中国科学院高能物理研究所
Institute of High Energy Physics

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CHINESE ACADEMY OF SCIENCES

# Production of the neutral hyper-nucleus ${}^{4}_{\Lambda\Lambda}n$ at SPD NICA


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SPD NICA

Physics programme for the first stage of the NICA SPD experiment

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# Outline

1. Motivation and possible existence of ${}^{4}_{\Lambda\Lambda}n$

2. Production mechanism for ${}^{4}_{\Lambda\Lambda}n$

3. Summary

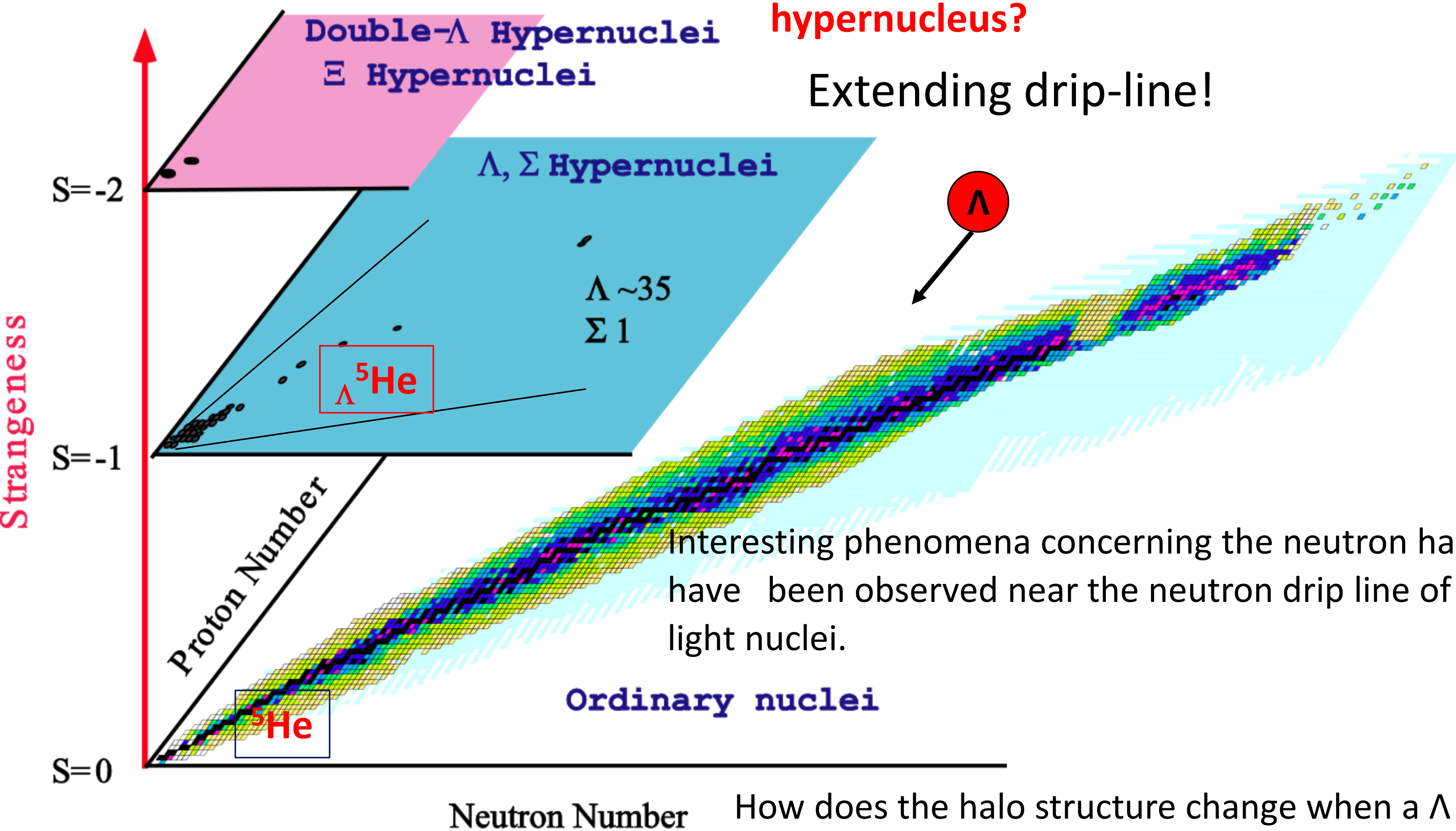
# 1. Motivation and possible existence of ${}^{4}_{\Lambda\Lambda}n$

# Nuclear chart with strangeness

**Multi-strangeness system such as Neutron star**

**What's the role played by the $\Lambda$-N interaction in the formation of neutral hypernucleus?**

Extending drip-line!




Interesting phenomena concerning the neutron halo have been observed near the neutron drip line of light nuclei.

How does the halo structure change when a Λ particle is injected into an unstable nucleus?

Slide from E. Hiyama

## Some hints from the first principle

With $m_\pi$=890 MeV at the SU(3) flavor symmetry point, L= 3.4, 4.5 and 6.7 fm, and b=0.145 fm, LQCD calculations are performed using an isotropic clover quark action [S. Beane et al, NPLQCD, PRC87, 034506 (2013)].

Local source adopted for 4-baryon system:

$$(\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8})_{J^\pi=0^+} \rightarrow \mathbf{1}\oplus\mathbf{27}\oplus\overline{\mathbf{28}},$$

$$(\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8})_{J^\pi=1^+} \rightarrow \mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\overline{\mathbf{35}},$$

$$(\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8})_{J^\pi=2^+} \rightarrow \mathbf{8}\oplus\mathbf{27},$$

$\boldsymbol{I = 1, J^\pi = 0^+}$: ${}^{4}_{\Lambda\Lambda}\mathrm{He}$, ${}^{4}_{\Lambda\Lambda}\mathrm{H}$, and $nn\Lambda\Lambda$

Baryon effective mass plot

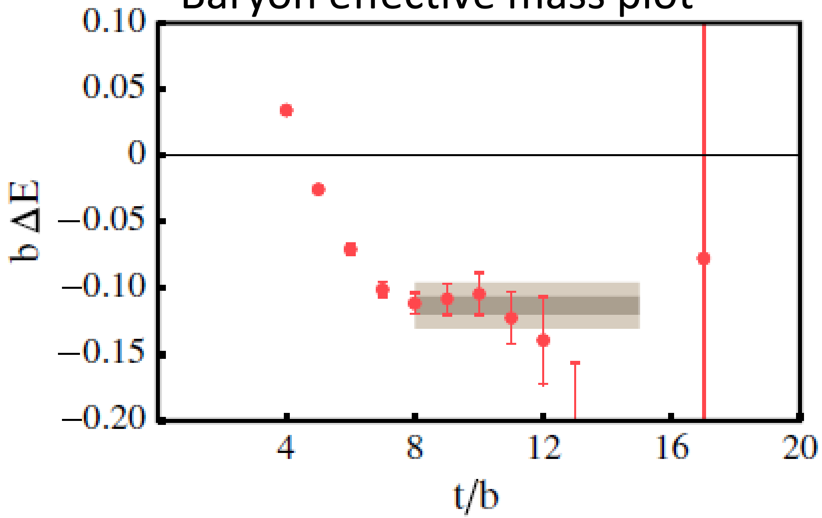


TABLE XVIII. Summary of the extracted ground-state binding energies of the nuclei and hypernuclei studied in this work.

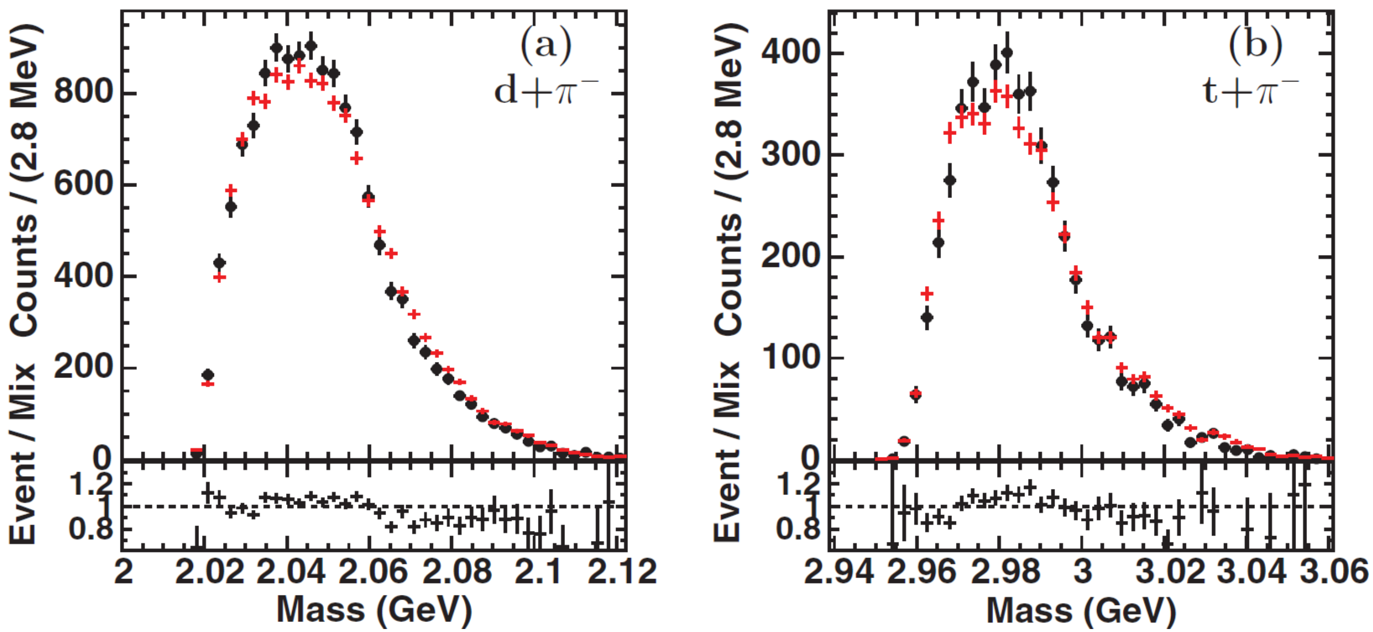
| State | $A$ | $s$ | $I$ | $J^\pi$ | SU(3) irrep | Binding energy [MeV] | $\sim B/A$ [MeV] |
|---|---|---|---|---|---|---|---|
| $d$ (deuteron) | 2 | 0 | 0 | $1^+$ | $\overline{\mathbf{10}}$ | 19.5(3.6)(3.1)(0.2) | 10 |
| $nn$ (dineutron) | 2 | 0 | 1 | $0^+$ | $\mathbf{27}$ | 15.9(2.7)(2.7)(0.2) | 8 |
| $n\Sigma$ | 2 | $-1$ | $\frac{3}{2}$ | $1^+$ | $\mathbf{10}$ | 5.5(3.4)(3.7)(0.0) | 3 |
| H (H-dibaryon) | 2 | $-2$ | 0 | $0^+$ | $\mathbf{1}$ | 74.6(3.3)(3.3)(0.8) | 37 |
| $n\Xi$ | 2 | $-2$ | 0 | $1^+$ | $\mathbf{8}_A$ | 37.7(3.0)(2.7)(0.4) | 19 |
| $^3$He, $^3$H | 3 | 0 | $\frac{1}{2}$ | $\frac{1}{2}^+$ | $\overline{\mathbf{35}}$ | 53.9(7.1)(8.0)(0.6) | 18 |
| $^3_\Lambda$H (hypertriton) | 3 | $-1$ | 0 | $\frac{1}{2}^+$ | $\overline{\mathbf{35}}$ | 53.9(7.1)(8.0)(0.6) | 18 |
| $^3_\Lambda$H (hypertriton) | 3 | $-1$ | 0 | $\frac{3}{2}^+$ | $\overline{\mathbf{10}}$ | 82(8)(12)(1) | 27 |
| $^3_\Lambda$He, $^3_\Lambda\tilde{\text{H}}$, $\boxed{nn\Lambda}$ | 3 | $-1$ | 1 | $\frac{1}{2}^+$ | $\mathbf{27}$ | 69(5)(12)(0) | 23 |
| $^3_\Sigma$He | 3 | $-1$ | 1 | $\frac{3}{2}^+$ | $\mathbf{27}$ | 55(6)(10)(1) | 18 |
| $^4$He | 4 | 0 | 0 | $0^+$ | $\overline{\mathbf{28}}$ | 107(12)(21)(1) | 27 |
| $^4_\Lambda$He, $^4_\Lambda$H | 4 | 0 | 0 | $0^+$ | $\overline{\mathbf{28}}$ | 107(12)(21)(1) | 27 |
| $^4_{\Lambda\Lambda}$He, $^4_{\Lambda\Lambda}$H, $\boxed{nn\Lambda\Lambda}$ | 4 | 0 | 0 | $0^+$ | $\mathbf{27}$ | 156(16)(21)(2) | 39 |

# Experimental evidence for neutral baryon system?

- **$N\Lambda$ binding is not supported by either experiment or theory investigations.**
- **Still controversial for $nn\Lambda$ system.**

- Marques et al, "**Detection of neutron clusters**", PRC65, 044006 (2002)
- HypHI Collaboration, "**Search for evidence of ${}^{3}_{\Lambda}n$ by observing $d + \pi^-$ and $t + \pi^-$ final states in …**", PRC88, 041001(R) (2013)

$$ {}^{3}_{\Lambda}n \to t^* + \pi^- \to d + n + \pi^- \qquad\qquad {}^{3}_{\Lambda}n \to t + \pi^- $$



# Can we tolerate a neutral baryon system at all within the present available nuclear models ?

- A. Gal and H. Garcilazo, Phys. Lett. B 736, 93 (2014), [arXiv:1404.5855 [nucl-th]].
- H. Garcilazo and A. Valcarce, Phys. Rev. C 89, 057001 (2014).
- E. Hiyama, S. Ohnishi, B.F. Gibson and T.A. Rijken, *Three-body structure of the nn$\Lambda$ system with $\Lambda$N-$\Sigma$N coupling*, PRC 89, 061302(R) (2014); arXiv:1405.2365 [nucl-th]

| $x$ | $a_{nn}$(fm) | $r_{\rm eff}$(fm) | $\epsilon_{nn}$(MeV) | $E_{^3{\rm H}}$(MeV) | $E_{^3_\Lambda n}$(MeV) |
|---|---|---|---|---|---|
| 1.0 | $-23.7$ | 2.78 | unbound | $-7.77$ | unbound |
| 1.13 | 25.1 | 2.40 | $-0.066$ | $-9.75$ | unbound |
| 1.35 | 6.88 | 1.96 | $-1.269$ | $-13.93$ | $-1.272$ |

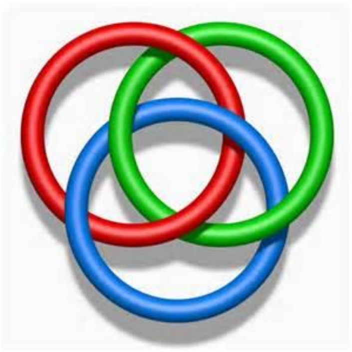
**Better understanding of N$\Lambda$ potential is required.**

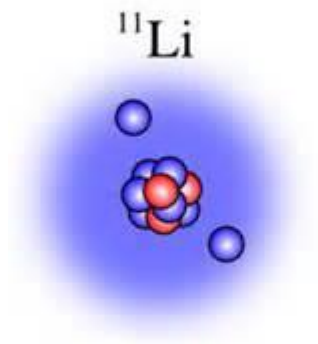
# Thomas collapse

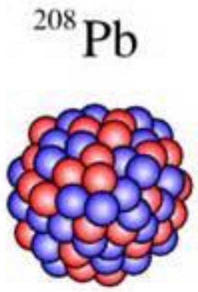
The ratio of 3-body to 2-body bound state energies, $E_3/E_2$, becomes very large if the range of the interaction decreases. Equivalently, for a given (short) range, $E_3/E_2 \rightarrow \infty$ if the coupling $g$ approaches (from above) the minimal value $g_2$ required to bind two particles.

The potential energy is written as $g\sum v(r_{ij})$ where $v$ is attractive or contains attractive parts.

The minimal coupling $g_3$ to bind three particles is smaller than $g_2$, so that for $g_3 < g < g_2$ there is a "Borromean" 3-body bound state whose two-body subsystems are unbound.





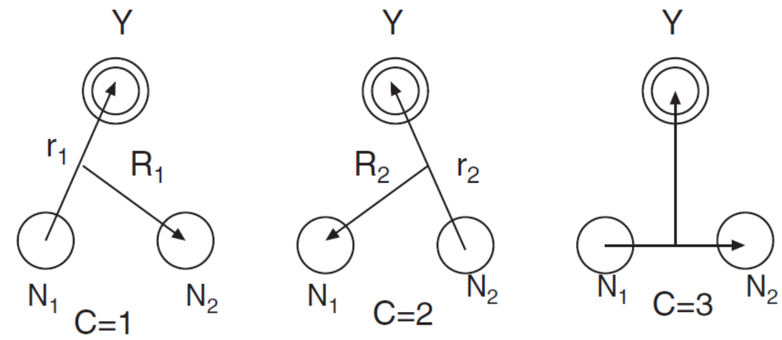



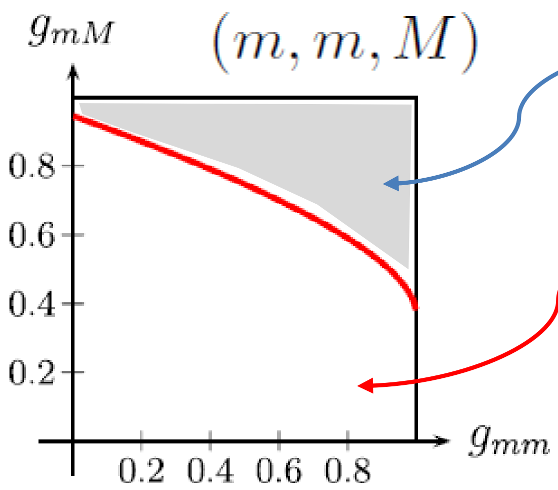
# Binding condition for a Borromean system:

Decompose the Hamiltonian $H$ into sub-Hamiltonians:

$$H = H_1 + H_2 + \cdots$$

$H$ is hardly negative if $H_i$ is positive.




Borromean sector $\{g_2\colon\ g_{31} = g_{12} < 1, g_{23} < 1\}$ with $g_3 < g < g_2$.

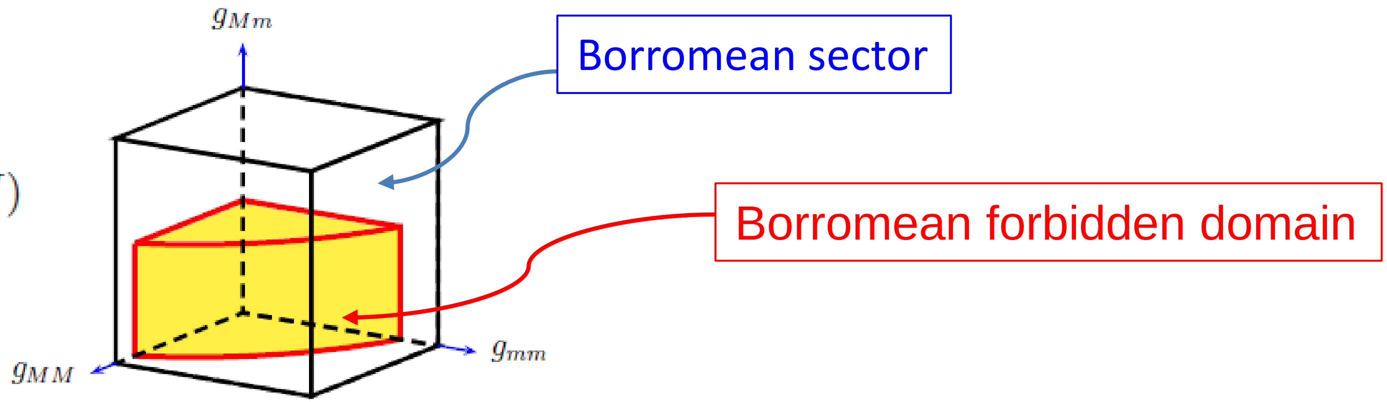
Borromean forbidden domain

- The scale is set such that the pair $(m_i, m_j)$ is bound for $g_{ij} > 1$. This drawing corresponds to $M/m$ = 1.2.
- Variants on cluster decompositions can lead to Borromean forbidden domain.

Richard and Fleck, PRL73, 1464 (1994); J.-L. Basdevant, A. Martin, J.-M. Richard, and T. T. Wu, Nucl. Phys. B 393, 111 (1993).

## Borromean binding for four-body system

$(m, m, M, M)$




- The Borromean allowed domain is larger than the case of three-body system.
- There exists fully Borromean four-body bound state in the case of screened Coulomb interactions (see L. Bertini et al., PRA69, 042504 (2004)).

**Possible binding due to 3-body interactions?**

## Three-body binding energy $E_3$ as a function of the effective range $r_{eff}$ :

$$
\begin{array}{ll}
-\,g\,\exp(-\mu\,r) & \text{(exponential)}\,, \\
-\,g\,\exp(-\mu\,r)/r & \text{(Yukawa)}\,, \\
g\exp[-2\,\mu\,(r-R)] - 2\,g\,\exp[-\mu\,(r-R)] & \text{(Morse)}
\end{array}
$$

- Fix the two-body scattering length at $a = -10$, i.e. just below the threshold for two-body binding. The coupling $g$ is fixed by reproducing the $nn$, $n\Lambda$, and $\Lambda\Lambda$ scattering lengths $a$ and effective range $r_{eff}$.

- Values (in fm) adopted for the scattering length and effective range parameter in two models: ESC08 and CEFT.

| | ESC08 | | CEFT | |
|---|---|---|---|---|
| Pair | $a$ | $r_{\rm reff}$ | $a$ | $r_{\rm eff}$ |
| $nn$ | $-16.51$ | 2.85 | $-18.9$ | 2.75 |
| $(n\Lambda)_{s=0}$ | $-2.7$ | 2.97 | $-2.9$ | 2.65 |
| $(n\Lambda)_{s=1}$ | $-1.65$ | 3.63 | $-1.51$ | 2.64 |
| $\Lambda\Lambda$ | $-0.88$ | 4.34 | $-1.54$ | 0.31 |

**ESC08 model of the Nijmegen-RIKEN group:**

- T. Fukuda *et al.*, in International Workshop on Strangeness Nuclear Physics (SNP12), Aug. 27–29, 2012, Osaka, Japan (unpublished).
- T. A. Rijken, M. M. Nagels, and Y. Yamamoto, Prog. Theor. Phys. Suppl. **185**, 14 (2010).
- T. A. Rijken, M. M. Nagels, and Y. Yamamoto, Few-Body Syst. **54**, 801 (2013).

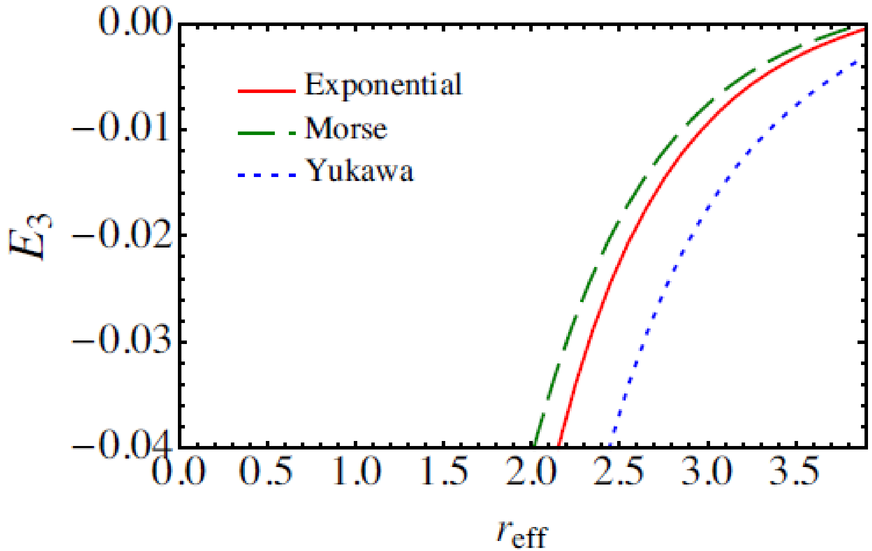
**chiral effective field theory (CEFT):**

- H. Polinder, J. Haidenbauer, and U.-G. Meissner, Phys. Lett. B **653**, 29 (2007).
- J. Haidenbauer, S. Petschauer, N. Kaiser, U.-G. Meissner, A. Nogga, and W. Weise, Nucl. Phys. A **915**, 24 (2013).

## Three-body binding energy $E_3$ as a function of the effective range $r_{eff}$ :

$$\boxed{\begin{array}{ll} -g\,\exp(-\mu\, r) & (\text{exponential})\ , \\ -g\,\exp(-\mu\, r)/r & (\text{Yukawa})\ , \\ g\exp[-2\,\mu\,(r-R)] - 2\,g\,\exp[-\mu\,(r-R)] & (\text{Morse}) \end{array}}$$

Fix the two-body scattering length at $a = -10$, i.e. just below the threshold for two-body binding.




- In the regime of weak binding a variational method is adopted.
- $E_3$ is a quick increasing function of $r_{eff}$, which means that models with a very large effective range cannot generate much Borromean binding.
- The amount of Borromean binding does not depend dramatically on the potential shape.

# Cautions:

- **The effects of the $\Lambda N \leftrightarrow \Sigma N$ transitions in the $\Lambda N$ interactions.**
- **The effects of the $\Lambda\Lambda \leftrightarrow \Xi N$ in the $\Lambda\Lambda$ interactions.**

- Experimental evidence for ${}^{4}_{\Lambda\Lambda}n$ is crucial for clarifying some of those long-standing puzzling questions.
- The study of ${}^{4}_{\Lambda\Lambda}n$ relies on the final experimental results to decide.

# 2. Production mechanism for ${}^{4}_{\Lambda\Lambda}n$

- How to organize the ($\Lambda\Lambda$nn) system?

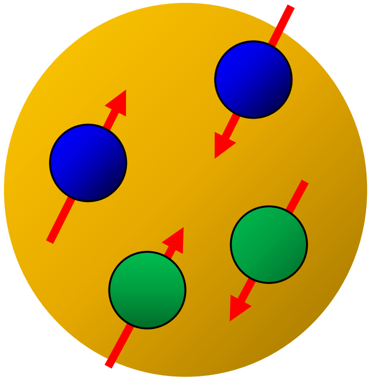
$$T = {}^{4}_{\Lambda\Lambda}\mathrm{n} = (\Lambda\Lambda\mathrm{nn})$$

**with** $I = 0, S = 0, L = 0$ .

**Thus,** $J = L + S = 0$.

**No Pauli blocking**

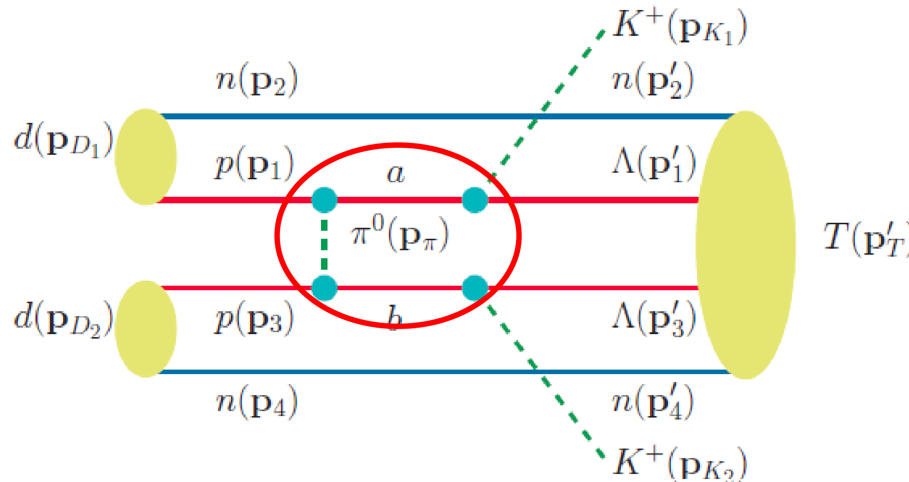
**Groundstate:** $J^P = 0^+$.



Wang, Richard and Zhao, PRC91, 014003 (2015); 1404.3473[nucl-th]

# Production mechanism for (ΛΛnn) in deuteron-deuteron collision

$$d + d \to K^+ + K^+ + T.$$



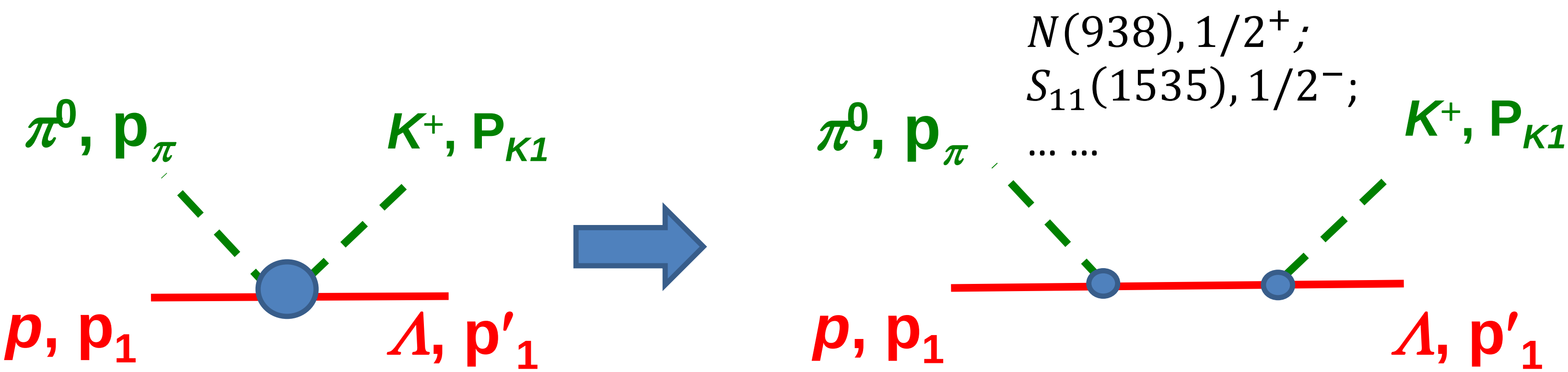

$$\begin{aligned}\mathcal{M} \;=\; & \int \psi_T^*(\boldsymbol{p}'_1, \boldsymbol{p}'_2, \boldsymbol{p}'_3, \boldsymbol{p}'_4; \boldsymbol{P}'_T)\psi_{K_1}^*(\boldsymbol{P}_{K_1})\psi_{K_2}^*(\boldsymbol{P}_{K_2})\hat{\mathcal{O}}(\boldsymbol{p}'_1, \boldsymbol{p}'_3, \boldsymbol{p}_1, \boldsymbol{p}_3, \boldsymbol{P}_{K_1}, \boldsymbol{P}_{K_2}) \\ & \times\; \psi_{d_1}(\boldsymbol{p}_1, \boldsymbol{p}_2; \boldsymbol{P}_{d_1})\psi_{d_2}(\boldsymbol{p}_3, \boldsymbol{p}_4; \boldsymbol{P}_{d_2})\delta(\boldsymbol{P}'_T + \boldsymbol{P}_{K_1} + \boldsymbol{P}_{K_2} - \boldsymbol{P}_{d_1} - \boldsymbol{P}_{d_2}) \\ & \times\; \delta(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{P}_{d_1})\delta(\boldsymbol{p}_3 + \boldsymbol{p}_4 - \boldsymbol{P}_{d_2})\delta(\boldsymbol{p}'_1 + \boldsymbol{p}'_2 + \boldsymbol{p}'_3 + \boldsymbol{p}'_4 - \boldsymbol{P}'_T) \\ & \times\; \delta(\boldsymbol{p}_2 - \boldsymbol{p}'_2)\delta(\boldsymbol{p}_4 - \boldsymbol{p}'_4)d\boldsymbol{p}_1 d\boldsymbol{p}_2 d\boldsymbol{p}_3 d\boldsymbol{p}_4 d\boldsymbol{p}'_1 d\boldsymbol{p}'_2 d\boldsymbol{p}'_3 d\boldsymbol{p}'_4\end{aligned}$$

**Elementary transition:** $\boldsymbol{p} + \boldsymbol{p} \to \boldsymbol{\Lambda} + \boldsymbol{\Lambda} + \mathbf{K}^+ + \mathbf{K}^+.$

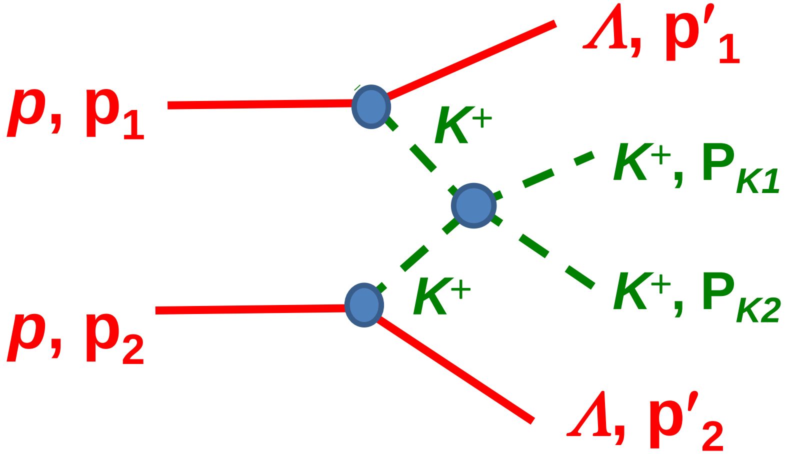
Wang, Richard and Zhao, PRC91, 014003 (2015); 1404.3473[nucl-th]

## Assumptions and approximations:

- The S-wave nuclear wavefunctions are dominant.
- The two $K^+$ production is via the elementary process, $pp \to \Lambda\Lambda K^+K^+$.
- The intermediate $S_{11}(1535)$ excitations is sizeable near threshold.

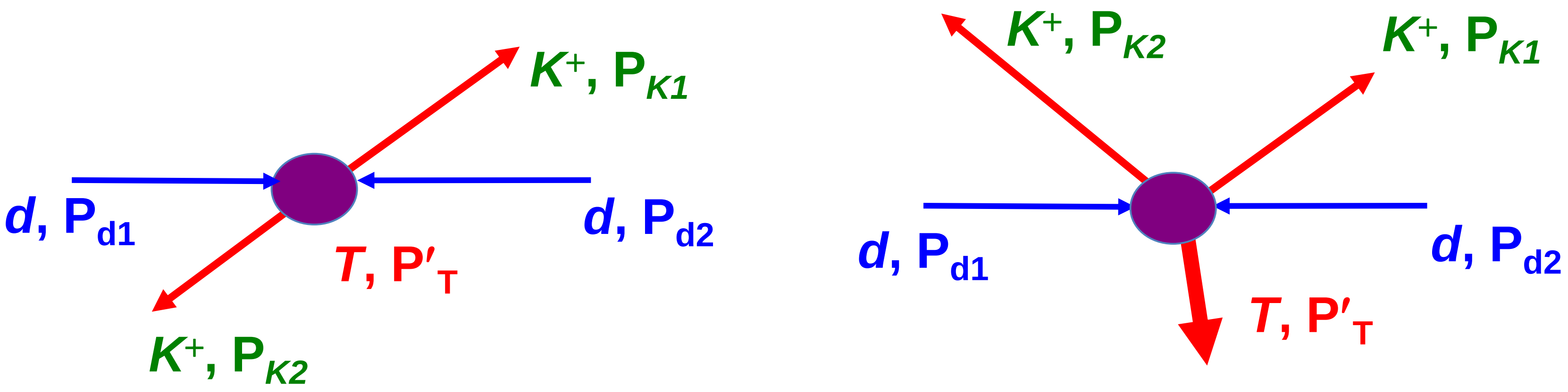



- Other t-channel exchanges and baryon resonance excitations can also contribute.



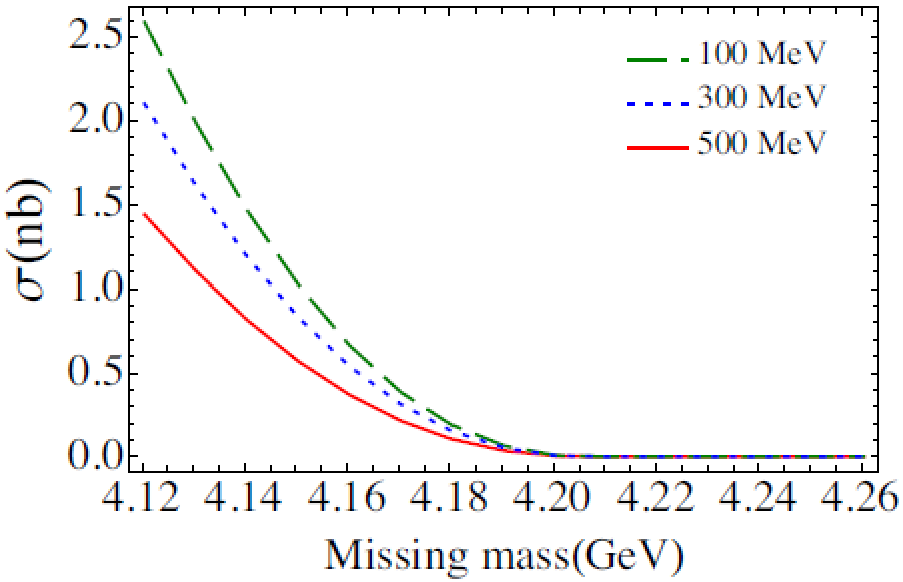
## Advantages of $d-d$ collisions:

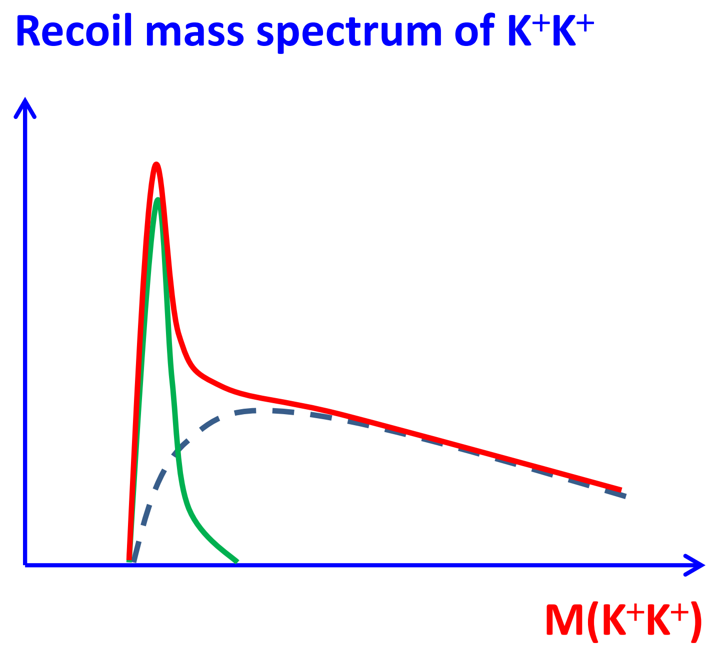
- Central production of $T(\Lambda\Lambda nn)$ is favored in order to have the largest wavefunction overlapping with the initial states;
- Relatively large momenta for the charged kaons;
- Smooth background in the missing mass spectrum;
- Even angular distributions for the background $K^+$ events.



- Missing mass spectrum for $(\Lambda\Lambda nn)$

$$d + d \to K^+ + K^+ + T.$$

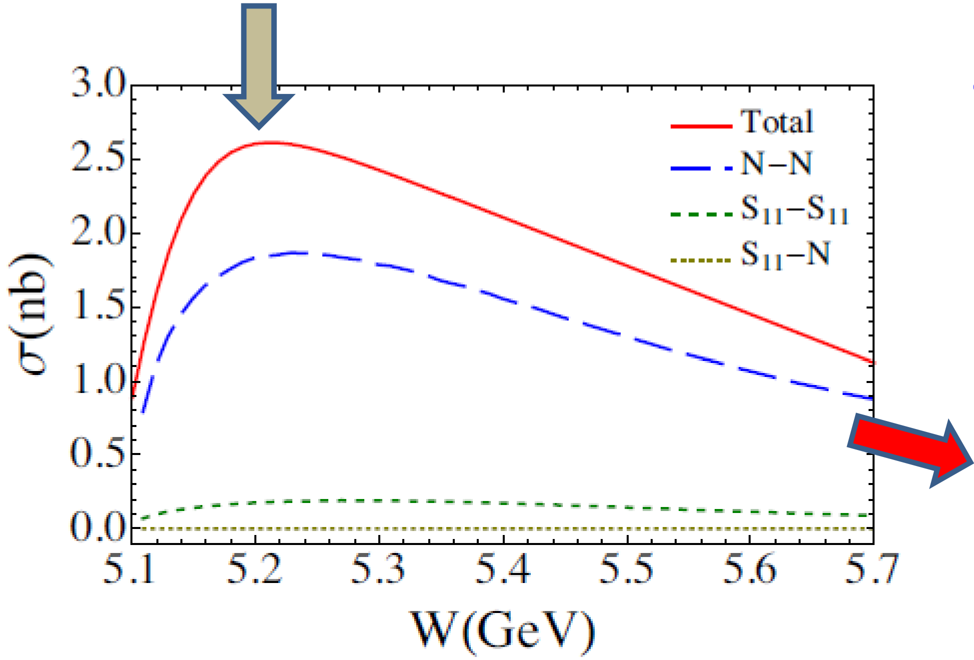



**By tagging the two $K^+$ in the final state, the production of the $(\Lambda\Lambda nn)$ will not suffer from nontrivial background. The signal will peak at the mass of the $(\Lambda\Lambda nn)$.**

Wang, Richard and Zhao, PRC91, 014003 (2015); 1404.3473[nucl-th]

- Total cross section for (ΛΛnn)




**The interference between the double nucleon Born term and double $S_{11}$(1535) excitation plays a significant role.**

With $L = 10^{27} \sim 2 \times 10^{30} cm^{-2}s^{-1}$ at $E_{cm} = 6.7 \sim 28.0$ GeV, respectively, we estimate the number of events as

$$N_{min} = \sigma \times L \times T \simeq 0.2\, nb \times 10^{27} cm^{-2}s^{-1} \times 1\, year \simeq 6.3$$
$$N_{max} = 0.2nb \times 2 \times 10^{30} cm^{-2}s^{-1} \times 1\, year \simeq 12000$$

Taking into account the efficiency corrections, about hundreds events are anticipated for one-year run.

# Other interesting topics at NICA

1. Search for H-dibaryon ($\Lambda\Lambda$) in the missing mass spectrum of $K^+K^+$

- $p + p \rightarrow \underbrace{\Lambda + \Lambda}_{\textbf{H-dibaryon?}} + K^+ + K^+$

2. Search for pentaquark $\Theta^+$

- $p + d \rightarrow \Lambda + \Lambda + \underbrace{n + K^+}_{\boldsymbol{\Theta^+ \rightarrow n + K^+}} + K^+$

# 3. Summary

1. The neutral hypernuclei, if exist, would bring important insights into the three-body nuclear force.
2. More elaborate model-building is needed for the study of neutral hypernucleus ${}^{4}_{\Lambda\Lambda}\mathbf{n}$. However, it may rely on the experimental measurements to decide its existence. The deuteron-deuteron collisions at NICA and FAIR can directly search for its signals.
3. The $pp$ and $pd$ collisions can also probe some exotic nuclear systems in the region around two kaon production threshold.

***Thanks for your attention!***