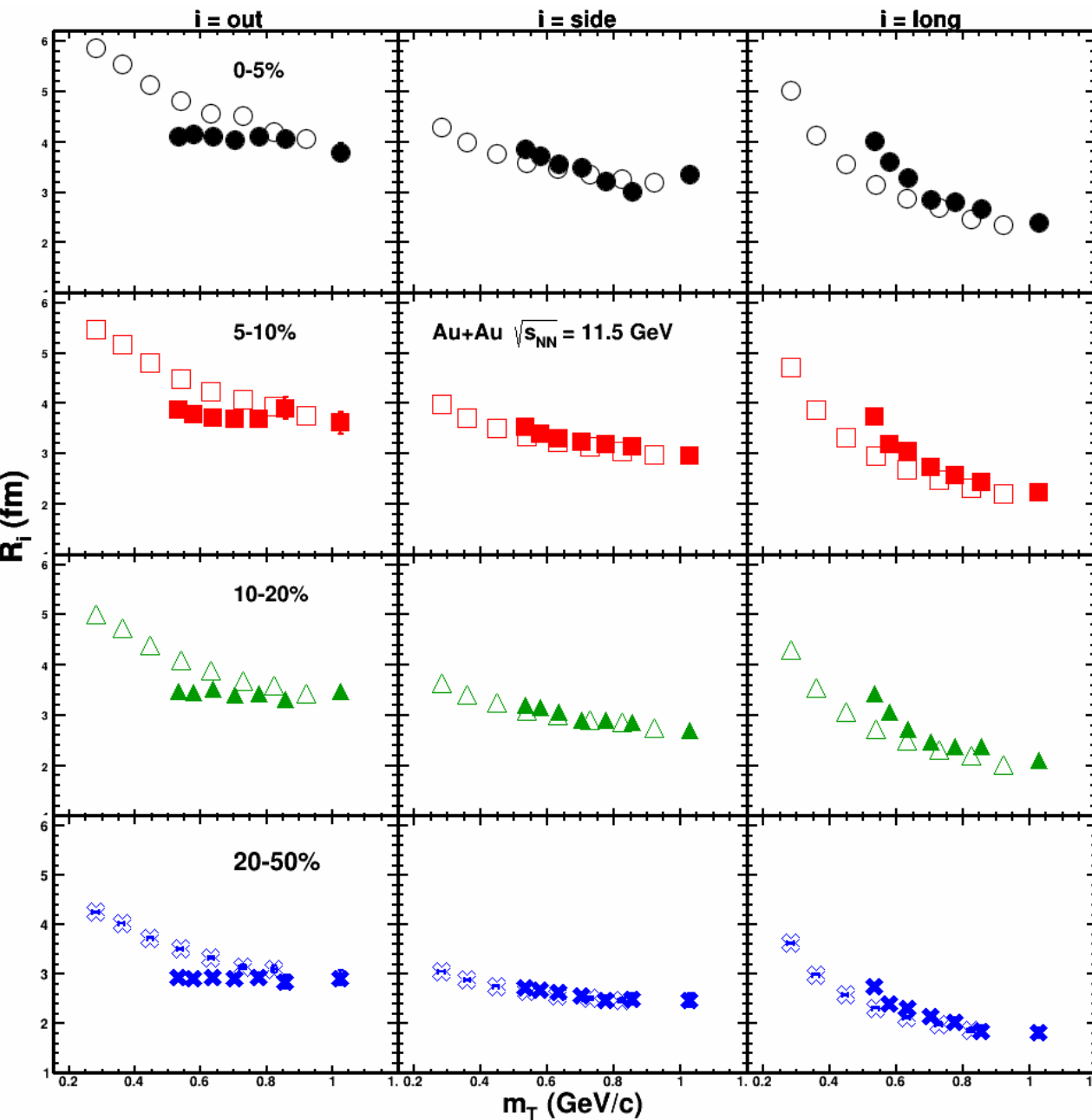
Au+Au, $\sqrt{s_{NN}}$ = 11.5 GeV

We observe the similar trends for AuAu 11.5 GeV as for 7.7 GeV
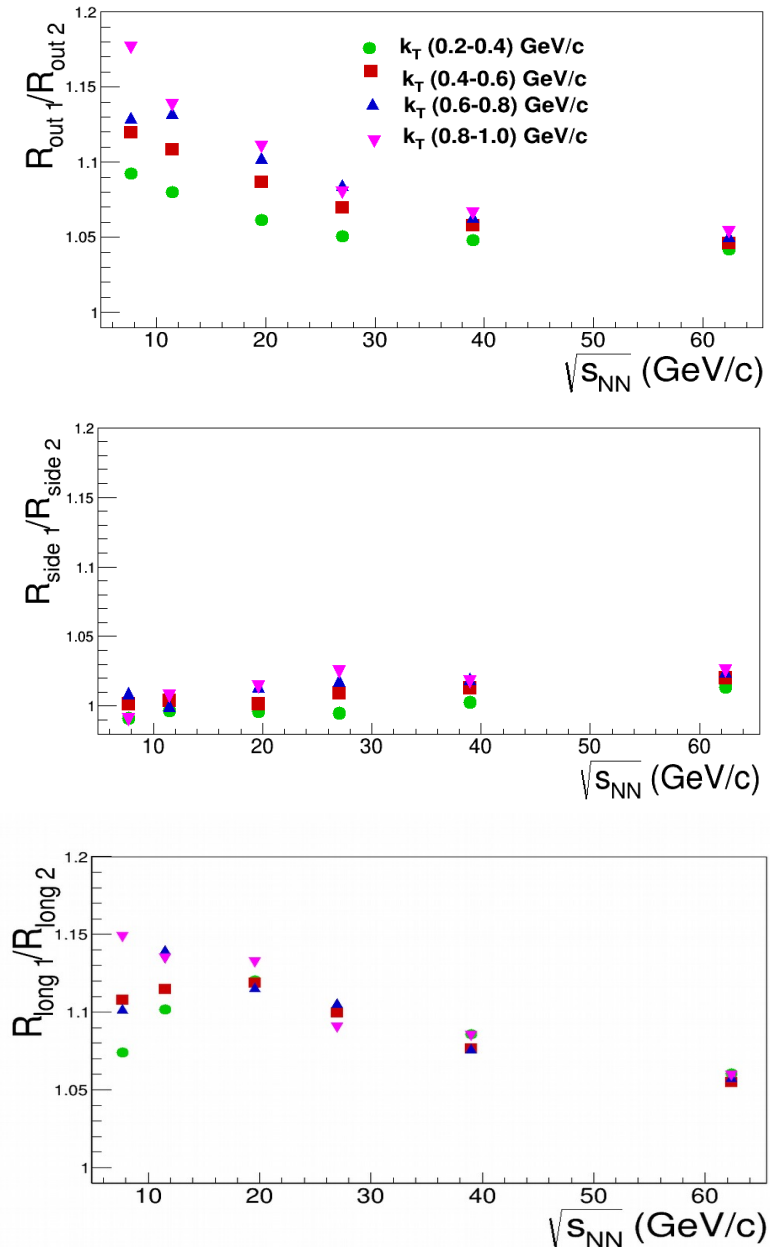
Pion and kaon radii with vHLLE model
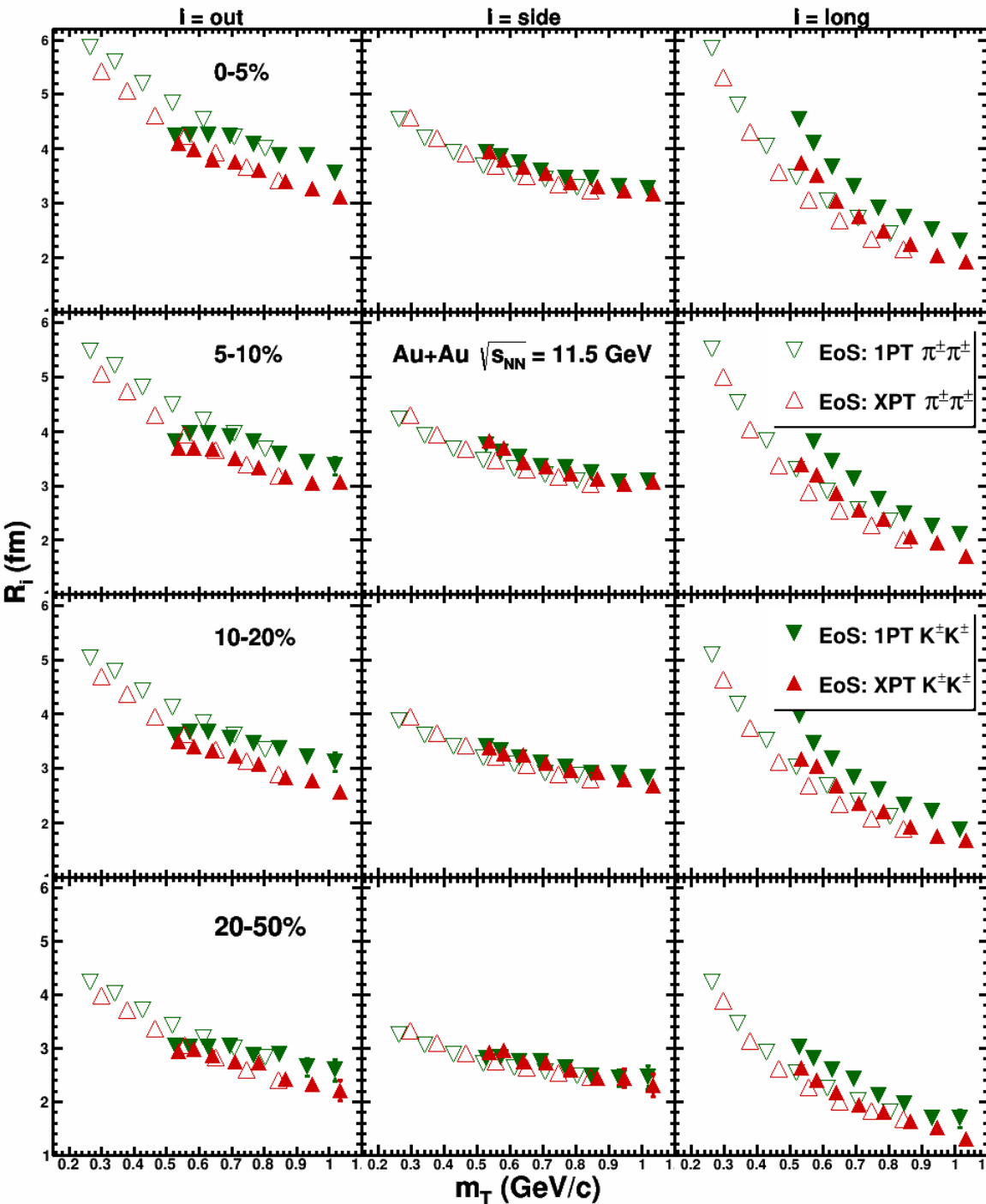
- Au+Au, $\sqrt{s_{NN}}$ = 11.5 GeV

- We observe the similar trends for AuAu 11.5 GeV as for 7.7 GeV
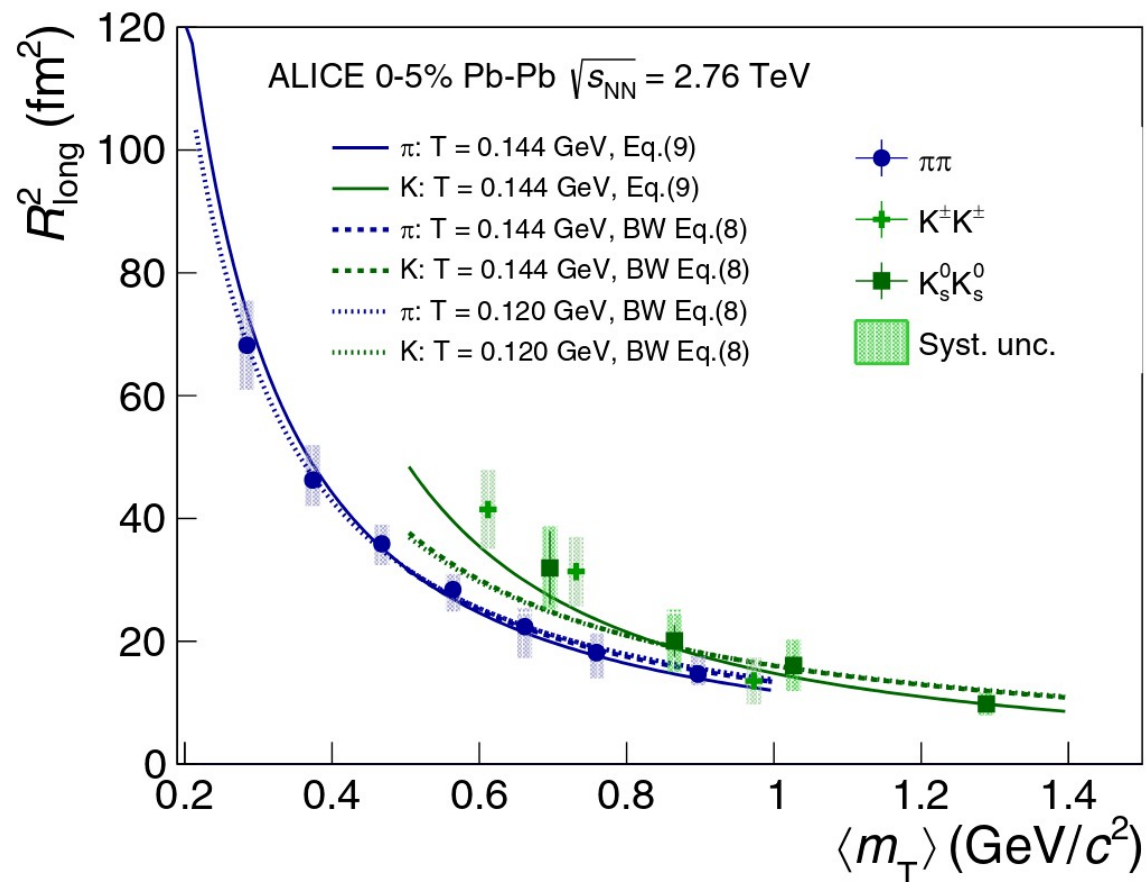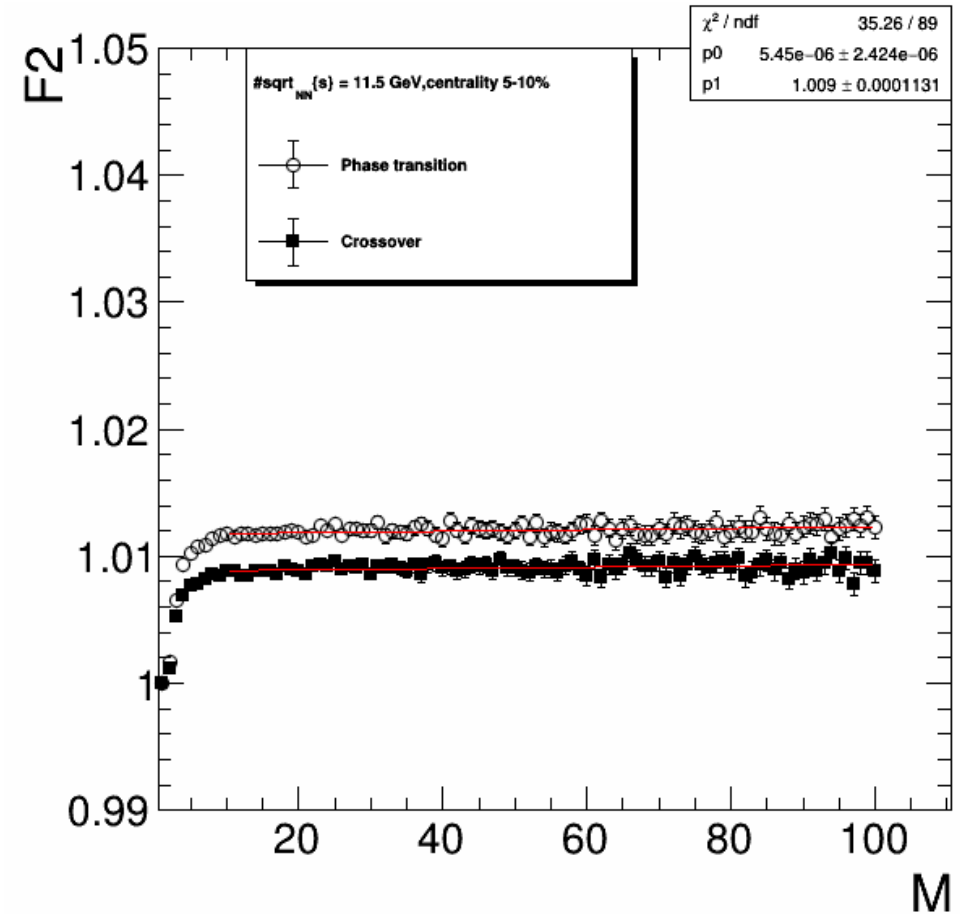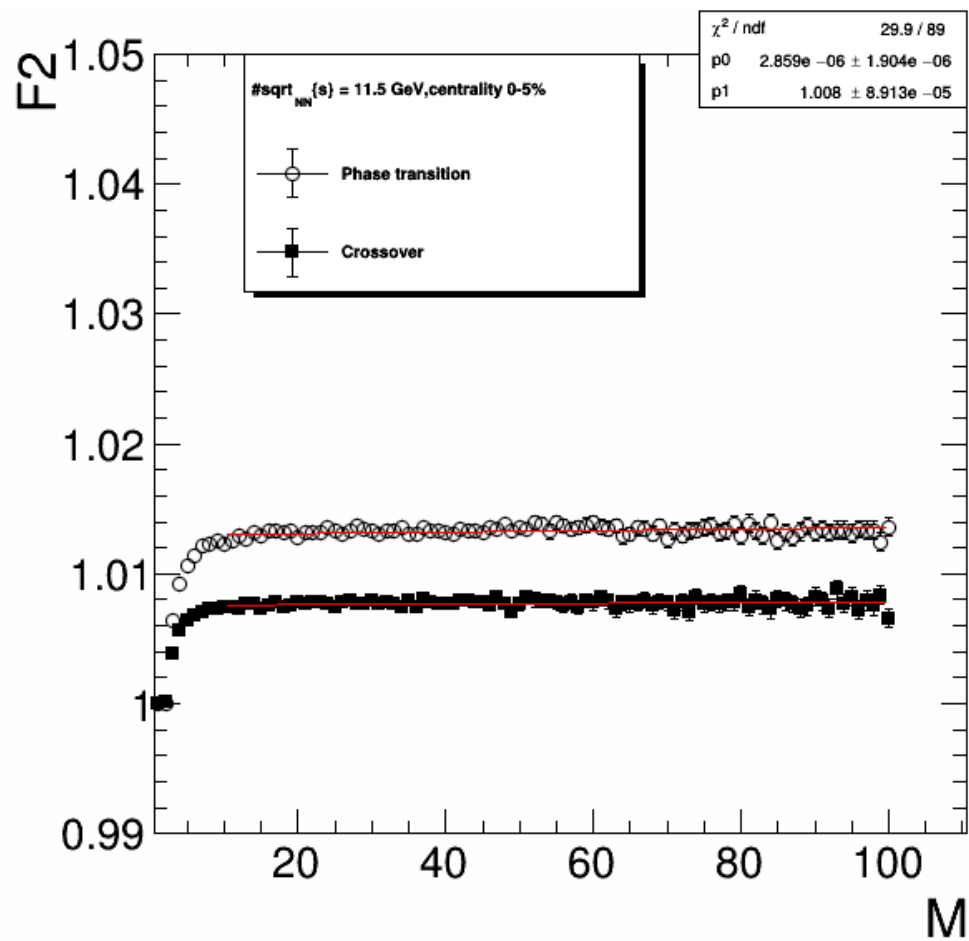
- Pion k_T divided into 4 bins
- R_{side} ratio practically coincide for both scenarios
- R_{out} and R_{long} ratios for 1PT EoS are greater than for XPT EoS and demonstrating a strong k_T-dependence at low energy

- The difference comes from a weaker transverse flow developed in the fluid phase with 1PT EoS as compared to XPT EoS and its longer lifetime in 1PT EoS

Emission delay in ALICE data

- ALICE kaon data in hydro-based parameterization: kaons emitted on average later than pions.

- It comes from rescattering via K* resonance

- $R_{long} \sim \tau / \sqrt{m_T}$

- Measured values: $\tau_\pi = 9.5 \pm 0.2$ fm/c
  $\tau_K = 11.6 \pm 0.1$ fm/c

ALICE 0-5% Pb-Pb $\sqrt{s_{NN}} = 2.76$ TeV

- π: T = 0.144 GeV, Eq.(9)
- K: T = 0.144 GeV, Eq.(9)
- π: T = 0.144 GeV, BW Eq.(8)
- K: T = 0.144 GeV, BW Eq.(8)
- π: T = 0.120 GeV, BW Eq.(8)
- K: T = 0.120 GeV, BW Eq.(8)

$\pi\pi$
$K^\pm K^\pm$
$K_s^0 K_s^0$
Syst. unc.

R_{long}^2 (fm^2) vs $\langle m_T \rangle$ (GeV/c^2)

Factorial Moments with vHLLE+UrQMD (11.5 GeV)

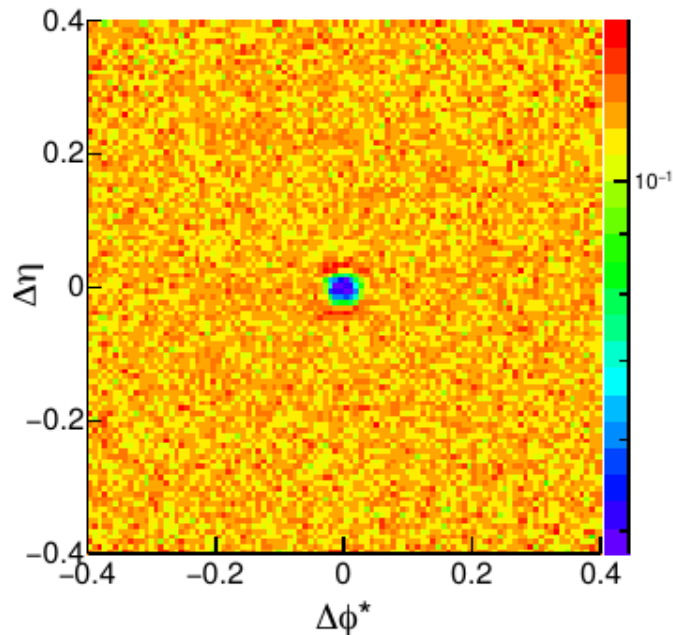Δη-Δφ* with MPD reconstructed tracks

$$\Delta\phi^* = \phi_1 - \phi_2 + \arcsin\left(\frac{z \cdot e \cdot B_z \cdot R}{2p_{T1}}\right) - \arcsin\left(\frac{z \cdot e \cdot B_z \cdot R}{2p_{T2}}\right)$$
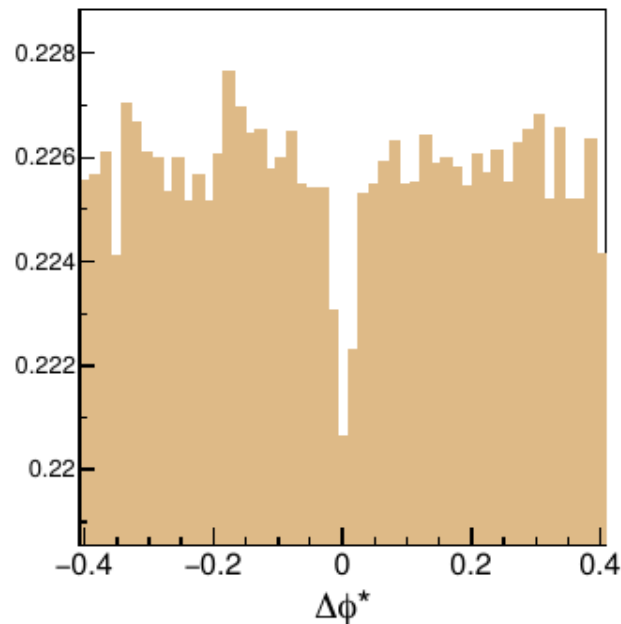
R is a given cylindrical radius
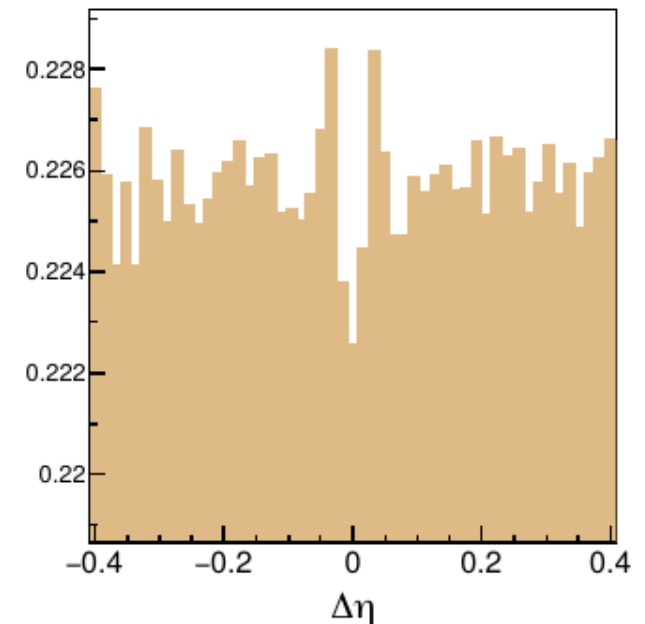Φ$_{1,2}$ are azimuthal angles of track at reconstructed vertex

Introduction: Factorial Moments (intermittency)

Factorial Moments
(see Olga Kodolova's talk)

It was proposed by A. Bialas and R. Peschanski (Nucl. Phys. B 273 (1986) 703) to study the dependence of the normalized factorial moments

$$F_i = M^{i-1} \times \left\langle \frac{\sum_{j=1}^{M} k_j \times (k_j - 1) \times ... \times (k_j - i + 1)}{N \times (N-1) \times ... \times (N-i+1)} \right\rangle$$

Note: there is a set of definitions of moments and cumulants.

of the rapidity distribution on the size δy (Δy/M, M is the number of bins, Δy is the size of the mid rapidity window):

  1. if fluctuations are purely statistical no variation of moments as a function of δy is expected

  2. Observation of variations indicates the presence of physics origin fluctuations

Intermittency (fluctuations of various different sizes in 1D, 2D and 3D phase space) have been studied at LEP, Tevatron, Protvino in ee, hh, hA, AA interactions at the various energies.

Au-Au, UrQMD+vHLLE
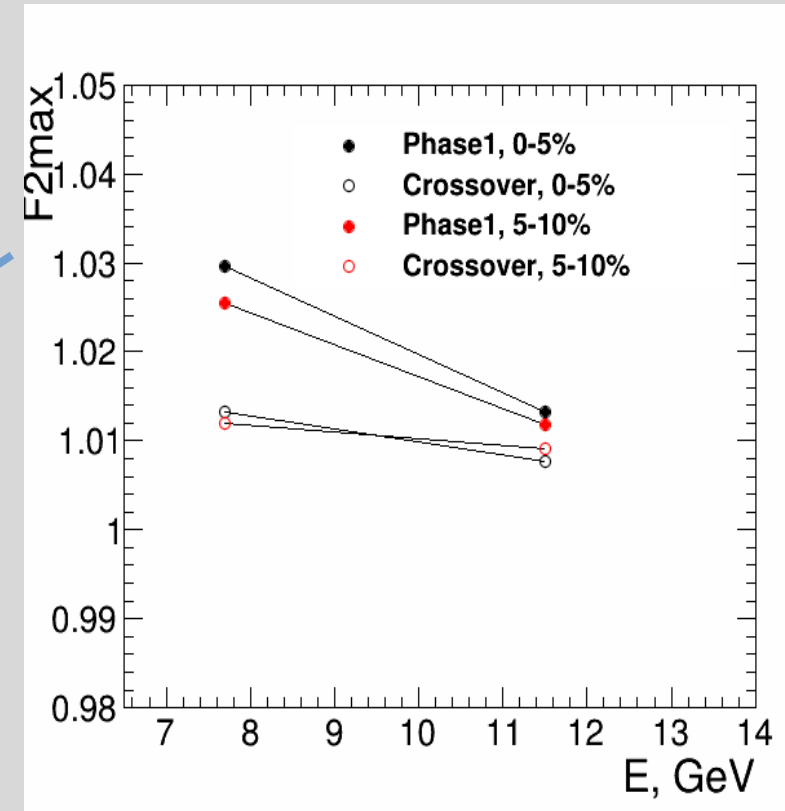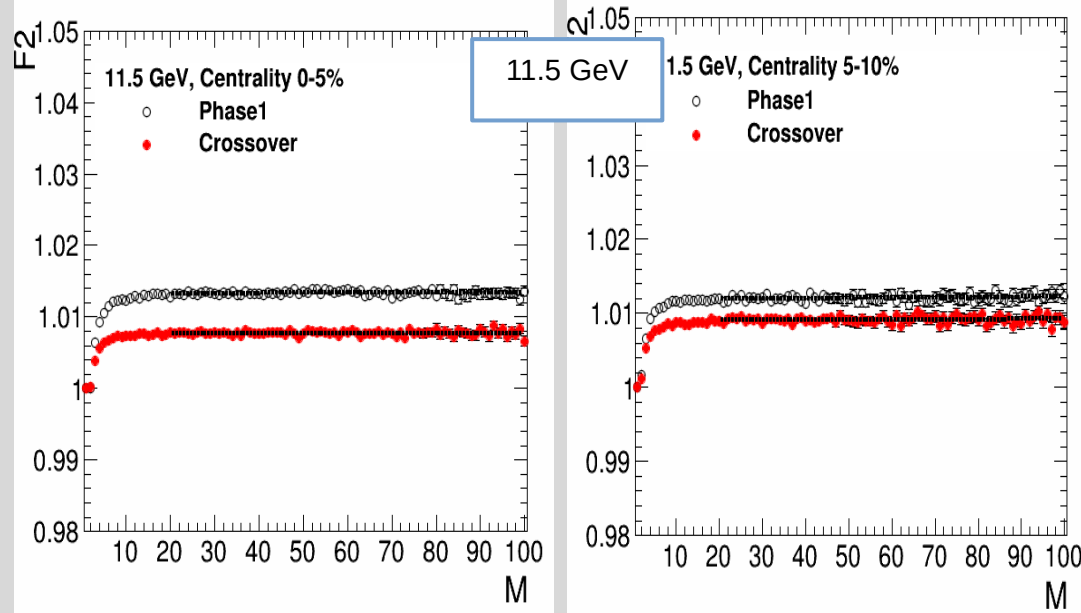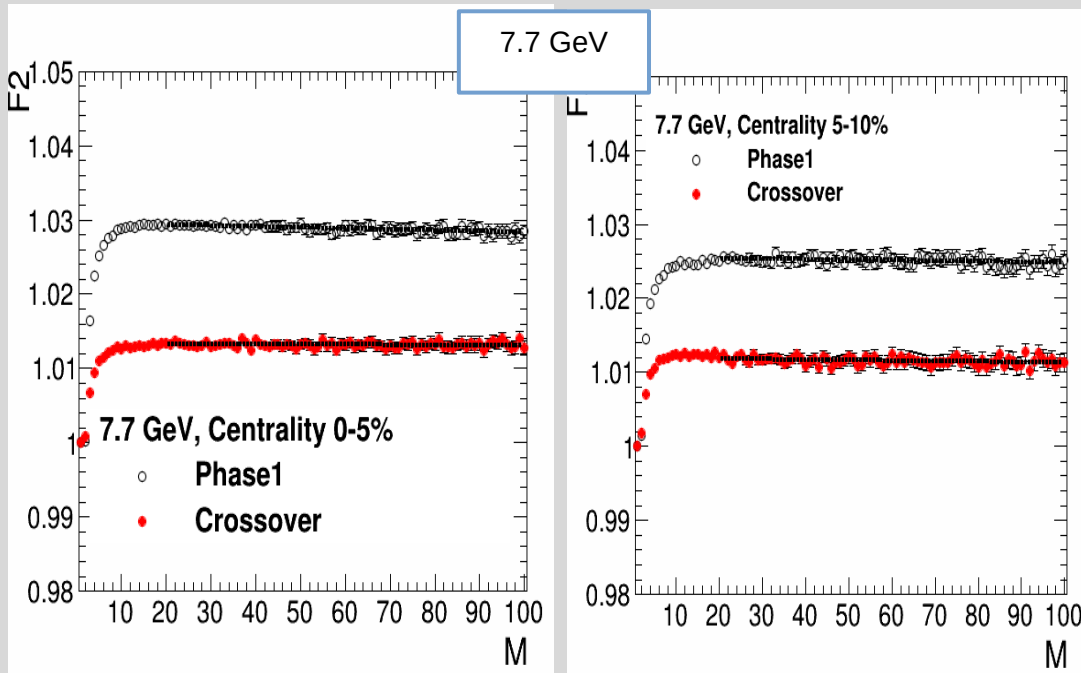
- ♦ Different energy dependence is expected for Crossover and 1st order phase transition

- ♦ There is a mild dependence on centrality for 1st order phase transition