



Dilepton production within GiBUU microscopic transport model

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Plan:

- Motivation: dileptons as a probe of hadron modifications in nuclear matter.
- Giessen Boltzmann-Uehling-Uhlenbeck (GiBUU) model: relativistic mean field, collision term, dilepton (e^+e^-) production channels.
- Spectral function of ρ -meson in nuclear matter. Collisional broadening.
- Heavy ion collisions at SIS18 energies (1-2 A GeV): comparison with HADES data on e^+e^- production. Deuteron puzzle.
- Opportunities at NICA-SPD: dilepton production, proton and pion production, hadron formation effect.
- Conclusions.

Based on [AL, U. Mosel, L. von Smekal, arXiv:2009.11702](#)

- Vector mesons (ρ, ω, ϕ) may experience spectral changes in nuclear matter: mass lowering due to disappearance of the scalar $\bar{q}q$ condensate with nuclear density ([G.E. Brown, M. Rho, PRL 66, 2720 \(1991\)](#); [T. Hatsuda, S.H. Lee, PRC 46, R34 \(1992\)](#)), and collisional broadening.
- Direct decays to dileptons $V \rightarrow l^+l^-$ allow to study the spectral function of vector mesons in hadronic medium. Lepton momentum is almost undistorted by interactions with hadrons.
- Heavy-ion collision (HIC) experiments at CERN/SPS (cf. [D. Adamova et al., PRL 91, 042301 \(2003\)](#); [G. Agakishiev et al., EPJC 41, 475 \(2005\)](#)) did not allow to make a clear distinction between in-medium mass shift and collisional broadening of ρ . Why? Educated guess: too quickly evolving system. No time for mesons to change their spectral properties.
- HADES experiment at SIS18 (see [P. Salabura and J. Stroth, 2020, arXiv:2005.14589](#) for the most recent overview) is designed to study e^+e^- signals from HIC at moderate energies of 1-2 A GeV. Here the evolution is relatively slow and in-medium modifications are more probable.

GiBUU model

- solves the coupled system of kinetic equations for the baryons ($N, N^*, \Delta, \Lambda, \Sigma, \dots$), corresponding antibaryons ($\bar{N}, \bar{N}^*, \bar{\Delta}, \bar{\Lambda}, \bar{\Sigma}, \dots$), and mesons (π, K, \dots)
- initializations for the lepton-, photon-, hadron-, and heavy-ion-induced reactions on nuclei

Open source code in Fortran 2003 downloadable from:

<https://gibuu.hepforge.org/trac/wiki>

Details of GiBUU: *O. Buss et al., Phys. Rep. 512, 1 (2012).*

$$\begin{aligned}
 & \text{Distribution function in phase space } (\mathbf{r}, \mathbf{p}^*) & \text{Number of sort "j" particles} = \int \frac{g_s^j d^3 r d^3 p^*}{(2\pi)^3} f_j^*(x, \mathbf{p}^*) \\
 (p_0^*)^{-1} \left[p_\mu^* \partial^\mu + (p_\mu^* \mathcal{F}_j^{\alpha\mu} + m_j^* \partial^\alpha m_j^*) \frac{\partial}{\partial p^{*\alpha}} \right] \overbrace{f_j^*(x, \mathbf{p}^*)} &= \underbrace{I_j[\{f^*\}]}_{\text{Collision term}}, & (*) \\
 \mu = 0, 1, 2, 3, \quad \alpha = 1, 2, 3, \quad j = N, \bar{N}, \Delta, \bar{\Delta}, \Lambda, \bar{\Lambda}, \pi, K, \dots & \quad x \equiv (t, \mathbf{r}) &
 \end{aligned}$$

$$m_j^* = m_j + S_j \quad \text{- effective mass, } S_j = g_{\sigma j} \sigma \quad \text{- scalar field,}$$

$$p^{*\mu} = p^\mu - V_j^\mu \quad \text{- kinetic four-momentum with effective mass shell constraint } p^{*\mu} p_\mu^* = m_j^{*2},$$

$$V_j^\mu = g_{\omega j} \omega^\mu + g_{\rho j} \tau_j^3 \rho^{3\mu} + q_j A^\mu \quad \text{- vector field,} \quad \tau_j^3 = +(-)1 \text{ for } j = p, \bar{n} (\bar{p}, n),$$

$$\mathcal{F}_j^{\mu\nu} = \partial^\mu V_j^\nu - \partial^\nu V_j^\mu \quad \text{- field tensor.}$$

- For momentum-independent fields Eq.(*) is equivalent to the BUU equation

$$(\partial_t + \nabla_{\mathbf{p}} \varepsilon_j \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}} \varepsilon_j \nabla_{\mathbf{p}}) f_j(x, \mathbf{p}) = I_j[\{f\}]$$

$$\varepsilon_j(x, \mathbf{p}) = V_j^0 + \sqrt{m_j^{*2} + \mathbf{p}_j^{*2}}, \quad f_j(x, \mathbf{p}) = f_j^*(x, \mathbf{p}^*).$$

Direct derivations of relativistic kinetic equation:

Yu.B. Ivanov, NPA 474, 669 (1987);

B. Blättel, V. Koch, U. Mosel, Rept. Prog. Phys. 56, 1 (1993).

Used non-linear Walecka model version NL2 from ***A. Lang et al., NPA 541, 507 (1992)***

Collision term of the GiBUU model

- Includes $2 \rightarrow 2$, $2 \leftrightarrow 3$, $2 \rightarrow 4$, and $3 \rightarrow 3$ transitions at low energies, and $2 \rightarrow N$ transitions at high energies (via PYTHIA and FRITIOF models) and for baryon-antibaryon annihilation (via statistical annihilation model);
- Pauli blocking for the outgoing nucleons;
- Cross sections of the time-reversed processes (e.g. $\Lambda K \rightarrow N\pi$) – by the detailed balance relation:

$$\sigma_{cd \rightarrow ab} = \sigma_{ab \rightarrow cd} \left(\frac{q_{ab}}{q_{cd}} \right)^2 \frac{(2J_a + 1)(2J_b + 1) \mathcal{S}_{ab}}{(2J_c + 1)(2J_d + 1) \mathcal{S}_{cd}},$$

q_{ab}, q_{cd} - c.m. momenta,

J_a, J_b - spins,

$$\mathcal{S}_{ab} = \begin{cases} 1 & \text{if a,b not identical} \\ \frac{1}{2} & \text{if a,b identical.} \end{cases}$$

Baryon-baryon collisions:

For $\sqrt{s} < 4$ GeV: $BB \rightarrow BB$ (elastic & inelastic), $NN \rightarrow NNM$ ($M = \pi, \omega, \phi$),
 $np \rightarrow d\eta$ (via $np\eta$ final state), $BB \rightarrow BYK$, $BB \rightarrow NNK\bar{K}$ ($B = N, R$).

For $\sqrt{s} > 4$ GeV: $BB \rightarrow X$ (PYTHIA 6).

Meson-baryon collisions:

For $\sqrt{s} < 2.2$ GeV: $\pi N \rightarrow R$, MN ($M = \pi, \omega, \phi, \rho, \sigma, \eta$), $M\Delta$ ($M = \pi, \eta, \rho$),
 $\pi N^*(1440)$, $K\Lambda$, $K\Sigma$, $\omega\pi N$, $\phi\pi N$, $K\bar{K}N$, $\Lambda K\pi$, $\Sigma K\pi$, $\pi\pi N$, $\pi\pi\pi N$;

$\omega N \rightarrow R$, πN , ωN , $\pi\pi N$, ΛK , ΣK ;

$\rho N \rightarrow R$, πN , ΛK , ΣK ;

$\sigma N \rightarrow R$, πN , σN ;

$\eta N \rightarrow R$, πN , ΛK , ΣK ;

$\phi N \rightarrow \phi N$, πN , $\pi\pi N$;

$KN \rightarrow KN$, $KN\pi$;

$\bar{K}N \rightarrow Y^*$, $\bar{K}N$, $Y\pi$, $Y^*\pi$, ΞK , $\Xi K\pi$;

$J/\psi N \rightarrow J/\psi N$, $\Lambda_c\bar{D}$, $\Lambda_c\bar{D}^*$, $NDD\bar{D}$;

$\pi\Delta \rightarrow R$, $K\Lambda$, $\Sigma\Lambda$;

$\rho\Delta \rightarrow R$;

$\eta\Delta \rightarrow \pi N$;

$\pi N^*(1440) \rightarrow R$;

$\pi Y(Y^*) \rightarrow Y^*$, $\bar{K}N$;

$\eta\Lambda \rightarrow \Lambda^*$;

$K\Lambda \rightarrow R$, πN , $\pi\Delta$;

For $\sqrt{s} > 2.2$ GeV: $MB \rightarrow X$ (PYTHIA 6 and JETSET)

Meson-meson collisions:

$MM \rightarrow R, K\bar{K}, K^*\bar{K}, K\bar{K}^*$ ($M = \pi, \eta, \eta', \sigma, \rho, \omega$).

Baryon-antibaryon collisions:

$\bar{B}B \rightarrow$ mesons by statistical annihilation model

E.S. Golubeva et al., NPA 537, 393 (1992);

I.A. Pshenichnov, PhD thesis, INR Moscow (1998)

or by string model (if $|Z_{\text{tot}}| > 1$ or total charm $\neq 0$ or total strangeness $\neq 0$),

$\bar{B}B \rightarrow \bar{B}B$ (EL and CEX), $\bar{N}N \leftrightarrow \bar{\Delta}N(\bar{N}\Delta)$ (for $\sqrt{s} < 2.38$ GeV)

or $\bar{B}B \rightarrow \bar{B}B +$ mesons by FRITIOF (for $\sqrt{s} > 2.38$ GeV),

$\bar{N}N \rightarrow \bar{\Lambda}\Lambda, \bar{N}(\bar{\Delta})N(\Delta) \rightarrow \bar{\Lambda}\Sigma(\bar{\Sigma}\Lambda), \bar{N}(\bar{\Delta})N(\Delta) \rightarrow \bar{\Xi}\Xi, \bar{N}N \rightarrow \bar{\Omega}\Omega,$
 $\bar{N}N \rightarrow J/\psi.$

3 \rightarrow 2 collisions: $NN\pi \rightarrow NN.$

3 \rightarrow 3 collisions: $NN\Delta \rightarrow NNN.$

3 \rightarrow N collisions: optional.

Dilepton production channels

Direct decays of vector mesons, $V \rightarrow e^+e^-$, with $V = \rho, \omega, \phi$,
partial widths are calculated based on the vector dominance model (VDM)

Y. Nambu, J.J. Sakurai, PRL 8, 79 (1962); O. Dumbrajs et al., NPB 216, 277 (1983)

$$\Gamma_{V \rightarrow e^+e^-}(m) = C_V \frac{m_V^4}{m^3} (1 + 2m_e^2/m^2) \sqrt{1 - 4m_e^2/m^2},$$

with $C_V = 4\pi\alpha^2/3f_V^2$ chosen to reproduce empirical $V \rightarrow e^+e^-$ width at pole mass,
 m – off-shell mass of decaying hadron.

Direct $\eta \rightarrow e^+e^-$ decay:

$$\Gamma_{\eta \rightarrow e^+e^-} = \Gamma_{\eta}^{\text{tot}} \text{BR}_{\eta \rightarrow e^+e^-}, \quad \Gamma_{\eta}^{\text{tot}} = 1.3 \text{ keV}, \quad \text{BR}_{\eta \rightarrow e^+e^-} = 7 \cdot 10^{-7}$$

upper limit from

**M. Tanabashi et al. (PDG),
PRD 98, 030001 (2018)**

$$\frac{d\Gamma_{A \rightarrow Be^+e^-}}{dm^2} = \Gamma_{A \rightarrow B\gamma^*} \frac{\alpha}{3\pi m^2} (1 + 2m_e^2/m^2) \sqrt{1 - 4m_e^2/m^2} ,$$

Meson Dalitz decays: $P \rightarrow \gamma e^+ e^-$, $P = \pi^0, \eta, \eta'$; $\omega \rightarrow \pi^0 e^+ e^-$

$$\Gamma_{A \rightarrow B\gamma^*}(m^2) = \eta_{\text{sym}} \Gamma_{A \rightarrow B\gamma} \left(\frac{q_{B\gamma^*}(m^2)}{q_{B\gamma^*}(0)} \right)^3 |F_{AB}(m^2)|^2 ,$$

$$\eta_{\text{sym}} = 2 \text{ for } P \rightarrow \gamma\gamma^*; \eta_{\text{sym}} = 1 \text{ for } \omega \rightarrow \pi^0\gamma^*$$

Eff. Lagrangian description:
R.H. Dalitz, Proc. Phys. Soc. A 64, 667 (1951);
A. Faessler et al., PRC 61, 035206 (2000)

$$q_{B\gamma^*}(m^2) = \frac{m_A^2 - m_B^2}{2m_A} \left[\left(1 + \frac{m^2}{m_A^2 - m_B^2} \right)^2 - \frac{4m_A^2 m^2}{(m_A^2 - m_B^2)^2} \right]^{1/2} \quad - \text{c.m. momentum of } B \text{ and } \gamma^* .$$

Form factors:

$$F_{\pi^0\gamma}(m^2) = 1 + b_\pi m^2 , \quad b_\pi = 5.5 \text{ GeV}^{-2} \quad \text{L.G. Landsberg, Phys. Rept. 128, 301 (1985)}$$

$$F_{\eta\gamma}(m^2) = (1 - m^2/\Lambda_\eta^2)^{-1} , \quad \Lambda_\eta^{-2} = 1.95 \text{ GeV}^{-2} \quad \text{R. Arnaldi et al. (NA60), PLB 677, 260 (2009)}$$

$$F_{\eta'\gamma}(m^2) = 1$$

$$F_{\omega\pi^0}(m^2) = \frac{\Lambda_\omega^2}{[(\Lambda_\omega^2 - m^2)^2 + \Lambda_\omega^2 \Gamma_\omega^2]^{1/2}} , \quad \Lambda_\omega = 0.65 \text{ GeV} , \quad \Gamma_\omega = 75 \text{ MeV}$$

E.L. Bratkovskaya, W. Cassing, NPA 619, 413 (1997)

Δ Dalitz decay: $\Delta(1232) \rightarrow Ne^+e^-$

M.I. Krivoruchenko, A. Faessler, PRD 65, 017502 (2002)

$$\Gamma_{\Delta \rightarrow N\gamma^*}(m^2) = \frac{\alpha}{16} \frac{(m_\Delta + m_N)^2}{m_\Delta^3 m_N^2} [(m_\Delta + m_N)^2 - m^2]^{1/2} [(m_\Delta - m_N)^2 - m^2]^{3/2} |F_{\Delta N}(m^2)|^2 ,$$

$F_{\Delta N}(m^2) \equiv F_{\Delta N}(0) = 3.029$, from the real photon decay width $\Gamma_{\Delta \rightarrow N\gamma^*}(0) = 0.66$ MeV.

$N^*(1520)$ Dalitz decay: effectively included via the two-step decay $N^* \rightarrow \rho N, \rho \rightarrow e^+e^-$.

Bremsstrahlung $pn \rightarrow pne^+e^-, pp \rightarrow ppe^+e^-$:

boson exchange model , including radiation from internal pion line

R. Shyam, U. Mosel, PRC 82, 062201 (2010)

Charged pion bremsstrahlung $\pi^\pm N \rightarrow \pi^\pm Ne^+e^-$: soft-photon approximation (SPA)

C. Gale, J.I. Kapusta, PRC 35, 2107 (1987); G. Wolf et al., NPA 517, 615 (1990)

ρ-meson spectral function

In vacuum:

$$\mathcal{A}(m^2) = \frac{m \Gamma_{\text{dec}}(m)/\pi}{(m^2 - m_\rho^2)^2 + m^2 \Gamma_{\text{dec}}^2(m)}, \quad \int_{4m_e^2}^{\infty} dm^2 \mathcal{A}(m^2) = 1.$$

$$\Gamma_{\text{dec}}(m) = \Gamma_{\rho \rightarrow \pi\pi}(m) + \Gamma_{\rho \rightarrow e^+e^-}(m)$$

$$\Gamma_{\rho \rightarrow \pi\pi}(m) = \Gamma_{\rho \rightarrow \pi\pi}^0 \left(\frac{q_{\text{cm}}(m)}{q_{\text{cm}}(m_\rho)} \right)^3 \frac{m_\rho}{m} \frac{1 + (q_{\text{cm}}(m_\rho)R)^2}{1 + (q_{\text{cm}}(m)R)^2}, \quad q_{\text{cm}}(m) = \sqrt{m^2/4 - m_\pi^2}$$

$$m_\rho = 775.5 \text{ MeV}, \Gamma_{\rho \rightarrow \pi\pi}^0 = 149.1 \text{ MeV}, R = 1 \text{ fm}$$

**D.M. Manley, E.M. Saleski,
PRD 45, 4002 (1992)**

In nuclear matter:

$$\Gamma_{\text{dec}} \rightarrow \Gamma_{\text{dec}} + \Gamma_{\text{coll}}, \quad \Gamma_{\text{coll}} = \gamma_{\text{Lor}} \langle v_{\rho N} \sigma_{\rho N} \rangle \rho_N,$$

$$\rho_N = \rho_n + \rho_p, \sigma_{\rho N} = (\sigma_{\rho n} + \sigma_{\rho p})/2, \quad v_{\rho N} = \sqrt{(qp_N)^2 - m^2 m_N^2} / q^0 p_N^0, \quad \gamma_{\text{Lor}} = q^0 / m$$

q and p_N – four momenta of ρ and nucleon, resp., $q^2 = m^2$, $p_N^2 = m_N^2$,

$\langle \dots \rangle$ – averaging over nucleon Fermi motion.

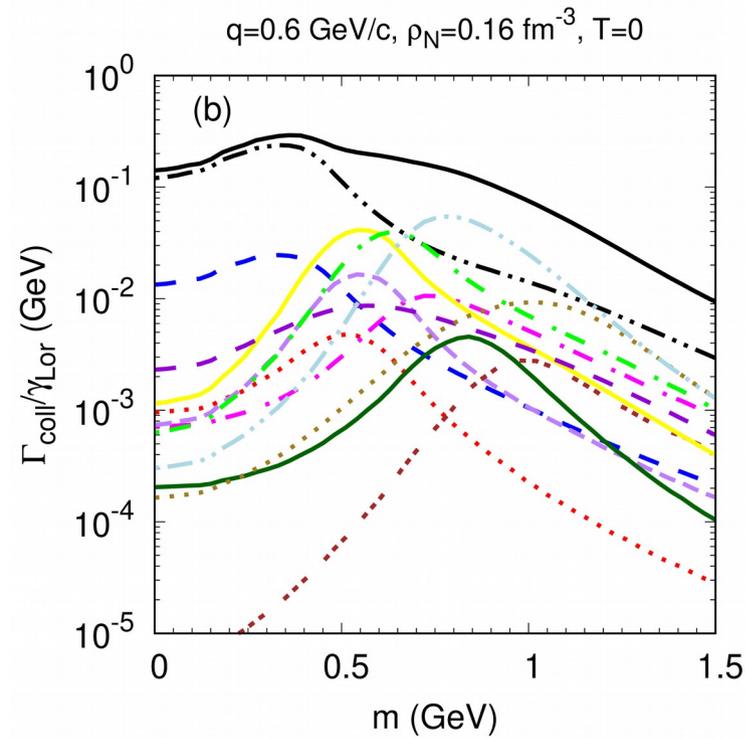
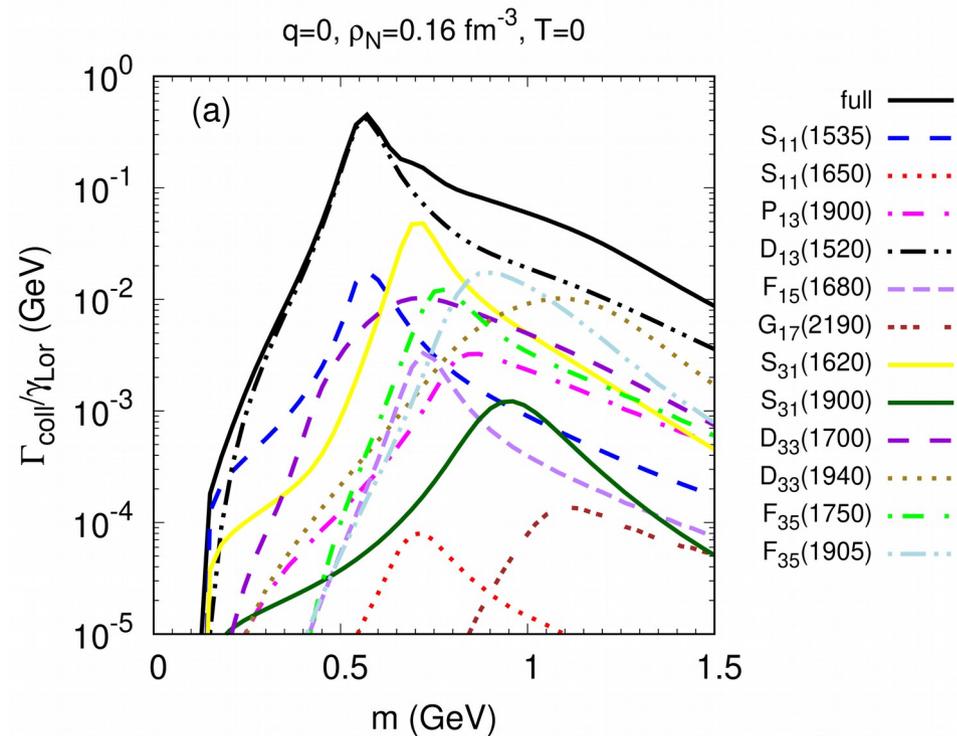
Collisional width of ρ -meson in the nuclear matter rest frame

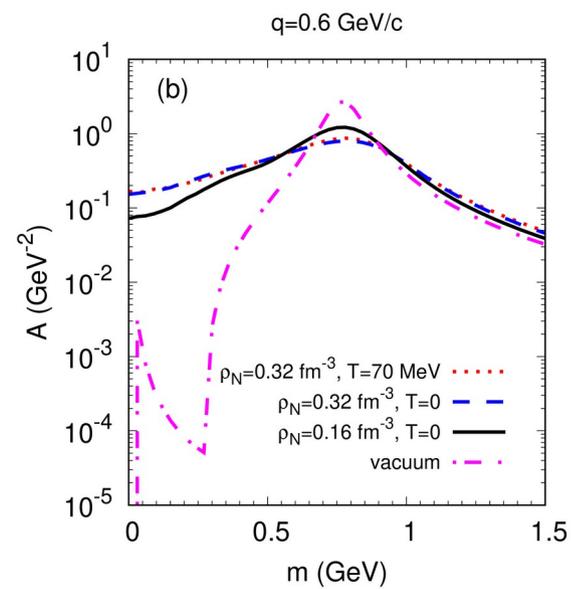
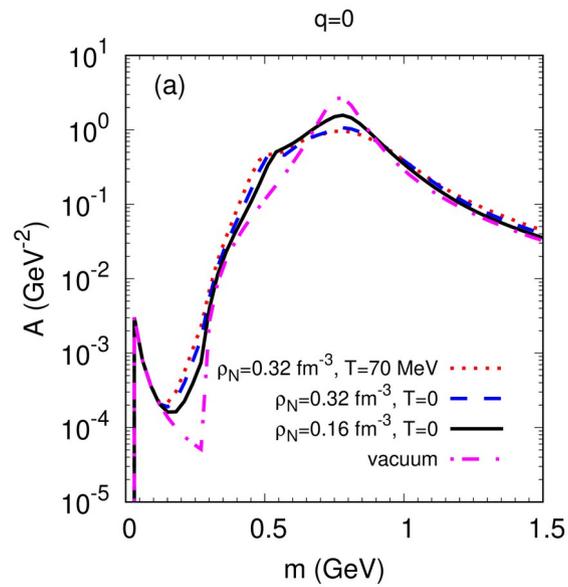
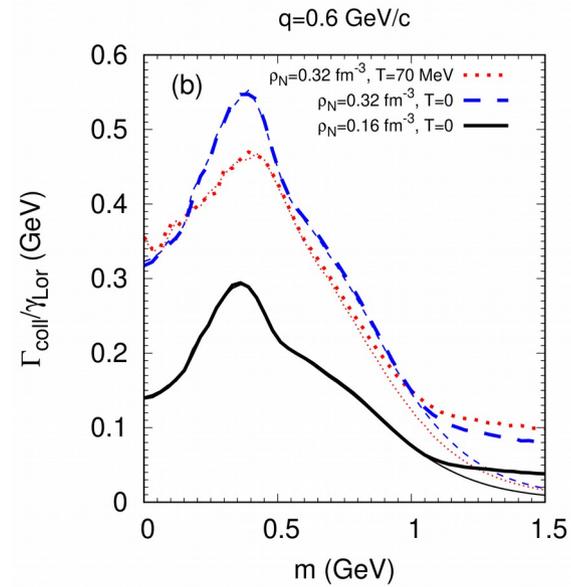
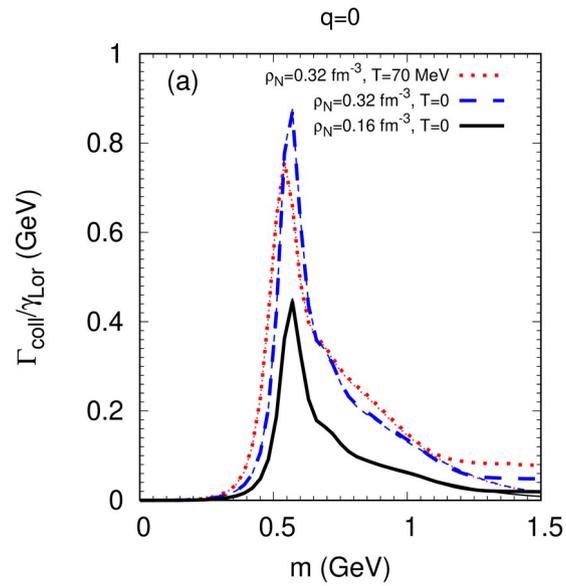
- Significant effect, comparable
to vacuum ρ decay width (149 MeV).

- Dominant contribution:

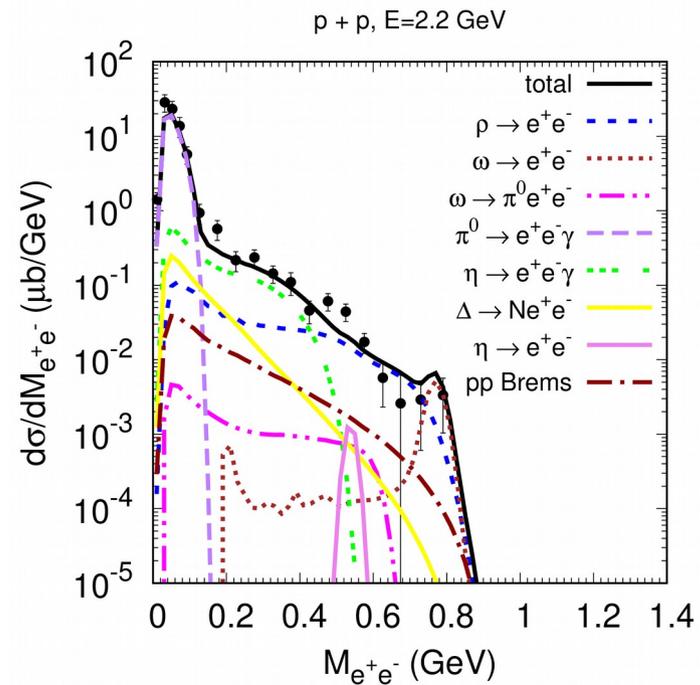
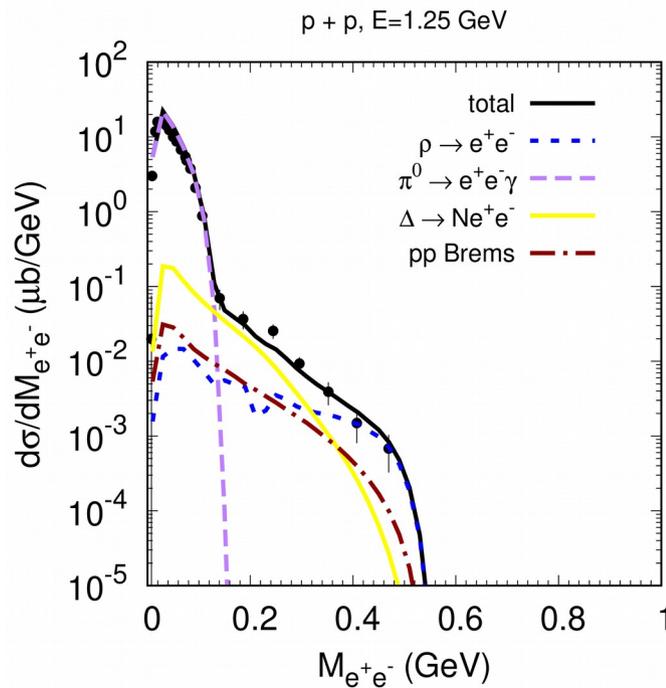


- At finite momentum, ρ becomes
broad even at very low inv. mass.





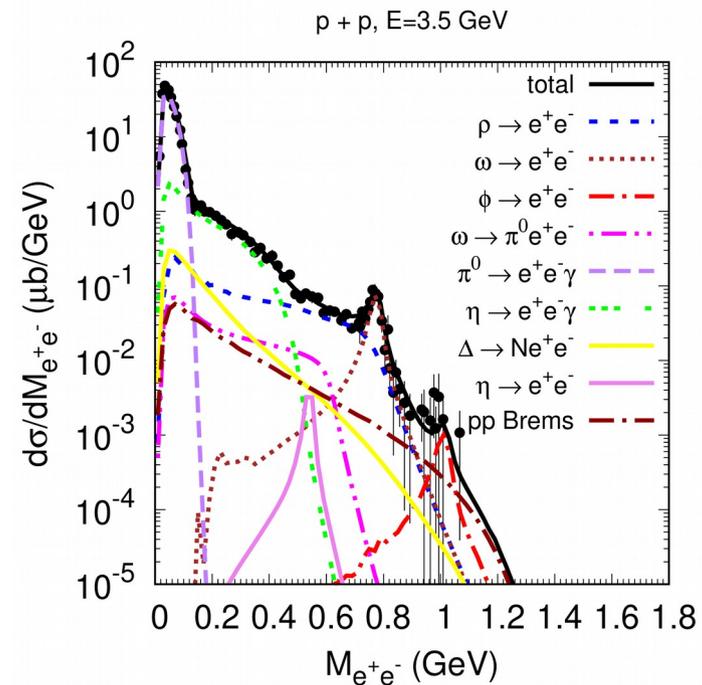
- Big effect of collisional width at finite momenta of ρ !

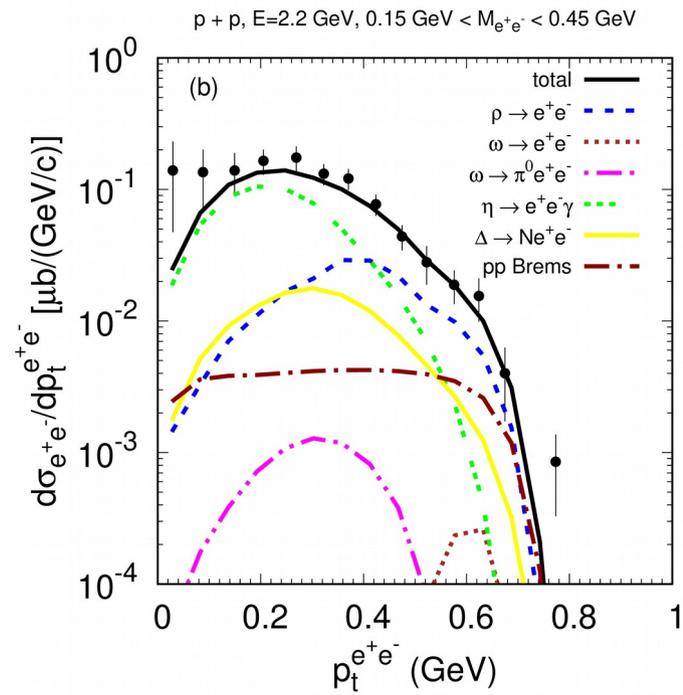
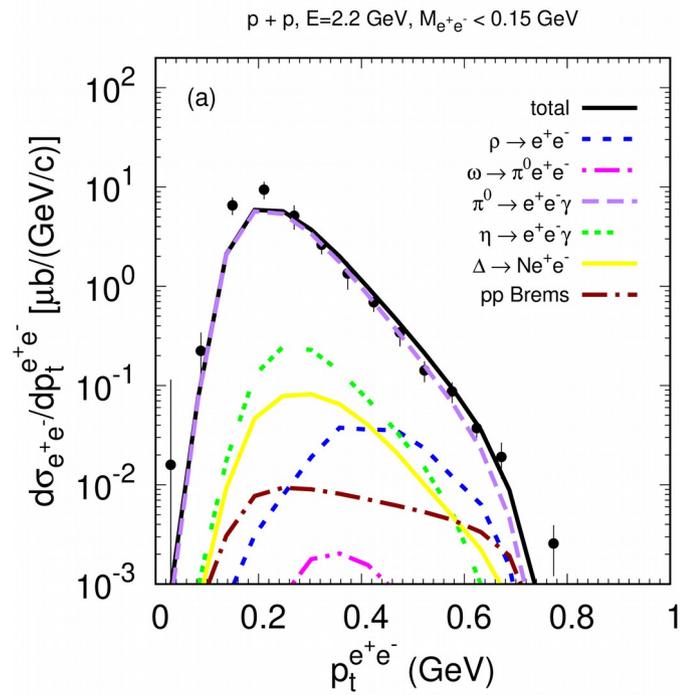


Data: [G. Agakishiev et al. \(HADES\), PLB 690 \(2010\) 118;](#)
[PRC 85 \(2012\) 054005;](#)
[EPJA 48 \(2012\) 64.](#)

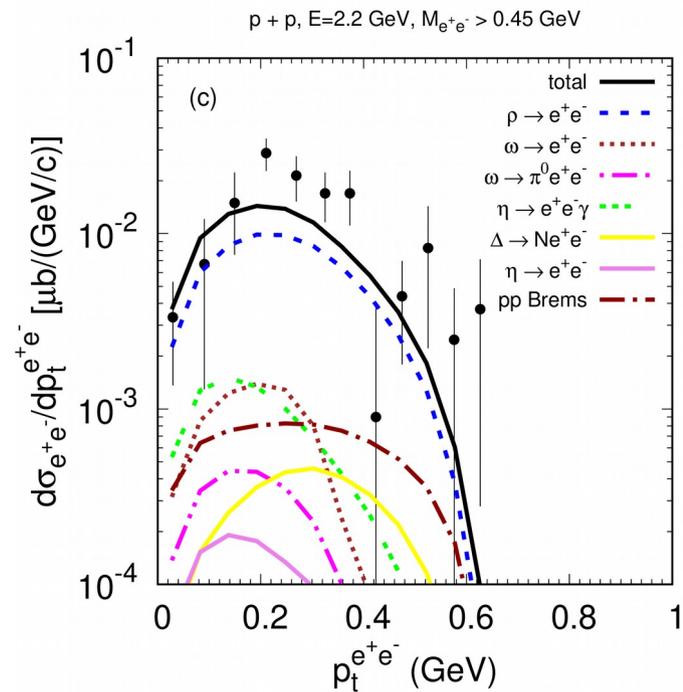
HADES acceptance filter and cuts
 on lepton's momenta and opening angle
 ($\Theta_{e^+e^-} > 9^\circ$) applied.

Good agreement in the entire $M_{e^+e^-}$ range.





Data: [G. Agakishiev et al. \(HADES\), PRC 85 \(2012\) 054005](#)



In the r.f. of deuteron: $\sigma_{pd \rightarrow e^+e^- p_{\text{spec}} X} \simeq \int d^3 p_n |\phi(p_n)|^2 \sigma_{pn \rightarrow e^+e^- X}(\sqrt{s_{pn}})$

Deuteron wave function of full Bonn model [R. Machleidt et al., Phys. Rept. 149, 1 \(1987\)](#)

- η Dalitz component is due to neutron Fermi motion:

$$E_{\text{thr}} = 1.256 \text{ GeV}$$

- Default calculation underpredicts dilepton yield with inv. mass above ≈ 0.2 GeV.

- Many possible reasons: insufficient brems., insufficient η , LC corrections to the DWF ...

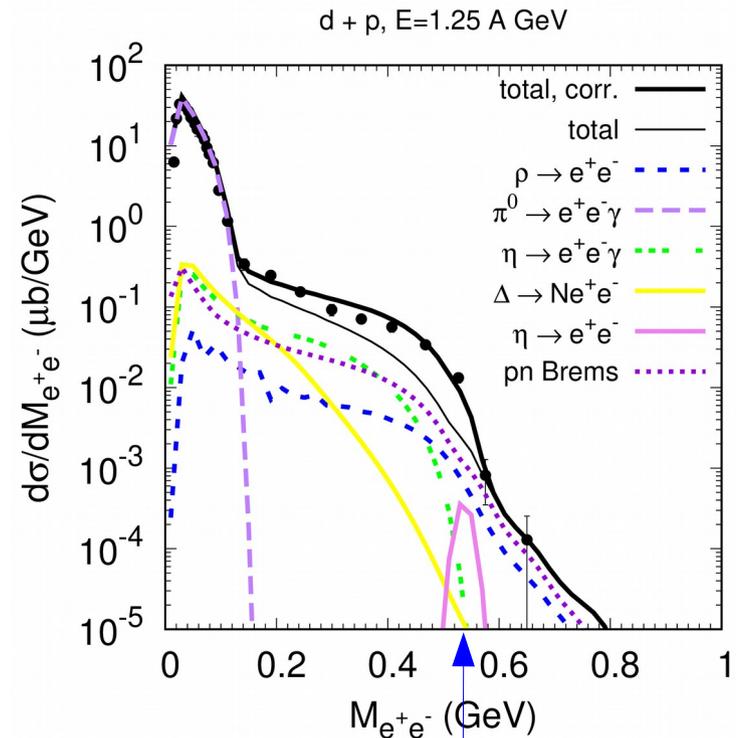
- Educated guess:

brems. need to be corrected
by dil. mass dependent factor

$$f(M) = C \frac{1 + wM^2/b^2}{(\exp[(a - M)/d] + 1)(\exp[(M - b)/d] + 1)} + 1,$$

with dilepton invariant mass M in GeV,
 $C = 1.5$, $d = 0.01$, $a = 0.10$, $b = 0.55$, and $w = 3.0$.

- Parameters determined by fit to the data

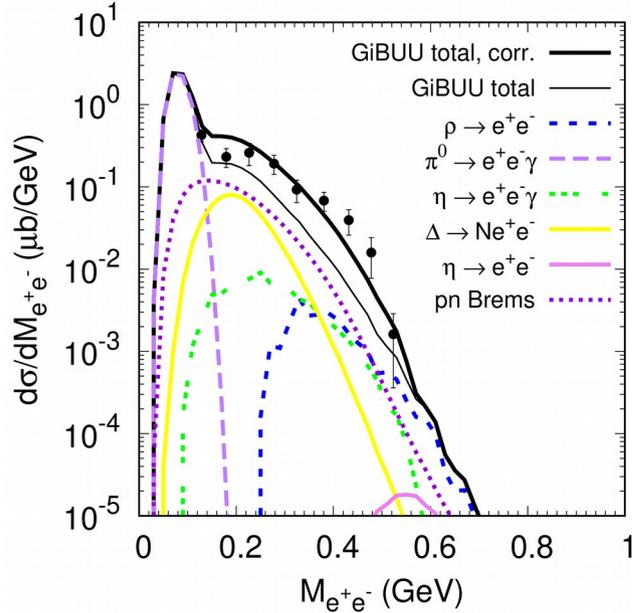


kin. limit for free np

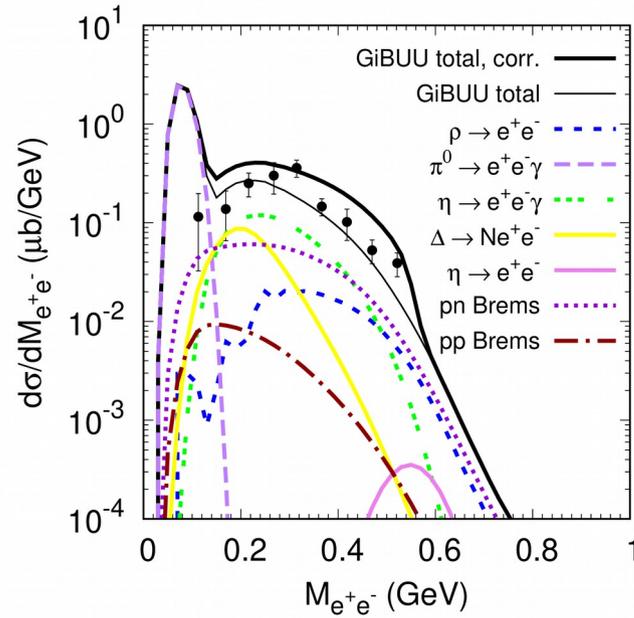
$$M_{e^+e^-}^{\text{max}} = \sqrt{s_{NN}} - 2m_N = 0.545 \text{ GeV}$$

Data: [G. Agakishiev et al. \(HADES\), PLB 690 \(2010\) 118](#)

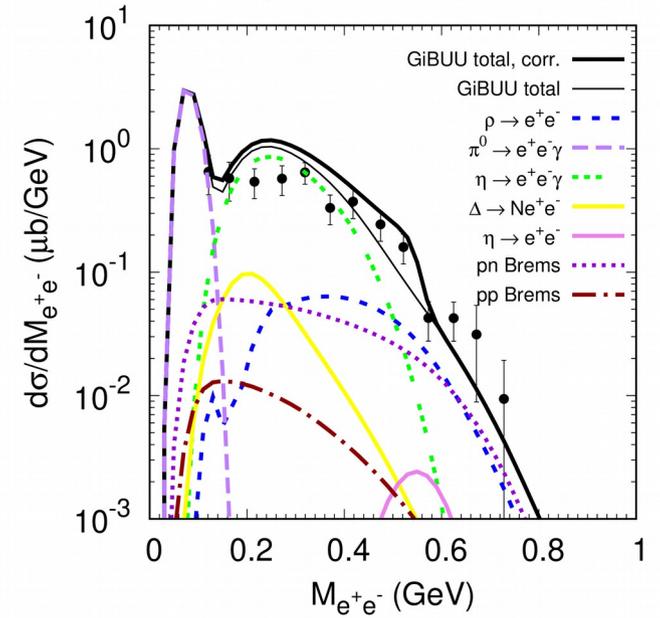
p + d, E=1.04 GeV, DLS



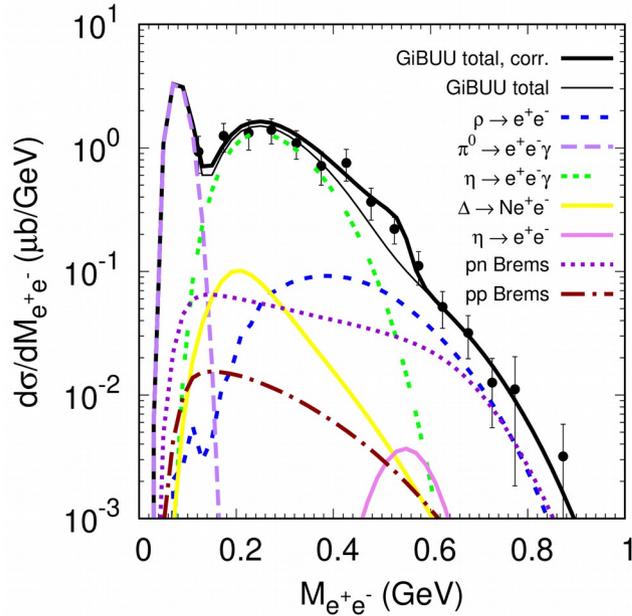
p + d, E=1.27 GeV, DLS



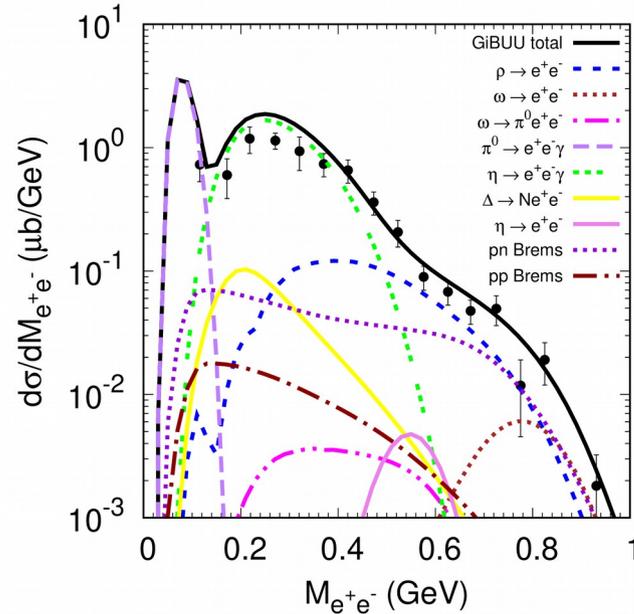
p + d, E=1.61 GeV, DLS



p + d, E=1.85 GeV, DLS

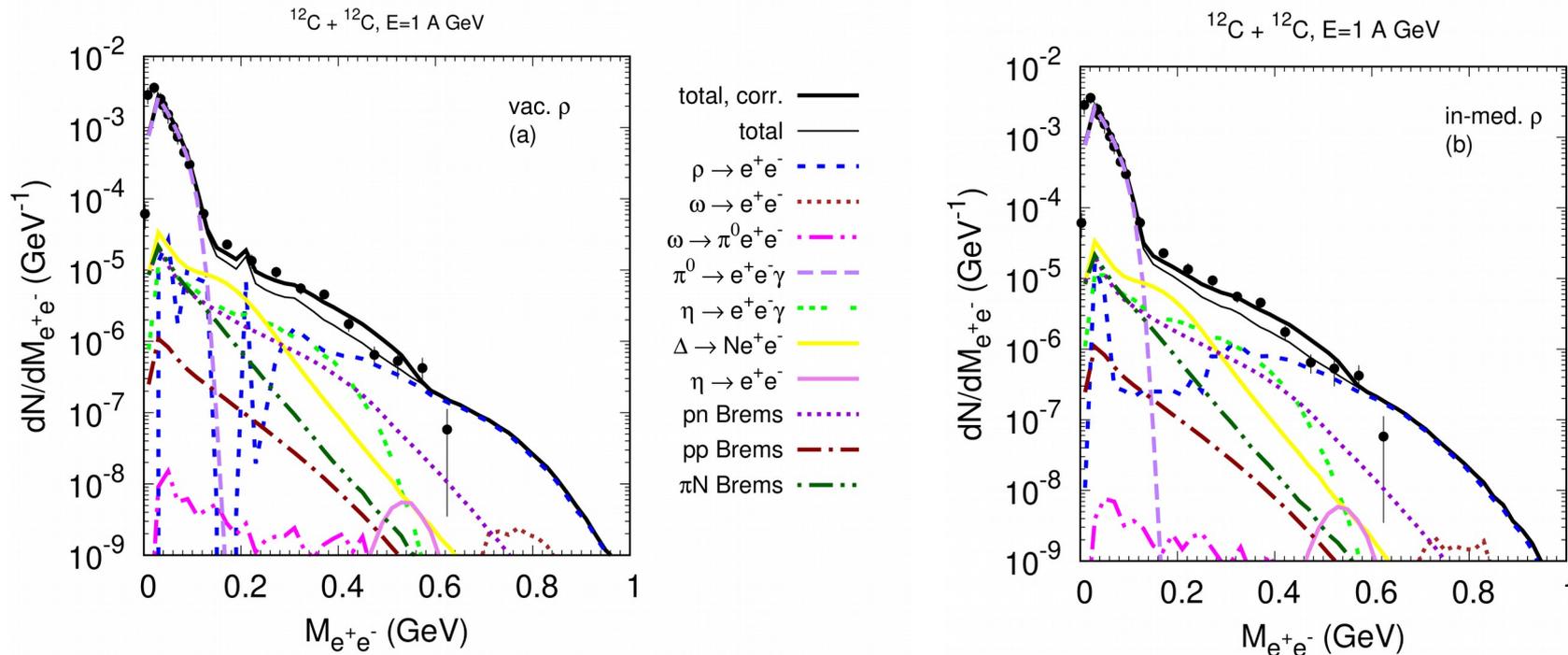


p + d, E=2.09 GeV, DLS



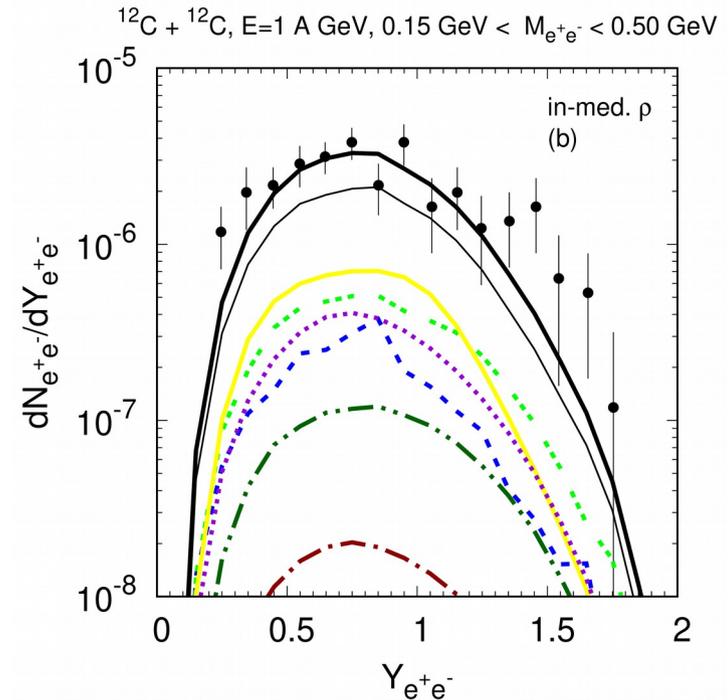
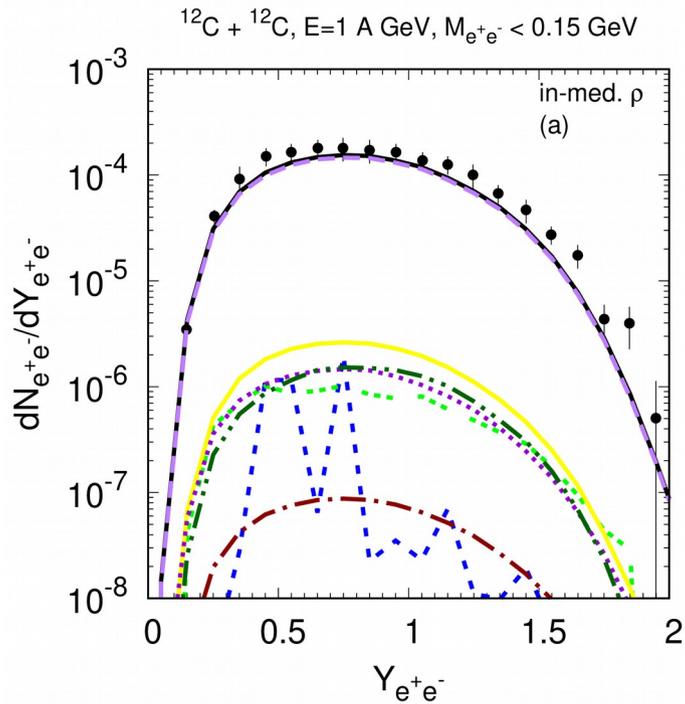
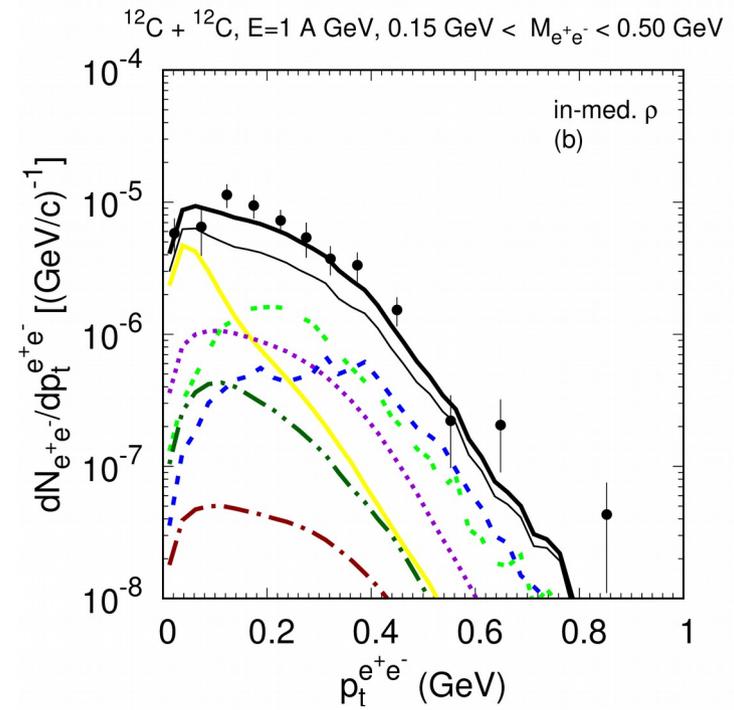
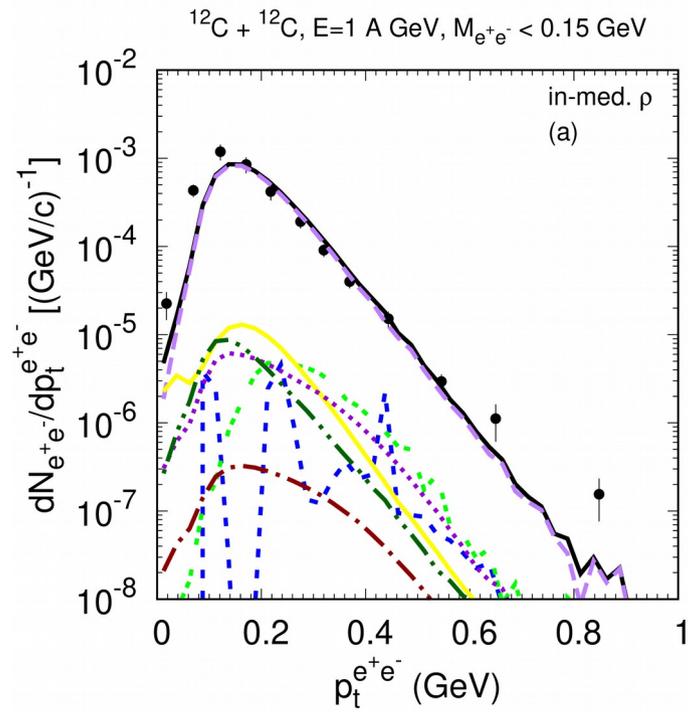
Data: [W.K. Wilson et al. \(DLS\), PRC 57, 1865 \(1998\)](#)

- At beam energies above ≈ 2 GeV correction of pn brems. is not needed.

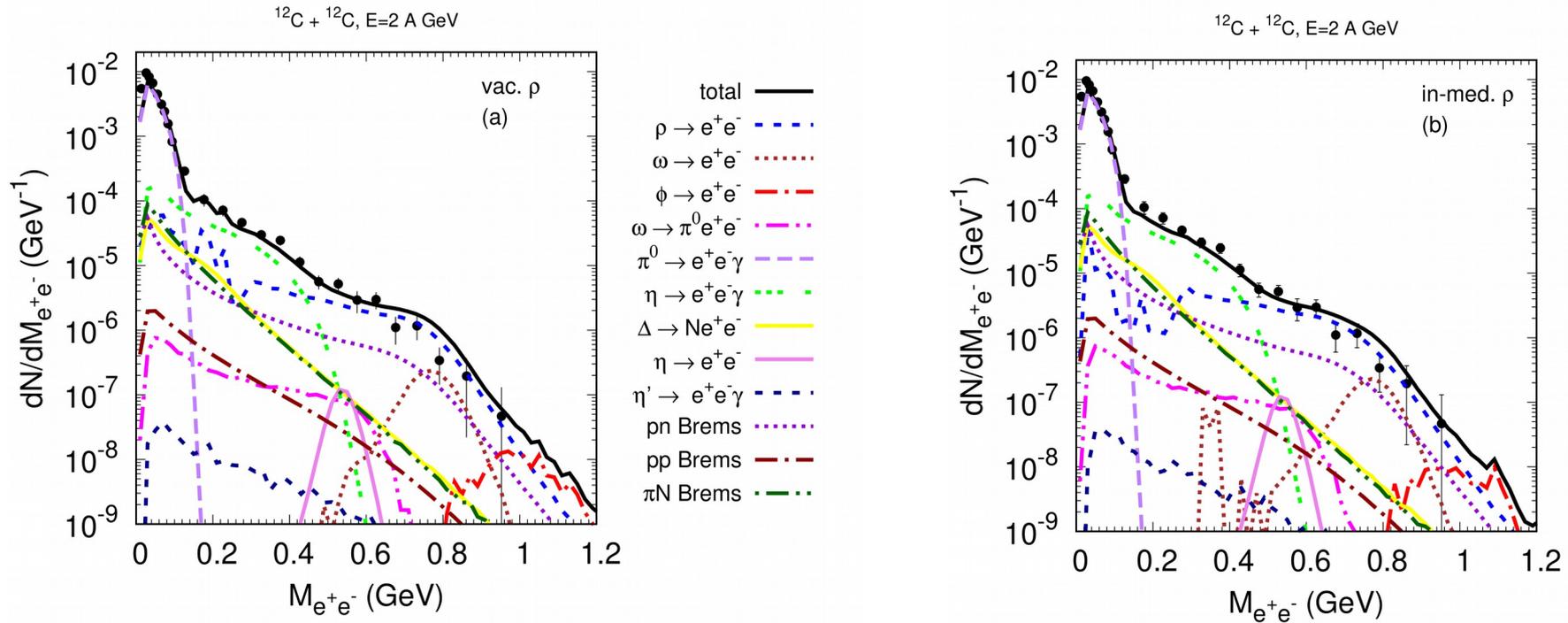


Data: **G. Agakishiev et al. (HADES),
PLB 663 (2008) 43**

- Several comparable contribs. in the interm. inv. mass region 0.2-0.4 GeV.
- Corrected pn brems. improves agreement at $E_{\text{beam}} = 1 \text{ A GeV}$.
- Collisional broadening of ρ : marginal effect for a light C+C system (reduced stat. fluctuations in $\rho \rightarrow e^+e^-$ component below 2π threshold).

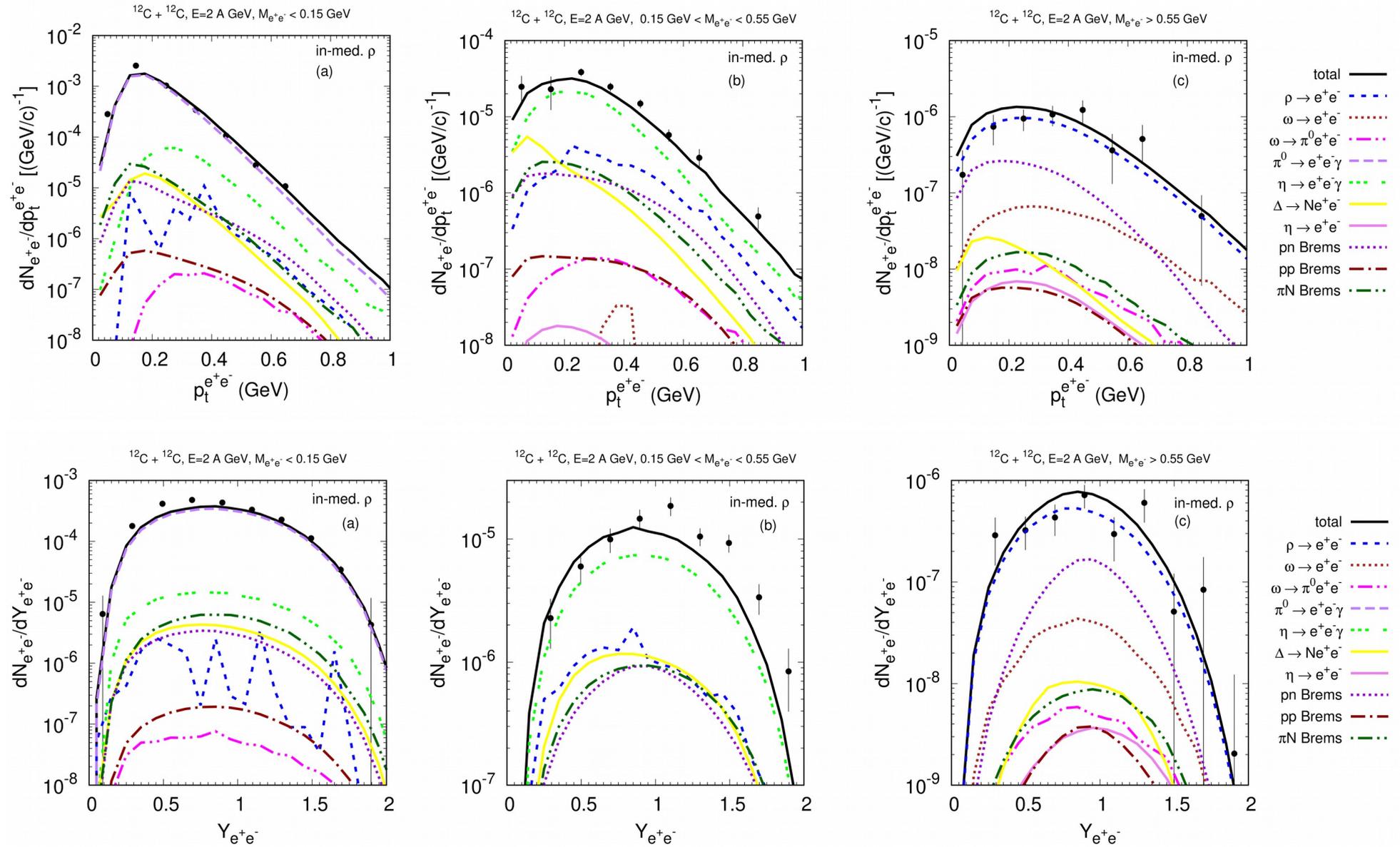


Data: *Y. Pachmayer (HADES), PhD thesis, Frankfurt U. (2008).*



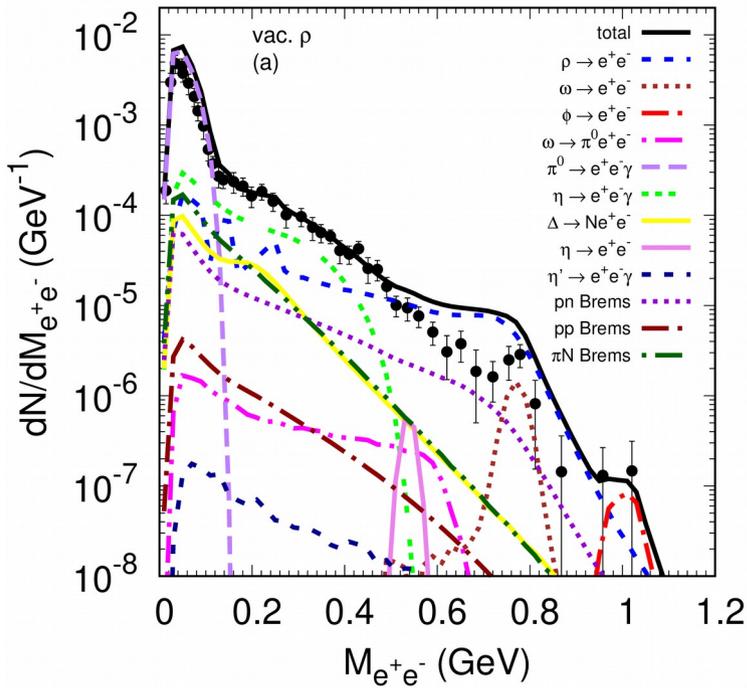
Data: [G. Agakishiev et al. \(HADES\), PRL 98 \(2007\) 052302](#)

- η Dalitz contrib. dominates in the interm. inv. mass range 0.2-0.4 GeV.
- very good agreement with data, no need to correct pn brems.
- ρ coll. broadening: again marginal effect (small system).

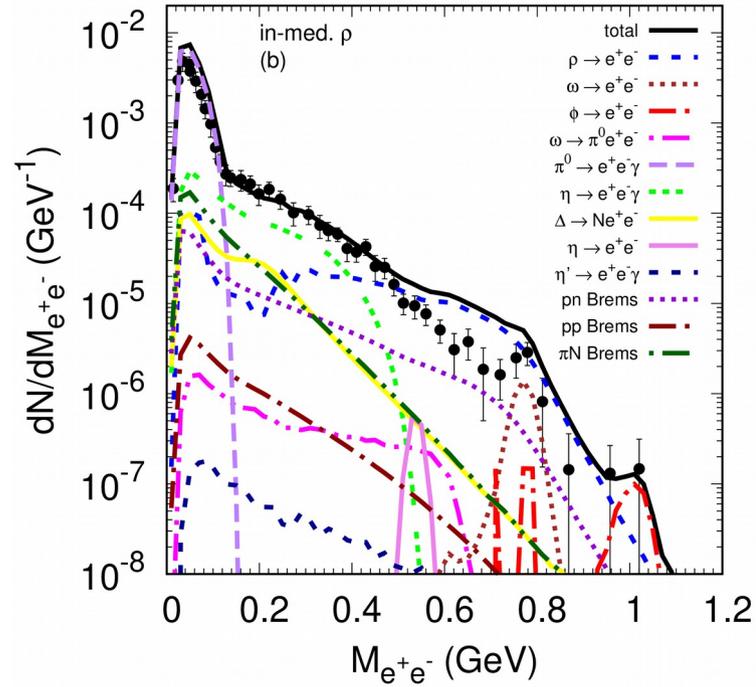


Data: *M. Sudol (HADES), PhD thesis, Frankfurt U. (2007).*

Ar + KCl, E=1.756 A GeV



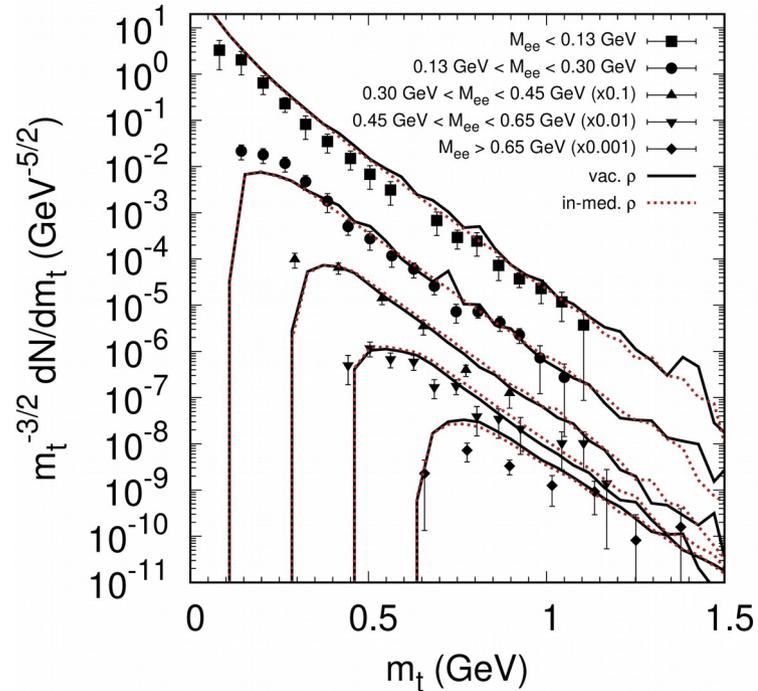
Ar + KCl, E=1.756 A GeV

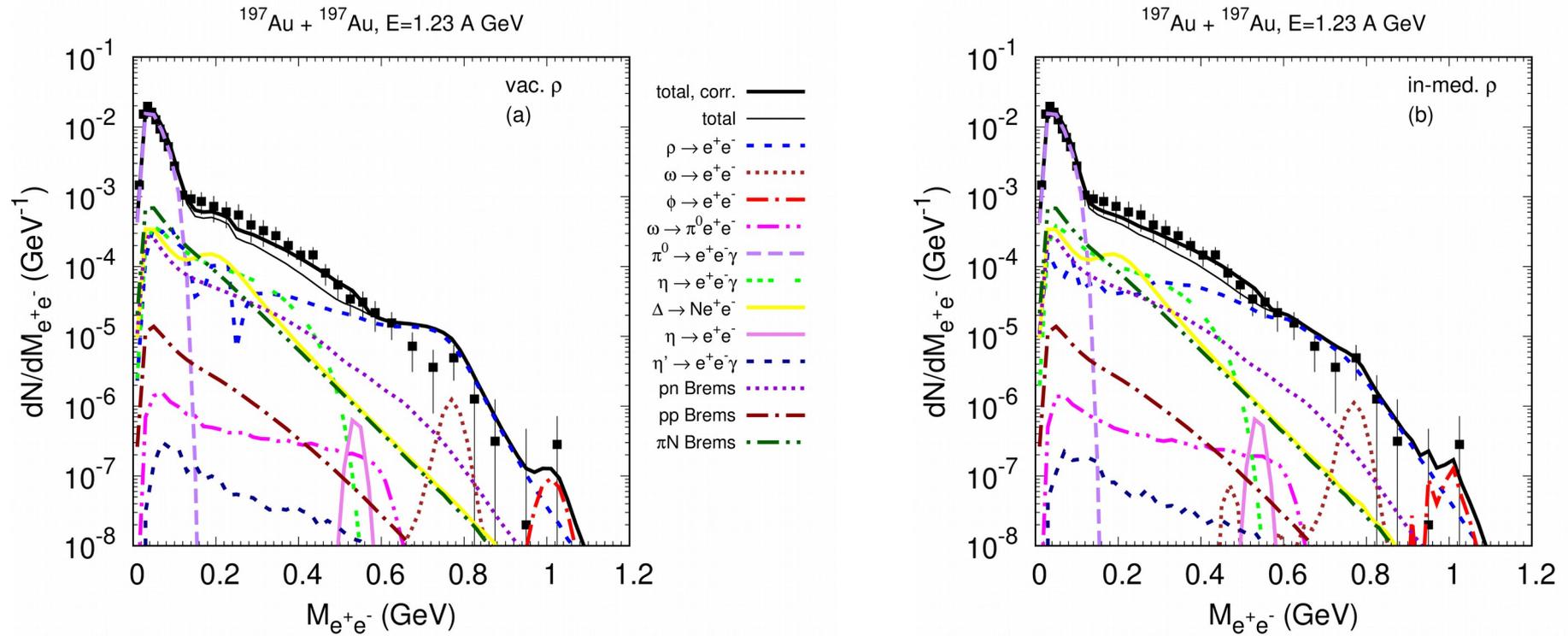


Data: [G. Agakishiev et al. \(HADES\), PRC 84 \(2011\) 014902](#)

- Overestimation at ρ on-shell mass.
- Collisional broadening of ρ improves agreement with data, but not enough.
- m_t -slopes well described.

Ar + KCl, E=1.756 A GeV

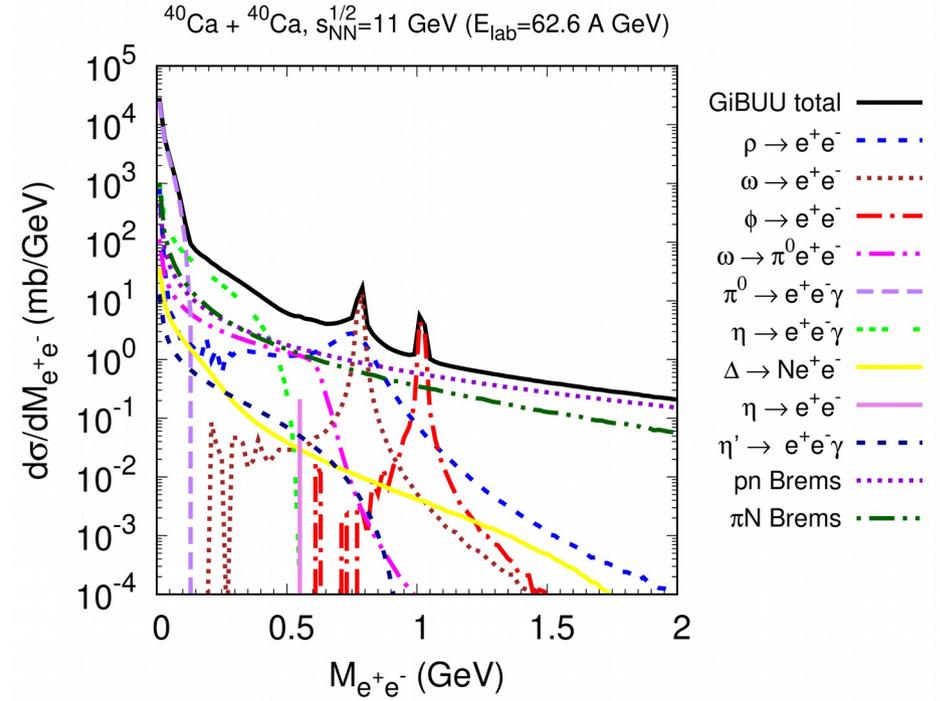
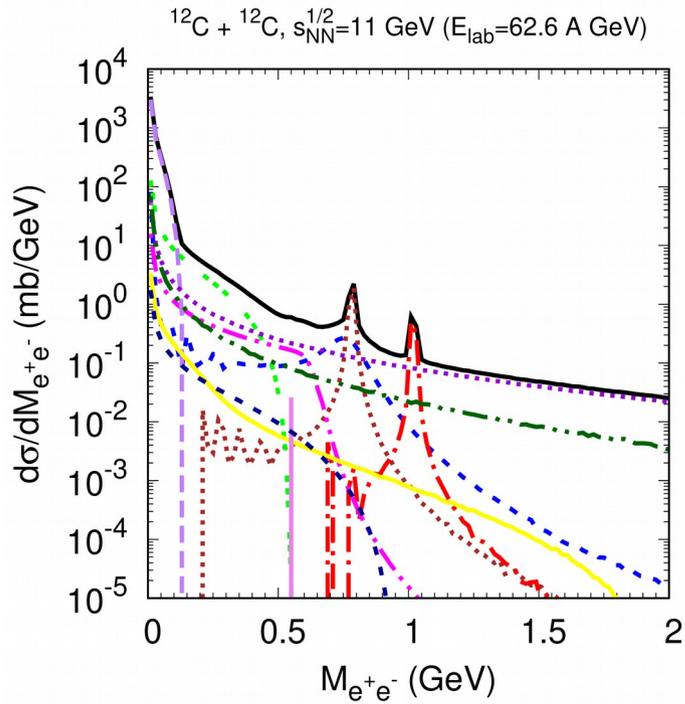




Data: [J. Adamczewski-Musch et al. \(HADES\), Nature Physics 15 \(2019\) 1040](#)

- Significant effect of ρ collisional broadening for the heavy Au+Au system. Improvement near the on-shell ρ mass.
- In the interm. inv. mass region 0.2-0.4 GeV corrected pn brems. needed.

- Higher energies: $\sqrt{s_{NN}} = 3.35 - 14 \text{ GeV}$ (C+C, Ca+Ca) $\leftrightarrow E_{\text{lab}} = 4 - 103 \text{ A GeV}$ for fixed target.
- PYTHIA is activated (at least in primary NN collisions).
- Hadronic formation effects.
- Mean field potentials turned-off: cascade simulation (with 'frozen' nuclei to avoid spurious particle emission).



- pn brems. dominates at $M_{e^+e^-} > 0.5$ GeV (calculated using SPA).

- pronounced ω and ϕ peaks.

- Hadrons created in a hard binary **exclusive** process emerge, first, as “prehadrons” that have a reduced transverse size, so-called point-like color singlet $q\bar{q}$ and qqq configurations (PLCs):

$$r_{\perp} \sim 1/Q$$

Color dipole – proton cross section in the pQCD limit ($r_{\perp} \rightarrow 0$):

$$\sigma_{q\bar{q}} \propto r_{\perp}^2 \sim 1/Q^2$$

➡ Color transparency (CT) – today's afternoon talk by [Mark Strikman](#).

- In **inclusive** $2 \rightarrow X$ processes CT is not proved theoretically. However, on phenomenological basis, most transport models include it.

Hadron formation length:

$$l_h \simeq \frac{2p_h}{|M_h^2 - M_{h'}^2|} \sim 0.4 - 0.6(\text{fm/GeV}) \cdot p_h[\text{GeV}]$$

Typically, h' is the 1st radially excited state of h .

(I) Based on JETSET-production-formation points (GiBUU default):

**K. Gallmeister, T. Falter,
PLB 630, 40 (2005)**

$$\sigma_{\text{eff}}(t)/\sigma_0 = X_0 + (1 - X_0) \frac{t - t_{\text{prod}}}{t_{\text{form}} - t_{\text{prod}}},$$

$$X_0 = r_{\text{lead}} a / Q^2, \quad a = 1 \text{ GeV}^2,$$

r_{lead} - the ratio (#of leading quarks)/(total # of quarks) in the prehadron,

Favored by the simultaneous analysis of HERMES and EMC data on fast forward production in DIS within GiBUU **K. Gallmeister, U. Mosel, NPA 801, 68 (2008).**

(II) Quantum diffusion model (QDM): **G.R. Farrar, H. Liu, L.L. Frankfurt, M.I. Strikman, PRL 61, 686 (1988)**

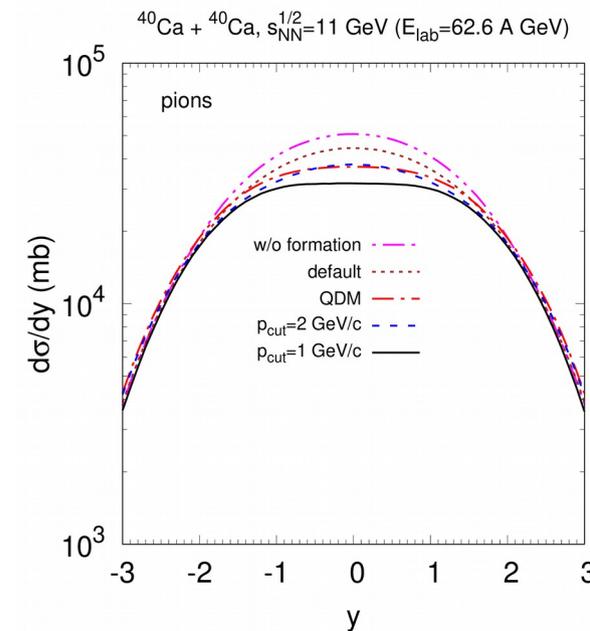
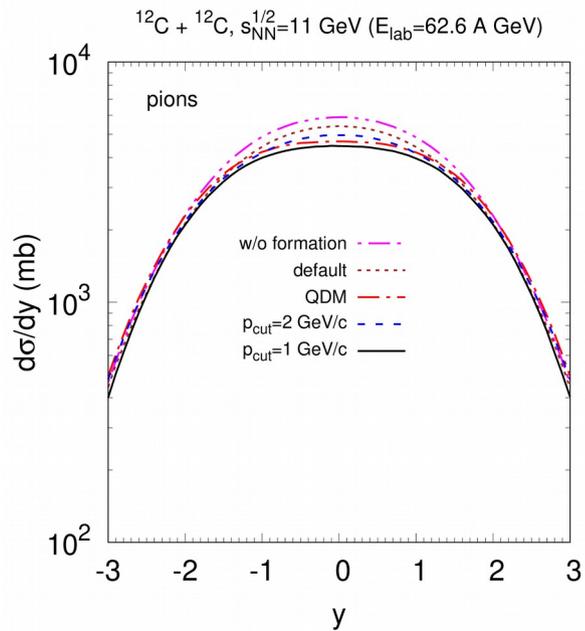
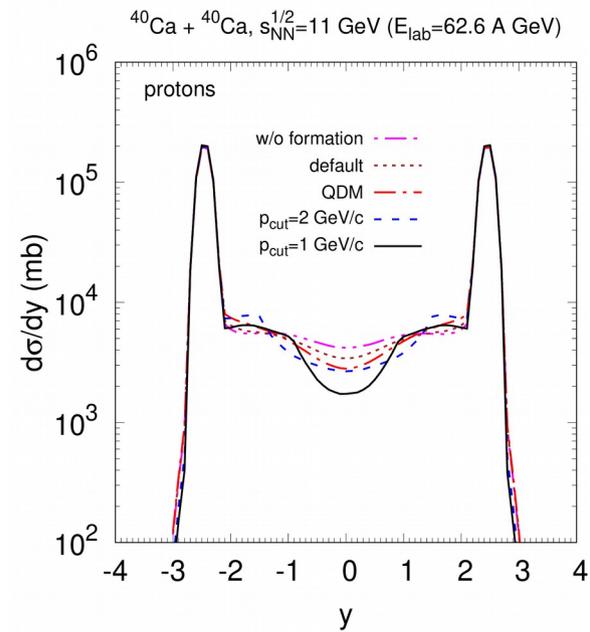
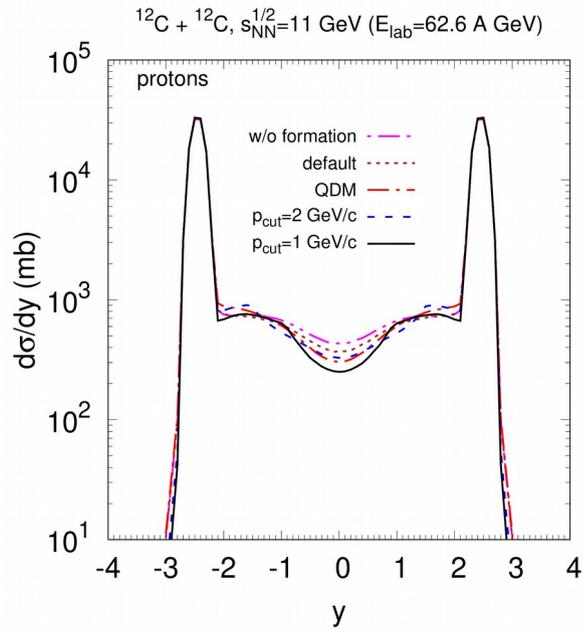
$$\sigma_{\text{eff}}(t)/\sigma_0 = X_0 + (1 - X_0) \frac{c(t - t_{\text{hard}})}{l_h},$$

(No direct way to derive X_0 in inclusive processes. Here we set $X_0=0$ for simplicity.)

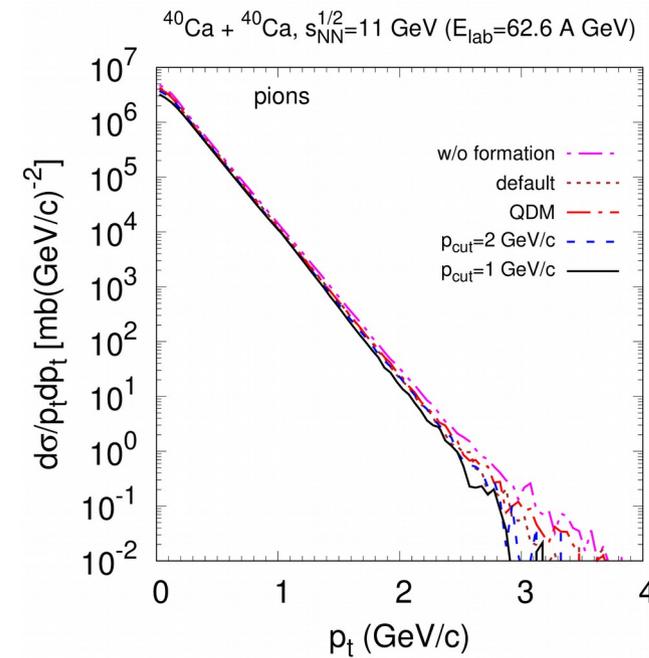
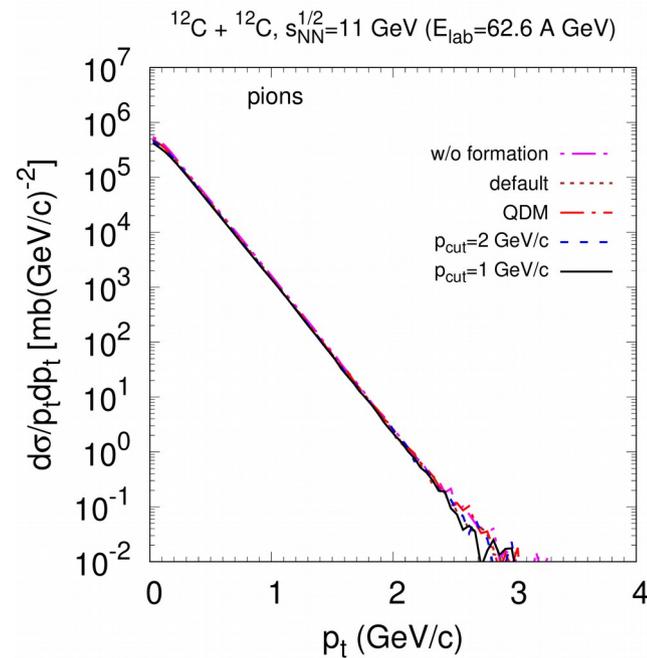
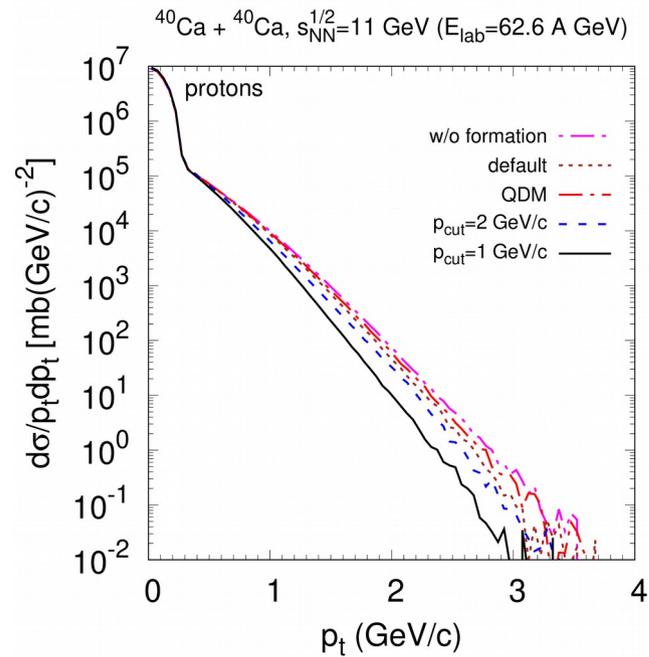
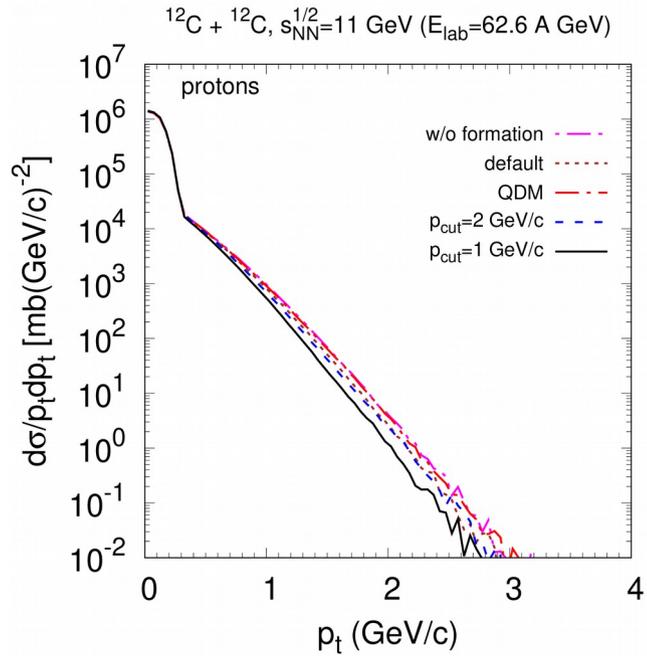
Favored by analysis of pion electroproduction at JLab **A. Larson, G. Miller, M. Strikman, PRC 74, 018201 (2006); AL, M. Strikman, M. Bleicher, PRC 93, 034618 (2016).**

(III) Cutoff: $\sigma_{\text{eff}}/\sigma_0 = \Theta(p_{\text{cut}} - p), \quad p_{\text{cut}} \sim 1 - 2 \text{ GeV}/c.$

Favored by analysis of low-energy ($E < 10 \text{ MeV}$) neutron production in μ^- DIS at 470 GeV on Pb (E665 experiment at Fermilab) **M. Strikman, M.G. Tverskoy, M.B. Zhalov, PLB 459, 37 (1999); AL, M. Strikman, PRC 101, 014617 (2020), arXiv:1812.08231.**



- less proton stopping and less pion production for more restrictive FSI ($p_{\text{cut}}=1$ GeV/c)



- less proton rescattering ➡ softer proton p_t spectra for restrictive FSI ($p_{\text{cut}}=1$ GeV/c)

Conclusions

- GiBUU successfully describes dilepton production in pp, dp, and AA at beam energies 1-2 A GeV. Collisional broadening of ρ improves agreement with HADES data for heavy colliding systems (Ar+KCl at 1.76 A GeV and Au+Au at 1.23 A GeV). Remaining underprediction in the intermediate inv. masses of e^+e^- (0.2-0.4 GeV) for d+p at 1.25 A GeV, C+C at 1 A GeV and Au+Au at 1.23 GeV is removed by correcting pn brems. component (same correction for all three systems).
- The reason for underprediction of e^+e^- yield at inv. mass above 0.2 GeV in d+p collisions at 1.25 A GeV remains unclear. At higher beam energies the discrepancy seems to disappear (DLS data). Good to have d+p data on e^+e^- production at NICA-SPD test the np channel.
- Dilepton inv. mass spectra from systems C+C and Ca+Ca at $\sqrt{s_{NN}}=11$ GeV show up sharp peaks at ω and ϕ on-shell masses on the smooth background (mostly due to pn brems.). Sharp peak of J/ψ is also expected (to be calculated).
- NICA-SPD energies are well suited to study hadron formation. Sensitive observables are rapidity- and p_t -spectra of protons and pions.

Backup

Lagrangian density:

$$\mathcal{L} = \sum_{j=N, \bar{N}} \bar{\psi}_j [\gamma_\mu (i\partial^\mu - g_{\omega j} \omega^\mu - g_{\rho j} \boldsymbol{\tau} \boldsymbol{\rho}^\mu - \frac{e}{2} (B_j + \tau^3) A^\mu) - m_N - g_{\sigma j} \sigma] \psi_j$$

$$+ \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^2 - \frac{1}{4} \mathbf{R}_{\mu\nu} \mathbf{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}^2 - \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} ,$$

$$B_N = 1, B_{\bar{N}} = -1, \quad \Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \quad \mathbf{R}_{\mu\nu} = \partial_\mu \boldsymbol{\rho}_\nu - \partial_\nu \boldsymbol{\rho}_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu ,$$

$$U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 .$$

G-parity (Walecka model): $g_{\sigma \bar{N}} = g_{\sigma N}, \quad g_{\omega \bar{N}} = -g_{\omega N}, \quad g_{\rho \bar{N}} = g_{\rho N} .$

Phenomenological couplings: $g_{\sigma \bar{N}} = \xi g_{\sigma N}, \quad g_{\omega \bar{N}} = -\xi g_{\omega N}, \quad g_{\rho \bar{N}} = \xi g_{\rho N}, \quad 0 < \xi \leq 1 .$

Lagrange equations of motion for meson fields:

$$\left\{ \begin{array}{l} (\partial_\mu \partial^\mu + m_\sigma^2) \sigma(x) + g_2 \sigma^2 + g_3 \sigma^3 = - \sum_{j=N, \bar{N}} g_{\sigma j} \rho_{Sj}(x) , \\ (\partial_\nu \partial^\nu + m_\omega^2) \omega^\mu(x) = \sum_{j=N, \bar{N}} g_{\omega j} J_{Bj}^\mu(x) , \\ (\partial_\nu \partial^\nu + m_\rho^2) \rho^{3\mu}(x) = \sum_{j=N, \bar{N}} g_{\rho j} J_{Ij}^\mu(x) , \\ \partial_\nu \partial^\nu A^\mu(x) = 4\pi \sum_{j=N, \bar{N}} J_{Qj}^\mu(x) . \end{array} \right.$$

$$\rho_{Sj}(x) = \langle \bar{\psi}_j(x) \psi_j(x) \rangle = \frac{2}{(2\pi)^3} \int \frac{d^3 p^*}{p^{*0}} m_j^* f_j(x, \mathbf{p}^*) ,$$

$$J_{aj}^\mu(x) = \langle \bar{\psi}_j(x) \gamma^\mu O_a \psi_j(x) \rangle = \frac{2}{(2\pi)^3} \int \frac{d^3 p^*}{p^{*0}} p^{*\mu} O_a f_j(x, \mathbf{p}^*) , \quad O_B = 1, \quad O_I = \tau^3, \quad O_Q = \frac{e}{2} (B_j + \tau^3) \equiv q_j .$$

Particles propagated by GiBUU

By default, all resonances are propagated
while for cross sections are used all resonances except those with $I=1/2$ one-star.

Nonstrange baryons

Name	ID	Mass	Width	Spin	Rating	Isospin	Strange	Charm	Stability	min.Mass
N	1	0.938	0.000	0.5	****	0.5	0	0	0	0.700
Δ	2	1.232	0.118	1.5	****	1.5	0	0	3	1.076
$P_{11}(1440)$	3	1.462	0.391	0.5	****	0.5	0	0	3	1.076
$S_{11}(1535)$	4	1.534	0.151	0.5	***	0.5	0	0	3	1.076
$S_{11}(1650)$	5	1.659	0.173	0.5	****	0.5	0	0	3	1.076
$S_{11}(2090)$	6	1.928	0.414	0.5	*	0.5	0	0	3	1.076
$D_{13}(1520)$	7	1.524	0.124	1.5	****	0.5	0	0	3	1.076
$D_{13}(1700)$	8	1.737	0.249	1.5	*	0.5	0	0	3	1.076
$D_{13}(2080)$	9	1.804	0.447	1.5	*	0.5	0	0	3	1.076
$D_{15}(1675)$	10	1.676	0.159	2.5	****	0.5	0	0	3	1.076
$G_{17}(2190)$	11	2.127	0.547	3.5	****	0.5	0	0	3	1.076
$P_{11}(1710)$	12	1.717	0.478	0.5	*	0.5	0	0	3	1.076
$P_{11}(2100)$	13	1.885	0.113	0.5	*	0.5	0	0	3	1.076
$P_{13}(1720)$	14	1.717	0.383	1.5	*	0.5	0	0	3	1.076
$P_{13}(1900)$	15	1.879	0.498	1.5	***	0.5	0	0	3	1.076
$F_{15}(1680)$	16	1.684	0.139	2.5	****	0.5	0	0	3	1.076
$F_{15}(2000)$	17	1.903	0.494	2.5	*	0.5	0	0	3	1.076
$F_{17}(1990)$	18	2.086	0.535	3.5	**	0.5	0	0	3	1.076
$S_{31}(1620)$	19	1.672	0.154	0.5	**	1.5	0	0	3	1.076
$S_{31}(1900)$	20	1.920	0.263	0.5	***	1.5	0	0	3	1.076
$D_{33}(1700)$	21	1.762	0.599	1.5	*	1.5	0	0	3	1.076
$D_{33}(1940)$	22	2.057	0.460	1.5	*	1.5	0	0	3	1.076
$D_{35}(1930)$	23	1.956	0.526	2.5	**	1.5	0	0	3	1.076
$D_{35}(2350)$	24	2.171	0.264	2.5	**	1.5	0	0	3	1.076
$P_{31}(1750)$	25	1.744	0.299	0.5	*	1.5	0	0	3	1.076
$P_{31}(1910)$	26	1.882	0.239	0.5	****	1.5	0	0	3	1.076
$P_{33}(1600)$	27	1.706	0.430	1.5	***	1.5	0	0	3	1.076
$P_{33}(1920)$	28	2.014	0.152	1.5	*	1.5	0	0	3	1.076
$F_{35}(1750)$	29	1.752	0.251	2.5	*	1.5	0	0	3	1.076
$F_{35}(1905)$	30	1.881	0.327	2.5	***	1.5	0	0	3	1.076
$F_{37}(1950)$	31	1.945	0.300	3.5	****	1.5	0	0	3	1.076

Name	ID	Mass	Width	Spin	Rating	Isospin	Strange	Charm	Stability	min.Mass
Λ	32	1.116	0.000	0.5	****	0.0	-1	0	0	1.076
Σ	33	1.189	0.000	0.5	****	1.0	-1	0	0	1.076
$\Sigma(1385)$	34	1.385	0.036	1.5	****	1.0	-1	0	3	1.254
$\Lambda(1405)$	35	1.405	0.050	0.5	****	0.0	-1	0	3	1.254
$\Lambda(1520)$	36	1.520	0.016	1.5	****	0.0	-1	0	3	1.254
$\Lambda(1600)$	37	1.600	0.150	0.5	***	0.0	-1	0	3	1.254
$\Lambda(1670)$	38	1.670	0.035	0.5	****	0.0	-1	0	3	1.254
$\Lambda(1690)$	39	1.690	0.060	1.5	****	0.0	-1	0	3	1.254
$\Lambda(1810)$	40	1.810	0.150	0.5	***	0.0	-1	0	3	1.254
$\Lambda(1820)$	41	1.820	0.080	2.5	****	0.0	-1	0	3	1.254
$\Lambda(1830)$	42	1.830	0.095	2.5	****	0.0	-1	0	3	1.254
$\Sigma(1670)$	43	1.670	0.060	1.5	****	1.0	-1	0	3	1.254
$\Sigma(1775)$	44	1.775	0.120	2.5	****	1.0	-1	0	3	1.254
$\Sigma(2030)$	45	2.030	0.180	3.5	****	1.0	-1	0	3	1.254
$\Lambda(1800)$	46	1.800	0.300	0.5	***	0.0	-1	0	3	1.254
$\Lambda(1890)$	47	1.890	0.100	1.5	****	0.0	-1	0	3	1.254
$\Lambda(2100)$	48	2.100	0.200	3.5	****	0.0	-1	0	3	1.254
$\Lambda(2110)$	49	2.110	0.200	2.5	***	0.0	-1	0	3	1.254
$\Sigma(1660)$	50	1.660	0.100	0.5	***	1.0	-1	0	3	1.254
$\Sigma(1750)$	51	1.750	0.090	0.5	***	1.0	-1	0	3	1.254
$\Sigma(1915)$	52	1.915	0.120	2.5	****	1.0	-1	0	3	1.254
Ξ	53	1.315	0.000	0.5	****	0.5	-2	0	0	1.254
Ξ^*	54	1.530	0.009	1.5	****	0.5	-2	0	3	1.254
Ω	55	1.672	0.000	1.5	****	0.0	-3	0	0	1.254

Charmed baryons

Name	ID	Mass	Width	Spin	Rating	Isospin	Strange	Charm	Stability	min.Mass
Λ_c	56	2.285	0.000	0.5	****	0.0	0	1	0	1.076
Σ_c	57	2.452	0.000	0.5	****	1.0	0	1	0	1.076
Σ_c^*	58	2.520	0.015	1.5	****	1.0	0	1	3	2.423
Ξ_c	59	2.466	0.000	0.5	***	0.5	-1	1	0	2.423
Ξ_c^*	60	2.645	0.004	1.5	***	0.5	-1	1	3	2.423
Ω_c	61	2.697	0.000	0.5	***	0.0	-2	1	0	2.423

Mesons

Name	ID	Mass	Width	Spin	Isospin	Strange	Charm	Stability	min.Mass
π	101	0.1380	0.0000	0.0	1.0	0	0	0	0.000
η	102	0.5478	0.0000	0.0	0.0	0	0	3	0.000
ρ	103	0.7755	0.1491	1.0	1.0	0	0	3	0.276
σ	104	0.8000	0.5000	0.0	0.0	0	0	3	0.276
ω	105	0.7826	0.0085	1.0	0.0	0	0	3	0.138
η'	106	0.9578	0.0002	0.0	0.0	0	0	3	0.000
ϕ	107	1.0194	0.0043	1.0	0.0	0	0	3	0.414
η_c	108	2.9800	0.0280	0.0	0.0	0	0	3	0.000
J/ψ	109	3.0969	0.0000	1.0	0.0	0	0	0	0.000
K	110	0.4960	0.0000	0.0	0.5	1	0	0	0.496
\bar{K}	111	0.4960	0.0000	0.0	0.5	-1	0	0	0.496
K^*	112	0.8920	0.0500	1.0	0.5	1	0	3	0.634
\bar{K}^*	113	0.8920	0.0500	1.0	0.5	-1	0	3	0.634
D	114	1.8670	0.0000	0.0	0.5	0	1	0	1.500
\bar{D}	115	1.8670	0.0000	0.0	0.5	0	-1	0	1.500
D^*	116	2.0070	0.0020	1.0	0.5	0	1	3	1.500
\bar{D}^*	117	2.0070	0.0020	1.0	0.5	0	-1	3	1.500
D_s^+	118	1.9690	0.0000	0.0	0.0	1	1	0	1.500
D_s^-	119	1.9690	0.0000	0.0	0.0	-1	-1	0	1.500
D_s^{*+}	120	2.1120	0.0010	1.0	0.0	1	1	3	1.500
D_s^{*-}	121	2.1120	0.0010	1.0	0.0	-1	-1	3	1.500
$f_2(1270)$	122	1.2754	0.1852	2.0	0.0	0	0	3	0.276

Modeling dilepton emission by “shining” method:

- Dilepton production is simulated perturbatively, i.e. it does not change the state of the system.
- At every time step Δt every resonance emits a e^+e^- pair that contributes to the total dilepton spectrum with a weight

$$w = \gamma_{Lor} \Gamma_{R \rightarrow e^+e^-} \Delta t$$

for direct decay (and similar for Dalitz decay).

- A resonance that survived until the end of time evolution emits a e^+e^- pair with a weight

$$w = \Gamma_{R \rightarrow e^+e^-} / \Gamma_R^{tot}$$

- At every $\pi^\pm N$, pn , and pp collision a e^+e^- pair is emitted with a weight

$$w = \sigma_{brems} / \sigma_{tot}$$

Charged pion bremsstrahlung $\pi^\pm N \rightarrow \pi^\pm N e^+ e^-$: soft-photon approximation (SPA)

C. Gale, J.I. Kapusta, PRC 35, 2107 (1987); G. Wolf et al., NPA 517, 615 (1990)

$$E \frac{d\sigma_{e^+e^-}}{d^3p dm} = \frac{\alpha^2}{6\pi^3} \frac{\bar{\sigma}_{\text{el}}(s)}{mE^2} \frac{R_2(s_2)}{R_2(s)}, \quad \bar{\sigma}_{\text{el}}(s) = \int_{-|t|_{\text{max}}}^0 dt \frac{-t}{m_\pi^2} \frac{d\sigma_{\text{el}}(s, t)}{dt} \simeq \frac{2q_{\text{cm}}^2(s)}{m_\pi^2} \sigma_{\text{el}}(s),$$

(E, \mathbf{p}) – four-momentum of the e^+e^- pair in the c.m. system of the colliding pion and nucleon,

$$m^2 = E^2 - \mathbf{p}^2,$$

$$q_{\text{cm}}(s) = [(s + m_\pi^2 - m_N^2)^2/4s - m_\pi^2]^{1/2},$$

$$s_2 = s + m^2 - 2\sqrt{s}E,$$

$$R_2(s) = 2q_{\text{cm}}(s)/\sqrt{s} - \text{two-body } (\pi N) \text{ phase space.}$$